

CITO 24/02

Information:

3 MARCH → NO CLASS (CARNIVAL)

COMPENSATION:

SATURDAY, 8 MARCH, 11:00 - 12:30

ROOM: TO BE CONFIRMED

TO BE CONFIRM.

# OPTIMIZATION PROBLEMS

(O.P.)

AN O.P. IS A PAIR  $(S, f)$  WHERE:

-  $S$  THE SET OF ALL SOLUTIONS, SEARCH SPACE

-  $f: S \rightarrow \mathbb{R}$  FUNCTION THAT QUANTIFIES THE QUALITY OF SOLUTIONS,  
FITNESS FUNCTION.

AN O.P. CAN BE A MAXIMIZATION PB. IF ITS OBJECTIVE IS  
TO FIND A SOLUTION  $x \in S$  SUCH THAT  $\forall y \quad f(y) \leq f(x)$

MINIMIZATION

$x$  IS CALLED  
GLOBAL OPTIMUM

$\forall y \quad f(y) \geq f(x)$


## OPTIMIZATION ALGORITHM

AN ITERATIVE ALGORITHM THAT, AT EACH STEP, OUTPUTS A SOLUTION.

$$\vec{c} = [s_1, s_2, \dots, s_m, \dots]$$

## NO FREE LUNCH THEOREM (INFORMAL)

THE AVERAGE PERFORMANCE OF ALL EXISTING OPT. ALG. IS  
THE SAME IF EVALUATED OVER ALL POSSIBLE EXISTING O.P.

$P(\vec{c} | f, m, A)$   PROB. OF FINDING A GLOBAL OPTIMUM  
+ TIME

## CONSEQUENCES NFL

- IT CANNOT EXIST AN O.A. BETTER THAN ALL THE OTHERS ON ALL O.P. (SUPER-ALGORITHM)
- GIVEN A PARTICULAR PROBLEM, THERE CANNOT BE ANY FORMAL PROCESS TO DECIDE WHAT IS THE BEST ALGORITHM THE ONLY WAY TO DO THAT IS TO EXPERIMENTALLY COMPARE OR BY MEANS OF EXPERIENCE. (INFORMAL METHODS)  
(JUSTIFICATION FOR THE EXISTANCE OF SCIKIT LEARN, WEKA, ...)
- WE NEED TO KNOW MANY O.A. !

# HILL CLIMBING

A WAY OF FINDING A SOLUTION IN A STEP-WISE REFINEMENT FASHION, TRYING TO IMPROVE FITNESS USING THE CONCEPT OF NEIGHBORHOOD

## NEIGHBORHOOD

FUNCTION THAT, APPLIED TO ANY SOLUTION  $x \in S$ ,  
RETURNS A SET OF SOLUTIONS  $N(x)$ ,  
(NEIGHBORS OF  $x$ )

# HILL CLIMBING

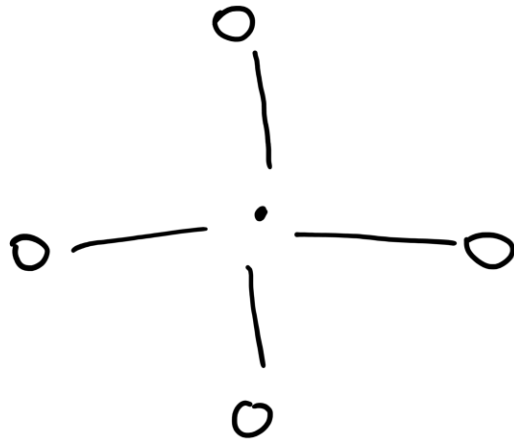
- INITIALIZE THE CURRENT SOLUTION  $x$  (TYPICALLY AT RANDOM)
- REPEAT
  - "GENERATE" A SOLUTION  $y$  FROM  $N(x)$  → THE BEST OF  $N(x)$
  - IF  $f(y)$  IS BETTER OR EQUAL THAN  $f(x)$   
THEN  $x := y$  ( $y$  BECOMES THE NEW CURR. SOL.)
- UNTIL  $\forall y \in N(x)$   $f(y)$  IS WORSE THAN  $f(x)$   
 $\Downarrow$   
 $x$  IS A LOCAL OPTIMUM

## EXAMPLE

FIND THE STUDENT IN THIS ROOM WITH THE  
MAX. STUDENT NUMBER

- $S = \{ \text{STUDENTS IN THIS ROOM} \}$
- $f(x) = \text{STUDENT NUMBER OF } x \quad \forall x \in S$
- MAXIMIZATION

- NEIGHBORHOOD



$$X = \text{LEO}$$

$$f(x) = 0.485$$

$$N(x) = \{\text{PHIL}, \text{SVEN}, \text{ALEX}\}$$

$$f(\text{PHIL}) = 0.518, \quad f(\text{SVEN}) = 0.503, \quad f(\text{ALEX}) = 15.99$$

$$y = \text{ALEX}$$

$$\underline{X = \text{ALEX}}$$



$X = \text{ALEX}$

$f(x) = 1599$

$N(x) = \{ \text{LEO}, \text{JULIA}, \text{SOPHIA}, \text{PEDRO} \}$

$f(\text{LEO}) = 0485$

$f(\text{JULIA}) = 0514$

$f(\text{SOPHIA}) = 0654$

$f(\text{PEDRO}) = 0495$

}  $y = \text{SOPHIA}$

STOP

RETURN ALEX

## DEF. (LOCAL OPTIMUM)

GIVEN AN O.P.  $(S, f)$  AND A NEIGHBORHOOD  $N$

A LOCAL OPTIMUM IS A SOLUTION  $x \in S$

SUCH THAT  $\forall y \in N(x)$   $f(y)$  IS WORSE THAN  $f(x)$

By ITS VERY DEFINITION, THE H.C. ALWAYS RETURNS A LOCAL OPTIMUM, WITH NO GUARANTEE THAT THIS WILL ALSO BE A GLOBAL OPTIMUM.

