CIFO 17/03/2025

- LAST CLASS : EXAMPLES OF O.P. (KNAPSACK, TSP)

- TWO CLASSES AGO : HILL CLIMBING, FITNESS LANDSC.

IDEA : ALLOWING THE ALGO. TO WORSEN FITNESS WITH SOME PROBABILITY

SIMULATED ANNEALING

ANNEALING

A CHEMISTRY EXPERIMENT WHOSE OBJECTIVE IS TO FIND THE SOLID STATE OF A MATERIAL WITH THE LOWEST POSSIBLE LEVEL OF (INTRA-MOLECULAR) ENERGY.

- BEGIN WITH THE MATERIAL IN A STATE i, WITH ENERGY E_i
- WE MODIFY SOME MOLECULAR BINDINGS TO FIND A NEW STATE j, WITH ENERGY E_j
- PROB. OF ACCEPTING THE NEW STATE j AS THE NEW CURRENT STATE:

$$P(\text{accept } j) = \begin{cases} 1 & \text{if } E_j \leq E_i \\ \exp\left(\dfrac{-|E_i - E_j|}{K_B \cdot T}\right) & \text{otherwise} \end{cases}$$

$K_B \cdot T$ → constant parameter

where:

- K_B BOLZMANN CONSTANT
- T TEMPERATURE
- $\exp(x) \rightsquigarrow e^x$

SIMULATED ANNEALING

CURRENT SOLUTION i , NEIGHBOR j

$$P(\text{accept } j) = \begin{cases} 1 & \text{if } f(j) \text{ BETTER OR EQUAL TO } f(i) \\ \exp\left(\dfrac{-|f(i) - f(j)|}{C}\right) & \text{OTHERWISE} \end{cases}$$

THE CONTROL PARAMETER START WITH A HIGH VALUE AND DECREASES STEADILY DURING THE EXECUTION

CONTROL PARAMETER

$$f(c) = \exp\left(-\frac{K}{c}\right) \qquad K > 0$$

DECREASE OF c

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TENDING TOWARDS ZERO, WITHOUT EVER ARRIVING AT ZERO

DECREASING c WE DECREASE THE PROB. OF ACCEPTING A WORSE NEIGHBOR THAN THE CURRENT SOLUTION.

SIMULATED ANNEALING

1. INITIALIZE THE CURRENT SOLUTION i (TYPICALLY RANDOM)
2. INITIALIZE L AND C
3. REPEAT UNTIL TERMINATION CONDITION
   - 3.1. REPEAT L TIMES
     - 3.1.1. CHOOSE A NEIGHBOR j OF i
     - 3.1.2. if $f(j)$ IS BETTER OR EQUAL THAN $f(i)$
       then ACCEPT j AS NEW CURRENT SOL.
       WITH PROB. = 1
       else ACCEPT j AS NEW CURRENT SOL.
       WITH PROB: $\exp\left(-\frac{|f(i)-f(j)|}{c}\right)$
   - 3.2 UPDATE c ~~(AND L)~~
4. RETURN THE SOLUTION WITH THE BEST FITNESS SO FAR

- TERMINATION CONDITION: EITHER A "SUFFICIENTLY GOOD" SOLUTION WAS FOUND OR A PREFIXED MAXIMUM NUMBER OF ITERATIONS WERE EXECUTED.

- UPDATE OF C: DECREASING C TENDING TO 0 WITHOUT EVER ARRIVING AT ZERO, FOR INSTANCE:

$$C = C / K \quad , \quad K > 1$$

- UPDATE OF L: TYPICALLY L CAN BE KEPT AS CONSTANT (THE BIGGER, THE BETTER)

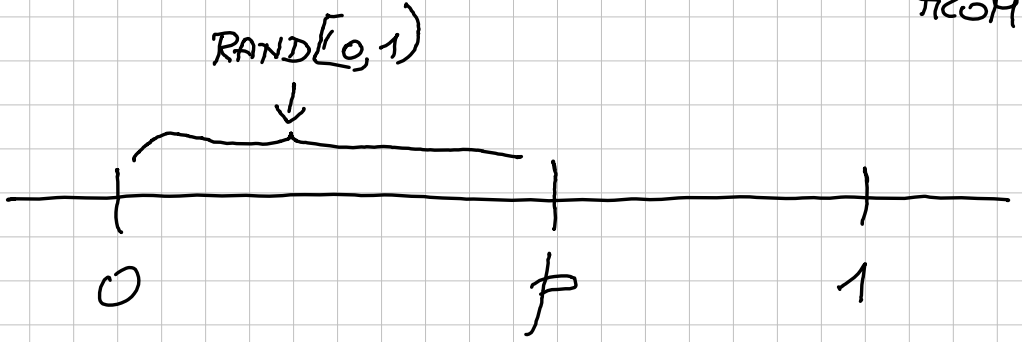
- CHOICE OF THE NEIGHBOR j: TYPICALLY j IS A RANDOM NEIGHBOR OF i

- HOW TO IMPLEMENT A PROBABILISTIC EVENT ?

HOW TO MAKE AN ACTION WITH PROBABILITY p ?

IF RAND $[0,1)$ < p
THEN MAKE THE ACTION
ELSE DON'T MAKE IT

RAND $[0,1)$
RANDOM NUMBER WITH UNIFORM DISTRIBUTION FROM $[0,1)$



EXAMPLE

- $S = \{ i \mid i \in \mathbb{N} \text{ AND } 0 \leq i \leq 15 \}$
- $\forall i \in S$, $f(i)$ = NUMBER OF 1s IN THE BINARY CODE OF i
- MAXIMIZATION
- NEIGHBORHOOD:

$$\forall i, j \in S, \quad i \in N(j) \iff |i - j| = 1$$

- START: $i = 5$ $\quad f(i) = 2$ $\quad bin(5) = 101$
- RANDOM NEIGHBOR OF i: $j = 6$

  $f(j) = 2$ $\quad bin(6) = 110$
- ACCEPT j $\quad i := j$

- $i = 6 \qquad f(i) = 2$
- RANDOM NEIGHBOR OF i : $j = 7$

  $f(j) = 3 \qquad bin(7) = 111$

- ACCEPT $i = 7$

- $i = 7 \quad , \quad f(i) = 3$
- RANDOM NEIGHBOR OF i : $j = 8$

  $f(j) = 1 \qquad bin(8) = 1000$

ASSUME THAT $C = 1$

$$P(\text{accept } j) = \exp\left(-\frac{|f(i) - f(j)|}{C}\right) =$$

$$= \exp\left(-\frac{|3-1|}{1}\right) = \exp(-2) = e^{-2} \approx 0.13$$

PARAMETERS OF S.A. :

- INITIAL VALUE OF C
- "SPEED" AT WHICH C IS DECREASED
- MAX. NUMBER OF ITERATIONS OF THE EXTERNAL LOOP
- L : NUMB. OF ITERATIONS OF INTERNAL LOOP

IT IS POSSIBLE TO PROVE THAT THE SIMULATED ANNEALING TENDS ASYMPTOTICALLY TOWARDS A GLOBAL OPTIMUM.

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- "INTELLIGENT" ALGORITHM