PATTERN RECOGNITION
Computer Science COMP - 644 B
Second Midterm Test - March 29, 2004

1) Bayes Decision Theory (discrete case) *(3 points)*

Let the features X = (x_1,x_2,...,x_d) be binary valued (1 or 0). Let p_{ij} denote the probability that feature x_i takes on the value 1 given class j. Assume that there are only two classes and that they are equally probable. Let the features be *conditionally independent* for both classes. Finally, assume that d is odd and that $p_{i1} = p > 1/2$ and $p_{i2} = 1 - p$, for all i. Show that the *optimal Bayes* decision rule becomes: decide class one if $x_1 + x_2 + ... + x_d > d/2$, and class two otherwise.

2) Nearest Neighbor Decision Rule Condensing *(3 points)*

The *relative neighborhood graph* of a set of n points S in space is defined as follows. The *vertices* in the graph are the n given points of S. Let L_{ij} denote the *intersection* of the two circles determined by two points x_i and x_j, such that they are centered at x_i and x_j, respectively, and each circle has radius equal to the distance between x_i and x_j. If no other points of S fall strictly inside L_{ij} then x_i and x_j are joined by an edge in the graph. In the *relative neighborhood graph editing* algorithm all data points in S that have all their relative neighbors in the same class are removed (in parallel) from S. The resulting condensed set (remaining points) is denoted by C. Prove or disprove that the relative neighborhood graph editing algorithm is *training-set-consistent.* Recall that training-set-consistent means that using C as the classifier the *nearest-neighbor* decision rule classifies all points in S correctly.

3) 2-Nearest Neighbor Decision Rule with a Reject Option *(4 points)*

Consider the 2-class problem. The 2-*NN* rule classifies a pattern by first finding its two nearest neighbors. If both neighbors belong to the same class then the pattern is classified into that class. Otherwise the pattern is rejected and no classification is made. Prove under reasonable assumptions (state them) that the asymptotic probability of misclassification of the 2-*Nearest-Neighbor* decision rule is no more than the Bayes probability of misclassification.