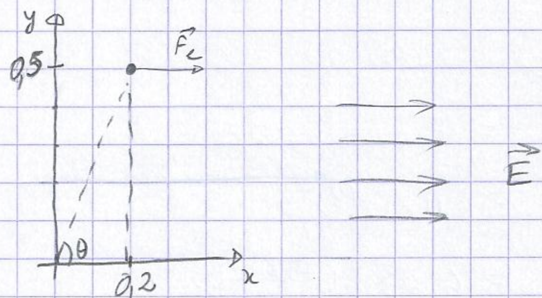
(1) Cap 3.2 : Energia Potencial do C.E.

1) $E = 250$ V/m $\quad +x \qquad q = 12{,}0\,\mu C = 12\times10^{-6}$ C

$(x,y) = (20{,}0\,;\,50{,}0)$ cm



a) $W(F_e) = \Delta E_p$

$$W = F \times \Delta r \times \cos\theta$$

$$= E\times q \times \sqrt{0{,}2^2+0{,}5^2} \times \frac{0{,}2}{\sqrt{0{,}2^2+0{,}5^2}} \qquad \cos\theta = \frac{0{,}2}{\sqrt{0{,}2^2+0{,}5^2}}$$

$$= 12\times10^{-6}\times 250\times 0{,}2$$

$$= 6\times10^{-4}\ J$$

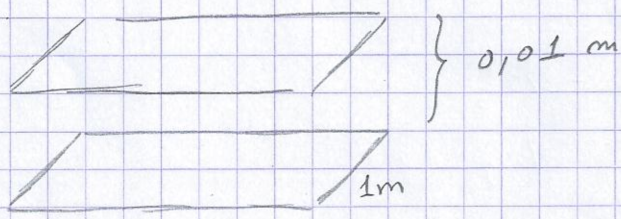
logo $\Delta E_p = 6\times10^{-4}$ J

b) $\Delta V = \dfrac{\Delta E_p}{q_0} = \dfrac{6\times10^{-4}}{12\times10^{-6}} = 50$ V

2) $l = 1{,}00$ m

$d = 1{,}00$ cm

$V = 10$ kV



$C = \varepsilon_0 \times \dfrac{A}{d}$ e $C = \dfrac{Q}{V}$

$C = \varepsilon_0 \times \dfrac{1^2}{0{,}01} = 100\,\varepsilon_0$

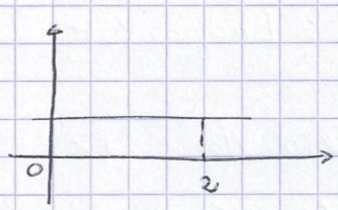
$Q = 100\,\varepsilon_0 \times 10\times10^3 = 10\times10^5\,\varepsilon_0$

logo $\sigma = \dfrac{Q}{A} = 10\times10^5 \times 8{,}85\times10^{-12} = 8{,}85\times10^{-6}$ C/m²

$V = Ed \iff E = \dfrac{V}{d} = \dfrac{10\times10^3}{0{,}01} = 1{,}0\times10^6$ V/m

3) $v_0 = 3,7 \times 10^6$ m/s

$v_f = 1,4 \times 10^5$ m/s $\quad x = 2,0$ m



$\Delta_{EP} = W(F_e)$

$= F \cdot \Delta x \cdot \cos 0°$

$= 3,11 \times 10^{-18} \times 2$

$= 6,23 \times 10^{-18}$ J

$F = m \cdot a$

$F = 9,109 \times 10^{-31} \times a$

$v_f^2 - v_i^2 = 2a\Delta x$

$a = \dfrac{(1,4\times10^5)^2 - (3,7\times10^6)^2}{2\times2}$

$a = -3,42 \times 10^{12}$

logo $F = 3,11 \times 10^{-18}$ N

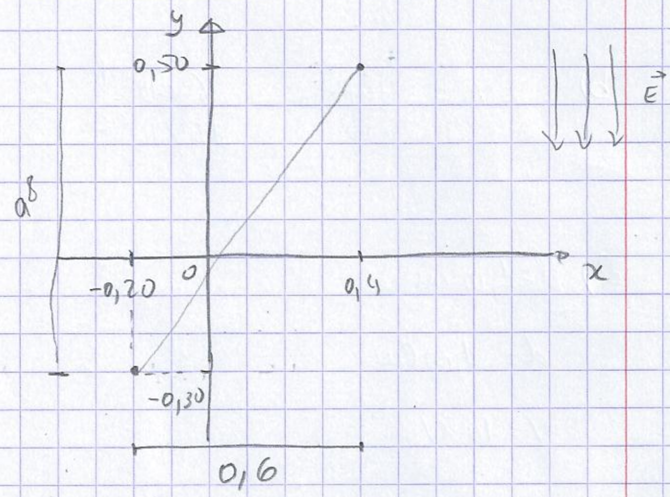
Assim, $\Delta V = \dfrac{\Delta_{EP}}{q_0} = \dfrac{6,23\times10^{-18}}{1,602\times10^{-19}} = 3,89 \times 10^1 = 38,9$ V/m ✓

O potencial é superior no primeiro ponto (a aceleração diminui) ?

4) $E = 325$ V/m ↓

$(x,y) = (-0,20\ ,\ -0,30)$ m

$(x,y) = (0,40\ ;\ 0,50)$ m



$\vec{E} = 0\hat{i} - 325\hat{j}$

$\vec{J} = V_x\hat{i} - V_y\hat{j}$

$\vec{E} = -\vec{\nabla} V$ logo

$E_y = -\dfrac{dV}{dy}$ (a componente em x é nula)

$dV = -E_y\, dy$

$V = \displaystyle\int_{-0,30}^{0,50} 325\, dy = 325\,[0,50 + 0,30]$

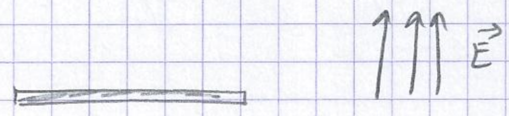
$= 260$ V

2

5) $\lambda = 40,0\ \mu C/m$

$\mu = 0,10\ kg/m$

$E = 100\ V/m$



a) $\Delta x = 2,0\ m$ $\quad$ $m = 0,10\ l\ (kg)$ $\quad$ $l \to$ comprimento

$v_0 = 0\ m/s$ $\quad$ $q = 40 \times 10^{-6}\ l\ (C)$

$F_e = q \cdot E$

$F_e = 40 \times 10^{-6}\ l \times 100$

$F_e = 0,004\ l\ (N)$

Mas $\quad F = m.a$

$0,004\ l = 0,10\ l\ a \iff a = 0,04\ m/s^2$

logo

$v_f^2 = 2a\Delta x$

$v_f = 2 \times 0,04 \times 2$

$v_f = 0,4\ m/s$

b) Os cálculos em a) não dependeram da direção do $\vec{E}$

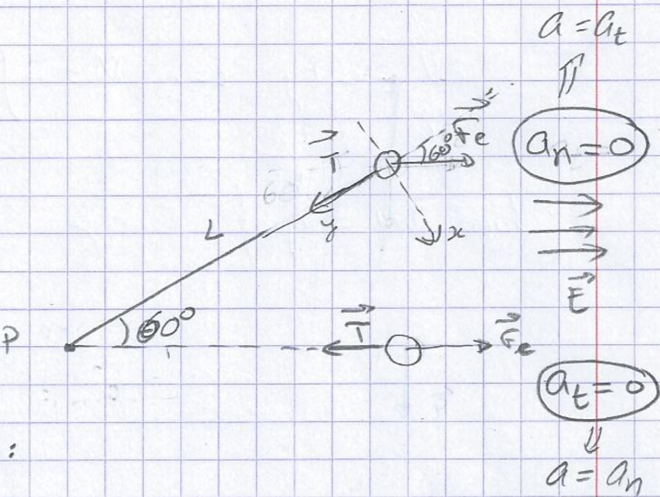
logo $\quad v = 4\ m/s$

c) Roda (provavelmente)

6) $q = 2,00\ \mu C$ $\quad$ Plano horizontal

$m = 0,010\ kg$ $\quad$ $E = 300\ V/m$

$L = 1,50\ m$ $\quad$ $v_i = 0\ m/s$



Nota:

$F_e = qE$ e $F = ma$

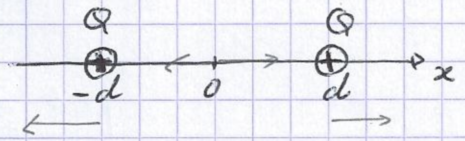
$$\begin{cases} T - F_e \cos 60^\circ = m a_n \\ F_e \operatorname{sen} 60^\circ = m a_t \end{cases} \iff \begin{cases} T = 2 \times 10^{-6} \times 300 \times \cos 60^\circ \\ \_\_\_\_ \end{cases}$$

$a = a_t \iff \begin{cases} T = 3 \times 10^{-4}\ N \\ a = 0,05\ m/s^2 \end{cases}$

$a = a_n = \dfrac{v^2}{L} \iff v = \sqrt{a_n \times L}$

$\iff v = 0,3\ m/s$

7)



a) As cargas são iguais $E = 0 \, V/m$

$$E = K_e \left[\frac{Q}{d^2} \times (-1) + \frac{Q}{d^2} \times (1) \right] = 0 \ V/m$$

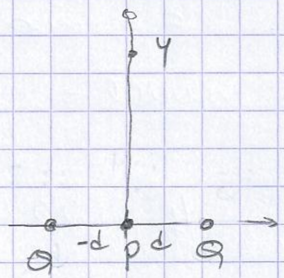
b) $V = K_e \sum_{i=1}^{N} \frac{q_i}{r_i}$

$$V = K_e \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} \right] = K_e \left[\frac{Q}{d} + \frac{Q}{d} \right] = 2 K_e \frac{Q}{d}$$

c) $W = U = qV$

$$= q \times 2 K_e \frac{Q}{d}$$

$$= 2 K_e \frac{qQ}{d}$$



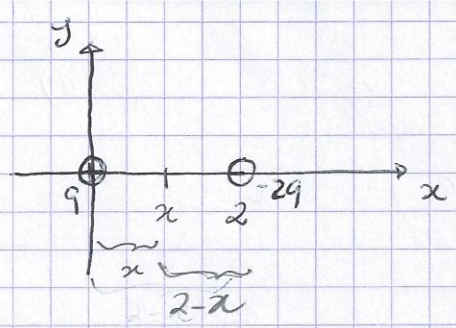
d) $\vec{E} = -\vec{\nabla} V$ $\qquad \vec{E} = E_y \hat{j}$ $\qquad (E_x = 0)$

$$\frac{dV}{dy} = -E_y \iff V = -\int_0^{\infty} E_y \, dy \iff V = \int_{\infty}^{0} E_y \, dy$$

logo $W = q \int_{\infty}^{0} E_y \, dy$

3)

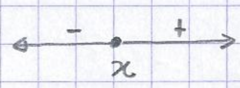
8) $q \; ; \; -2q$



a) $E = 0$

$E_y = 0$

$E_x = k_e \left[\frac{q}{x^2} - \frac{2q}{(2-x)^2} \right]$



$E=0 \Leftrightarrow \frac{\cancel{q}}{x^2} = \frac{2\cancel{q}}{(2-x)^2} \Leftrightarrow 2x^2 = (2-x)^2 \Leftrightarrow$

32	2
16	2
8	2
4	2
2	2
1	

$\Leftrightarrow 2x^2 = 4 - 4x + x^2$

$\Leftrightarrow x^2 + 4x - 4 = 0 \Leftrightarrow x = \frac{-4 \pm \sqrt{16+16}}{2}$

$\Leftrightarrow x = \frac{-4 \pm \sqrt{32}}{2} \Leftrightarrow x = \frac{-4 \pm 4\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow x = -2 \pm 2\sqrt{2} \quad \wedge \quad x > 0$

$\Rightarrow x = -2 + 2\sqrt{2}$ m

b) $V = k_e \sum_{i=1}^{2} \frac{q_i}{r_i}$

$\Leftrightarrow 0 = k_e \left[\frac{q}{x} + \frac{-2q}{2-x} \right]$

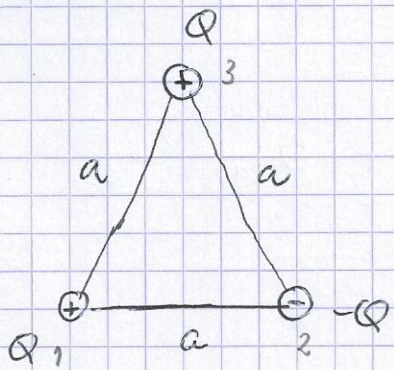
$\Leftrightarrow \frac{\cancel{q}}{x} = \frac{2\cancel{q}}{2-x} \Leftrightarrow$

$\Leftrightarrow 2 - x = 2x \Leftrightarrow$

$\Leftrightarrow -3x = -2$

$\Leftrightarrow x = \frac{2}{3}$ m

9)



a) $U = k_e \sum_{i=1}^{3}\sum_{j>i}^{3} \frac{q_i q_j}{r_{ij}} =$

$$= k_e\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

$$= k_e\left[\cancel{\frac{-Q^2}{a}} + \cancel{\frac{Q^2}{a}} + \frac{-Q^2}{a}\right] = -k_e \cdot \frac{Q^2}{a}$$

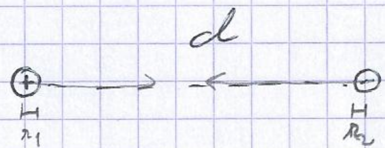
b)

$$V = k_e \sum_{i=1}^{3} \frac{q_i}{r_i} = k_e\left[\cancel{\frac{Q}{r}} + \cancel{\frac{-Q}{r}} + \frac{Q}{r}\right]$$

$$= k_e \cdot \frac{Q}{r} = k_e \cdot Q \cdot \frac{\sqrt{3}}{a}$$

$$\cos 30^\circ = \frac{\frac{a}{2}}{r} \Leftrightarrow r = \frac{a}{2} \times \frac{2}{\sqrt{3}} \Leftrightarrow r = \frac{a\sqrt{3}}{3}$$

10) r_1 e r_2
m_1 e m_2
$-q_1$ e q_2



a) $\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$

$$m_1\vec{v}_1 - 0 + m_2\vec{v}_2 - 0 = 0 \Leftrightarrow \vec{v}_2 = -\frac{m_1}{m_2}\vec{v}_1$$

$$\Rightarrow v_2 = \frac{m_1}{m_2} v_1$$

$\Delta E_p = \Delta E_c$

$$E_{p_f} - E_{p_i} = E_{c_f} - \underbrace{E_{c_i}}_{0}$$

$$K\frac{-q_1q_2}{d} - K\frac{-q_1q_2}{r_1+r_2} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$-Kq_1q_2\left(\frac{1}{d} - \frac{1}{r_1+r_2}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}\cancel{m_2}\cdot\frac{m_1^2}{m_2^{\cancel{2}}}v_1^2$$

$$Kq_1q_2\left(\frac{1}{r_1+r_2} - \frac{1}{d}\right) = \frac{1}{2}m_1v_1^2\left(\frac{1}{1_{(m_2)}} + \frac{m_1}{m_2}\right)$$

$$\sqrt{\frac{2Kq_1q_2\left(\frac{1}{r_1+r_2} - \frac{1}{d}\right)\times m_2}{m_1m_2 + m_1^2}} = v_1$$

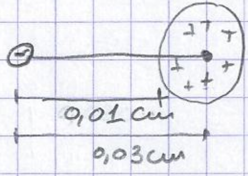
4

11) $Q = 1{,}00\, nC = 1\times10^{-9}\, C$

$m_e = 9{,}1\times10^{-31}\, kg$
$q_e = 1{,}6\times10^{-19}\, C$

$r = 0{,}03\, cm$

$R_{esf} = 0{,}02\, cm$



$\Delta E_p = \Delta E_c$

$E_{p_f} - E_{p_i} = \frac{1}{2} m v^2 - 0$

$$K \frac{q_1 Q}{0{,}02} - K \frac{q_1 Q}{0{,}03} = \frac{1}{2} m v^2$$

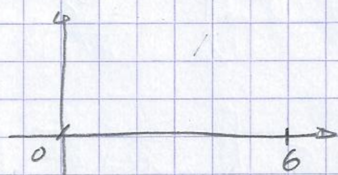
$$8{,}99\times10^{9} \times 1{,}6\times10^{-19} \times 1\times10^{-9} \left(\frac{1}{0{,}02} - \frac{1}{0{,}03}\right) = \frac{1}{2} \times 9{,}1\times10^{-31} \times v^2$$

$v^2 = 5{,}27\times10^{13}$

$v = 7{,}26\times10^{6}\, m/s$

12) $x=0$; $x=6$ $\quad V = ax+b$

$a = 10\, V/m$ e $b = -7{,}0\, V$



a) $V = ax+b$

$V = 10x - 7$

$V(0) = -7{,}0\, V$

$V(3) = 30 - 7 = 23\, V$

$V(6) = 60 - 7 = 53\, V$

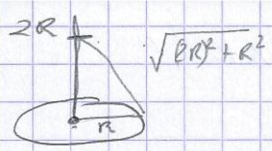
b) $\vec{E} = -\vec{\nabla} V$

$\vec{E}x = (10x-7)'\ \hat{\imath}$

$\vec{E}x = -10\ \hat{\imath}$

13)

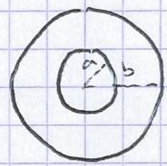
Q



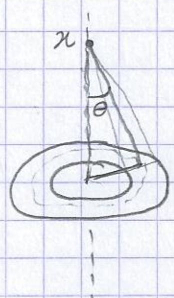
$$V_B - V_A = KQ\left[\frac{1}{R} - \frac{1}{\sqrt{4R^2+R^2}}\right]$$

$$= KQ\left(\frac{1}{R} - \frac{1}{R\sqrt{5}}\right) = \frac{KQ}{R}\left(1 - \frac{1}{\sqrt{5}}\right)$$

14)



σ superficial

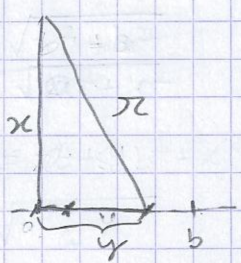


a) $V = K_e \int \frac{1}{r}\, dq$

$dq = \sigma\, dA$

$A = \pi R^2$

$V = K_e \int \frac{1}{r}\, \sigma\, dA$



$$V = K_e \int_0^{b-a} \frac{1}{\sqrt{x^2+y^2}}\, \sigma \times 2\pi y\, dy$$

$r = \sqrt{x^2+y^2}$

$$V = K_e \times 2\pi \times \sigma \int_a^b \frac{y}{\sqrt{x^2+y^2}}\, dy$$

$a \leq y \leq b$

$dA = 2\pi R\, dR$

$R = y$

$$V = K_e \times 2\pi\sigma \left[\sqrt{x^2+y^2}\right]_a^b$$

$$\left((x^2+y^2)^{1/2}\right)' = \frac{1}{2} \times 2y \times (x^2+y^2)^{-1/2}$$

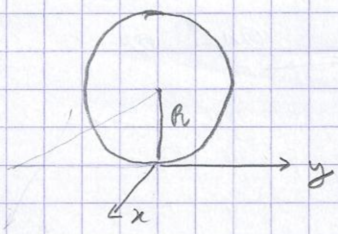
$$V = K_e \times 2\pi\sigma \left[\sqrt{x^2+b^2} - \sqrt{x^2+a^2}\right] \text{//}$$

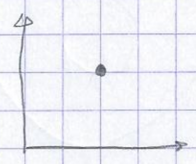
b) $\vec{E} = -\nabla V$

$$E = K_e 2\pi\sigma \left[\frac{x}{\sqrt{x^2+b^2}} - \frac{x}{\sqrt{x^2+a^2}}\right] \text{//}$$

(5) 15) R

Q

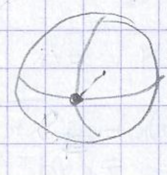




$$V = k_e \frac{Q}{R} \quad \text{(trivial)}$$

16) $r = 0{,}300\ m$

$V = 7{,}50\ kV = 7500\ V$



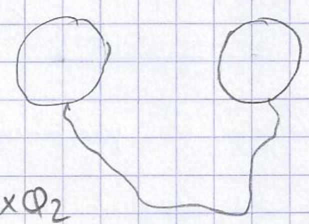
$$V = k_e \frac{Q}{R} \Leftrightarrow Q = \frac{7500 \times 0{,}3}{8{,}99 \times 10^9} \Leftrightarrow Q = 2{,}5 \times 10^{-7}\ C$$

$$n_e q_e = Q \Leftrightarrow n_e = \frac{2{,}5 \times 10^{-7}}{1{,}6 \times 10^{-19}} \Leftrightarrow n_e = 1{,}56 \times 10^{12}$$

17) $Q = 20\ \mu C = 20 \times 10^{-6}\ C$

a) $r_1 = 4{,}00\ cm = 0{,}04\ m$

$r_2 = 6{,}00\ cm = 0{,}06\ m$



$$\begin{cases} \dfrac{Q_1}{R_1} = \dfrac{Q_2}{R_2} \\ Q = Q_1 + Q_2 \end{cases} \quad \begin{cases} 0{,}06 \times Q_1 = 0{,}04 \times Q_2 \\ \_\_\_\_ \end{cases}$$

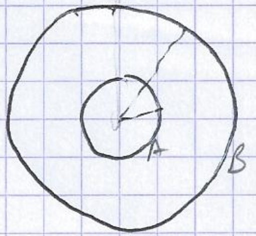
$$\begin{cases} Q_1 = \dfrac{2}{3} Q_2 \\ 20 = \dfrac{2}{3} Q_2 + \dfrac{3}{3} Q_2 \end{cases} \quad \begin{cases} Q_1 = 8\ \mu C \\ Q_2 = 12\ \mu C \end{cases}$$

logo $E_1 = k_e \dfrac{8 \times 10^{-6}}{0{,}04^2} = 4{,}5 \times 10^7\ V/m$

$E_2 = k_e \dfrac{12 \times 10^{-6}}{0{,}06^2} = 3{,}0 \times 10^7\ V/m$ ✓

b) $V_1 = k_e \left[\dfrac{8 \times 10^{-6}}{0{,}04}\right] = 1{,}8 \times 10^6\ V$ e $V_2 = k_e \left[\dfrac{12 \times 10^{-6}}{0{,}06}\right] = 1{,}8 \times 10^6\ V$

Iguais!)

18) 

Ora o campo elétrico no interior é zero ⇒ a carga é zero

No equilíbrio não há carga na esfera interior. Se existir alguma, vai criar um campo elétrico para distribuir essa carga para a esfera exterior. Toda a carga está na esfera exterior. $Q_A = 0$ e $Q_B = Q$ (está no livro)

19) $l = 5\ mm = 5 \times 10^{-3}\ m$ (está no livro)

a)

20) $r = 0{,}08\ m$

a) $\Delta V = E d \Leftrightarrow E = \frac{15}{0{,}08} = 187{,}5\ V/m$ para baixo
V aumenta para cima
($E = V'$)

b) O potencial é positivo logo a carga é positiva

c) $\vec{E} = \frac{\vec{F}}{q}$ como a carga é negativa e o campo ↑
a força é para

d) $r = 0{,}09\ m$

$V_P = ?$ 1 cm

$E = 187{,}5$

$\Delta V = 187{,}5 \times 0{,}09 = 16{,}875\ V$

$V = k_e \frac{q}{r}$

$16{,}875 = 8{,}99 \times 10^{9} \times \frac{q}{0{,}09}$

$q = 1{,}69 \times 10^{-10}\ C$