Quádricas $\rightarrow$ "superfícies" em $\mathbb{R}^3$ definidas por equações do 2º grau nas variáveis x, y e z.

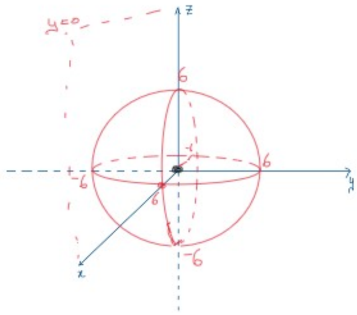
6

(a) $x^2 + y^2 + z^2 = 36$

$\rightarrow z = 0 \rightsquigarrow x^2 + y^2 = 36$

$\rightarrow x = 0 \longrightarrow y^2 + z^2 = 36$

$\rightarrow y = 0 \longrightarrow x^2 + z^2 = 36$



Superfície esférica de $C(0,0,0)$ e raio $\sqrt{36} = 6$

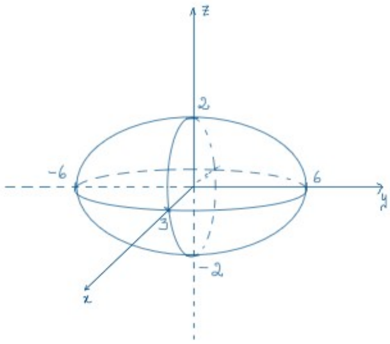
$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$ Superfície esférica $C(x_0, y_0, z_0)$ e raio r

(b) $4x^2 + y^2 + 9z^2 = 36 \iff \frac{4x^2}{36} + \frac{y^2}{36} + \frac{9z^2}{36} = 1 \iff \boxed{\frac{x^2}{9} + \frac{y^2}{36} + \frac{z^2}{4} = 1}$ elipsóide

$\boxed{z = 0}$ xOy $\longrightarrow 4x^2 + y^2 = 36 \iff \frac{4x^2}{36} + \frac{y^2}{36} = 1 \iff \frac{x^2}{9} + \frac{y^2}{36} = 1$ elipse semi-eixos $a = 3$ e $b = 6$

$\boxed{x = 0}$ yOz $\longrightarrow \frac{y^2}{36} + \frac{z^2}{4} = 1$ elipse de semi-eixos 6 e 2

$\boxed{y = 0} \longrightarrow \frac{x^2}{9} + \frac{z^2}{4} = 1$ " " " 3 e 2



elipsóide

$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} + \frac{(z - z_0)^2}{c^2} = 1$ elipsóide com centro $C(x_0, y_0, z_0)$ e semi-eixos a, b e c

(c) $x^2 + \frac{y^2}{4} - z^2 - 1 = 0 \iff x^2 + \frac{y^2}{4} = z^2 + 1$

$\boxed{z = 0}$ xOy $\longrightarrow x^2 + \frac{y^2}{4} = 1$ elipse de semi-eixos $\underline{a = 1}$ e $\underline{b = 2}$

$\boxed{z = \pm 1} \longrightarrow x^2 + \frac{y^2}{4} = \underline{\underline{2}}$ elipses de semi-eixos $a = \sqrt{2}$ e $b = \sqrt{8} = 2\sqrt{2}$ $C(0, 0, \pm 1)$

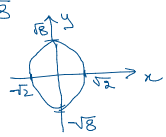
x^2 . . 2 $\rightarrow x = 0$: $\frac{y^2}{4} = 2 \Rightarrow y^2 = 8 \Rightarrow y = \pm\sqrt{8}$ ...

$\boxed{z=\pm 1} \rightarrow \quad x^2+\frac{y^2}{4}=2$ elipses de semi-eixos $a=\sqrt{2}$ e $b=\sqrt{8}=2\sqrt{2}$ $\quad C(0,0,\pm 1)$

$\frac{x^2}{2}+\frac{y^2}{8}=1$

$x=0: \quad \frac{y^2}{4}=2 \Rightarrow y^2=8 \Rightarrow y=\pm\sqrt{8}$

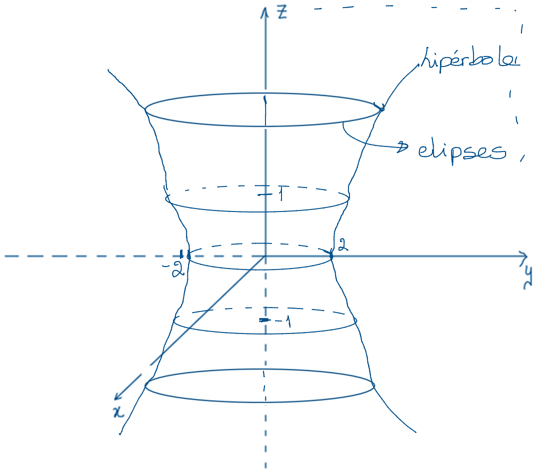
$y=0: \quad x^2=2 \Rightarrow x=\pm\sqrt{2}$



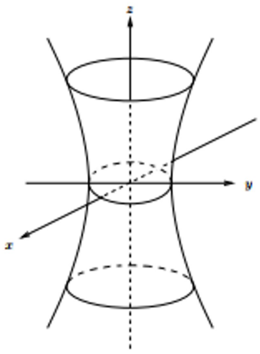
$\boxed{z=\pm 2} \rightarrow \quad x^2+\frac{y^2}{4}=5$ elipses de semi-eixos $a=\sqrt{5}$ e $b=\sqrt{20}$ $\quad C(0,0,\pm 2)$

$\boxed{x=0} \rightarrow \quad \frac{y^2}{4}-z^2=1$ hipérbole

$\boxed{y=0} \rightarrow \quad x^2-z^2=1$ hipérbole



Hiperbolóide de 1 folha
ao longo do eixo dos zz



$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Hiperbolóide de 1 folha ao longo do eixo dos zz

$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Hiperbolóide de 1 folha ao longo do eixo dos yy

$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Hiperbolóide de 1 folha ao longo do eixo dos xx

(f) $\frac{x^2}{4}-y^2+\frac{z^2}{9}=1$ Hiperbolóide de 1 folha ao longo do eixo dos yy

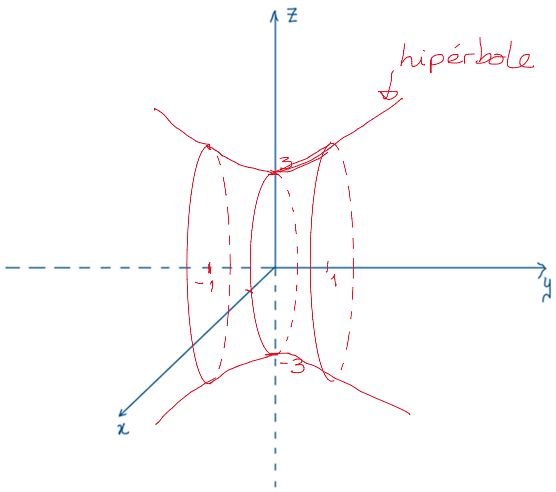
$\frac{x^2}{4}+\frac{z^2}{9}=y^2+1$

• $\underset{xOz}{\boxed{y=0}} \rightarrow \quad \frac{x^2}{4}+\frac{z^2}{9}=1$ elipse semi-eixos 2 e 3 $\quad C(0,0,0)$

- $\boxed{y=\pm 1} \longrightarrow \frac{x^2}{4}+\frac{z^2}{9}=2$ elipses $C_1(0,1,0)$ ou $C_2(0,-1,0)$ com semi-eixos $\sqrt{8}=2\sqrt{2}$ e $\sqrt{18}=3\sqrt{2}$

- $\boxed{y=\pm 2} \longrightarrow \frac{x^2}{4}+\frac{z^2}{9}=5$ elipses $C_1(0,2,0)$ ou $C_2(0,-2,0)$ com semi-eixos $\sqrt{20}=2\sqrt{5}$ e $\sqrt{45}=3\sqrt{5}$

- $\boxed{x=0} \longrightarrow -y^2+\frac{z^2}{9}=1$
hipérbole



(d) $x^2+y^2-z^2=-1 \Longrightarrow \underbrace{x^2+y^2}_{\geq 0} = \underbrace{z^2-1}_{\therefore\ \geq 0}$

$-x^2-y^2+z^2=1$

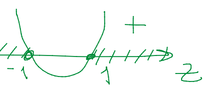
hiperbolóide de duas folhas

$\underbrace{z^2-1\geq 0}_{\text{ineq. 2º grau}} \Rightarrow$

$\boxed{z=\pm 1}$ $\quad x^2+y^2=0 \Rightarrow x^2=0 \wedge y^2=0 \Rightarrow x=0 \wedge y=0$

$P_1(0,0,1)$ ou $P_2(0$

$z \geq 1 \vee z \leq -1$



$, 0, -1)$