2. Data Points: $[3 \quad 10 \quad 30]$

R matrix after E step: $R = \begin{bmatrix} 1 & 0 \\ 0.4 & 0.6 \\ 0 & 1 \end{bmatrix}$

a) The likelihood function we are trying to optimize is given by:

$$p(x) = \sum_{k=1}^{K} \pi_k \mathcal{N}(x \mid \mu_k, \sigma_k)$$

The log of this likelihood function is given by:

$$\ln p(x \mid \pi, \mu, \Sigma) = \sum_{n=1}^{N} \ln \left\{ \sum_{k=1}^{K} \pi_k \mathcal{N}(x_n \mid \mu_k, \Sigma_k) \right\}$$

Important Equations:

- Setting derivatives of $\ln p(X \mid \pi, \mu, \Sigma)$, with respect to μ_k, to zero, we obtain: $\mu_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma_{nk} x_n$,

with $N_k = \sum_{n=1}^{N} \gamma_{nk}$, the

effective number of points assigned to cluster k.

- Setting derivatives of $\ln p(X \mid \pi, \mu, \Sigma)$, with respect to Σ_k, to zero, we obtain:

$$\Sigma_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma_{nk} (x_n - \mu_k)(x_n - \mu_k)^T$$

- Setting the derivatives of $\ln p(X|\pi, \mu, \Sigma)$ with respect to π_k, to zero, we obtain : $\pi_k = \frac{N_k}{N}$.

b) New values of π_1 and π_2 :

$$\pi_k^{new} = \frac{N_k}{N} \quad ; \quad N_k = \sum_{n=1}^{N} q_{nk}$$

For π_1 :

$$\pi_1^{new} = \frac{N_1}{N} \; ; \; N = 3 \; ; \; N_1 = (1 + 0.4 + 0) = 1.4 \; ; \; \pi_1 = \frac{1.4}{3} = \boxed{\frac{7}{15}}$$

$$\pi_2^{new} = \frac{N_2}{N} \; ; \; N = 3 \; ; \; \pi_2 = \frac{1.6}{3} = \boxed{\frac{8}{15}}$$

$$N_2 = (0 + 0{,}6 + 1) = 1.6$$

c) New values of μ_1 and μ_2:

$$\mu_k^{new} = \frac{1}{N_k} \sum_{n=1}^{N} r_{nk} x_n$$

$$\mu_1^{new} = \frac{1}{N_1} (1 \times 3 + 0{,}4 \times 10 + 0 \times 30) = \frac{7}{1.4} = \boxed{5}$$

$N_1 = 1.4$

$$\mu_2^{new} = \frac{1}{N_2} \left(0 \times 3 + 0,6 \times 10 + 1 \times 30 \right) = \frac{36}{1.6} = \boxed{22,5}$$

$$N_2 = 1.6$$