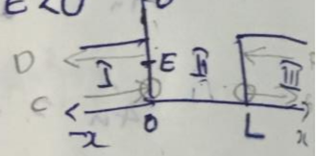
(2) Finite Potential well:

Less find out wave fn for potential well with finite height U, L width and contains a particle whose energy $E < U$.

classically, the particle bounce inside the box.

QMly, the particle bounce back but it has a probability of penetrating the barrier & enter into I & III even though $E < U$



In region I & III, S-eqn is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$$

or $$\frac{d^2\psi}{dx^2} - k^2\psi = 0 \quad \begin{cases} x<0 \\ x>L \end{cases}$$

where $k = \frac{\sqrt{2m(E-U)}}{\hbar}$

the solns are $\psi_I = Ce^{kx} + De^{-kx}$

$$\psi_{III} = Fe^{kx} + Ge^{-kx}$$

ψ_I & ψ_{III} must be finite.

so, As $x \to -\infty$ $e^{-kx} \to \infty$

$x \to \infty$ $e^{kx} \to \infty$

$\psi_I^{-\infty} = C.0 + D\infty$

$\therefore D = 0$

$\psi_{III}^{\infty} = F.\infty + G.0$

$\therefore F = 0$

In order to become, ψ_I & ψ_{III} exist,

$\therefore D = F = 0$

we've $\psi_{I} = Ce^{kx}$

$\psi_{III} = Ge^{-kx}$

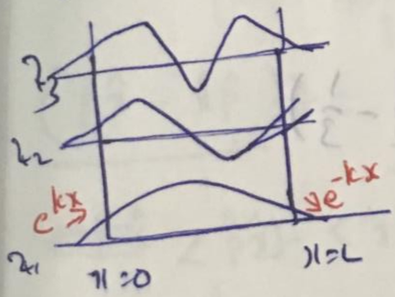
In region II,

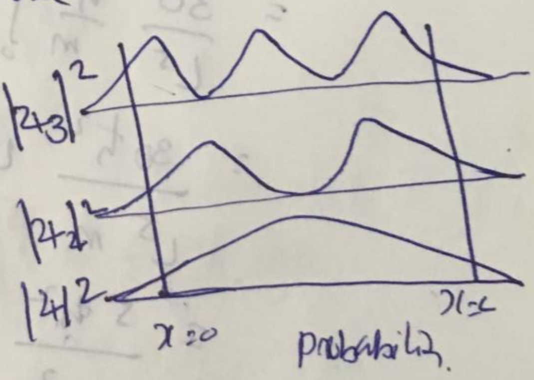
$$\psi_{II} = A \sin \frac{\sqrt{2mE}}{\hbar} x + B \cos \left(\frac{\sqrt{2mE}}{\hbar} x \right)$$

In infinite case, $\psi = 0$ @ boundries. But here

$\psi_{II} = C$ at $x=0$ & $\psi_{II} = Ge^{-kL}$ @ $x = L$

For either soln ψ & $d\psi/dx$ must be continuous at $x=0$ & $x=L$. ∴ The ψ @ $x=0$ $x=L$ have certain value. the waves fn. are





the wave length donot fit inside becoz its long ⇒ the momentum & hence E_n will lower than that of infinite well. The particle bounce to & forth and it has a probability to penetrate the walls in both directions.

If $E > V$, full transmission $T \sim 1$ continuous state

Not bound state.

The deeper & broader the well, greater the no. of bound states.