Exc 3.

$Y, p(x) = e^{-x}, \quad x \geq 0$

Z, Uniform probability distribution (density) in $[0, \beta]$

$$h(Z) = -\int_0^\beta \frac{1}{\beta} \log\left(\frac{1}{\beta}\right) dx = \log(\beta)$$

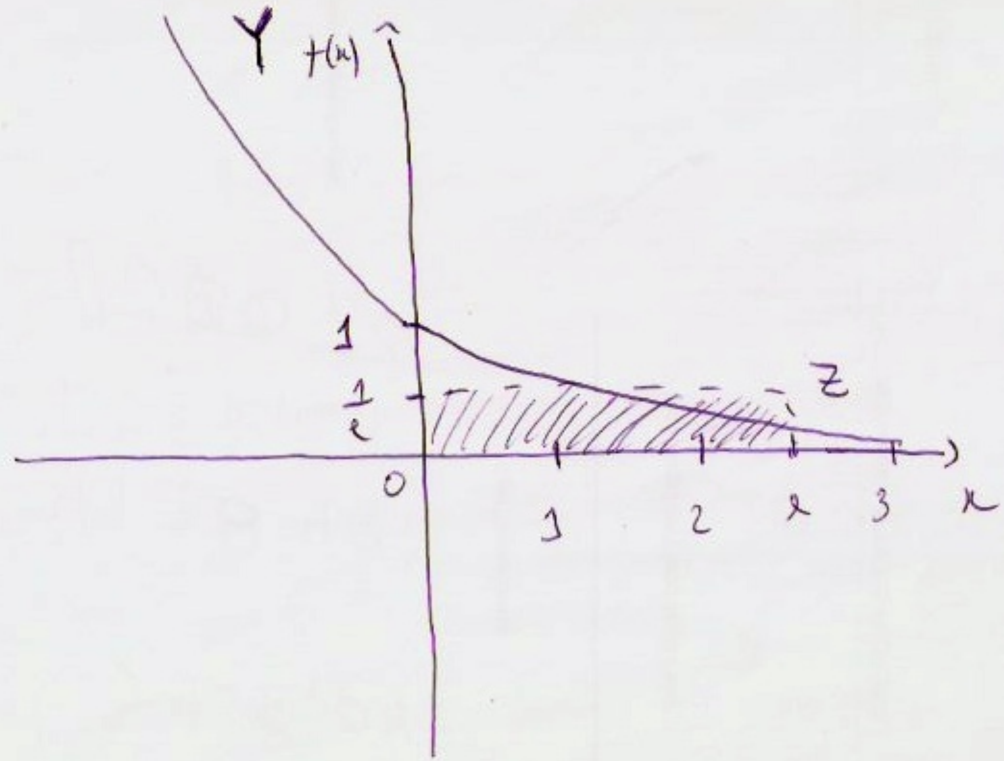
$$h(Y) = -\int e^{-x} \log(e^{-x})\, dx$$

$$= -\int -x\, e^{-x} \log(e)\, dx$$

$$= -\log(e) \int -x\, e^{-x}\, dx$$

- We have to evaluate this integral in the interval $[0, -\infty[$ (see next page)

Sketch of Distribution



CA :

$$\int -x e^{-x} \, dx = e^{-x} x + e^{-x}, \text{ by parts.}$$

Now, we evaluate in in the interval $[0, +\infty[$:

$$\left[e^{-x} x + e^{-x} \right]_0^{+\infty} = e^{-x} x + e^{-x} \Big|_{x = +\infty} - e^{-x} x + e^{-x} \Big|_{x=0}$$

$$= \lim_{x \to +\infty} \left(e^{-x} x + e^{-x} \right) - (1) = 0 - 1 = \underline{\underline{-1}}$$

- Hence :

$$h(Y) = -\log(e)(-1) = \log(e)$$

Now :

$$\log(\beta) = \log(e) \Leftrightarrow \boxed{\beta = e}$$