TP4_11_03_2021

11 de março de 2021 16:56

6

(d) $x^2 + y^2 - z^2 = -1$ $\Rightarrow$ $\underbrace{x^2 + y^2}_{\geq 0} = \underbrace{z^2 - 1}_{\geq 0}$

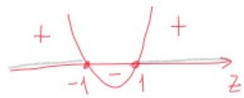
implica

$-x^2 - y^2 + z^2 = 1$

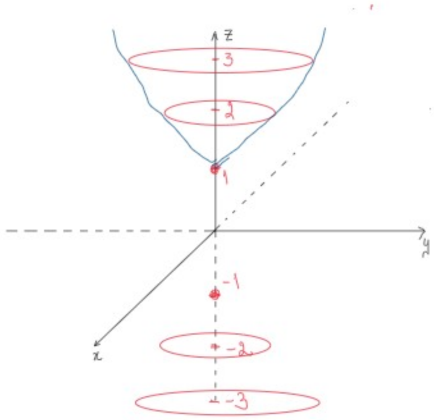
Hiperbolóide de 2 folhas

Concluímos que

$z^2 - 1 \geq 0$

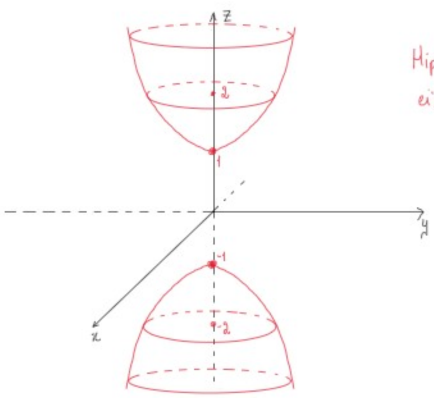


$z \leq -1 \ \vee \ z \geq 1$



- $\cancel{z = 0}$
- $z = 1$ ou $z = -1$: $x^2 + y^2 = 0 \Rightarrow x^2 = 0 \wedge y^2 = 0$
  $\Rightarrow x = 0 \wedge y = 0$
  $\downarrow$ $P_1(0,0,1)$ $\quad$ $\downarrow$ $P_2(0,0,-1)$
- $z = 2$ ou $z = -2$: $x^2 + y^2 = 3$ eq. circunferência de raio $\sqrt{3}$
  $\downarrow$ $C(0,0,2)$ $\quad$ $\downarrow$ $C(0,0,-2)$
- $z = 3$ ou $z = -3$: $x^2 + y^2 = 8$ eq. circunf. $r = \sqrt{8}$

- $x = 0$: $-\cancel{x^2}_{0} - y^2 + z^2 = 1$ $\quad$ $-y^2 + z^2 = 1$ hipérbole
- $y = 0$ $\quad$ $-x^2 + z^2 = 1$ hipérbole

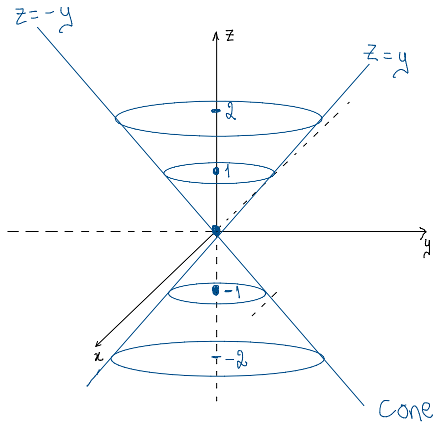


Hiperbolóide de duas folhas ao longo do eixo dos zz

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(h) $4x^2 + y^2 - z^2 = 0$ Cone

$\Rightarrow \; 4x^2 + y^2 = z^2$



- $\boxed{z=0}$ (xOy) $\longrightarrow 4x^2 + y^2 = 0 \Rightarrow 4x^2 = 0 \wedge y^2 = 0 \Rightarrow \underline{x=0} \wedge \underline{y=0}$

  $P(0,0,0)$

- $\boxed{z=\pm 1} \longrightarrow 4x^2 + y^2 = 1 \Leftrightarrow \dfrac{x^2}{\frac{1}{4}} + y^2 = 1 \Leftrightarrow \dfrac{x^2}{\left(\frac{1}{2}\right)^2} + y^2 = 1$

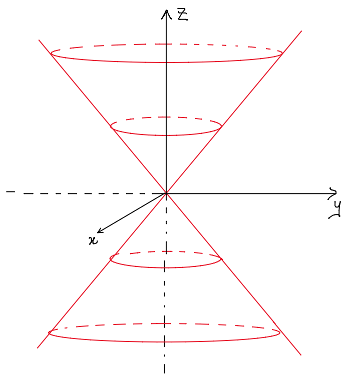
  elipse com semieixos $a = \frac{1}{2}$ e $b = 1$

- $\boxed{z = \pm 2} \longrightarrow 4x^2 + y^2 = 4 \Leftrightarrow x^2 + \dfrac{y^2}{4} = 1$

  elipse com semi-eixos $a = 1$ e $b = 2$

Cone elíptico ao longo do eixo dos zz

- $\boxed{x=0}$ (yOz) $\Rightarrow y^2 = z^2 \Leftrightarrow y = z \vee y = -z \Rightarrow z = y \vee z = -y$



cone elíptico ao longo do eixo dos zz

(i) $\dfrac{x^2}{4} - y^2 + \dfrac{z^2}{9} = 0 \quad \Rightarrow \quad \dfrac{x^2}{4} + \dfrac{z^2}{9} = y^2$

- $\boxed{y=0} \quad \dfrac{x^2}{4} + \dfrac{z^2}{9} = 0 \Rightarrow x = z = 0 \quad P(0,0,0)$

- $\boxed{y = \pm 1} \quad \dfrac{x^2}{4} + \dfrac{z^2}{9} = 1$ elipses $C(0,1,0)$ ou $C(0,-1,0)$ e semi-eixos $a=2$ e $b=3$

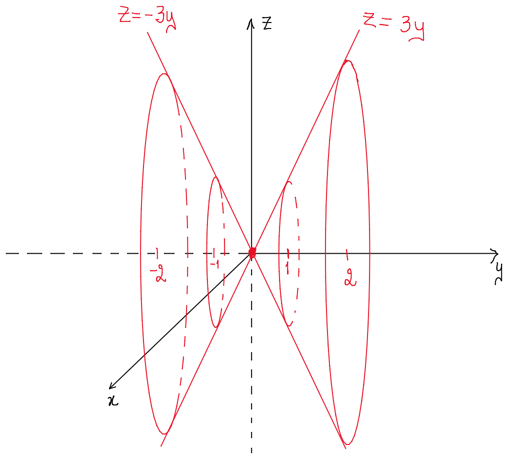
- $\boxed{y = \pm 2} \quad \dfrac{x^2}{4} + \dfrac{z^2}{9} = 4 \Leftrightarrow \dfrac{x^2}{16} + \dfrac{z^2}{36} = 1$ elipses $C(0,2,0)$ ou $C(0,-2,0)$ e Semi-eixos $a=4$ e $b=6$

- $\boxed{x=0} \quad \dfrac{z^2}{9} = y^2 \Leftrightarrow \left(\dfrac{z}{3}\right)^2 = y^2 \Leftrightarrow \dfrac{z}{3} = y \vee \dfrac{z}{3} = -y \Leftrightarrow z = 3y \vee z = -3y$

  (duas retas)

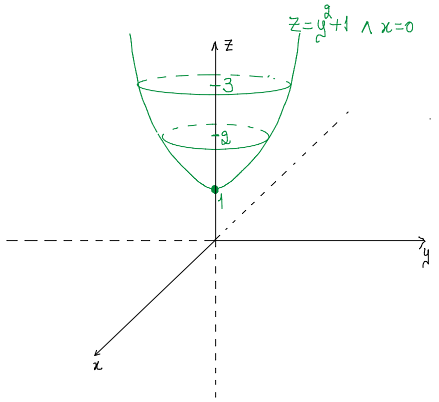
- $\boxed{z=0} \quad \dfrac{x^2}{4} = y^2 \Rightarrow \dfrac{x}{2} = y \vee \dfrac{x}{2} = -y \Rightarrow y = \dfrac{x}{2} \vee y = -\dfrac{x}{2}$

  (duas retas)



(k) $z = \underbrace{x^2+y^2+1}_{\geqslant 1}$

Parabolóide circular ao longo do eixo dos zz



$\boxed{z=1} \longrightarrow \cancel{1} = x^2+y^2+\cancel{1} \Rightarrow x^2+y^2=0 \Rightarrow x=y=0$

$P(0,0,1)$

$\boxed{z=2} \longrightarrow 2 = x^2+y^2+1 \Rightarrow x^2+y^2=1$

Circunferência de centro $C(0,0,2)$ e raio 1

$\boxed{z=3} \longrightarrow 3 = x^2+y^2+1 \Rightarrow x^2+y^2=2$

Circunferência de centro $C(0,0,3)$ e raio $\sqrt{2}$

$\boxed{x=0}$ yOz $\longrightarrow z = y^2+1$ parábola com concavidade voltada para cima e vértice $V(0,0,1)$

$\boxed{y=0} \longrightarrow z = x^2+1$ parábola

(l) $y = \underbrace{x^2+2z^2}_{\geqslant 0}$

Se $\boxed{y=0} \longrightarrow 0 = x^2+2z^2 \Rightarrow P(0,0,0)$

Se $\boxed{y=1} \longrightarrow 1 = x^2+2z^2 \Rightarrow x^2+2z^2=1 \Rightarrow x^2+\dfrac{z^2}{\frac{1}{2}}=1 \Rightarrow x^2+\dfrac{z^2}{\left(\frac{1}{\sqrt{2}}\right)^2}=1$

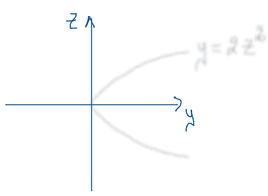
$x=0 \to 2z^2=1 \Rightarrow z^2=\frac{1}{2} \Rightarrow z=\pm\sqrt{\frac{1}{2}}$

$z=0 \to x^2=1 \Rightarrow x=\pm 1$

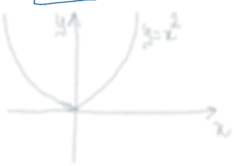
elipse com semi-eixos $a=1$ e $c=\frac{1}{\sqrt{2}}$

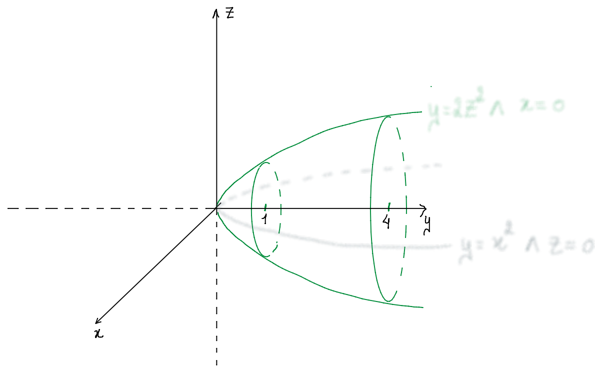
Se $\boxed{y=4} \longrightarrow 4 = x^2+2z^2 \Rightarrow 1 = \dfrac{x^2}{4}+\dfrac{z^2}{2}$ elipse com semi-eixos $a=2$ $c=\sqrt{2}$

Se $\boxed{x=0}$ plano yOz $\longrightarrow y = 2z^2$ parábola

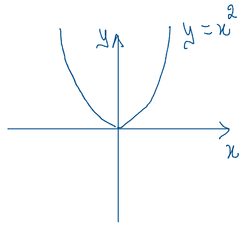


Se $\boxed{z=0}$ $\rightarrow$ $y = x^2$ parábola



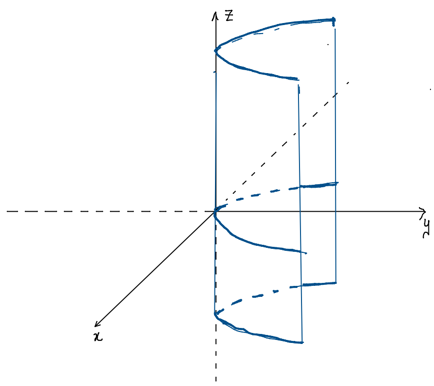


(r) $y = x^2$



no plano xOy $(z=0)$

Qualquer que seja o valor de z, temos sempre que desenhar a parábola $y=x^2$.



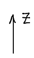
cilindro parabólico

(s) $z = -x^2 + 2y^2$ Parabolóide hiperbólico ("sela do cavalo")

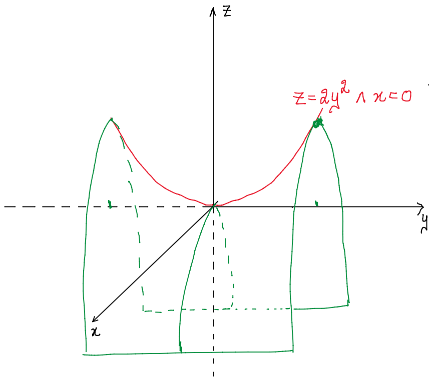
Se $\boxed{x=0}$ $\rightarrow$ $\boxed{z = 2y^2}$ parábola concavidade voltada para cima

Se $\boxed{y=0}$ $\rightarrow$ $\boxed{z = -x^2}$ parábola " " " baixo

Se $z=0$ $\rightarrow$ $-x^2+2y^2=0 \Rightarrow x^2 = 2y^2 \Rightarrow x^2 = (\sqrt{2}\,y)^2 \Rightarrow \underbrace{x=\sqrt{2}\,y \vee x=-\sqrt{2}\,y}_{\text{retas}}$



retas



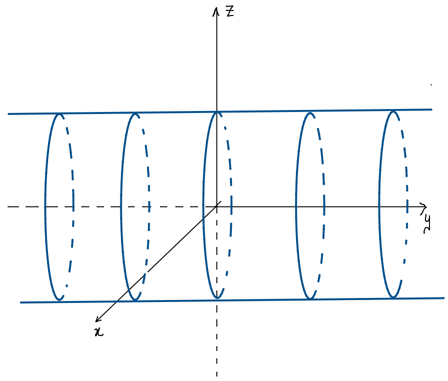
$y=\pm 4 \rightarrow z = -x^2 + 2.16$

$z = -x^2 + \underline{\underline{32}}$ parábola

(m) $4x^2 + z^2 = 1 \quad \Leftrightarrow \quad \dfrac{x^2}{\frac{1}{4}} + z^2 = 1 \quad \Leftrightarrow \quad \dfrac{x^2}{\left(\frac{1}{2}\right)^2} + z^2 = 1$

Qualquer que seja o valor de y estamos sempre perante uma elipse de semi-eixos $a=\frac{1}{2}$ e $c=1$.

Trata-se de um **cilindro elíptico ao longo do eixo dos yy**.



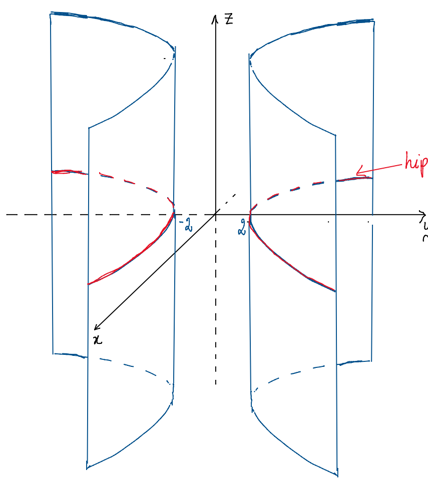
(o) $y^2 - x^2 = 4 \quad \Leftrightarrow \quad \dfrac{y^2}{4} - \dfrac{x^2}{4} = 1 \quad \Leftrightarrow \quad \dfrac{y^2}{2^2} - \dfrac{x^2}{2^2} = 1$

Qualquer que seja o valor de z, estamos sempre perante uma **hipérbole ao longo do eixo dos yy**

No plano $z=0$ (xOy)

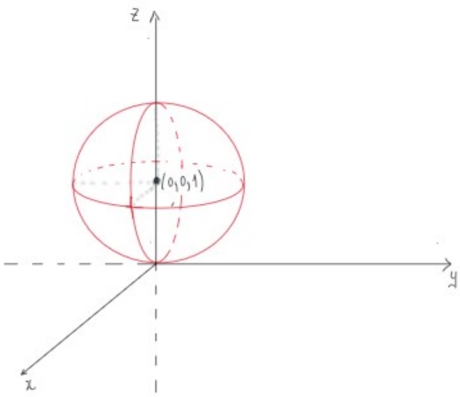
Se $x=0 \quad \dfrac{y^2}{2^2} = 1 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

Se $y=0 \quad -\dfrac{x^2}{4} = 1$ impossível



(p) $x^2+y^2+z^2-2z=0 \iff x^2+y^2+(z^2-2z+1)=1 \iff x^2+y^2+(z-1)^2=1$

Superfície esférica de centro $(0,0,1)$ e raio 1.

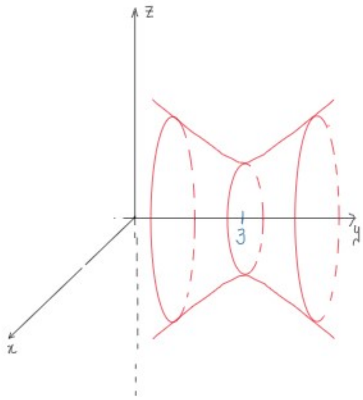


(q) $x^2-y^2+6y+z^2=10 \iff x^2-(y^2-6y)+z^2=10$

$\iff x^2-(y^2-6y+9)+z^2=10-9$

$\iff x^2-(y-3)^2+z^2=1$

hiperbolóide de 1 folha



(t) $z^2=9 \iff z=-3 \lor z=3$

As variáveis x e y são livres