TP4_15_04_2021_18h

TP1 antecipar aula de 4ªF 21/04/2021 $\longrightarrow$ 16/04/2021 : 11h $\to$ 12h30

TP4 " $\longrightarrow$ 16/04/2021 : 14h $\to$ 15h30

33. Seja $f : A \subset \mathbb{R}^2 \longrightarrow \mathbb{R}$ uma função diferenciável em A e $g : D \subset \mathbb{R}^3 \longrightarrow \mathbb{R}$ definida pela expressão $g(x, y, z) = x^3 f\left(\sin \frac{x}{z}, \cos \frac{y}{z}\right)$. Mostre que existe um valor real a para o qual vale a igualdade

$$x\boxed{\frac{\partial g}{\partial x}} + y\boxed{\frac{\partial g}{\partial y}} + z\boxed{\frac{\partial g}{\partial z}} = a\, g\ .$$

(1) (2) (3) $a \in \mathbb{R}$

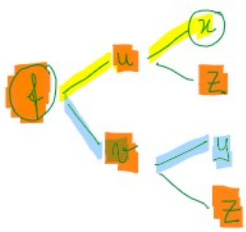
produto

$$g(x,y,z) = x^3 \cdot f(u, v)$$

$\cancel{f(x,y,z)}$ $\quad f\big(\underbrace{\sin \tfrac{x}{z}}_{u}, \underbrace{\cos \tfrac{y}{z}}_{v}\big)$

$$\frac{\partial g}{\partial x} = \frac{\partial}{\partial x}(x^3) \cdot f(u,v) + x^3 \cdot \frac{\partial f}{\partial x}$$

(derivada do produto)



$$= 3x^2 \cdot f(u,v) + x^3 \cdot \frac{\partial f}{\partial u} \frac{\partial u}{\partial x}$$

$$= 3x^2 \cdot f(u,v) + x^3 \frac{\partial f}{\partial u} \cdot \frac{1}{z} \cos\left(\frac{x}{z}\right) \quad (1)$$

$\to u = \sin\left(\frac{x}{z}\right)$

$$\frac{\partial u}{\partial x} = \frac{1}{z} \cdot \cos\left(\frac{x}{z}\right)$$

$$\frac{\partial u}{\partial z} = x \cdot \left(-\frac{1}{z^2}\right) \cos\left(\frac{x}{z}\right)$$

$$\frac{\partial g}{\partial y} = x^3 \frac{\partial f}{\partial y} = x^3 \cdot \frac{\partial f}{\partial v} \frac{\partial v}{\partial y} = x^3 \frac{\partial f}{\partial v}\left(-\frac{1}{z} \sin\left(\frac{y}{z}\right)\right) \quad (2)$$

$\to v = \cos\left(\frac{y}{z}\right)$

$$\frac{\partial v}{\partial y} = -\frac{1}{z} \sin\left(\frac{y}{z}\right)$$

$$\frac{\partial v}{\partial z} = -y \cdot \left(-\frac{1}{z^2}\right) \sin\left(\frac{y}{z}\right)$$

$$\frac{\partial g}{\partial z} = x^3 \cdot \frac{\partial f}{\partial z} = x^3 \cdot \left(\frac{\partial f}{\partial u} \underbrace{\frac{\partial u}{\partial z}}_{-\frac{x}{z^2}\cos\left(\frac{x}{z}\right)} + \frac{\partial f}{\partial v} \underbrace{\frac{\partial v}{\partial z}}_{+\frac{y}{z^2}\sin\left(\frac{y}{z}\right)}\right)$$

$$= -\frac{x^4}{z^2} \cos\left(\frac{x}{z}\right) \frac{\partial f}{\partial u} + \frac{x^3 y}{z^2} \sin\left(\frac{y}{z}\right) \frac{\partial f}{\partial v} \quad (3)$$

$$x \cdot \frac{\partial g}{\partial x} + y \frac{\partial g}{\partial y} + z \frac{\partial g}{\partial z} \overset{?}{=} a \cdot g$$

$$x \cdot \frac{\partial g}{\partial x} + y \frac{\partial g}{\partial y} + z \frac{\partial g}{\partial z} = x \cdot \left(3x^2 \cdot f(u,v) + x^3 \frac{\partial f}{\partial u} \cdot \frac{1}{z} \cos\left(\frac{x}{z}\right)\right) +$$

$$+ y \cdot x^3 \frac{\partial f}{\partial v}\left(-\frac{1}{z} \sin\left(\frac{y}{z}\right)\right) +$$

$$+ \; z \cdot \left(-\frac{x^4}{z^2}\cos\left(\frac{x}{z}\right)\frac{\partial f}{\partial u} + \frac{x^3 y}{z^2}\sin\left(\frac{y}{z}\right)\frac{\partial f}{\partial v}\right)$$

$$= 3x^3 f(u,v) + \cancel{\frac{x^4}{z}\cos\left(\frac{x}{z}\right)\frac{\partial f}{\partial u}} - \cancel{\frac{y x^3}{z}\sin\left(\frac{y}{z}\right)\frac{\partial f}{\partial v}} -$$

$$- \cancel{\frac{x^4}{z}\cos\left(\frac{x}{z}\right)\frac{\partial f}{\partial u}} + \cancel{\frac{x^3 y}{z}\sin\left(\frac{y}{z}\right)\frac{\partial f}{\partial v}}$$

$$= 3\,x^3 \cdot f = 3 \cdot g$$

$a = 3$

35 a) $z = \underbrace{xy}_{} + \underbrace{f(\overset{u}{x^2+y^2})}_{}$ Soma

? $y \cdot \overset{(1)}{\boxed{\frac{\partial z}{\partial x}}} - x \cdot \overset{(2)}{\boxed{\frac{\partial z}{\partial y}}} = y^2 - x^2$?

$f \longrightarrow u \begin{cases} x \\ y \end{cases}$

$\boxed{u = x^2 + y^2}$

$\overset{x\;x}{x^2} + \overset{y\;y}{y^2}$

$$\frac{\partial z}{\partial x} = y + \underbrace{\frac{\partial f}{\partial x}}_{\frac{df}{du}\cdot\frac{\partial u}{\partial x}} = y + 2x\, f'(u) \quad (1)$$

$\underbrace{\frac{df}{du}}_{f'(u)} \cdot \underbrace{\frac{\partial u}{\partial x}}_{2x}$

$\begin{matrix} x \\ y \end{matrix} > x^2+y^2 \longrightarrow f(x^2+y^2)$

$$\frac{\partial z}{\partial y} = x + \underbrace{\frac{\partial f}{\partial y}}_{} = x + 2y\, f'(u) \quad (2)$$

$$\frac{df}{du}\underbrace{\frac{\partial u}{\partial y}}_{2y} = f'(u)\cdot 2y$$

$$y \cdot \overset{(1)}{\boxed{\frac{\partial z}{\partial x}}} - x \cdot \overset{(2)}{\boxed{\frac{\partial z}{\partial y}}} = y\cdot\left(y + 2x\, f'(u)\right) - x\cdot\left(x + 2y\, f'(u)\right)$$

$$= y^2 + \cancel{2xy\, f'(u)} - x^2 - \cancel{2xy\, f'(u)} = y^2 - x^2 \;//$$

$$x^2 + y^2 + z^2 = 4$$

$$x^2 + y^2 + z^2 - 4 = 0 \iff z^2 = 4 - x^2 - y^2 \Rightarrow z = \pm\sqrt{4 - x^2 - y^2}$$

$$\boxed{F(x,y,z) = 0} \iff z = f_1(x,y)$$

$$\boxed{F(x,y,z)=0} \iff z = f_1(x,y)$$

$$xy + z + y^2 = 1 \Leftrightarrow \quad z = \underbrace{1 - xy - y^2}_{f_1(x,y)}$$

$$\Leftrightarrow \quad x = \frac{1 - z - y^2}{y} = f_2(y,z)$$

Teorema da função implícita $\qquad F(x,y,z)=0 \qquad P_0 = (x_0, y_0, z_0) \in D$

Se

(i) $F(P_0) = 0$

(ii) $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial y}$ e $\frac{\partial F}{\partial z}$ funções contínuas num conjunto D

(iii) $\boxed{\frac{\partial F}{\partial z}(P_0) \neq 0}$ $\left(\text{ou } \frac{\partial F}{\partial x}(P_0) \neq 0 \quad \text{ou} \quad \frac{\partial F}{\partial y}(P_0) \neq 0 \right)$

então a equação $F(x,y,z)=0$ define implicitamente z como função de (x,y) em D

" " x " " " (y,z)

" " y " " " (x,z)

(39) S: $x^3 + xy + yz = 11$

$\underbrace{x^3 + xy + yz - 11}_{F(x,y,z)} = 0$ $\qquad$? a eq. $F(x,y,z)=0$ define (implicitamente) x como função de (y,z) numa vizinhança do ponto $\underbrace{(2,1,1)}_{P_0}$?

(i) $F(P_0) = 0$

$$2^3 + 2.1 + 1.1 - 11 = 0$$

$$8 + 2 + 1 - 11 = 0$$

$$11 - 11 = 0 \checkmark$$

(ii) $\frac{\partial F}{\partial x} = 3x^2 + y$

$\frac{\partial F}{\partial y} = x + z$

$\frac{\partial F}{\partial z} = y$

são Funções polinomiais, donde contínuas em qualquer subconjunto aberto de $\mathbb{R}^3$

(iii) $\frac{\partial F}{\partial x}(P_0) = 3x^2 + y \Big|_{P_0} = 3 \cdot 2^2 + 1 = 13 \neq 0$

$P_0(2,1,1)$

Pelo T. da função implícita concluímos que

equação $F(x,y,z) = 0$ define implicitamente $\underbrace{x \text{ como função de } (y,z)}$ numa vizinhança do ponto.

$\boxed{x = f(y,z)}$ $\rightarrow$ $x = f(y_0, z_0)$

$x < \begin{matrix} y \\ z \end{matrix}$

$x = x(y,z)$

$$\frac{\partial x}{\partial y}(y_0,z_0) = -\frac{\frac{\partial F}{\partial y}(P_0)}{\frac{\partial F}{\partial x}(P_0)} = -\frac{x+z\big|_{(2,1,1)}}{13} = -\frac{2+1}{13} = -\frac{3}{13}$$

$P_0(2,1,1)$, x_0, y_0, z_0

$$\frac{\partial x}{\partial z}(y_0,z_0) = -\frac{\frac{\partial F}{\partial z}(P_0)}{\frac{\partial F}{\partial x}(P_0)} = -\frac{y\big|_{(2,1,1)}}{13} = -\frac{1}{13}$$

$F(x,y,z)$

$F(x_0,y_0,z_0)$

y_0, z_0

$\rightarrow \frac{\partial x}{\partial y}(1,1)$

$\rightarrow \frac{\partial x}{\partial z}(1,1)$