Exc. 3 c)

So, we have: $p(x|y_0) = \frac{e^{K_0 \cos(x-\mu_0)}}{(2\pi I(K_0))}$ and $p(x|y_1) = \frac{e^{K_1 \cos(x-\mu_1)}}{(2\pi I(K_1))}$

The goal is to show that: $p(y_0|x) = \frac{1}{\left(1 + e^{w_0 + w_1 \sin(x-\theta)}\right)}$

First of all: $p(C_i|x) = \frac{P(x|C_i)P(C_i)}{P(x)}$, $P(x) = \sum_{i=1}^{2} P(x|C_i)P(C_i)$

So,

$$P(y_0|x) = \frac{P(x|y_0)P(y_0)}{P(x|y_0)P(y_0) + P(x|y_1)P(y_1)}$$

Assuming that $P(y_0) = P(y_1)$:

$$P(y_0|x) = \frac{P(x|y_0)\cancel{P(y_0)}}{\cancel{P(y_0)}\left(P(x|y_0) + P(x|y_1)\right)}$$

$$P(y_0|x) = \frac{P(x|y_0)}{P(x|y_0) + P(x|y_1)}$$

$$= \frac{\frac{e^{K_0 \cos(x-\mu_0)}}{2\pi I(K_0)}}{\frac{e^{K_0 \cos(x-\mu_0)}}{2\pi I(K_0)} + \frac{e^{K_1 \cos(x-\mu_1)}}{2\pi I(K_1)}}$$

$$= \frac{\dfrac{e^{K_0 \cos(x-\mu_0)}}{2\pi\ell(K_0)}}{\dfrac{2\pi\ell(K_1)\, e^{K_0\cos(x-\mu_0)} + 2\pi\ell(K_0)\, e^{K_1\cos(x-\mu_1)}}{2\pi\ell(K_0)\; 2\pi\ell(K_1)}}$$

$$= \frac{e^{K_0\cos(x-\mu_0)}}{2\pi\cancel{\ell(K_0)}} \cdot \frac{\cancel{2\pi\ell(K_0)}\, 2\pi\ell(K_1)}{2\pi\ell(K_1)\, e^{K_0\cos(x-\mu_0)} + 2\pi\ell(K_0)\, e^{K_1\cos(x-\mu_1)}} =$$

$\Rightarrow$ Divide numerator and denominator by: $2\pi\ell(K_1)\, e^{K_0\cos(x-\mu_0)}$

$$\frac{1}{1 + \dfrac{2\pi\ell(K_0)\, e^{K_1 \cdot \cos(x-\mu_1)}}{2\pi\ell(K_1)\, e^{K_0\cos(x-\mu_0)}}} = \frac{1}{1 + \dfrac{\ell(K_0)}{\ell(K_1)}\, e^{K_1\cos(x-\mu_1) - K_0\cos(x-\mu_0)}}$$

$$= \frac{1}{1 + e^{\ln\left(\frac{\ell(K_0)}{\ell(K_1)}\right) + K_1\cos(x-\mu_1) - K_0\cos(x-\mu_0)}}$$

$\Rightarrow$ If we assume that $\mu_1 = \mu_0 = \mu$

$$= \frac{1}{1 + e^{\ln\left(\frac{\ell(K_0)}{\ell(K_1)}\right) + (K_1 - K_0)\cos(x-\mu)}}$$

$\Rightarrow \cos\left(x - \frac{\pi}{2}\right) = \sin(x) \; ; \; \sin\left(x + \frac{\pi}{2}\right) = \cos(x)$

$$= \frac{1}{1 + e^{\ln\left(\frac{\ell(k_0)}{\ell(k_1)}\right) + (k_1 - k_0)\sin\left(x - \mu + \frac{\pi}{2}\right)}}$$

$$= \frac{1}{1 + e^{w_0 + w_1 \sin(x - \vartheta)}}$$

, where:

$$w_0 = \ln\left(\frac{\ell(k_0)}{\ell(k_1)}\right)$$

$$w_1 = k_1 - k_0$$

$$\vartheta = \mu - \frac{\pi}{2},$$