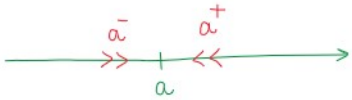
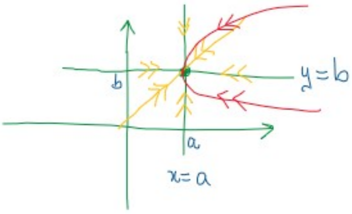
TP4_25_03_2021 18h

25 de março de 2021



$f(x)$ $\qquad \lim\limits_{x\to a} f(x)$

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$$



$$\lim_{\substack{(x,y)\to(a,b)\\ x=a}} f(x,y) \qquad \lim_{\substack{(x,y)\to(a,b)\\ y=b}} f(x,y)$$

Nota 2 *Na prática,*

$$\lim_{X\to X_0} f(X) = b \quad \textit{se} \quad |f(X)-b| \xrightarrow[X\to X_0,\ X\neq X_0]{} 0$$

$X=(x,y) \qquad X_0=(x_0,y_0)$

Quando nos pedirem para provar que $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = b$

14. A7 Mostre que: (a) $\lim\limits_{(x,y)\to(0,0)} \underbrace{\frac{x^2(y+1)+y^2}{x^2+y^2}}_{\left(\frac{0}{0}\right)} = 1$

$$\left|\frac{x^2(y+1)+y^2}{x^2+y^2} - \frac{1(x^2+y^2)}{(x^2+y^2)}\right| = \left|\frac{x^2y+\cancel{x^2}+\cancel{y^2}-\cancel{x^2}-\cancel{y^2}}{x^2+y^2}\right| = \left|\frac{x^2y}{x^2+y^2}\right| = \frac{x^2|y|}{x^2+y^2} \leq$$

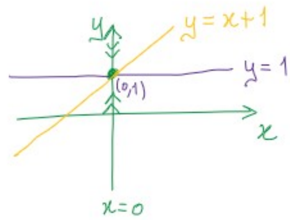
$$\leq \frac{\cancel{x^2}|y|}{\cancel{x^2}} = |y| \xrightarrow[(x,y)\to(0,0)]{} 0$$

A, B e C forem positivos $\qquad \dfrac{A}{B+C} \leq \dfrac{A}{B}$

Concluímos que $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$

(b) não existe $\lim\limits_{(x,y)\to(0,1)} \underbrace{\frac{x(y-1)}{x^2+y^2-2y+1}}_{f(x,y)} \quad \left(\frac{0}{0}\right) \qquad f(x,y) = \dfrac{x(y-1)}{x^2+(y-1)^2}$

Ideia: $y-1=x$
$y=x+1$



Calculemos alguns limites restritos

$$\lim_{\substack{(x,y)\to(0,1)\\ x=0}} f(x,y) = \lim_{y\to 1} \frac{0.(y-1)}{0^2+(y-1)^2} = \lim_{y\to 1} \frac{0}{(y-1)^2} = \lim_{y\to 1} 0 = 0 \,//$$

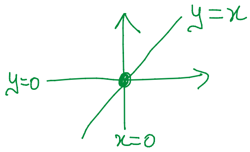
$$\lim_{\substack{(x,y)\to(0,1)\\ y=1}} f(x,y) = \lim_{x\to 0} \frac{x\cdot 0}{x^2+0^2} = \lim_{x\to 0} \frac{0}{x^2} = \lim_{x\to 0} 0 = 0$$

$$\lim_{\substack{(x,y)\to(0,1)\\ y=x+1}} f(x,y) = \lim_{x\to 0} \frac{x\cdot x}{x^2+x^2} = \lim_{x\to 0} \frac{\cancel{x^2}}{2\cancel{x^2}} = \frac{1}{2}$$

Como encontramos dois limites restritos com valores diferentes $\left(0 \neq \frac{1}{2}\right)$ concluímos que não existe limite.

(15)

(g) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2y^2+(x-y)^2}$ $\left(\frac{0}{0}\right)$



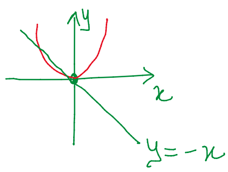
$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}} f(x,y) = \lim_{y\to 0} \frac{0}{0+(-y)^2} = \lim_{y\to 0} \frac{0}{y^2} = \lim_{y\to 0} 0 = 0$$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=x}} f(x,y) = \lim_{x\to 0} \frac{x^2\cdot x^2}{x^2\cdot x^2+0^2} = \lim_{x\to 0} \frac{x^4}{x^4} = 1$$

$\neq$

$$\nexists \lim_{(x,y)\to(0,0)} f(x,y)$$

(m) $\displaystyle\lim_{(x,y)\to(0,0)} \underbrace{\frac{x^3}{x^2-y}}_{f(x,y)}$ $\left(\frac{0}{0}\right)$



Ideia: $\dfrac{x^3}{x^2-y}$ $(0,0)$

$\uparrow$

$y = x^2 - x^3$

$\cancel{\dfrac{x^3}{0}}$

$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}} f(x,y) = \lim_{y\to 0} \frac{0^3}{0^2-y} = \lim \underbrace{\frac{0}{-y}}_{=0} = 0$$

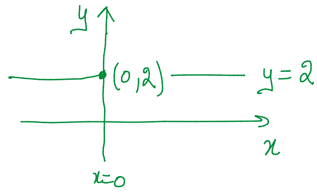
$$\lim_{\substack{(x,y)\to(0,0)\\ y=0}} f(x,y) = \lim_{x\to 0} \frac{x^3}{x^2-0} = \lim_{x\to 0} x = 0$$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=x^2-x^3}} f(x,y) = \lim_{x\to 0} \frac{x^3}{\cancel{x^2}-\cancel{x^2}+x^3} = \lim 1 = 1$$

$$\nexists \lim_{(x,y)\to(0,0)} f(x,y)$$

(f) $\underbrace{\lim_{(x,y)\to(0,2)} \frac{x\sin^2(y-2)}{x^2+y^2-4y+4}}_{f(x,y)}$ $\left(\frac{0}{0}\right)$

$$f(x,y) = \frac{x \cdot \sin^2(y-2)}{x^2 + (y-2)^2}$$



$$\lim_{\substack{(x,y)\to(0,2)\\ x=0}} f(x,y) = \lim_{y\to 2} \frac{0}{0+(y-2)^2} = \lim_{y\to 2} 0 = 0$$

$\sin h \approx h$ (h pequeno)

$$\lim_{\substack{(x,y)\to(0,2)\\ y=2}} f(x,y) = \lim_{x\to 0} \frac{x \cdot 0}{x^2+0^2} = \lim_{x\to 0} \frac{0}{x^2} = \lim_{x\to 0} 0 = 0$$

$$\left| f(x,y) - 0 \right| = \left| \frac{x \cdot \sin^2(y-2)}{x^2+(y-2)^2} \right| = \frac{|x| \sin^2(y-2)}{x^2+(y-2)^2} \leq \frac{|x| \cdot \sin^2(y-2)}{(y-2)^2} \longrightarrow 0$$

$$\xrightarrow{(x,y)\to(0,2)} |0| \cdot 1 = 0$$

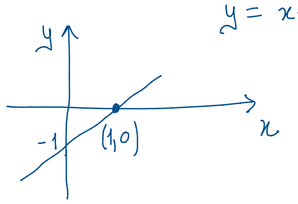
$$\lim_{x\to 0} \frac{\sin^2 x}{x^2} = 1 \qquad \lim_{u\to 0} \frac{\sin^2 u}{u^2} = 1$$

Portanto, concluímos que $\lim_{(x,y)\to(0,2)} f(x,y) = 0$

$$\lim_{u\to 0} \frac{\sin u}{u} = 1$$

(h) $\underbrace{\lim_{(x,y)\to(1,0)} \frac{y\ \mathrm{sen}(x-1)}{|x-1|+|y|}}_{f(x,y)}$ $\left(\frac{0}{0}\right)$

$y = x-1$ $\quad (1,0)$



$$\lim_{\substack{(x,y)\to(1,0)\\ y=x-1}} f(x,y) = \lim_{x\to 1} \frac{(x-1)\ \mathrm{sen}(x-1)}{|x-1|+|x-1|}$$

$$= \lim_{x\to 1} \frac{(x-1)\ \mathrm{sen}(x-1)}{2|x-1|}$$

$$= \pm \frac{1}{2} \lim_{x\to 1} \mathrm{sen}(x-1) = 0$$

$$\lim_{x\to 1} \frac{\mathrm{sen}(x-1)}{x-1} = 0$$

$$\frac{x-1}{|x-1|} = \begin{cases} \dfrac{x-1}{x-1} = 1 & \text{se } x-1>0 \\[2mm] \dfrac{x-1}{-(x-1)} = -1 & \text{se } x-1<0 \end{cases}$$

$$\lim_{\substack{(x,y)\to(1,0)\\ y=0}} f(x,y) = 0$$

$$\left| f(x,y) - 0 \right| = \left| \frac{y \cdot \operatorname{sen}(x-1)}{|x-1|+|y|} - 0 \right| = \left| \frac{y \operatorname{sen}(x-1)}{|x-1|+|y|} \right| = \frac{|y \cdot \operatorname{sen}(x-1)|}{|x-1|+|y|} =$$

$|A \cdot B| = |A| \cdot |B|$

$$= \frac{|y| \cdot |\operatorname{sen}(x-1)|}{|x-1|+|y|} \leq \frac{\cancel{|y|} \cdot |\operatorname{sen}(x-1)|}{\cancel{|y|}} = |\operatorname{sen}(x-1)| \xrightarrow[(x,y)\to(1,0)]{} 0$$

Fica provado que $\lim_{(x,y)} f(x,y) = 0$