① Cap. 1.1.

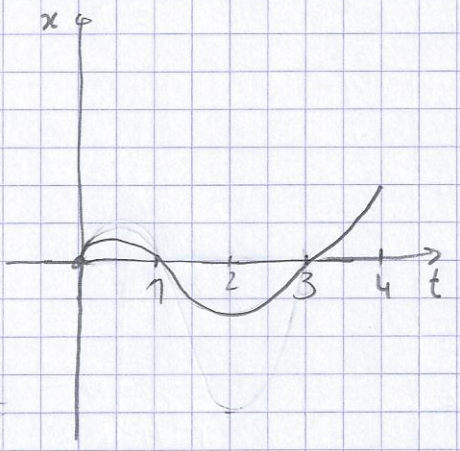
1) $x = 3{,}0t - 4{,}0t^2 + t^3$ $\quad x \to m \quad t \to s$

a) $x(1) = 3 - 4 + 1 = 0$ m

$x(2) = 6 - 16 + 8 = -2$ m

$x(3) = 9 - 36 + 27 = 0$ m

$x(4) = 12 - 64 + 64 = 12$ m



b) $\frac{dx}{dt} = 3 - 8t + 3t^2$

$3 - 8t + 3t^2 = 0 \iff$

$\iff 3t^2 - 8t + 3 = 0 \iff$

$\iff t = \frac{8 \pm \sqrt{64 - 36}}{6}$

$42 \times 3 = 36$

$64 - 36 = 28 = 2^2 \cdot 7$

$\iff t = \frac{8 \pm \sqrt{28}}{6} \iff t = \frac{8 \pm 2\sqrt{7}}{6} \iff t = \frac{4}{3} \pm \frac{1}{3}\sqrt{7}$

$\iff t \simeq 0{,}45$ v $t = 2{,}22$

$x(0{,}45) = 0{,}63$ m $\qquad x(2{,}22) = -2{,}11$

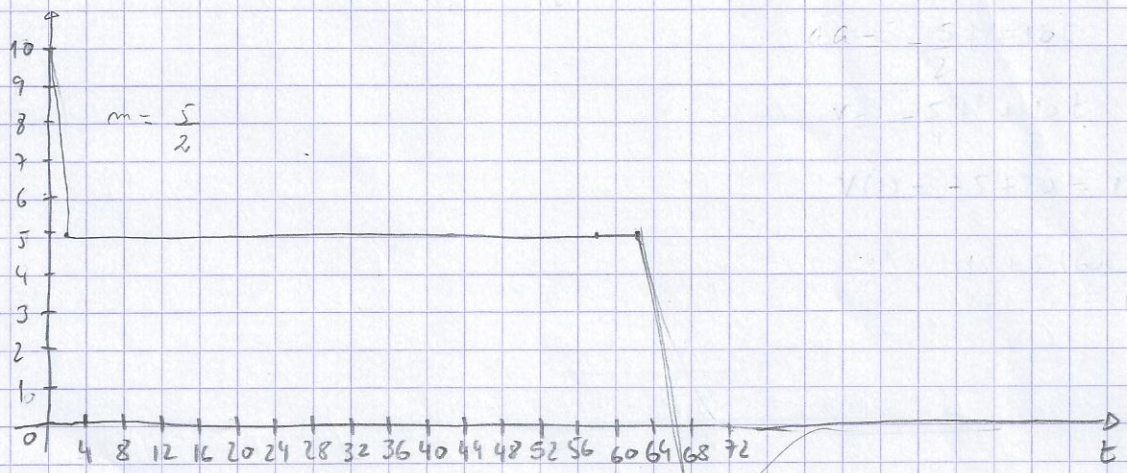
$d = 0{,}63 + 0{,}63 + 2{,}11 + 2{,}11 + 12 = 17{,}48$ m

c) $v_m = \frac{x(4) - x(2)}{4 - 2} = \frac{12 + 2}{2} = \frac{14}{2} = 7$ m/s

d) $v(t) = 3{,}0 - 8{,}0t + 3t^2$

2) $t = 0$ s v $a = 10$ m/s^2 $\qquad t = 2$ s $\quad a = 5$ m/s^2

a)



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b) máximo em $t = 60$ s $\quad v = v_0 + 60a = v_0 + 300 = 15 + 300 = 315$ m/s

$a = -\frac{5}{2}t + 10$

$v = -\frac{5}{4}t^2 + 10t$

$v(2) = -5 + 20 = 15$ m/s

$v(72) = 0$

$0 = 315 + 10a \iff a = -31{,}5$ m/s^2

3) $a = 4\ m/s^2$ $t = 4\ s$ $v = v_0 + at = 0 + 4t$

$v = 16 - 8(t - 14)$
$= 16 - 8t + 112$
$= 128 - 8t$

a)

b) $d_1 = \int_0^4 4t\,dt = \left[2t^2\right]_0^4 = 2 \times 16 - 0 = 32\ m$

$d_2 = \int_4^{14} 16\,dt = \left[16t\right]_4^{14} = 224 - 64 = 160\ m$

$d_3 = \int_{14}^{16} (-8t + 128)\,dt = \left[-4t^2 + 128t\right]_{14}^{16} = 16\ m$

$d = d_1 + d_2 + d_3 = 32 + 160 + 16 = 208\ m$

4) A: $v_0 = 0$, $a = 2\ m/s^2$

B: $t_0 = 4\ s$, $a = 3\ m/s^2$

$v_A = \int_0^t 2\,dt = 2t\ m/s$

$v_B = \int_4^t 3\,dt = \left[3t\right]_4^t = 3t - 12$

$d_A = \int_0^t 2t\,dt = t^2$

$d_B = \int_4^t (3t - 12)\,dt = \left[\frac{3t^2}{2} - 12t\right]_4^t$

$= \frac{3t^2}{2} - 12t - 24 + 48$

$= \frac{3}{2}t^2 - 12t + 24$

$d_A = d_B \Leftrightarrow$

$\Leftrightarrow \frac{2}{2}t^2 = \frac{3}{2}t^2 - 12t + 24$

$\Leftrightarrow \frac{1}{2}t^2 - 12t + 24 = 0$

$\Leftrightarrow t^2 - 24t + 48 = 0$

$\Leftrightarrow t = \frac{24 \pm \sqrt{576 - 192}}{2}$

$\Leftrightarrow t = \frac{24 \pm \sqrt{384}}{2}$

$\Leftrightarrow t = \frac{24 \pm 19{,}6}{2}$

$\Leftrightarrow t = 2{,}2$ (crossed out, < 4) $\vee$ $t = 21{,}8$

logo $d = 21{,}8^2 = 475{,}2\ m$

(2) 5) $17m \to 2s$
$24m \to 2s$

a) $v = v_0 + at$, a é constante

$x = x_0 + v_0 t + \frac{1}{2}at^2$

1º parte

$17 = 2v_0 + 2a$

$V = v_0 + 2a$

2ª parte

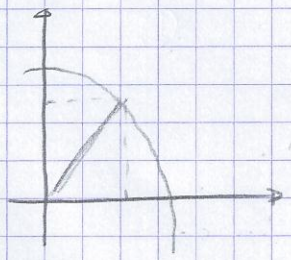
$24 = 2V + 2a$
$12 = V + a$
$V = 12 - a$

logo $\begin{cases} 12 - a = v_0 + 2a \\ 17 = 2v_0 + 2a \end{cases} \Leftrightarrow \begin{cases} v_0 = 12 - 3a \\ 17 = 24 - 6a + 2a \end{cases}$

$\Leftrightarrow \begin{cases} - \\ -7 = -4a \end{cases} \quad \begin{cases} v_0 = 12 - 3 \times 1{,}75 \\ a = 1{,}75 \end{cases} \Leftrightarrow \begin{cases} V = 6{,}75 \ m/s \\ a = 1{,}75 \ m/s^2 \end{cases}$

6) $x = 2 \operatorname{sen} wt \qquad y = 2 \cos wt$

a)



$r^2 = x^2 + y^2$

$(2 \operatorname{sen} wt)^2 + (2 \cos wt)^2 = x^2 + y^2$

$4 \operatorname{sen}^2(wt) + 4 \cos^2(wt) = x^2 + y^2$

$4 \underbrace{(\operatorname{sen}^2 wt + \cos^2 wt)}_{1} = x^2 + y^2$

$\therefore x^2 + y^2 = 4 \qquad r = \sqrt{4} = 2$

b) $\vec{v} = (+2w \cos(wt) \ , \ -2w \operatorname{sen}(wt))$

$|\vec{v}| = \sqrt{4w^2 \cos^2(wt) + 4w^2 \operatorname{sen}^2(wt)}$

$= \sqrt{4w^2 \times 1}$

$= 2w \ cm/s$

c) movimento circular uniforme
$a_t = 0$

c) $\vec{a} = (-2w^2 \operatorname{sen}(wt), 2w^2 \cos(wt))$

$|\vec{a}| = \sqrt{4w^4} = 2w^2 \quad cm/s$

$a_n = \frac{v^2}{r} = \frac{4w^2}{2} = 2w^2 \quad cm/s$

$a = \sqrt{a_t^2 + a_n^2}$

$2w^2 = \sqrt{a_t^2 + 4w^4} \Leftrightarrow 4w^4 = a_t^2 + 4w^4 \Leftrightarrow a_t = 0 \ cm/s^2$

7)

8) $\vec{a} = (4 - t^2)\,\hat{i}$ m/s^2 $t = 3s$ $v = 2\,m/s$ $x = 9m$

$$\Rightarrow a = \sqrt{(4-t^2)^2 + 0^2} \Leftrightarrow a = 4 - t^2 \quad m/s^2$$

$$\Rightarrow v = \int (4 - t^2)\,dt = 4t - \frac{t^3}{3} + K \quad m/s \quad , K \in \mathbb{R}$$

mas $v(3) = 2 \Leftrightarrow 2 = 4 \times 3 - \frac{3^3}{3} + K$

$$\Leftrightarrow 2 = 12 - 9 + K$$

$$\Leftrightarrow K = 2 - 3 \Leftrightarrow K = -1$$

log: $v = 4t - \frac{t^3}{3} - 1 \quad m/s$

$$\Rightarrow x(t) = \int \left(4t - \frac{t^3}{3} - 1\right) dt$$

$$= 2t^2 - \frac{t^4}{12} - t + K \quad , K \in \mathbb{R}$$

mas $x(3) = 9 \Leftrightarrow 2 \times 9 - \frac{81}{12} - 3 + K = 9$

$$\Leftrightarrow 18 - 6{,}75 - 3 + K = 9$$

$$\Leftrightarrow 8{,}25 + K = 9 \Leftrightarrow K = 0{,}75$$

$$x(t) = 2t^2 - \frac{t^4}{12} - t + 0{,}75 \quad m$$

(3) a) $a = 2-6t \quad m/s^2 \qquad v_0 = 1\ m/s \qquad t = 1\,s \Rightarrow x(1) \neq 0$

$$v = \int (2-6t)\,dt = 2t - 3t^2 + K \quad , K \in \mathbb{R}$$

mas $v(0) = 1$

$K = 1 \Rightarrow v(t) = 2t - 3t^2 + 1 \quad m/s$

$$x(t) = \int (2t - 3t^2 + 1)\,dt$$

$$= t^2 - t^3 + t + K \quad , K \in \mathbb{R}$$

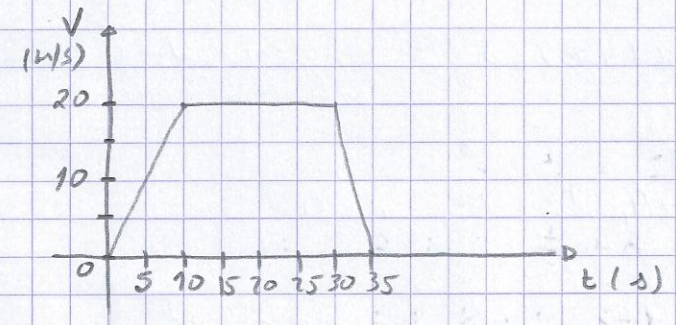
mas $x(1) = 0$

$1 - 1 + 1 + K = 0 \Leftrightarrow K = -1 \Rightarrow x(t) = t^2 - t^3 + t - 1 \quad m$

b)

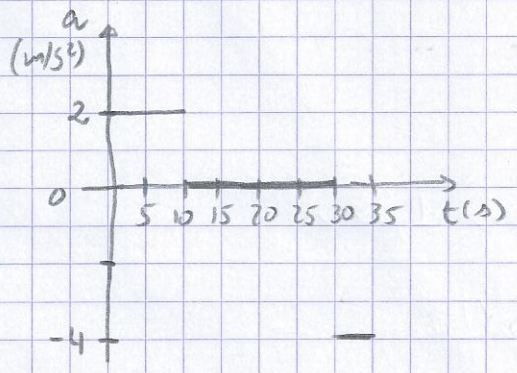
$$v = \begin{cases} 2t & se \quad 0 < t < 10 \\ 20 & se \quad 10 < t < 30 \\ 140 - 4t & se \quad 30 < t < 35 \end{cases}$$

a)



b) $a(t) = \frac{\partial v}{\partial t}$

$$a(t) = \begin{cases} 2 & se \quad 0 < t < 10 \\ 0 & se \quad 10 < t < 30 \\ -4 & se \quad 30 < t < 35 \end{cases}$$



c) $x(t) = \int 2t\,dt = t^2$ (...)

$x(t) = \int 20\,dt = 20t$

$x(t) = \int (140 - 4t)\,dt = 140t - 2t^2$

11) $v = 1 + 6t^2$ $\quad v \in (cm/s^*)$ $\quad x(2) = 20\,cm$

a) $a(t) = \frac{dv}{dt} = 12t \Rightarrow a(t) = 12t \ cm/s^2$

$x(t) = \int v(t)\,dt = t + 2t^3 + K \ , K \in \mathbb{R}$

mas $\quad x(2) = 20$

$2 + 2 \times 8 + K = 20$

$K = 20 - 18$

$K = 2 \Rightarrow x(t) = 2t^3 + t + 2 \ cm$

b) $v(0) = ?$

$v(0) = 1 \ cm/s$

$x(0) = 2 \ cm$

12) $v_0 = 10 \ m/s$ $\quad x(4,25) = 0$ $\quad g = 9,8 \ m/s^2$

a) $a = -9,8 \ m/s^2$

$v = \int (-9,8)\,dt = -9,8t + K \ , K \in \mathbb{R}$

$v(0) = 10 \Leftrightarrow K = 10$

$y = \int (10 - 9,8t)\,dt = 10t - 4,9t^2 + K \ , K \in \mathbb{R}$

mas $\quad y(4,25) = 0 \quad 10 \times 4,25 - 4,9 \times 4,25^2 + K = 0$

$K = 46$

log $\quad y = 46 + 10t - 4,9t^2$

$y' = 10 - 9,8t$

$y' = 0 \Leftrightarrow t = \frac{10}{9,8} \approx 1,02 \ s$

$y(1,02) = 51,1 \ m$ $\quad$ altura viva.

b) $y(0) = 46 \ m$

c) $v = -9,8t + 10 \Rightarrow v(4,25) = -9,8 \times 4,25 + 10 = -31,65 \ m/s$

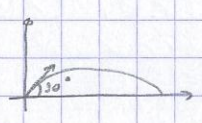
(4)

13) A: ↑

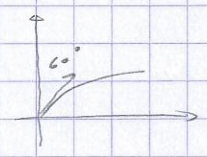
B: 

(A) $a(t) = -9,8 \ m/s^2$

$v(t) = -9,8t + v_0 \quad m/s$

(B) 

14) $v_0 = 100 \ m/s \qquad \theta = 60°$



$\cos 60° = \frac{1}{2}$

$\text{sen}\, 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{x}{100}$

a) $\vec{a} = -9,8\,\hat{j}$

$R = \frac{v_0^2}{g} \text{sen}(2\theta)$

$R = \frac{100^2}{9,8} \text{sen}(120°) \simeq 884 \ m$

b) $h = \frac{v_0^2 \, \text{sen}^2 60}{2g} = \frac{100^2 \times 0,75}{2 \times 9,8} \simeq 383 \ m$

c) $\vec{v_0} = 100 \cos 60° \, \vec{i} + 100 \text{sen} 60° \, \vec{j}$

$= 50\vec{i} + 86,6\vec{j}$

$r = r_0 + v_0 t - \frac{1}{2} g t^2$

$\vec{v} = 50\vec{i} + (-9,8 \times 10 + 86,6)\vec{j} \qquad |\vec{v}| = \sqrt{50^2 + 11,4^2} = 51,3 \ m/s$

$\vec{r} = 50t\,\vec{i} + (86,6t - 4,9t^2)\vec{j}$

$r(10) = 500\vec{i} + 376\vec{j} \qquad h(10) = 376 \ m$

15) $r = 1,49 \times 10^{11} \ m$

$d = 2 \times \pi \times 1,49 \times 10^{11} = 9,4 \times 10^{11}$

$t = 365,25 \times 24 \times 60 \times 60 = 31\,557\,600 \ s$

$v = \frac{9,4 \times 10^{11}}{31\,557\,600} = 29,7 \times 10^3 \ m/s$

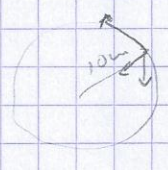
$a_c = \frac{v^2}{r} = \frac{(29,7 \times 10^3)^2}{1,49 \times 10^{11}} \simeq 0,0059 = 5,9 \times 10^{-3} \ m/s^2$

16) $t = 27{,}3$ dias $\quad r = 384\,000$ km $= 3{,}84 \times 10^5 \times 10^3 = 3{,}84 \times 10^8$ m

$$v = \frac{2\pi r}{T} = \frac{2 \times \pi \times 3{,}84 \times 10^8}{27{,}3 \times 24 \times 60^2} = 1022{,}9 \text{ m/s}$$

$$a = \frac{1022{,}9^2}{3{,}84 \times 10^8} = 0{,}002{,}72 = 2{,}72 \times 10^{-3} \text{ m/s}^2$$

17) $r = 10$ m $\quad s(t) = 2\cos(0{,}2t) \quad t = 0 \Rightarrow s = 0$



a) $\vec{v} = 0{,}4 \operatorname{sen}(0{,}2t)\, \hat{u}_t$

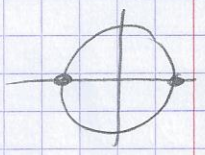
b) $\vec{a}_n = \frac{0{,}16 \operatorname{sen}^2(0{,}2t)}{10}\, \hat{u}_n = 0{,}016 \operatorname{sen}^2(0{,}2t)\, \hat{u}_n$

$\vec{a}_t = -0{,}08 \cos(0{,}2t)\, \vec{u}_t$

$\vec{a} = -0{,}08 \cos(0{,}2t)\, \hat{u}_t + 0{,}016 \operatorname{sen}^2(0{,}2t)\, \hat{u}_n$

c) $a_n = +0{,}016 \operatorname{sen}^2(0{,}2) = 6{,}32 \times 10^{-4}$ m/s^2

$a_t = -0{,}08 \cos(0{,}2) = -0{,}078$ m/

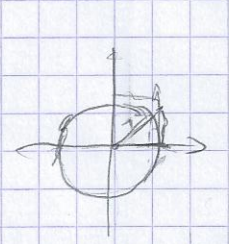


d) $0{,}4 \operatorname{sen}(0{,}2t) = 0$

$\operatorname{sen}(0{,}2t) = 0 \iff 0{,}2t = k\pi \quad , k \in \mathbb{Z}_0^+$

$\iff t = 5k\pi \quad , k \in \mathbb{Z}_0^+$

18) $r = 1{,}0$ m $\quad \underbrace{s(t)}_{\theta(t)} = 2t - t^2$



a)

$x(t) = r\cos(2t - t^2) + k_1$

$y(t) = r \operatorname{sen}(2t - t^2) + k_2$

$x(0) = 0 \iff \cos 0 + k_1 = 0 \iff k_1 = -1$

$y(0) = 0 \iff \operatorname{sen} 0 + k_2 = 0 \iff k_2 = 0$

Logo $\quad x(t) = -1 + \cos(2t - t^2)$,

$y(t) = \operatorname{sen}(2t - t^2)$

(5)

b) $\vec{v}(t) = ?$

$v_x = -(2-2t)\operatorname{sen}(2t-t^2)$

$v_y = (2-2t)\cos(2t-t^2)$

$|v| = \sqrt{(2-2t)^2 \times 1} = |2-2t|$

c) $a_x = -(-2)\operatorname{sen}(2t-t^2) + \left(-(2-2t)\right) \times (2-2t)\cos(2t-t^2)$

$= 2\operatorname{sen}(2t-t^2) - (2-2t)^2\cos(2t-t^2)$

$a_y = -2\cos(2t-t^2) + (2-2t) \times \left(-(2-2t)\operatorname{sen}(2t-t^2)\right)$

$= -2\cos(2t-t^2) - (2-2t)^2\operatorname{sen}(2t-t^2)$

d) $t = 0{,}5$ s

$a_t = \frac{\partial v}{\partial t} = (2-2t)' = -2 \Rightarrow a_t(0{,}5) = -2 \text{ m/s}^2$

$a_c = \frac{v^2}{x} = \frac{(2-2t)^2}{1} = (2-2t)^2 \Rightarrow$

$\Rightarrow a_c(1) = (2-2\times 0{,}5)^2 = (2-1)^2 = 1 \text{ m/s}^2$

e) $x(2) = -1 + \cos(4-4) = 0$

$y(2) = \operatorname{sen}(0) = 0$

$s = 0$; $P(0,0)$

Note: $x'(t) = 2-2t$

$2-2t = 0 \Leftrightarrow t = 1$ ← máximo

$x(1) = 2-1 = 1$

$d = 1+1 = 2\text{m}$

$30° \longrightarrow 2s$

19) $v = 50\ cm/s = 0{,}5\ m/s \quad 30° \quad \Delta t = 2s$

a) $x(t) = r\cos(wt)$

$y(t) = r\,sen(wt)$

$x^2 + y^2 = r$ ✓

$30° \longrightarrow \frac{\pi}{6} \longrightarrow 2s$

$\frac{\pi}{12} \longrightarrow 1s$

$w = \frac{\pi}{12}$

$x(t) = r\cos\left(\frac{\pi}{12}t\right)$ e $y(t) = r\,sen\left(\frac{\pi}{12}t\right)$

$V_x(t) = -\frac{\pi}{12} r\,sen\left(\frac{\pi}{12}t\right)$ e $V_y(t) = \frac{\pi}{12} r\cos\left(\frac{\pi}{12}t\right)$

$|\vec{v}| = \sqrt{\left(\frac{\pi}{12} r\right)^2 \times 1}$

$\Leftrightarrow \quad 0{,}5 = \frac{\pi}{12} r \Leftrightarrow r = \frac{6}{\pi} \Leftrightarrow r \simeq 1{,}9\ m$

b)

$a_c = \frac{v^2}{r} = \frac{0{,}5^2}{1{,}9} = 0{,}13\ m/s^2$

20) $r = 2m \qquad x^2 + y^2 = 4$

$w = 3t + 1$

a) $v = wR = 6t + 2$

$\vec{a}_t = 6\,\hat{u}_t \qquad \vec{a}_n = \frac{v^2}{R} = \frac{(6t+2)^2}{2} = \frac{36t^2 + 24t + 4}{2}$

$= (18t^2 + 12t + 2)\,\hat{u}_n \Rightarrow \vec{a}_n(1) = (18 + 12 + 2)\,\hat{u}_n$

$= 32\,\hat{u}_n$

- $\vec{a}(1) = 6\hat{u}_t + 32\hat{u}_n$
- $a(1) = \sqrt{36 + 1024} = 32{,}6\ m/s^2$

$tg(\phi) = \frac{32}{6} \Leftrightarrow \phi = 79{,}4°$