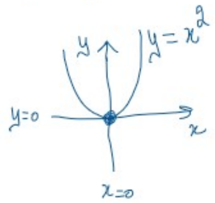
18. Seja f uma função contínua tal que

$$\text{Se } (x,y) \in \mathbb{R}^2 \backslash \{(0,0)\}, \text{ então } f(x,y) = \frac{x^2 y}{x^4 + y^2}.$$

(a) Mostre que não existe $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

(b) Existirá uma extensão contínua de f a $\mathbb{R}^2$?



a) $$\lim_{\substack{(x,y)\to(0,0)\\ y=0}} f(x,y) = \lim_{x\to 0} \frac{x^2 \cdot 0}{x^4 + 0^2} = \lim_{x\to 0} \underbrace{\frac{0}{x^4}}_{=0} = 0$$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=x^2}} f(x,y) = \lim_{x\to 0} \frac{x^2 \cdot x^2}{x^4 + (x^2)^2} = \lim_{x\to 0} \frac{\cancel{x^4}}{2\cancel{x^4}} = \frac{1}{2}$$

$0 \neq \frac{1}{2}$

$\Downarrow$

Não existe limite

b) $$g(x,y) = \begin{cases} \boxed{\dfrac{x^2 y}{x^4 + y^2} \quad \text{se} \quad (x,y) \neq (0,0)} \\ \boxed{g(0,0)} \quad \text{se} \quad (x,y) = (0,0) \end{cases}$$

definição de f

??

f é contínua em $\mathbb{R}^2 \backslash \{(0,0)\}$

?? g é contínua em $\mathbb{R}^2$??

extensão de f a $\mathbb{R}^2$

Por definição g contínua em $(0,0)$ se $\underbrace{\lim\limits_{(x,y)\to(0,0)} g(x,y)}_{} = g(0,0)$

$$= \lim_{(x,y)\to(0,0)} f(x,y)$$

este limite não existe

Sendo assim concluímos que é impossível ter uma extensão de f a $\mathbb{R}^2$ que seja contínua.

$u \to$ função de x

20. Para cada caso, calcule as derivadas parciais $\frac{\partial z}{\partial x}$ e $\frac{\partial z}{\partial y}$ no ponto indicado:

$u \to$ função de x

$(e^u)' = u' e^u$

$(\cos u)' = -u' \operatorname{sen} u$

$f(x) \qquad f'(x) = \frac{df}{dx}$

$f(x,y) \qquad \cancel{f'(x,y)} \qquad \frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y}$

derivadas parciais

(b) $z = e^{4x} \cos(2x + y)$, $(0, \pi)$

$$\frac{\partial z}{\partial x} = 4e^{4x} \cdot \cos(2x+y) + e^{4x} \cdot \left(-2 \sin(2x+y)\right)$$

$$= 4e^{4x} \cdot \cos(2x+y) - 2e^{4x} \sin(2x+y)$$

$$\frac{\partial z}{\partial x}(0,\pi) = 4e^0 \cos(2.0+\pi) - 2e^{4.0} \sin(2.0+\pi)$$

$$= 4.(-1) - 2.0 = -4$$

$$\frac{\partial z}{\partial y} = 0 \cdot \cos(2x+y) + e^{4x} \cdot \left(-\sin(2x+y)\right)$$

Trato x como uma constante

$$\cos(2x+y) \xrightarrow{\frac{\partial}{\partial x}} -(2+0) \sin(2x+y)$$

$$\cos(2x + 1000)$$

$$\frac{\partial z}{\partial y}(0,\pi) = 0 \cdot \cos(2.0+\pi) + e^{4.0}\left(-\sin(2.0+\pi)\right)$$

$$= 0 + 0 = 0$$

$$\cos(2x+y) \xrightarrow{\frac{\partial}{\partial y}} -(0+1) \sin(2x+y)$$

$$\cos(2.5+y)$$

(d) $z = \dfrac{y}{x}$, $(1, -1)$

$$z = \frac{1}{x} \cdot y$$

$(K.f)' = K.f'$

K constante

$$\left.\frac{\partial z}{\partial x}\right|_{(1,-1)} = \left. y \cdot \left(-\frac{1}{x^2}\right)\right|_{(1,-1)} = \left. -\frac{y}{x^2}\right|_{(1,-1)} = 1$$

$$\frac{\partial z}{\partial y} = \frac{1}{x} \cdot 1 = \frac{1}{x}$$

$$\frac{\partial z}{\partial x}(1,-1) = \left. -\frac{y}{x^2}\right|_{(1,-1)} = -\frac{-1}{1^2} = 1$$

$$\frac{\partial z}{\partial y}(1,-1) = \left.\frac{\partial z}{\partial y}\right|_{(1,-1)} = \left.\frac{1}{x}\right|_{(1,-1)} = \frac{1}{1} = 1$$

21 (a) $f(x,y) = \sin(x^3 y + 2x^2)$

$(\sin u)' = u' \cos u$

$$\frac{\partial f}{\partial x} = (3yx^2 + 4x) \cdot \cos(x^3 y + 2x^2)$$

$$(x^3 . 5)' = 3x^2 . 5 = 15x^2$$

$$\frac{\partial f}{\partial y} = x^3 \cdot \cos\left(x^3 y + 2x^2\right) \qquad \frac{\partial}{\partial x}\left(x^3 \cdot y\right) = 3x^2 \cdot y$$

22. Calcule as derivadas parciais de segunda ordem das funções seguintes:

$$f(x) \longrightarrow f'(x) \longrightarrow f''(x)$$

$$f(x,y) \xrightarrow{\frac{\partial}{\partial x}} \frac{\partial f}{\partial x} = f_x \xrightarrow{\frac{\partial}{\partial x}} \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx}$$

$$\frac{\partial f}{\partial x} \xrightarrow{\frac{\partial}{\partial y}} \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} \longrightarrow f_{xy}$$

$$f(x,y) \xrightarrow{\frac{\partial}{\partial y}} \frac{\partial f}{\partial y} = f_y \xrightarrow{\frac{\partial}{\partial x}} \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = f_{yx}$$

$$\frac{\partial f}{\partial y} \xrightarrow{\frac{\partial}{\partial y}} \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = f_{yy}$$

(a) $f(x, y, z) = x^4 y^3 \cos(z^2)$

$$\frac{\partial f}{\partial x} = 4x^3 \cdot y^3 \cos(z^2)$$

$$\xrightarrow{\frac{\partial}{\partial x}} \frac{\partial^2 f}{\partial x^2} = 12\, x^2 y^3 \cos(z^2)$$

$$\xrightarrow{\frac{\partial}{\partial y}} \frac{\partial^2 f}{\partial y \partial x} = 4x^3 . 3y^2 \cdot \cos(z^2) = 12\, x^3 y^2 \cos(z^2)$$

$$\xrightarrow{\frac{\partial}{\partial z}} \frac{\partial^2 f}{\partial z \partial x} = 4x^3 . y^3 \cdot \left(-2z \sin(z^2)\right) = -8\, x^3 y^3 z \sin(z^2)$$

$$\frac{\partial f}{\partial y} = 3 . x^4 . y^2 \cos(z^2)$$

$$\xrightarrow{\frac{\partial}{\partial x}} \frac{\partial^2 f}{\partial x \partial y} = 3 \cdot \underbrace{4x^3}\, y^2 \cdot \cos(z^2) = 12\, x^3 y^2 \cos(z^2)$$

$$\xrightarrow{\frac{\partial}{\partial y}} \frac{\partial^2 f}{\partial y^2} = 6\, x^4 y \cos(z^2)$$

$$\xrightarrow{\frac{\partial}{\partial z}} \frac{\partial^2 f}{\partial z \partial y} = -3 . x^4 . y^2 \cdot 2z \sin(z^2) = -6\, x^4 y^2 z \sin(z^2)$$

$$\frac{\partial f}{\partial z} = -x^4 . y^3 \cdot \underbrace{2z}\, \underbrace{\sin(z^2)}$$

$$\xrightarrow{\frac{\partial}{\partial x}} \frac{\partial^2 f}{\partial x \partial z} = -4x^3 y^3 \cdot 2z \sin(z^2) = -8\, x^3 y^3 z \sin(z^2)$$

$$\xrightarrow{\frac{\partial}{\partial y}} \frac{\partial^2 f}{\partial y \partial z} = -x^4 \cdot 3y^2 \cdot 2z \sin(z^2) = -6\, x^4 y^2 z \sin(z^2)$$

$$\xrightarrow{\frac{\partial}{\partial z}} \frac{\partial^2 f}{\partial z^2} = -x^4 . y^3 . \left(2 . \sin(z^2) + 2z . 2z \cos(z^2)\right)$$