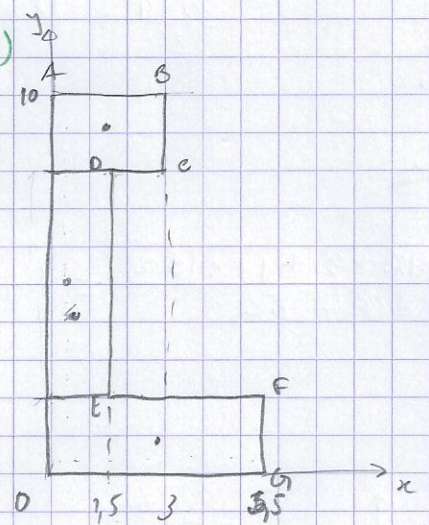
1.4 → Dinâmica de um sistema de partículas

1)



$$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{M}$$

$A_1 = 3 \times 2 = 6\ cm^2$

$A_2 = 1,5 \times 6 = 9\ cm^2$

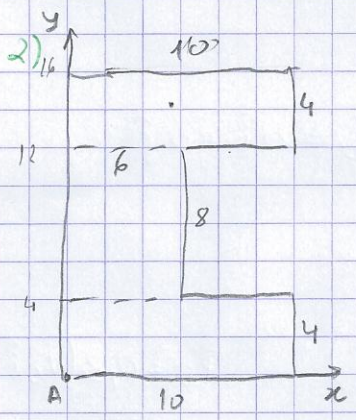
$A_3 = 5,5 \times 2 = 11\ cm^2$

$$r(x) = \frac{1,5 \times 6 + 0,75 \times 9 + 2,75 \times 11}{6+9+11} = 1,77\ cm$$

$$r(y) = \frac{1 \times 11 + 5 \times 9 + 9 \times 6}{6+9+11} = 4,23\ cm$$

$\vec{r}_{cm} = 1,77\,\hat{i} + 4,23\,\hat{j}$ cm

2)



$A_1 = 10 \times 4 = 40\ cm^2$ $\quad A_3 = 40\ cm^2$

$A_2 = 6 \times 8 = 48\ cm^2$

$$r(x) = \frac{5 \times 40 + 3 \times 48 + 5 \times 40}{40+48+40} = 4,25\ cm$$

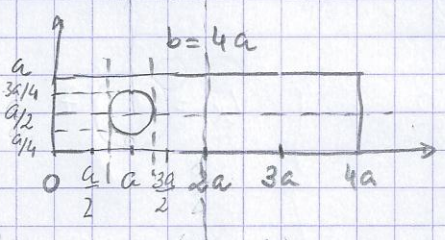
$$r(y) = \frac{2 \times 40 + 8 \times 48 + 14 \times 40}{40+48+40} = 8\ cm$$

$\vec{r} = 4,25\,\hat{i} + 8\,\hat{j}$ Qualquer ponto da reta vertical que contenha o centro da mesa

$\frac{4+12}{2} = \frac{16}{2} = 8$

$\frac{12+16}{2} = \frac{28}{2} = 14$

3) $b = 4a$ $\quad d = \frac{a}{2}$



$a - \frac{a}{2} = \frac{a}{2}$

$a - \frac{a}{4}$

$a + \frac{a}{4}$

$r(y) = \frac{a}{2}$

$$r(x) = \frac{a \times \frac{32-\pi}{16} a^2 + 3a \times 2a^2}{\frac{32-\pi}{16} a^2 + 2a^2}$$

$$= \frac{7,80 a^3}{3,80 a^3} a = 2,05\,a$$

$A_1 = 2a \times a - \pi \times \left(\frac{a}{4}\right)^2$

$= 2a^2 - \frac{a^2}{16}\pi$

$= \frac{32-\pi}{16} a^2$

$A_2 = 2a \times a = 2a^2$

$\vec{r} = 2,05a\,\hat{i} + \frac{a}{2}\,\hat{j}$ (cm)

4) $C = 20cm$
$l = a\ cm$

$\frac{20+4}{2} = 12$

inicial $\quad r(y) = \frac{a}{2}$ e $r(x) = 10\ cm$

final $\quad r(y) = \frac{a}{2}$ e $r(x) = \frac{3 \times 2a \times 2 + 12 \times 18a}{2a + 2a + 18a} = 10,2\ cm$

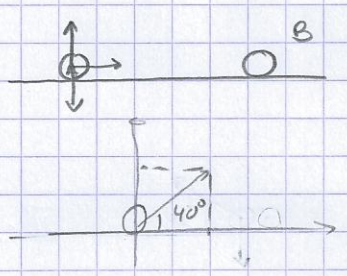
$\Delta x = 10,2 - 10 = 0,2\ cm$

5) $m_A = 0,2\ kg$

$\vec{v}_A = 0,4\ m/s \rightarrow v_A = 0,2\ m/s \quad \theta = 40°$

$m_B = 0,3\ kg$

$v_B = 0\ m/s$



a) $\vec{p} = m\vec{v}$

antes do choque: $\vec{p}_A = 0,2 \times 0,4\,\hat{i} + 0\hat{j} \qquad \vec{p}_B = 0\hat{i} + 0\hat{j}$

$= 0,08\,\hat{i}$

depois do choque: $\vec{p}_A = 0,2\left(0,2\cos 40°\,\hat{i} + 0,2\,\text{sen}\,40°\,\hat{j}\right) \qquad \vec{p}_B = 0,3\left(v_x\hat{i} + v_y\hat{j}\right)$

$= 0,03\hat{i} + 0,026\hat{j} \qquad = 0,3v_x\hat{i} + 0,3v_y\hat{i}$

$\Delta p_A = -0,05\hat{i} + 0,026\hat{j}$

$\Delta p_B = 0,3v_x\hat{i} + 0,3v_y\hat{i}$

$\Delta\vec{p}_A + \Delta\vec{p}_B = \vec{0}$

$$\begin{cases} -0,05 + 0,3v_x = 0 \\ 0,026 + 0,3v_y = 0 \end{cases} \quad \begin{cases} v_x = 0,17 \\ v_y = -0,09 \end{cases}$$

$\vec{v} = 0,19\ m/s$

$\theta = \text{tg}^{-1}\left(\frac{0,09}{0,17}\right) = 27,9°$

b) $\Delta v_A = (0,2\cos 40 - 0,4)\hat{i} + 0,2\,\text{sen}\,40\,\hat{j} = -0,25\hat{i} + 0,13\hat{j}$

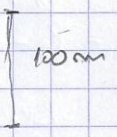
$\Delta v_B = 0,17\hat{i} - 0,09\hat{j}$

$\Delta p_A = -0,05\hat{i} + 0,026\hat{j}$

$\Delta p_B = 0,3 \times 0,17\hat{i} - 0,3 \times 0,09\hat{j} = 0,05\hat{i} - 0,027\hat{j}$

(2) 6)

a) $m_1 = 1\,g = 0{,}001\,kg$ $\quad m_2 = 1\,kg$ $\quad m_3 = 10^6\,kg$



$v = +gt = +9{,}8t$

$x = x_0 + v_0 t - \frac{1}{2}gt^2$

$-100 = -\frac{1}{2}gt^2 \Leftrightarrow t = 4{,}5$

$v = 9{,}8 \times 4{,}5 = 44{,}1\ m/s$

① $p = mv$

$p = 0{,}001 \times 44{,}1$

$p = 4{,}41 \times 10^{-2}\ kg\,m/s$

② $p = 1 \times 44{,}1 = 44{,}1\ kg\,m/s$

③ $p = 10^6 \times 44{,}1 = 44{,}1 \times 10^6\ kg\,m/s$

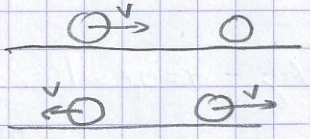
b) ① $4{,}41 \times 10^{-2} = 5{,}98 \times 10^{24} \cdot v \Leftrightarrow v = 7{,}37 \times 10^{-27}\ m/s$

② $44{,}1 = 5{,}98 \times 10^{24} v \Leftrightarrow v = 7{,}37 \times 10^{-24}\ m/s$

③ $44{,}1 \times 10^6 = 5{,}98 \times 10^{24} v \Leftrightarrow v = 7{,}37 \times 10^{-8}\ m/s$

7) $v_A = 0{,}5\ m/s \rightarrow v_A = 0{,}1\ m/s$

$v_B = 0{,}3\ m/s$



$m'_A = m_A + 1$

$v = 0{,}5\ m/s \rightarrow v_A = 0\ m/s$

$v_B = 0{,}5\ m/s$

1º caso

$\Delta\vec{p}_A + \Delta\vec{p}_B = \vec{0}$

$m_A(-0{,}1) - m_A(0{,}5) + m_B(0{,}3) - 0 = 0$

$-0{,}6 m_A = -0{,}3 m_B$

$m_A = 0{,}5 m_B$ (*)

2º caso

$\Delta\vec{p}_A + \Delta\vec{p}_B = \vec{0}$

$(m_A + 1) \times 0 - (m_A + 1) \times 0{,}5 + m_B \times 0{,}5 - m_B \times 0 = 0$

$-0{,}5 m_A - 0{,}5 + 0{,}5 m_B = 0$ $\quad \downarrow *$

$-0{,}5 \times 0{,}5 m_B + 0{,}5 m_B = 0{,}5$

$0{,}75 m_B = 0{,}5$

$m_B = 2\ kg$ logo $m_A = 0{,}5 \times 2 = 1\ kg$

8) $p_A = p_0 - bt$ (p_{0A}) após colisão

a) $\Delta p_A + \Delta p_B = 0$ $\quad p_{0B} = 0$

$p_0 - bt - p_0 + p_B = 0 \Leftrightarrow p_B = bt$

b) $p_{0B} = -p_0$

$\Delta p_A + \Delta p_B = 0$

$p_0 - bt - p_0 + p_B - (-p_0) = 0 \Leftrightarrow p_B = -p_0 + bt$

9) $m_A = 0{,}8\ kg$
$m_B = 0{,}016\ kg$ $\quad v = 700\ m/s$

$\Delta p_A + \Delta p_B = 0$

$0{,}8v + 11{,}2 = 0$

$v = -14\ m/s$

$\Delta p_A = p_{Af} - p_{Ai}$

$= m_A v - 0 = 0{,}8v$

$\Delta p_B = p_{Bf} - p_{Bi}$

$= m_B \times v - 0 = 0{,}016 \times 700 = 11{,}2$

10) $m = 1{,}5\ kg$ $\quad v = 0{,}2\ m/s$
$t = 0{,}1$

a) $\Delta p = p_{Af} - p_{Ai} = 0 - 1{,}5 \times 0{,}2 = -0{,}3\ kg\,m/s$

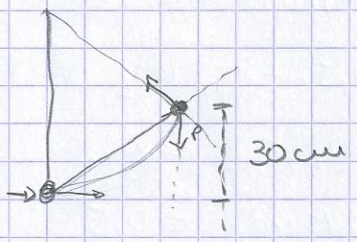
$F = ma = -1{,}5 \times \frac{0{,}2}{0{,}1} = -3\ N$ $\qquad a = \frac{v_f - v_i}{\Delta t} = \frac{0 - 0{,}2}{0{,}1} = -2\ m/s^2$

b) $\Delta p = p_{Af} - p_{Ai} = 1{,}5 \times (-0{,}1) - 1{,}5 \times 0{,}2 = -0{,}45\ kg\,m/s$

$a = \frac{v_f - v_i}{\Delta t} = \frac{-0{,}1 - 0{,}2}{0{,}1} = -3\ m/s^2$

$F = 1{,}5 \times (-3) = -4{,}5\ N$

11) $m_1 = 30\ g$ projétil
$m_2 = 3{,}0\ kg$ corpo
$h = 30\ cm$



a) $\Delta p_A + \Delta p_B = 0$

$\Delta p_A = (m_1 + m_2) v_f - m_1 v_i$

$\Delta p_B = m_2 v_f - 0$

$E_C = E_p$

$\frac{1}{2} m v^2 = mgh$

$v^2 = 2gh$

$v = \sqrt{2gh}$

$(m_1 + m_2) v_f - m_1 v_i = 0$

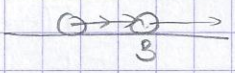
$v_i = \frac{m_1 + m_2}{m_1} v_f$

$v_i = \frac{m_1 + m_2}{m_1} \times \sqrt{2gh}$ (m/s)

b) $v_i = \frac{30 \times 10^{-3} + 3}{30 \times 10^{-3}} \times \sqrt{2 \times 9{,}8 \times 0{,}3} = 245\ m/s$

③

12) $m = 5{,}0\ kg$ $\quad m_B$



$v_f = \frac{1}{5} v_i \qquad \Delta p_A + \Delta p_B = 0$

velocidade relativa mantém-se

$v_{i_A} - v_{i_B} = v_{f_B} - v_{f_A}$

$\frac{5}{5} v_{i_A} - 0 = v_{f_B} - \frac{1}{5} v_{i_A} \Leftrightarrow v_{f_B} = \frac{6}{5} v_{i_A}$

$\Delta p_A + \Delta p_B = 0$

$m_A v_{f_A} - m_A v_{i_A} + m_B v_{f_B} - m_B v_{i_B} = 0$

$5 \times \frac{1}{5} v_{i_A} - 5 \times v_{i_A} + m_B \times \frac{6}{5} v_{i_A} - 0 = 0$

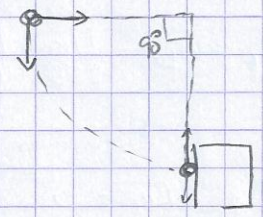
$\frac{6}{5} m_B = 4$

$m_B = 3{,}33\ kg$

13) $m = 0{,}5\ kg$
$l = 0{,}70\ m$
$\theta = 90°$
$m_B = 2{,}5\ kg$
$v_{i_B} = 0$
elástica

$v_{i_A} - v_{i_B} = v_{f_B} - v_{f_A}$

$v_{i_A} = v_{f_B} - v_{f_A}$ (2)



a) $\Delta p_A + \Delta p_B = 0$

$m_A v_{f_A} - m_A v_{i_A} + m_B v_{f_B} - m_B v_{i_B} = 0$

$m_A v_{f_A} - m_A v_{i_A} + m_B v_{f_B} = 0$

$0{,}5 v_{f_A} - 0{,}5 v_{i_A} + 2{,}5 v_{f_B} = 0$ (1)(4)

Nota: $E_c = E_p$

$\frac{1}{2} m v^2 = mgh$

$v = \sqrt{2gh}$

$v = \sqrt{2 \times 9{,}8 \times 0{,}70}$

$v = 3{,}7\ m/s$

v_{i_A}

logo (2) $v_{f_B} = 3{,}7 + v_{f_A}$

e (2)(3)

$0{,}5 v_{f_A} - 0{,}5 \times 3{,}7 + 2{,}5 (3{,}7 + v_{f_A}) = 0$

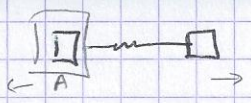
$\Leftrightarrow 3 v_{f_A} = 1{,}85 - 9{,}25$

$\Leftrightarrow v_{f_A} = -2{,}5\ m/s$

b) $v_{f_B} = 3{,}7 - 2{,}5 = 1{,}2\ m/s$

14) $m_A = 2m_B$

$E = 60\,J$



$v_{iA} = v_{iB} = 0$

$$m_A v_{fA} = m_B v_{fB}$$

$$2m_B v_{fA} = m_B v_{fB}$$

$$v_{fA} = \frac{1}{2} v_{fB}$$

$$60 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$60 = \frac{1}{2} 2m_B \times \left(\frac{1}{4} v_B^2\right) + \frac{1}{2} m_B v_B^2$$

$$60 = \frac{3}{4} m_B v_B^2 \iff m_B v_B^2 = 80 \Rightarrow \frac{1}{2} m_B v_B^2 = 40\,J$$

$$E_{C_B} = 40\,J$$

Logo $E_{C_A} = 60 - 40 = 20\,J$

15) m_1 e m_2 Perfeitamente inelástica → objetos mantêm-se juntos

$v_{2i} = 0$

a) $$E_{C_1} + E_{C_2} = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1i}^2$$

b) $v_{2f} = v_{1f}$

Ora

$$\Delta p_1 + \Delta p_2 = 0$$

$$p_{1f} - p_{1i} + p_{2f} - p_{2i} = 0$$

$$m_1 v^2 - m_1 v_{i_1}^2 + m_2 v^2 = 0$$

$$(m_1 + m_2) v^2 = m_1 v_{i_1}^2$$

$$v^2 = \frac{m_1 v_{i_1}^2}{m_1 + m_2}$$

$$\frac{1}{2} m_1 \frac{m_1}{m_1 + m_2} v_{i_1}^2 + \frac{1}{2} m_2 \frac{m_1}{m_1 + m_2} v_{i_2}^2$$

(9) (16) m_A m_B $v_{i_A} = 0$ m/s

$v_{i_B} = v$ m/s $\rightarrow v_{f_B} = \frac{v}{2}$ m/s

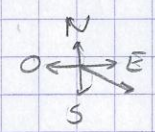
A ← B

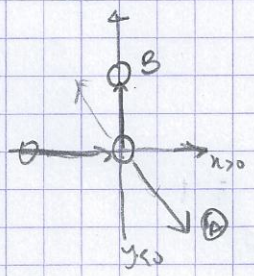
B ↑ 90°

a) $\Delta\vec{p}_A + \Delta\vec{p}_B = \vec{0}$

$m_A(v_x, v_y) - m_A(0,0) + m_B\left(0, \frac{v}{2}\right) - m_B(v, 0) = 0$

$$\begin{cases} m_A v_x - m_B v = 0 \\ m_A v_y + m_B \frac{v}{2} = 0 \end{cases} \qquad \begin{cases} v_x = + \frac{m_B}{m_A} v \\ v_y = - \frac{m_B}{m_A} \frac{v}{2} \end{cases}$$





$$tg\left(\frac{-\frac{v}{2}}{v}\right) = tg\left(-\frac{1}{2}\right) = -26,57°$$

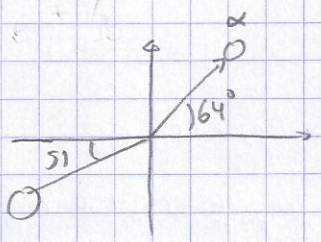
$$= 90° - 26,57°$$

$$= 63,44° \checkmark$$

b) NÃO

(17)

$m_0 = 4m_\alpha$ $\overset{\alpha}{O}$ $\bigcirc$ $v_i = 0$



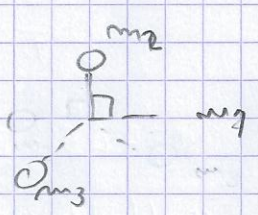
(OX)
$$\begin{cases} P_{inicial} = m_\alpha v_{\alpha_i} + m_0 v_{0_i} = m_\alpha v_{\alpha_i} \\ P_{final} = m_\alpha v_\alpha \cos 64° - m_0 v_0 \cos 51° \\ m_\alpha v_{\alpha_i} = m_\alpha v_{\alpha_f} \cos 64° - 4 m_{0_f} v_{0_f} \cos 51° \\ v_{\alpha_i} = v_{\alpha_f} \cos 64° - 4 \cos 51° v_{0_f} \end{cases}$$

(OY)
$$\begin{cases} P_i = 0 \\ P_f = m_\alpha v_\alpha \operatorname{sen} 64° - m_0 v_0 \operatorname{sen} 51° \end{cases} \qquad \begin{cases} 0 = v_\alpha \operatorname{sen} 64 - 4 v_0 \operatorname{sen} 51 \\ \frac{v_\alpha}{v_0} = \frac{4 \operatorname{sen} 51}{\operatorname{sen} 64} = 3,46 \end{cases}$$

(18) $m_1 = 17 \times 10^{-27}$ kg $\quad v_1 = 6{,}0 \times 10^6$ m/s $\quad \theta = 90°$

$m_2 = 8{,}0 \times 10^{-27}$ kg $\quad v_2 = 8{,}0 \times 10^6$ m/s

a) $m_3 = 12 \times 10^{-27}$ kg



Após a explosão $\quad p_1 + p_2 + p_3 = 0$

$\vec{v_1} = (6{,}0 \times 10^6 ; 0) \quad \vec{v_2} = (0 ; 8 \times 10^6)$

OX

$p_{3x} = -6 \times 10^6 \times 17 \times 10^{-27} = -1{,}02 \times 10^{-19}$

OY $\quad p_{3y} = -8 \times 10^6 \times 8{,}0 \times 10^{-27} = -6{,}4 \times 10^{-20}$

$p_3 = -1{,}02 \times 10^{-19}\, i + 6{,}4 \times 10^{-20}\, j$

b) $\vec{p_3} = m_3 \vec{v_3}$

$\vec{v_3} = -8{,}5 \times 10^6\, i - 5{,}3 \times 10^6\, j$

$|\vec{v_3}| = 10{,}0 \times 10^6$ m/s

$E_c = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2$

$= 3{,}06 \times 10^{-13} + 2{,}56 \times 10^{-13} + 6 \times 10^{-13}$

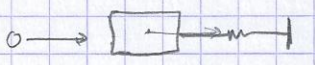
$= 1{,}1 \times 10^{-12}$ J

20) $m_A = 20$ g $\quad v_i = V$

$m_B = 980$ g

$K = 1000$ N/m

$\Delta x = 10$ cm



$\Delta p_A + \Delta p_B = 0$

$m_A v_f - m_A V + m_B v_f - m_B v_i = 0$

$0{,}02 v_f - 0{,}02 V + 0{,}98 v_f = 0$

$v_f = 0{,}02 V$

$F = -K \Delta x$

$F_e = -1000 \times 0{,}1$

$F_e = -100$ N

$W = F \cdot \Delta x \cdot \cos 0°$

$W = 100 \times 0{,}1 = 10$ J

logo $\quad \frac{1}{2} m v^2 = 10 \Leftrightarrow v^2 = \frac{20}{1} \Leftrightarrow v = \sqrt{20} \Leftrightarrow v \approx 4{,}5$ m/s

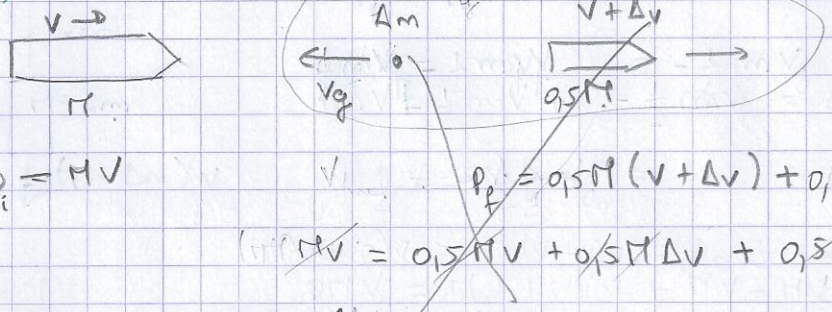
b) $4{,}5 = 0{,}02 V \Leftrightarrow V = 225$ m/s

c) $E_{ci} - E_{cf} = \frac{1}{2} \times 0{,}02 \times 225 - \frac{1}{2} \times 0{,}02 \times 4{,}5 = 2{,}205$ J

(5) 23) $v_f = 3 \times 10^3$ m/s

$v_g = 5 \times 10^3$ m/s

a)



$P_i = MV$ $\quad P_f = 0,5M(V+\Delta v) + 0,5M(V - v_g)$

$MV = 0,5MV + 0,5M\Delta v + 0,5MV - 0,5Mv_e$

$\Delta v = v_e$

$v_f = v_i + v_e \ln \frac{M_i}{M_f} = 6466$ m/s

b) $F_{ea} = v_g \times 50 = 2,5 \times 10^5$ N ✓

50 kg/s

24) $F_{ees} = 24 \times 10^6$ N $\quad v_g = 3000$ m/s

a) $F = v_e \times \frac{\Delta M}{dt} \Leftrightarrow 24 \times 10^6 = 3000 \times \Delta m$

$\Leftrightarrow \frac{\Delta M}{dt} = 8000$ kg/s

b) $M = 0,9Q$

$0 = M(V+\Delta v) + \Delta m(V - v_e)$

$\Delta v = 3000 \times \frac{0,10}{0,90} = 333,33$ m/s

$v_f = v_i + v_e \ln\left(\frac{1}{0,9}\right) = 0 + 3000 \times \ln\left(\frac{10}{9}\right) = 316,08$ m/s

$v_{mix} = 333,33 + 316,08 = 649,41$

$t = 0$

$\frac{dM}{dt} = 8000$ kg/s

8000 kg → 1 s

0,1M → t

$t = \frac{8000}{0,1M} = \frac{80000}{M}$

$F_n - P = ma \quad a = 0$

$M + \Delta M = 2448979$ $\quad M_f$

$F_n - P = 0$

$F_n = P$

$24 \times 10^6 = M'g$

$M = 2448979$

8000 kg → 1 s

2448979 kg → x

$\frac{1}{2} 2448979 v^2 =$