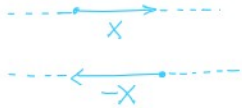
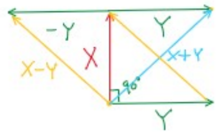
$\|X\| = \sqrt{<X,X>}$

4. Sendo $X, Y \in \mathbb{R}^n$ mostre (e interprete geometricamente para $n = 2$) que:

Em $\mathbb{R}^2$:

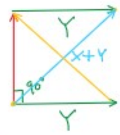
(b) $\overbrace{X \perp Y}^{a} \Leftrightarrow \overbrace{\|X - Y\| = \|X + Y\|}^{b}$





$\|X - Y\| = \sqrt{<X-Y, X-Y>}$

$\|X + Y\| = \sqrt{<X+Y, X+Y>}$



$\|X - Y\| = \|X + Y\| \Leftrightarrow$

$\sqrt{<X-Y, X-Y>} = \sqrt{<X+Y, X+Y>} \Leftrightarrow$

$<X-Y, X-Y> = <X+Y, X+Y> \Leftrightarrow$

As diagonais de um retângulo têm o mesmo comprimento

$<X, \cancel{X}> - \underbrace{<X,Y>} - \underbrace{<Y,X>} + <\cancel{Y},Y> = <\cancel{X},X> + <X,Y> + <Y,X> + <\cancel{Y},Y> \Leftrightarrow$

$\Leftrightarrow \quad -<X,Y> - <X,Y> = <X,Y> + <X,Y>$

propriedade comutativa do produto escalar

$\Leftrightarrow \quad -2<X,Y> = 2<X,Y>$

$\Leftrightarrow \quad 4<X,Y> = 0 \Leftrightarrow <X,Y> = 0 \Leftrightarrow$

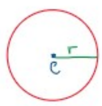
$\Leftrightarrow \quad X \perp Y$

Cónicas → curvas no plano xOy representadas por uma equação do 2º grau nas variáveis x e y:

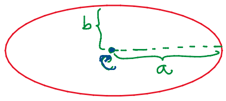
$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

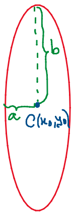
onde A, B, C, D, E, F são números reais.

- Circunferências: $\boxed{(x - x_0)^2 + (y - y_0)^2 = r^2}$ $\quad C(x_0, y_0)$ raio r



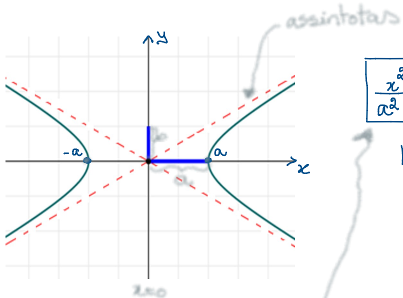
- Elipses: $\boxed{\dfrac{(x - x_0)^2}{a^2} + \dfrac{(y - y_0)^2}{b^2} = 1}$ $\quad C(x_0, y_0)$ semi-eixos a e b





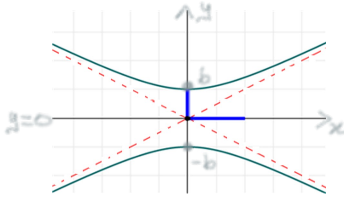
• Hipérboles : $\boxed{\dfrac{(x-x_0)^2}{a^2} - \dfrac{(y-y_0)^2}{b^2} = 1}$ ou $\boxed{-\dfrac{(x-x_0)^2}{a^2} + \dfrac{(y-y_0)^2}{b^2} = 1}$

$C(x_0, y_0)$ $\qquad$ $C(x_0, y_0)$



$\boxed{\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1}$

hipérbole ao longo do eixo dos xx



$\boxed{-\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1}$

hipérbole ao longo do eixo dos yy

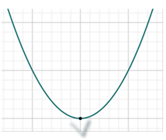
$y=0 \longrightarrow \dfrac{x^2}{a^2} = 1 \Leftrightarrow x^2 = a^2$
$\Leftrightarrow x = \pm a$

$x=0 \longrightarrow \underbrace{-\dfrac{y^2}{b^2}}_{\text{negativo}} = 1$ impossível

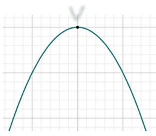
$x=0 \longrightarrow \dfrac{y^2}{b^2} = 1 \Rightarrow y^2 = b^2 \Rightarrow y = \pm b$

$y=0 \longrightarrow \underbrace{-\dfrac{x^2}{a^2}}_{\text{negativo}} = 1$ impossível

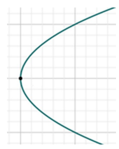
• Parábolas : $\boxed{y = ax^2 + bx + c}$ * ou $\boxed{x = ay^2 + by + c}$



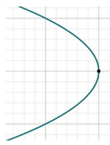
$a > 0$



$a < 0$



$a > 0$



$a < 0$

$x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$V \rightarrow \left(-\dfrac{b}{2a}, -\dfrac{\Delta}{4a}\right)$ $\qquad \Delta = b^2 - 4ac$

** $\boxed{(x-x_0)^2 = 4p(y-y_0)} \longrightarrow V(x_0, y_0)$

$\boxed{(y-y_0)^2 = 4p(x-x_0)} \longrightarrow V(x_0, y_0)$

5. **A2** Identifique e represente geometricamente no plano as "curvas" de equações:

(a) $4x^2 + 9y^2 = 36$;

$\dfrac{4x^2}{36} + \dfrac{9y^2}{36} = 1$

$\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

$\dfrac{x^2}{3^2} + \dfrac{y^2}{2^2} = 1$

(b) $(x-3)^2 = y - 1$;

parábola $\underline{V(3,1)}$

$y = (x-3)^2 + 1$

$y = x^2 - 6x + 9 + 1$

$y = x^2 - 6x + 10$ $\quad \cup$

(c) $y = x^2 - 4x$;

$a = 1 > 0$ $\quad \cup$

Zeros: $x^2 - 4x = 0$

$x(x-4) = 0$

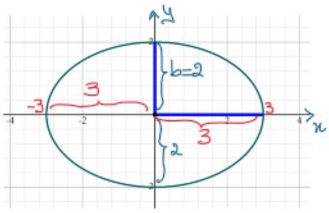
$x = 0 \lor x = 4$

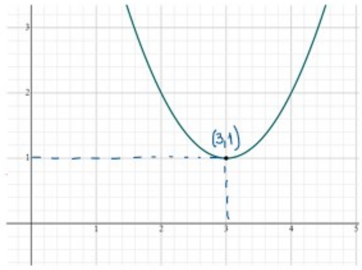
$V\left(2, -4\right)$

$y = x^2 - 6x + 10$ ∪

$$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$

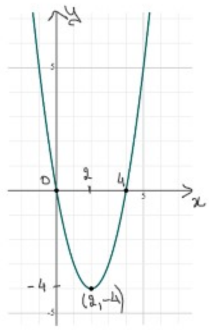
elipse com $C(0,0)$ e semi-eixos $a = 3$, $b = 2$





$V\left(\underset{\uparrow\, x}{2}, \underset{\uparrow}{-4}\right)$

$y = 2^2 - 4.2 = 4 - 8 = -4$



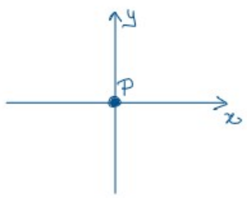
(d) $x^2 = -y^2$;

$x^2 + y^2 = 0$

$\Rightarrow x = y = 0$

$P(0,0)$

(cónica degenerada)



(e) $x^2 - 2x + 4y^2 - 8y + 1 = 0$;

feito na Teórica

(m) $1 + 4x^2 - y^2 = 0$ hipérbole

$4x^2 - y^2 = -1$

$-4.x^2 + y^2 = 1$

$-\dfrac{x^2}{\frac{1}{4}} + y^2 = 1$

$-\dfrac{x^2}{\left(\frac{1}{2}\right)^2} + y^2 = 1$

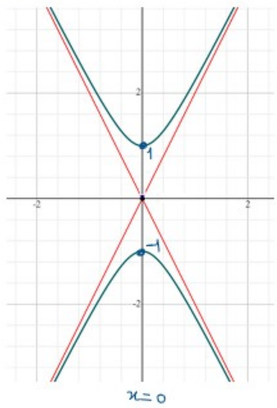
$$\frac{x^2}{\frac{1}{4}} = \frac{\frac{x^2}{1}}{\frac{1}{4}} = 4x^2$$

$$\boxed{-\frac{x^2}{a^2} + \frac{y^2}{1^2} = 1}$$

hipérbole com centro $(0,0)$ ao longo do eixo dos yy

$x = 0 \rightsquigarrow y^2 = 1 \Rightarrow y = \pm 1$

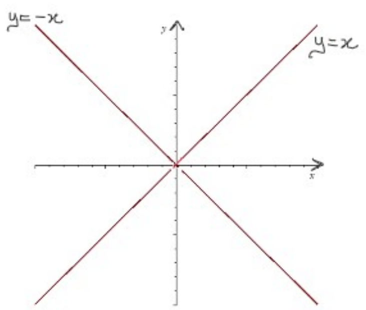
$y = 0$ (eixo dos xx) $\longrightarrow$ impossível



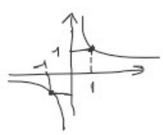
$x^2 = y^2$

$\Rightarrow x = y \lor x = -y$

$\Rightarrow y = x \lor y = -x$



$y = \dfrac{1}{x}$



(j) $xy = 9$;

$y = \dfrac{9}{\ }$

(k) $|x| = |y|$;

$x = y \lor x = -y$

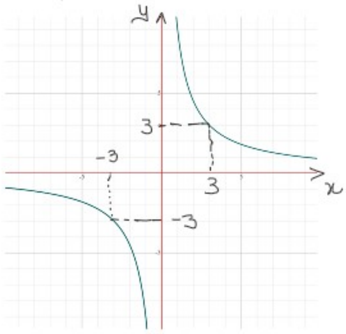
(l) $x = \underbrace{-\overbrace{|y-1|}^{\text{nº positivo}}}_{}$; nº negativo

$|y-1| = -x$

(j) $xy = 9$;

$y = \frac{9}{x}$

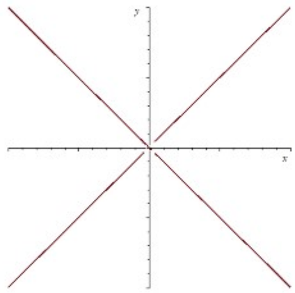
Hipérbole

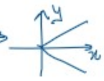


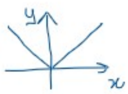
(k) $|x| = |y|$;

$x = y \lor x = -y$

$y = x \lor y = -x$



$x = |y| \rightarrow$ 

$y = |x| \rightarrow$ 

(l) $x = -|y-1|$;

$|y-1| = -x$

$y-1 = -x \lor y-1 = -(-x)$

$y = -x+1 \lor y = x+1$

2º Q ou 3º Q
porque x negativo