Statistics — Probability vs. Odds

September 19, 2014

Probability and odds are two basic statistic terms to describe the likeliness that an event will occur. They are often used interchangeably in causal conversation or even in published material. However, they are not mathematically equivalent because they are looking at likeliness in different contexts. In everyday conversation when numbers or values aren't given, the two terms are synonymous . If an event has a high probability, then it has high odds for happening. The incorrect usage arises when a person ascribes a mathematical value to either the odds or probability they are discussing. Hopefully, if you aren't quite sure what the exact mathematical difference is, this will clear it up for you.

Probability is defined as the fraction of desired outcomes in the context of every possible outcome with a value between 0 and 1, where 0 would be an impossible event and 1 would represent an inevitable event. Probabilities are usually given as percentages. [ie. 50% probability that a coin will land on HEADS.] Odds can have any value from zero to infinity and they represent a ratio of desired outcomes versus the field. Odds are a ratio, and can be given in two different ways: 'odds in favor' and 'odds against'. 'Odds in favor' are odds describing the if an event will occur, while 'odds against' will describe if an event will not occur. If you are familiar with gambling, 'odds against' are what Vegas gives as odds. More on that later. For the coin flip odds in favor of a HEADS outcome is 1:1, not 50%.

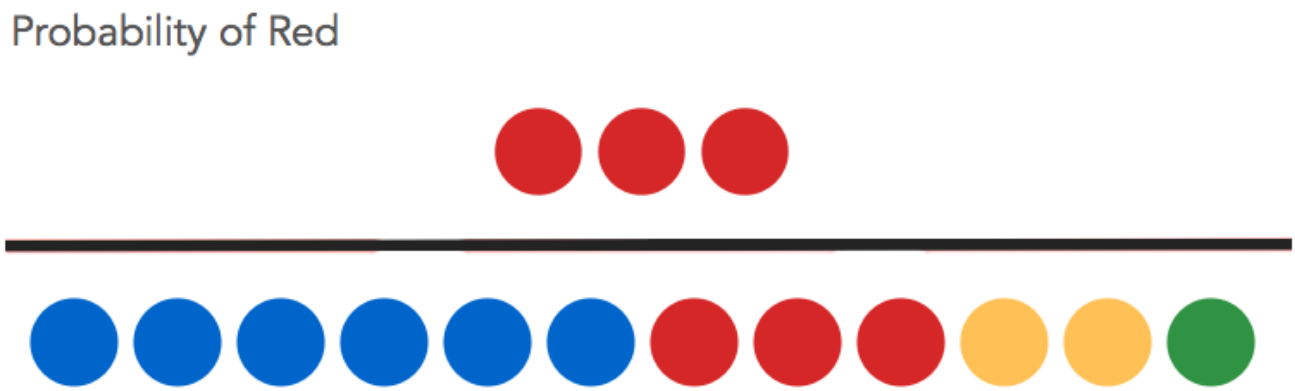
Visual Math

Simple probability of event A occurring is mathematically defined as:

$$P(A) = \frac{Number\ of\ Event\ A}{Total\ Number\ of\ Events}$$

The best way to illustrate this is with the classic marbles-in-a-bag example. The graphic below depicts all the marbles in an opaque bag that one marble will be pulled out of. There are 6 blue, 3 red, 2 yellow, and 1 green for a total of 12 marbles in the bag.

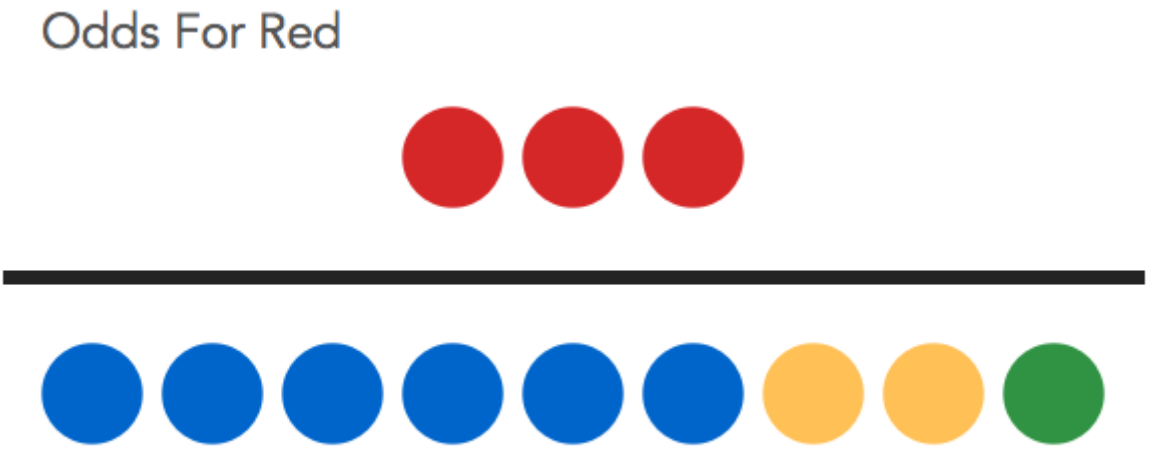
The probability of pulling a red marble would be calculated by taking the total number of red marbles and dividing it by the total number of marbles.



OR

$$P(RED) = \frac{3\ RED\ marbles}{12\ TOTAL\ marbles} = 25\%.$$

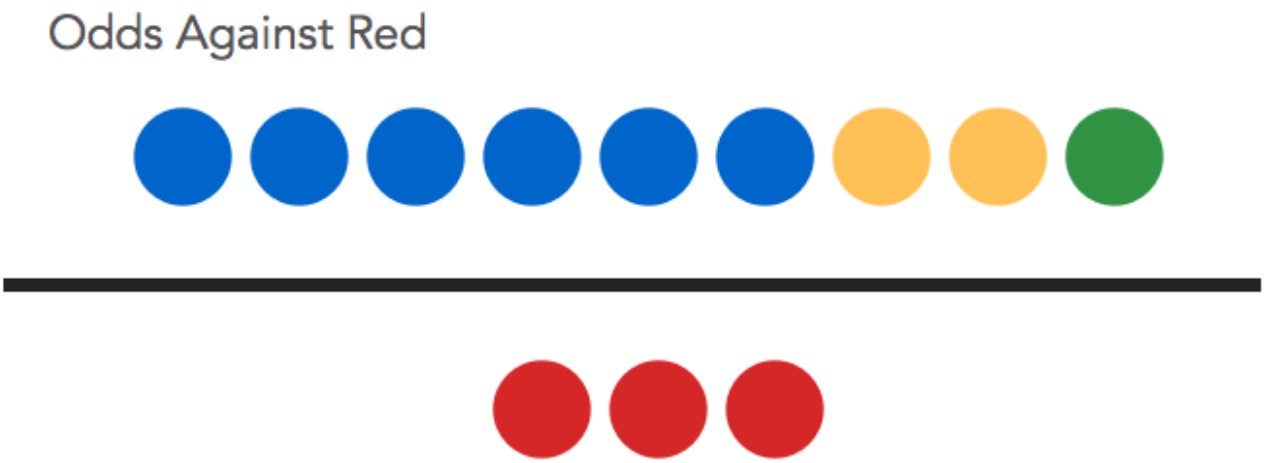
Notice that the probability calculation includes the red marbles in the denominator of the calculation, because probability considers the context of the entire event space. Odds, on the other hand, are the ratio of favorable outcomes to unfavorable outcomes. The denominator contains ONLY the marbles that aren't the favorable outcomes. Odds uses the contexts of good outcomes and bad outcomes. Written as fractions, these two values are completely different. Probability is 1/4 while odds in favor are 1/3. You can see how mistakenly interchanging the terms could give the wrong information. The 'odds in favor' of RED would be mathematically calculated by



OR

$$Odds\_Favor(RED) = \frac{3\ RED\ marbles}{9\ NOT\ RED\ marbles} = 1:3.$$

To find 'odds against' you would simply flip odds in favor upside down and this describes the odds of the event not occurring.



OR

$$Odds\_Against(RED) = \frac{9\ NOT\ RED\ marbles}{3\ RED\ marbles} = 3:1$$

Gambling

'Odds against' are commonly are used in the context of gambling. When you hear that the Seattle Seahawks Vegas odds to win the Super Bowl are 5:1 [Retrieved 9/19/2014], the 5:1 is referring to the 'odds against' Seattle winning the Super Bowl. Using some quick math we could determine the probability of Seattle winning the Super Bowl would be 1/6 or 16.7%.

Vegas odds are technically payoff odds, because they describe the payout if you were to win the bet. The payout on the Seahawks would win you $5 for every $1 bet on the Seattle winning the Super Bowl. They aren't true odds, since no one is really sure what the true odds are, because you can't simply count and weigh the possibilities like with the bag of marbles. The payoff will increase when the event becomes less likely. If you could create a reliable predictive model that told you the Seahawks actually had a 20% probability to win the Super Bowl, you could bet on the Seahawks, knowing that their actual probability to win is better than what Vegas is giving them. And if you made enough bets like this you could beat Vegas.

Mathematical Relationship

I stated earlier that probability and odds were colloquially interchangeable when values aren't given. This is true, because the two are mathematically related. Odds can be computed from probability and probability from odds.

$$P(A) = \frac{Odds\_Favor(A)}{1+Odds\_Favor(A)}$$

$$Odds\_Favor(A) = \frac{P(A)}{1-P(A)}$$

Using the RED marble example [P(RED) = 1/4 and Odds_Favor(RED) = 1/3] we can demonstrate how these are equivalent:

$$P(RED) = \frac{1/3}{1+1/3} = \frac{1/3}{4/3} = \frac{1}{4}$$

$$Odds\_Favor(RED) = \frac{1/4}{1-1/4} = \frac{1/4}{3/4} = \frac{1}{3}$$