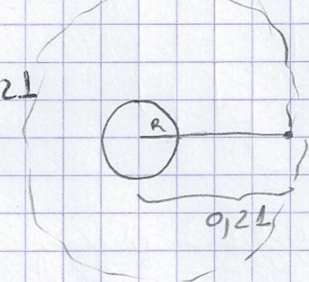
3.4 → Capacidade eléctrica

1)

1) $R = 12\,cm = 0,12\,m$ $\qquad \varepsilon_0 = 8,85 \times 10^{-12}$

$E = 4,9 \times 10^4\ V/m \qquad d = 21\,cm = 0,21$



a) Pela lei de Gauss,

$$E.A = \frac{Q}{\varepsilon_0} \Leftrightarrow 4,9\times10^4 \cdot 4\pi \times 0,21^2 = \frac{Q}{\varepsilon_0}$$

$Q = 2,4\times10^{-7}$ $\qquad Q = \sigma A$

$$\sigma = \frac{2,4\times10^{-7}}{4\pi\times 0,12^2} = 1,33\times10^{-6}\ C/m^2$$

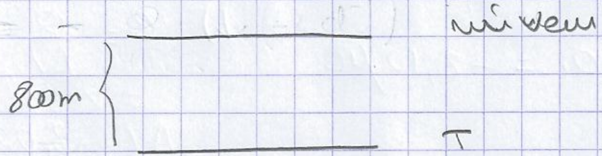
b) $C = \frac{Q}{\Delta V}$ $\qquad \Delta V = K_e \frac{Q}{R}$

logo $C = \frac{Q}{K_e \frac{Q}{R}} \Leftrightarrow C = \frac{R}{K_e} = \frac{0,12}{8,99\times10^9} = 1,33\times10^{-11}\ F$

2) $d = 800\,m$

$A = 1\,Km^2 = 1\times10^6\,m^2$

$E = 3\times10^6\ V/m$



Placas:

$$C = \varepsilon_0 \frac{A}{d} = 8,85\times10^{-12} \times \frac{1\times10^6}{800} = 1,11\times10^{-8}\ F$$

$$\Delta V = Ed = 3\times10^6 \times 800 = 2,4\times10^9\ V$$

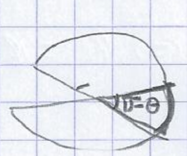
Por sua vez:

$$C = \frac{Q}{\Delta V} \Leftrightarrow Q = 1,11\times10^{-8} \times 2,4\times10^9 = 26,64\ C$$

3) N placas $\qquad$ Área da sobreposição: $\theta = \pi \rightarrow A = 0$

R $\qquad$ se $\theta = 0 \rightarrow A = \frac{\pi R^2}{2}$

$d \rightarrow d/2$



logo $A = \frac{(\pi-\theta)R^2}{2}$

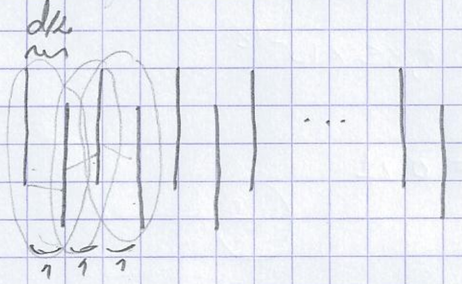
$\theta = 0$ máx C.

Placas

$$C = \varepsilon_0 \frac{A}{d} = \varepsilon_0 \times \frac{(\pi-\theta)R^2}{2(d/2)} = \frac{\varepsilon_0(\pi-\theta)R^2}{d}$$

$$A = \frac{\pi-\theta}{2\cancel{\pi}} \times \cancel{\pi} R^2 = \frac{(\pi-\theta)R^2}{2}$$

Quantos pares de placas?



N placas
+
N placas

cada duas placas produz 3 condensadores
Tem no total $2N-1$

logo $C = (2N-1) \times \dfrac{\varepsilon_0(\pi-\theta)R^2}{d}$ (confirmado!)

4) $L = 50\,m$

$d_i = 2{,}88\,mm$

$Q_i = 8{,}10\,\mu C$

$d_e = 7{,}27\,mm$

$Q_e = -8{,}10\,\mu C$

a) Num fio:

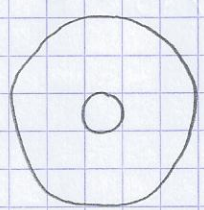
$$C = \frac{L}{2K_e \ln\left(\frac{d_e}{d_i}\right)} = 2{,}68 \times 10^{-9}\,C$$

b) $C = \dfrac{Q}{\Delta V} \Leftrightarrow \Delta V = \dfrac{8{,}10 \times 10^{-6}}{2{,}68 \times 10^{-9}}$

$\Delta V = 3{,}022 \times 10^{3}\,V = 3022\,V$

5) $C = 20\mu F = 20 \times 10^{-6}\,F$

2R
R



$R = ?$

$C = \dfrac{Q}{\Delta V}$

$\Delta V = V_R - V_{2R} = \dfrac{K_e Q}{R} - \dfrac{K_e Q}{2R} =$

$= K_e Q \left(\dfrac{1}{R_{(2)}} - \dfrac{1}{2R}\right) = \dfrac{K_e}{2R} Q$

logo, $C = \dfrac{Q}{\frac{K_e Q}{2R}} \Leftrightarrow C = \dfrac{2R}{K_e} \Leftrightarrow R = \dfrac{8{,}99 \times 10^9 \times 20 \times 10^{-6}}{2}$

$\Leftrightarrow R = 89900\,m$

logo,

$V = \dfrac{4}{3}\pi(2R)^3 - \dfrac{4}{3}\pi R^3 = \dfrac{28\pi R^3}{3} = 2{,}13 \times 10^{16}\,m^3$

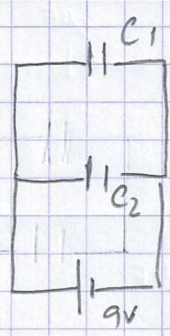
(confirmado)

2

6) $C_1 = 5{,}0\ \mu F = 5 \times 10^{-6}\ F$

$C_2 = 12\ \mu F = 12 \times 10^{-6}\ F$

$\Delta V = 9V$



a) $C_{eq} = C_1 + C_2$

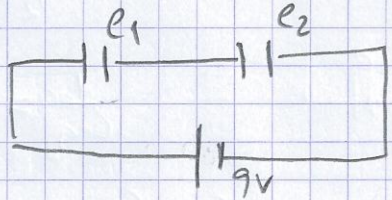
$C_{eq} = 17 \times 10^{-6}\ F$

b) É igual à voltagem da pilha: $\Delta V = 9V$

c) $C_1 = \frac{Q_1}{\Delta V} \Leftrightarrow Q_1 = 5 \times 10^{-6} \times 9 = 45 \times 10^{-6}\ C$

$C_2 = \frac{Q_2}{\Delta V} \Leftrightarrow Q_2 = 12 \times 10^{-6} \times 9 = 108 \times 10^{-6}\ C$

7) Série



a) $\frac{1}{C_{eq}} = \frac{1}{\underset{(C_2)}{C_1}} + \frac{1}{\underset{(C_1)}{C_2}} \Leftrightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 3{,}53 \times 10^{-6}\ F$

b)

c) $Q_{total} = Q_1 = Q_2$

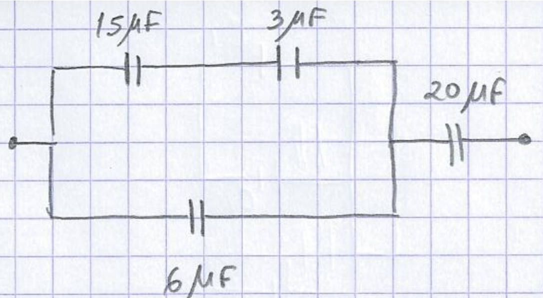
$C_{eq} = \frac{Q}{\Delta V} \Leftrightarrow 3{,}53 \times 10^{-6} = \frac{Q}{9}$

$\Leftrightarrow Q = 3{,}18 \times 10^{-5}\ C$

Como os condensadores estão em série, têm a mesma carga, logo, $Q_1 = Q_2 = 3{,}18 \times 10^{-5}\ C$

b) $\Delta V_1 = \frac{Q_1}{C_1} = 6{,}36\ V$ $\Delta V_2 = \frac{Q_2}{C_2} = 2{,}65\ V$

8) a)

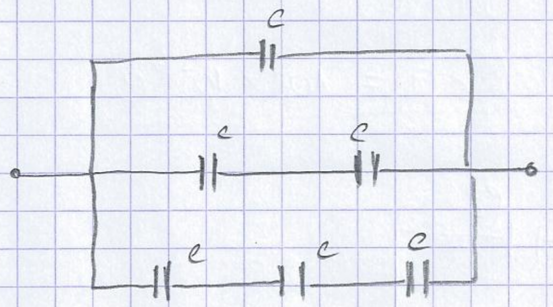


$$C_{eq_1} = \frac{15 \times 3}{15+3} = 2{,}5\,\mu F$$

$$C_{eq_2} = 2{,}5 + 6 = 8{,}5\,\mu F$$

$$C_{eq} = \frac{8{,}5 \times 20}{28{,}5} = 5{,}96\,\mu F$$

b)



$$C_{eq_1} = C$$

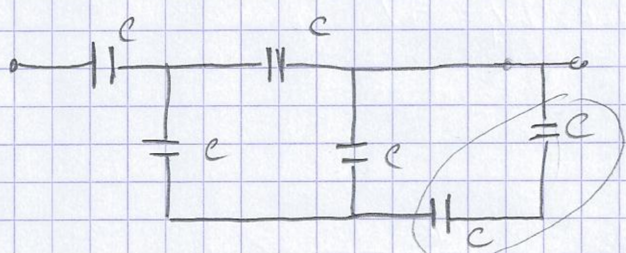
$$C_{eq_2} = \frac{C \times C}{2C} = \frac{C}{2}$$

$$\frac{1}{C_{eq_3}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \Leftrightarrow$$

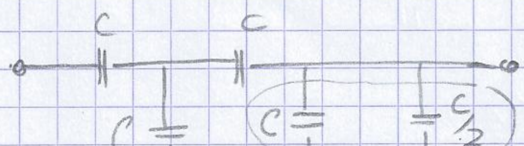
$$\Leftrightarrow C_{eq_3} = \frac{C}{3}$$

$$C_{eq} = \underset{(6)}{C} + \underset{(3)}{\frac{C}{2}} + \underset{(2)}{\frac{C}{3}} = \frac{11}{6} C \quad (F)$$

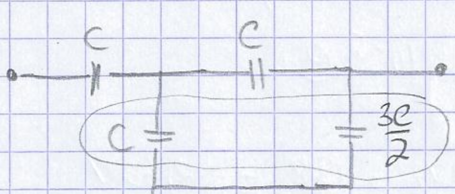
c)



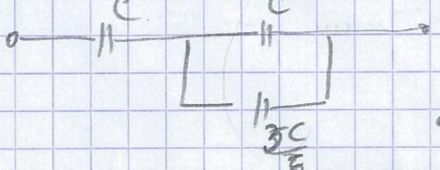
$$C_{eq_1} = \frac{C \times C}{2C} = \frac{C}{2}$$



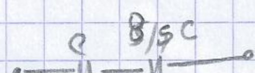
$$C_{eq} = C + \frac{C}{2} = \frac{3C}{2}$$



$$C_{eq} = \frac{C \times \frac{3}{2} C}{C + \frac{3}{2} C} = \frac{\frac{3}{2} C}{\frac{5}{2}} = \frac{3}{5} C$$



$$C_{eq} = C + \frac{3}{5} C = \frac{8}{5} C$$

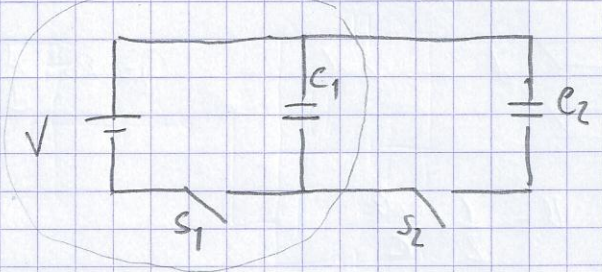


$$C_{eq} = \frac{C \times \frac{8}{5} C}{C + \frac{8}{5} C} = \frac{8/5}{13/5} C = \frac{8}{13} C$$

3) a) $C_1 = 6\,\mu F$

$C_2 = 3\,\mu F$

$V = 20\,V$



1º) S_1 fecha e carrega C_1

2º) S_1 abre e C_1 fica ligado a C_2

$$C_1 = \frac{Q}{\Delta V} \Leftrightarrow Q = 6\times10^{-6}\times 20 = 120\times10^{-6}\,F$$

$$C_{eq} = C_1 + C_2 = (6+3)\times10^{-6} = 9\times10^{-6}\,F$$

$$Q = Q_1 + Q_2 \Leftrightarrow 120\times10^{-6} = Q_1 + Q_2$$

Como estão em paralelo a diferença de Potencial é igual:

$$\Delta V = \frac{Q}{C} \Rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\Rightarrow \frac{120\times10^{-6} - Q_2}{6\times10^{-6}} = \frac{Q_2}{3\times10^{-6}}$$

$$\Leftrightarrow 120\times10^{-6} - Q_2 = 2Q_2$$

$$\Leftrightarrow Q_2 = 40\times10^{-6}\,C$$

e,

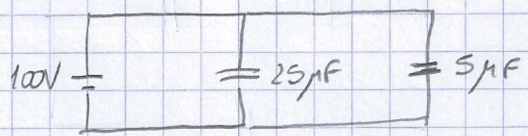
$$Q_1 = (120-40)\times10^{-6} = 80\times10^{-6}\,C$$

10) $C_1 = 25\,\mu F$

$C_2 = 5\,\mu F$

$\Delta V = 100\,V$

a)



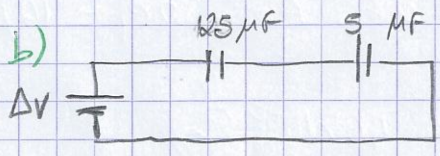
$$U = \frac{1}{2}C(\Delta V)^2$$

$C_{eq} = 25 + 5 = 30\,\mu F$

$\Delta V = 100\,V$ pois estão em paralelo

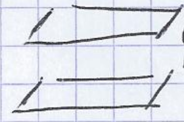
$$\Rightarrow U = \frac{1}{2}\times 30\times10^{-6}\times 100^2 = 0,15\,J$$

b)



$$C_{eq} = \frac{25\times5}{30} = 4,17\,\mu F$$

$$0,15 = \frac{1}{2}\times 4,17\times10^{-6}\times \Delta V^2 \Leftrightarrow \Delta V = 268\,V$$

11) 

$d \rightarrow 2d$ $\qquad C_i = \varepsilon_0 \frac{A}{d}$

$$U = \frac{1}{2} \times \frac{Q^2}{C}$$

$$C_i = \varepsilon_0 \frac{A}{d} \quad e \quad C_f = \varepsilon_0 \frac{A}{2d} \quad \Rightarrow \quad C_f = \frac{1}{2} C_i$$

Como a bateria foi desligada, o condensador libera energia → aumenta para o dobro. $U_f = 2U_i$

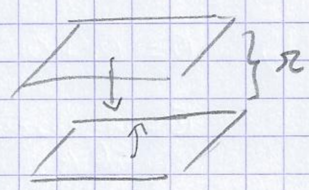
12) A
Q

$F = \frac{Q^2}{2\varepsilon_0 A}$?

$$U = \frac{1}{2}\frac{Q^2}{C} \qquad C = \varepsilon_0 \frac{A}{d}$$

$$U = \frac{1}{2}\frac{Q^2 \times d}{\varepsilon_0 A}$$

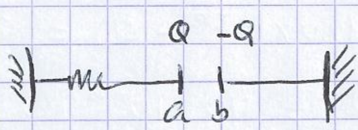
$$U = \frac{1}{2\varepsilon A} Q^2 \times d$$



Nota $\quad U = W(F) = F \cdot d \cdot \cos 0°$

$$\frac{1}{2\varepsilon A} Q^2 \times d = F \cdot d \quad \Rightarrow \quad F = \frac{Q^2}{2\varepsilon_0 A} \checkmark$$

13) K



$F = K\Delta x$ $\qquad$ Lei de Hooke

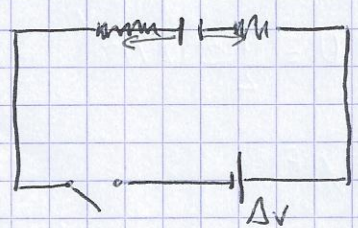
$\Delta x = \frac{F}{K}$ $\qquad$ Pela alínea 12) $\quad F = \frac{Q^2}{2\varepsilon_0 A}$

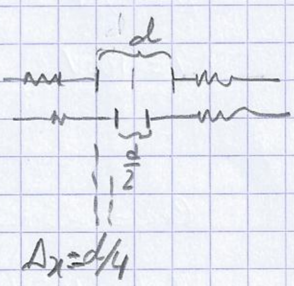
logo, $\quad \Delta x = \frac{Q^2}{2K\varepsilon_0 A}$ $\qquad$ (K → constante da mola)

14) $V = 100\,V$

$d = 8\,mm \rightarrow \frac{d}{2}$

$C = 8\mu F$





a) $C_1 = \varepsilon_0 \frac{A}{d}$ e $C_2 = \varepsilon_0 \frac{A}{d/2} = 2\varepsilon_0 \frac{A}{d} = 2C_1 = 16\mu F$

logo, $\quad C = \frac{Q}{\Delta V} \Leftrightarrow Q = 16 \times 10^{-6} \times 100 = 1{,}6 \times 10^{-3}\, C$ //

b) $F = K\Delta x \Leftrightarrow F = 2 \times 10^{-3} K$ $\quad$ mas $\quad F = \frac{Q^2}{2\varepsilon_0 A}$ $\quad$ então →

$\Delta x = d/4 = 8/4 = 2\,mm$

4

$$\Leftrightarrow 2\times10^{-3}\,K = \frac{Q^2}{2\cancel{\varepsilon_0}\times\frac{Cd}{\cancel{\varepsilon_0}}} \qquad C = \varepsilon_0\frac{A}{d} \Leftrightarrow A = \frac{Cd}{\varepsilon_0}$$

$$\Leftrightarrow 2\times10^{-3}\,K = \frac{(1{,}6\times10^{-3})^2}{2\times16\times10^{-6}\times4\times10^{-3}}$$

$$\Leftrightarrow K = 1{,}0\times10^{4}\ N/m$$

15) C

$V =$ constante

a) $U = \frac{1}{2}\frac{Q^2}{C}$ mas $C = \frac{Q}{V} \Leftrightarrow Q = CV$

$$U = \frac{1}{2}\frac{C^2V^2}{C} = \frac{1}{2}CV^2$$

b) $d' = 3d$

$$C = \frac{A}{\varepsilon d}\ , \quad U_f = \frac{1}{2}\left(\frac{A}{\varepsilon_0\times3d}\right)V^2 = \frac{1}{6}\times\frac{A}{\varepsilon_0 d}V^2$$

$$= \frac{1}{6}CV^2 = \frac{1}{3}U_i$$

c) $U_i = pE = dQE \qquad C_f = \frac{1}{3}C_i$ $\qquad p \rightarrow$ momento

$$U_f = -p_f E = -3dQE = -3pE = 3U_i$$

d) Depois de desligar o condensador descarrega a energia

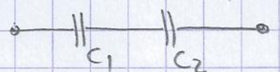
logo o $C_i = \frac{1}{3}C_f \Rightarrow$ Energia passa para o triplo

$U_f = 3U_i$

16) C_1 e C_2

$\mathcal{E}$

$Q = Q_1 = Q_2$



f.e.m $\rightarrow$ força eletromotriz

$\Delta V = \mathcal{E} - RI$

Mas $I = 0$ (desligado)

$\therefore \Delta V = \mathcal{E}$

a) (i) $C_{eq} = \frac{C_1C_2}{C_1+C_2}$; $C_{eq} = \frac{Q}{\Delta V}$

$$Q = Q_1 = Q_2 = \frac{C_1C_2}{C_1+C_2}\,\mathcal{E}$$

ii) $\Delta V_1 = \frac{Q}{C_1} = \frac{\frac{C_1 C_2}{C_1+C_2}\mathcal{E}}{C_1} = \frac{C_2}{C_1+C_2}\mathcal{E}$

$\Delta V_2 = \frac{Q}{C_2} = \frac{\frac{C_1 C_2}{C_1+C_2}\mathcal{E}}{C_2} = \frac{C_1}{C_1+C_2}\mathcal{E}$

iii) $W_1 = \frac{1}{2} C_1 (\Delta V_1)^2 = \frac{1}{2} \cdot C_1 \cdot \frac{C_2^2}{(C_1+C_2)^2}\mathcal{E}^2 = \frac{C_1 C_2^2}{(C_1+C_2)^2} \cdot \frac{\mathcal{E}^2}{2}$

$W_2 = \frac{1}{2} C_2 (\Delta V_2)^2 = \frac{1}{2} \cdot C_2 \cdot \frac{C_1^2}{(C_1+C_2)^2} \cdot \mathcal{E}^2 = \frac{C_2 \cdot C_1^2}{(C_1+C_2)^2} \cdot \frac{\mathcal{E}^2}{2}$

b) $W' = \frac{1}{2} C_{eq} \cdot (\Delta V)^2 = \frac{1}{2} \cdot \frac{C_1 C_2}{C_1+C_2}\mathcal{E}^2$

$W_1 + W_2 = \frac{\mathcal{E}}{2}\left(\frac{C_1 C_2^2 + C_2 C_1^2}{(C_1+C_2)^2}\right) =$

$= \frac{C_1 C_2 \mathcal{E}^2}{2}\left(\frac{C_2 + C_1}{(C_1+C_2)^{\cancel{2}}}\right) = \frac{C_1 C_2 \mathcal{E}}{2} \times \frac{1}{C_1+C_2}$

$= \frac{C_1 C_2}{C_1+C_2} \cdot \frac{\mathcal{E}^2}{2} = W'$

17) $A = 1,75\ cm^2$ $\qquad \varepsilon_r = 2,1$

$l = 0,040\ mm$

$C = \varepsilon_r \cdot \frac{\varepsilon_0 A}{l} = \frac{2,10 \times 8,85 \times 10^{-12} \times 1,75 \times 10^{-4}}{0,040 \times 10^{-3}} = 8,13 \times 10^{-11}\ F$

$\Delta V_{máx} = E_{máx}\, d = 60 \times 10^6 \times 0,040 \times 10^{-3} = 2400\ V$

$\downarrow$

Rigidez dielétrica (ver tabela)

(5) (18) $A = 5\ cm^2$

ar: $\varepsilon_r = 1{,}00059$ $\quad$ R.d. $= 3\times10^6$ V/m

$$Q_{máx} = C\Delta V_{máx}$$
$$= C\, E_{máx}\, d$$
$$= \frac{\varepsilon_r \varepsilon_0 A}{\cancel{d}} \cdot E_{máx}\, \cancel{d}$$
$$= \varepsilon_r \varepsilon_0 A . E_{máx}$$

ar: $Q = 1{,}00059 \times 8{,}85\times10^{-12} \times 5\times10^{-4} \times 3\times10^{6} = 1{,}33\times10^{-8}$ F

Polyester: $Q = 2{,}56 \times 8{,}85\times10^{-12} \times 5\times10^{-4} \times 24\times10^{6} = 2{,}72\times10^{-7}$ F

(confirmado!)

(19) $\Delta V = 12V$

$Q = 48\mu C$

a) $C = \frac{48\times10^{-6}}{12} = 4\times10^{-6}$ F

b) $C' = \varepsilon_r \frac{\varepsilon_0 A}{d} \Leftrightarrow C' = \varepsilon_r \times C = 2{,}1\times 4\times10^{-6} = 8{,}4\times10^{-6}$ F (b)

c) $Q = 48\mu C$

$C = \frac{Q}{\Delta V} \Leftrightarrow 8{,}4\times10^{-6} = \frac{48\times10^{-6}}{\Delta V} \Leftrightarrow \Delta V = 5{,}71\ V$

d) $U_i = \frac{1}{2}\cdot\frac{Q^2}{C} = \frac{1}{2}\cdot\frac{(48\times10^{-6})^2}{4\times10^{-6}} = 2{,}88\times10^{-4}$ J

$U_f = \frac{1}{2}\cdot\frac{(48\times10^{-6})^2}{8{,}4\times10^{-6}} = 1{,}37\times10^{-4}$ J

$\frac{U_i}{U_f} = 2{,}1 = \varepsilon_r$ do teflon!

20) $V_b - V_a = -\int_a^b E_r\, dr = -k_e Q\int_a^b \frac{dr}{r^2} = k_e Q\left[\frac{1}{r}\right]_a^b = k_e Q\left(\frac{1}{b}-\frac{1}{a}\right)$

$V_c - V_b = k_e Q\left(\frac{1}{c}-\frac{1}{b}\right)$

$$C = \frac{\cancel{Q}}{k_e \cancel{Q}\left(\frac{1}{c}-\frac{1}{b}\right) + k_e \cancel{Q}\left(\frac{1}{b}-\frac{1}{a}\right)} = \frac{1}{\frac{(1/c - 1/b)}{4\pi\varepsilon_2\,(\varepsilon_1)} + \frac{(1/b - 1/a)}{4\pi\varepsilon_1\,(\varepsilon_2)}} = \frac{4\pi\varepsilon_1\varepsilon_2}{\varepsilon_1\left(\frac{1}{c}-\frac{1}{b}\right) + \varepsilon_2\left(\frac{1}{b}-\frac{1}{a}\right)}$$