Appendix B

Constrained Benchmark Functions

A constrained optimization problem involves the minimization of $f(x)$ over all x such that $x \in \Gamma \in R^n$, where Γ is the feasible set and n is the problem dimension. We use x^* to represent the optimizing value of x, and $f(x^*)$ is the constrained minimum value of $f(x)$:

$$\begin{aligned} x^* &= \arg\min_x f(x) \\ \text{such that } & g_i(x) \le 0 \text{ for } i \in [1, m] \\ \text{and } & h_j(x) = 0 \text{ for } j \in [1, p] \end{aligned} \quad [B.1]$$

This problem includes $(m + p)$ constraints, m of which are inequality constraints and p of which are equality constraints. In this section, we only show simple constrained benchmarks, while detailed information and evaluation metrics for EA competitions at the 2006 and 2010 *IEEE Congress on Evolutionary Computation* can be found in Liang *et al.* [LIA 06] and Mallipeddi and Suganthan [MAL 10].

The G01 Function

$$\begin{aligned} f(x) &= 5\sum_{i=1}^{4} x_i - 5\sum_{i=1}^{4} x_i^2 - \sum_{i=5}^{13} x_i \\ g_1(x) &= 2x_1 + 2x_2 + x_{10} + x_{11} - 10 \le 0 \\ g_2(x) &= 2x_1 + 2x_3 + x_{10} + x_{11} - 10 \le 0 \\ g_3(x) &= 2x_2 + 2x_3 + x_{11} + x_{12} - 10 \le 0 \end{aligned} \quad [B.2]$$

Evolutionary Computation with Biogeography-based Optimization, First Edition.
Haiping Ma and Dan Simon.

$$g_4(x) = -8x_1 + x_{10} \le 0$$
$$g_5(x) = -8x_2 + x_{11} \le 0$$
$$g_6(x) = -8x_3 + x_{12} \le 0$$
$$g_7(x) = -2x_4 - x_5 + x_{10} \le 0$$
$$g_8(x) = -2x_6 - x_7 + x_{11} \le 0$$
$$g_9(x) = -2x_8 - x_9 + x_{12} \le 0$$

where $0 \le x_i \le 1\ (i = 1,...,9), 0 \le x_i \le 100\ (i = 10,11,12)$ and $0 \le x_{13} \le 1$. The optimum solution is $f(x^*) = -15$.

The G02 Function

$$f(x) = -\left| \frac{\sum_{i=1}^{n} \cos^4(x_i) - 2\prod_{i=1}^{n} \cos^2(x_i)}{\sqrt{\sum_{i=1}^{n} i x_i^2}} \right|$$
$$g_1(x) = 0.75 - \prod_{i=1}^{n} x_i \le 0 \qquad [B.3]$$
$$g_2(x) = \sum_{i=1}^{n} x_i - 0.75n \le 0$$

where $n = 20$ and $0 < x_i \le 10\ (i = 1,...,n)$. The optimum solution is $f(x^*) = -0.80361910412559$.

The G03 Function

$$f(x) = -(\sqrt{n})^n \prod_{i=1}^{n} x_i$$
$$h(x) = \sum_{i=1}^{n} x_i^2 - 1 = 0 \qquad [B.4]$$

where $n = 10$ and $0 \le x_i \le 1\ (i = 1,...,n)$. The optimum solution is $f(x^*) = -1.00050010001000$.

The G04 Function

$$
\begin{aligned}
& f(x)=5.3578547x_3^{\,2}+0.8356891x_1x_5+37.293239x_1-40792.141 \\
& g_1(x)=85.334407+0.0056858x_2x_5+0.0006262x_1x_4-0.0022053x_3x_5-92\le 0 \\
& g_2(x)=-85.334407-0.0056858x_2x_5-0.0006262x_1x_4+0.0022053x_3x_5\le 0 \\
& g_3(x)=80.51249+0.0071317x_2x_5+0.0029955x_1x_2+0.0021813x_3^{\,2}-110\le 0 \\
& g_4(x)=-80.51249-0.0071317x_2x_5-0.0029955x_1x_2-0.0021813x_3^{\,2}+90\le 0 \\
& g_5(x)=9.300961+0.0047026x_3x_5+0.0012547x_1x_3+0.0019085x_3x_4-25\le 0 \\
& g_6(x)=-9.300961-0.0047026x_3x_5-0.0012547x_1x_3-0.0019085x_3x_4+20\le 0
\end{aligned}
\qquad [B.5]
$$

where $78 \le x_1 \le 102, 33 \le x_2 \le 45$ and $27 \le x_i \le 45\ (i=3,4,5)$. The optimum solution is $f(x^*)=-3.066553867178332e+004$.

The G05 Function

$$
\begin{aligned}
& f(x)=3x_1+0.000001x_1^3+2x_2+(0.000002/3)x_2^3 \\
& g_1(x)=-x_4+x_3-0.55\le 0 \\
& g_2(x)=-x_3+x_4-0.55\le 0 \\
& h_3(x)=1000\sin(-x_3-0.25)+1000\sin(-x_4-0.25)+894.8-x_1=0 \\
& h_4(x)=1000\sin(x_3-0.25)+1000\sin(x_3-x_4-0.25)+894.8-x_2=0 \\
& h_5(x)=1000\sin(x_4-0.25)+1000\sin(x_4-x_3-0.25)+1294.8=0
\end{aligned}
\qquad [B.6]
$$

where $0 \le x_1 \le 1200, 0 \le x_2 \le 1200, -0.55 \le x_3 \le 0.55$ and $-0.55 \le x_4 \le 0.55$. The optimum solution is $f(x^*)=5126.4967140071$.

The G06 Function

$$
\begin{aligned}
& f(x)=(x_1-10)^3+(x_2-20)^3 \\
& g_1(x)=-(x_1-5)^2-(x_2-5)^2+100\le 0 \\
& g_2(x)=(x_1-6)^2+(x_2-5)^2-82.81\le 0
\end{aligned}
\qquad [B.7]
$$

where $13 \le x_1 \le 100$ and $0 \le x_2 \le 100$. The optimum solution is $f(x^*)=-6961.81387558015$.

The G07 Function

$$\begin{aligned}
f(x) &= x_1^2 + x_2^2 + x_1 x_2 - 14x_1 - 16x_2 + (x_3 - 10)^2 \\
&\quad + 4(x_4 - 5)^2 + (x_5 - 3)^2 + 2(x_6 - 1)^2 + 5x_7^2 \\
&\quad + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 + (x_{10} - 7)^2 + 45 \\
g_1(x) &= -105 + 4x_1 + 5x_2 - 3x_7 + 9x_8 \le 0 \\
g_2(x) &= 10x_1 - 8x_2 - 17x_7 + 2x_8 \le 0 \\
g_3(x) &= -8x_1 + 2x_2 + 5x_9 - 2x_{10} - 12 \le 0 \\
g_4(x) &= 3(x_1 - 2)^2 + 4(x_2 - 3)^2 + 2x_3^2 - 7x_4 - 120 \le 0 \\
g_5(x) &= 5x_1^2 + 8x_2 + (x_3 - 6)^2 - 2x_4 - 40 \le 0 \\
g_6(x) &= x_1^2 + 2(x_2 - 2)^2 - 2x_1 x_2 + 14x_5 - 6x_6 \le 0 \\
g_7(x) &= 0.5(x_1 - 8)^2 + 2(x_2 - 4)^2 + 3x_5^2 - x_6 - 30 \le 0 \\
g_8(x) &= -3x_1 + 6x_2 + 12(x_9 - 8)^2 - 7x_{10} \le 0
\end{aligned} \tag{B.8}$$

where $-10 \le x_i \le 10 \; (i = 1,...,10)$. The optimum solution is $f(x^*) = 24.30620906818$.

The G08 Function

$$\begin{aligned}
f(x) &= -\frac{\sin^3(2\pi x_1)\sin(2\pi x_2)}{x_1^3(x_1 + x_2)} \\
g_1(x) &= x_1^2 - x_2 + 1 \le 0 \\
g_2(x) &= 1 - x_1 + (x_2 - 4)^2 \le 0
\end{aligned} \tag{B.9}$$

where $0 \le x_1 \le 10$ and $0 \le x_2 \le 10$. The optimum solution is $f(x^*) = -0.0958250414180359$.

The G09 Function

$$\begin{aligned}
f(x) &= (x_1-10)^2+5(x_2-12)^2+x_3^{\,4}+3(x_4-11)^2 \\
&\quad +10x_5^{\,6}+7x_6^{\,2}+x_7^{\,4}-4x_6x_7-10x_6-8x_7 \\
g_1(x) &= -127+2x_1^{\,2}+3x_2^{\,4}+x_3+4x_4^{\,2}+5x_5 \le 0 \\
g_2(x) &= -282+7x_1+3x_2+10x_3^{\,2}+x_4-x_5 \le 0 \\
g_3(x) &= -196+23x_1+x_2^{\,2}+6x_6^{\,2}-8x_7 \le 0 \\
g_4(x) &= 4x_1^{\,2}+x_2^{\,2}-3x_1x_2+2x_3^{\,2}+5x_6-11x_7 \le 0
\end{aligned} \qquad \text{[B.10]}$$

where $-10 \le x_i \le 10$ for $i = 1,\cdots,7$. The optimum solution is $f\left(\vec{x}^*\right) = 680.630057374402$.

The G10 Function

$$\begin{aligned}
f(x) &= x_1+x_2+x_3 \\
g_1(x) &= -1+0.0025(x_4+x_6) \le 0 \\
g_2(x) &= -1+0.0025(x_5+x_7-x_4) \le 0 \\
g_3(x) &= -1+0.01(x_8-x_5) \le 0 \\
g_4(x) &= -x_1x_6+833.33252x_4+100x_1-83333.333 \le 0 \\
g_5(x) &= -x_2x_7+1250x_5+x_2x_4-1250x_4 \le 0 \\
g_6(x) &= -x_3x_8+1250000+x_3x_5-2500x_5 \le 0
\end{aligned} \qquad \text{[B.11]}$$

where $100 \le x_1 \le 10000, 1000 \le x_i \le 10000$ $(i = 2,3)$ and $10 \le x_i \le 1000$ $(i = 4,\ldots,8)$. The optimum solution is $f\left(x^*\right) = 7049.24802052867$.

The G11 Function

$$\begin{aligned}
f(x) &= x_1^{\,2}+(x_2-1)^2 \\
h(x) &= x_2-x_1^{\,2} = 0
\end{aligned} \qquad \text{[B.12]}$$

where $-1 \le x_1 \le 1$ and $-1 \le x_2 \le 1$. The optimum solution is $f\left(x^*\right) = 0.7499$.

The G12 Function

$$
\begin{aligned}
f(x) &= -(100-(x_1-5)^2-(x_2-5)^2-(x_3-5)^2)/100 \\
g(x) &= (x_1-p)^2+(x_2-q)^2+(x_3-r)^2-0.0625 \le 0
\end{aligned}
\tag{B.13}
$$

where $0 \le x_i \le 10\ (i=1,2,3)$ and $p,q,r = 1,2,\cdots,9$. The optimum solution is $f(x^*) = -1$.

The G13 Function

$$
\begin{aligned}
f(x) &= e^{x_1x_2x_3x_4x_5} \\
h_1(x) &= x_1^2+x_2^2+x_3^2+x_4^2+x_5^2-10=0 \\
h_2(x) &= x_2x_3-5x_4x_5=0 \\
h_3(x) &= x_1^3+x_2^3+1=0
\end{aligned}
\tag{B.14}
$$

where $-2.3 \le x_i \le 2.3\ (i=1,2)$ and $-3.2 \le x_i \le 3.2\ (i=3,4,5)$. The optimum solution is $f(x^*) = 0.053941514041898$.

The G14 Function

$$
\begin{aligned}
f(x) &= \sum_{i=1}^{10} x_i\left(c_i+\ln\frac{x_i}{\sum_{j=1}^{10}x_j}\right) \\
h_1(x) &= x_1+2x_2+2x_3+x_6+x_{10}-2=0 \\
h_2(x) &= x_4+2x_5+x_6+x_7-1=0 \\
h_3(x) &= x_3+x_7+x_8+2x_9+x_{10}-1=0
\end{aligned}
\tag{B.15}
$$

where $0 < x_i \le 10\ (i=1,...,10)$ and $c_1=-6.089$, $c_2=-17.164$, $c_3=-34.054$, $c_4=-5.914$, $c_5=-24.721$, $c_6=-14.986$, $c_7=-24.1$, $c_8=-10.708$, $c_9=-26.662$, $c_{10}=-22.179$. The optimum solution is $f(x^*) = -47.7648884594915$.

The G15 Function

$$
\begin{aligned}
f(x) &= 1000 - x_1^2 - 2x_2^2 - x_3^2 - x_1x_2 - x_1x_3 \\
h_1(x) &= x_1^2 + x_2^2 + x_3^2 - 25 = 0 \\
h_2(x) &= 8x_1 + 14x_2 + 7x_3 - 56 = 0
\end{aligned}
\qquad [B.16]
$$

where $0 \le x_i \le 10\ (i = 1,2,3)$. The optimum solution is $f(x^*) = 961.715022289961$.

The G16 Function

$$
\begin{aligned}
f(x) &= 0.000117y_{14} + 0.1365 + 0.00002358y_{13} \\
&\quad + 0.000001502y_{16} + 0.0321y_{12} + 0.004324y_5 \\
&\quad + 0.0001\frac{c_{15}}{c_{16}} + 37.48\frac{y_2}{c_{12}} - 0.0000005843y_{17} \\
g_1(x) &= \frac{0.28}{0.72}y_5 - y_4 \le 0 \\
g_2(x) &= x_3 - 1.5x_2 \le 0 \\
g_3(x) &= 3496\frac{y_2}{c_{12}} - 21 \le 0 \\
g_4(x) &= 110.6 + y_1 - \frac{62212}{c_{17}} \le 0 \\
g_5(x) &= 213.1 - y_1 \le 0 \\
g_6(x) &= y_1 - 405.23 \le 0 \\
g_7(x) &= 17.505 - y_2 \le 0 \\
g_8(x) &= y_2 - 1053.6667 \le 0 \\
g_9(x) &= 11.275 - y_3 \le 0 \\
g_{10}(x) &= y_3 - 35.03 \le 0 \\
g_{11}(x) &= 214.228 - y_4 \le 0 \\
g_{12}(x) &= y_4 - 665.585 \le 0 \\
g_{13}(x) &= 7.458 - y_5 \le 0 \\
g_{14}(x) &= y_5 - 584.463 \le 0 \\
g_{15}(x) &= 0.961 - y_6 \le 0
\end{aligned}
\qquad [B.17]
$$

$$g_{16}(x) = y_6 - 265.916 \le 0$$
$$g_{17}(x) = 1.612 - y_7 \le 0$$
$$g_{18}(x) = y_7 - 7.046 \le 0$$
$$g_{19}(x) = 0.146 - y_8 \le 0$$
$$g_{20}(x) = y_8 - 0.222 \le 0$$
$$g_{21}(x) = 107.99 - y_9 \le 0$$
$$g_{22}(x) = y_9 - 273.366 \le 0$$
$$g_{23}(x) = 922.693 - y_{10} \le 0$$
$$g_{24}(x) = y_{10} - 1286.105 \le 0$$
$$g_{25}(x) = 926.832 - y_{11} \le 0$$
$$g_{26}(x) = y_{11} - 1444.046 \le 0$$
$$g_{27}(x) = 18.766 - y_{12} \le 0$$
$$g_{28}(x) = y_{12} - 537.141 \le 0$$
$$g_{29}(x) = 1072.163 - y_{13} \le 0$$
$$g_{30}(x) = y_{13} - 3247.039 \le 0$$
$$g_{31}(x) = 8961.448 - y_{14} \le 0$$
$$g_{32}(x) = y_{14} - 26844.086 \le 0$$
$$g_{33}(x) = 0.063 - y_{15} \le 0$$
$$g_{34}(x) = y_{15} - 0.386 \le 0$$
$$g_{35}(x) = 71084.33 - y_{16} \le 0$$
$$g_{36}(x) = y_{16} - 140000 \le 0$$
$$g_{37}(x) = 2802713 - y_{17} \le 0$$
$$g_{38}(x) = y_{17} - 12146108 \le 0$$

where

$$y_1 = x_2 + x_3 + 41.6$$
$$c_1 = 0.024x_4 - 4.62$$
$$y_2 = \frac{12.5}{c_1} + 12$$

$$c_2 = 0.0003535x_1^2 + 0.5311x_1 + 0.08705y_2x_1$$
$$c_3 = 0.052x_1 + 78 + 0.002377y_2x_1$$
$$y_3 = \frac{c_2}{c_3}$$
$$y_4 = 19y_3$$
$$c_4 = 0.04782(x_1 - y_3) + \frac{0.1956(x_1 - y_3)^2}{x_2} + 0.6376y_4 + 1.594y_3$$
$$c_5 = 100x_2$$
$$c_6 = x_1 - y_3 - y_4$$
$$c_7 = 0.950 - \frac{c_4}{c_5}$$
$$y_5 = c_6c_7$$
$$y_6 = x_1 - y_5 - y_4 - y_3$$
$$c_8 = 0.995(y_5 + y_4)$$
$$y_7 = \frac{c_8}{y_1}$$
$$y_8 = \frac{c_8}{3798}$$
$$c_9 = y_7 - \frac{0.0663y_7}{y_8} - 0.3153$$
$$y_9 = \frac{96.82}{c_9} + 0.321y_1$$
$$y_{10} = 1.29y_5 + 1.258y_4 + 2.29y_3 + 1.71y_6$$
$$y_{11} = 1.71x_1 - 0.452y_4 + 0.580y_3$$
$$c_{10} = \frac{12.3}{752.3}$$
$$c_{11} = (1.75y_2)(0.995x_1)$$
$$c_{12} = 0.995y_{10} + 1998$$
$$y_{12} = c_{10}x_1 + \frac{c_{11}}{c_{12}}$$
$$y_{13} = c_{12} - 1.75y_2$$
$$y_{14} = 3623 + 64.4x_2 + 58.4x_3 + \frac{146312}{y_9 + x_5}$$

$$c_{13} = 0.995y_{10} + 60.8x_2 + 48x_4 - 0.1121y_{14} - 5095$$
$$y_{15} = \frac{y_{13}}{c_{13}}$$
$$y_{16} = 148000 - 331000y_{15} + 40y_{13} - 61y_{15}y_{13}$$
$$c_{14} = 2324y_{10} - 28740000y_2$$
$$y_{17} = 14130000 - 1328y_{10} - 531y_{11} + \frac{c_{14}}{c_{12}}$$
$$c_{15} = \frac{y_{13}}{y_{15}} - \frac{y_{13}}{0.52}$$
$$c_{16} = 1.104 - 0.72y_{15}$$
$$c_{17} = y_9 + x_5$$

and where $704.4148 \le x_1 \le 906.3855$, $68.6 \le x_2 \le 288.88$, $0 \le x_3 \le 134.75$, $193 \le x_4 \le 287.0966$ and $25 \le x_5 \le 84.1988$. The optimum solution is $f\left(x^*\right) = -1.90515525853479$.

The G17 Function

$$f(x) = f_1(x_1) + f_2(x_2)$$
$$f_1(x_1) = \begin{cases} 30x_1 & 0 \le x_1 < 300 \\ 31x_1 & 300 \le x_1 \le 400 \end{cases} \qquad \text{[B.18]}$$
$$f_2(x_2) = \begin{cases} 28x_2 & 0 \le x_2 < 100 \\ 29x_2 & 100 \le x_2 < 200 \\ 30x_2 & 200 \le x_2 < 1000 \end{cases}$$

$$h_1(x) = -x_1 + 300 - \frac{x_3x_4}{131.078}\cos(1.48477 - x_6) + \frac{0.90798x_3^{\ 2}}{131.078}\cos(1.47588)$$
$$h_2(x) = -x_2 - \frac{x_3x_4}{131.078}\cos(1.48477 + x_6) + \frac{0.90798x_4^{\ 2}}{131.078}\cos(1.47588)$$
$$h_3(x) = -x_5 - \frac{x_3x_4}{131.078}\sin(1.48477 + x_6) + \frac{0.90798x_4^{\ 2}}{131.078}\sin(1.47588)$$
$$h_4(x) = 200 - \frac{x_3x_4}{131.078}\sin(1.48477 - x_6) + \frac{0.90798x_3^{\ 2}}{131.078}\sin(1.47588)$$

where $0 \le x_1 \le 400$, $0 \le x_2 \le 1000$, $340 \le x_3 \le 420$, $340 \le x_4 \le 420$, $-1000 \le x_5 \le 1000$ and $0 \le x_6 \le 0.5236$. The optimum solution is $f\left(x^*\right) = 8853.53967480648$.

The G18 Function

$$\begin{aligned}
&f\left(x\right) = -0.5(x_1 x_4 - x_2 x_3 + x_3 x_9 - x_5 x_9 + x_5 x_8 - x_6 x_7) \\
&g_1\left(x\right) = x_3^{\,2} + x_4^{\,2} - 1 \le 0 \\
&g_2\left(x\right) = x_9^{\,2} - 1 \le 0 \\
&g_3\left(x\right) = x_5^{\,2} + x_6^{\,2} - 1 \le 0 \\
&g_4\left(x\right) = x_1^{\,2} + (x_2 - x_9)^2 - 1 \le 0 \\
&g_5\left(x\right) = (x_1 - x_5)^2 + (x_2 - x_6)^2 - 1 \le 0 \\
&g_6\left(x\right) = (x_1 - x_7)^2 + (x_2 - x_8)^2 - 1 \le 0 \\
&g_7\left(x\right) = (x_3 - x_5)^2 + (x_4 - x_6)^2 - 1 \le 0 \\
&g_8\left(x\right) = (x_3 - x_7)^2 + (x_4 - x_8)^2 - 1 \le 0 \\
&g_9\left(x\right) = x_7^{\,2} + (x_8 - x_9)^2 - 1 \le 0 \\
&g_{10}\left(x\right) = x_2 x_3 - x_1 x_4 \le 0 \\
&g_{11}\left(x\right) = -x_3 x_9 \le 0 \\
&g_{12}\left(x\right) = x_5 x_9 \le 0 \\
&g_{13}\left(x\right) = x_6 x_7 - x_5 x_8 \le 0
\end{aligned} \qquad [B.19]$$

where $-10 \le x_i \le 10\ (i = 1,...,8)$ and $0 \le x_9 \le 20$. The optimum solution is $f\left(x^*\right) = -0.866025403784439$.

The G19 Function

$$\begin{aligned}
&f\left(x\right) = \sum_{j=1}^{5}\sum_{i=1}^{5} c_{ij} x_{(10+i)} x_{(10+j)} + 2\sum_{j=1}^{5} d_j x_{(10+j)}^3 - \sum_{i=1}^{10} b_i x_i \\
&g_j\left(x\right) = -2\sum_{i=1}^{5} c_{ij} x_{(10+i)} - 3 d_j x_{(10+j)}^2 - e_j + \sum_{i=1}^{10} a_{ij} x_i \le 0 \qquad j = 1,\cdots,5
\end{aligned} \qquad [B.20]$$

where $b = [-40, -2, -0.25, -4, -4, -1, -40, -60, 5.1]$ and the remaining data is given in Table B.1. The bounds are $0 \le x_i \le 10\ (i = 1,\ldots,15)$. The optimum solution is $f\left(x^*\right) = 32.6555929502463$.

j	1	2	3	4	5
e_j	–15	–27	–36	–18	–12
c_{1j}	30	–20	–10	32	–10
c_{2j}	–20	39	–6	–31	32
c_{3j}	–10	–6	10	–6	–10
c_{4j}	32	–31	–6	39	–20
c_{5j}	–10	32	–10	–20	30
d_j	4	8	10	6	2
a_{1j}	–16	2	0	1	0
a_{2j}	0	–2	0	0.4	2
a_{3j}	–3.5	0	2	0	0
a_{4j}	0	–2	0	–4	–1
a_{5j}	0	–9	–2	1	–2.8
a_{6j}	2	0	–4	0	0
a_{7j}	–1	–1	–1	–1	–1
a_{8j}	–1	–2	–3	–2	–1
a_{9j}	1	2	3	4	5
a_{10j}	1	1	1	1	1

Table B.1. *Data set for benchmark function G19*

The G20 Function

$$
\begin{aligned}
& f(x)=\sum_{i=1}^{24} a_i x_i \\
& g_i(x)=\frac{(x_i+x_{(i+12)})}{\sum_{j=1}^{24} x_j+e_i} \leq 0 \quad i=1,2,3 \\
& g_i(x)=\frac{(x_{(i+3)}+x_{(i+15)})}{\sum_{j=1}^{24} x_j+e_i} \leq 0 \quad i=4,5,6 \\
& h_i(x)=\frac{x_{(i+12)}}{b_{(i+12)} \sum_{j=13}^{24} \frac{x_j}{b_j}}-\frac{c_i x_i}{40 b_i \sum_{j=1}^{12} \frac{x_j}{b_j}}=0 \quad i=1, \cdots, 12 \\
& h_{13}(x)=\sum_{i=1}^{24} x_i-1=0 \\
& h_{14}(x)=\sum_{i=1}^{12} \frac{x_i}{d_i}+k \sum_{i=13}^{24} \frac{x_i}{b_i}-1.671=0
\end{aligned}
\tag{B.21}
$$

where $k=(0.7302)(530)(14.7/40)$ and the data set is detailed in Table B.2. The bounds are $0 \leq x_i \leq 10\ (i=1,\ldots,24)$. This solution is a little infeasible and no feasible solution is found so far.

i	a_i	b_i	c_i	d_i	e_i
1	0.0693	44.094	123.7	31.244	0.1
2	0.0577	58.12	31.7	36.12	0.3
3	0.05	58.12	45.7	34.784	0.4
3	0.2	137.4	14.7	92.7	0.3
4	0.26	120.9	84.7	82.7	0.6
5	0.55	170.9	27.7	91.6	0.3
7	0.06	62.501	49.7	56.708	–
8	0.1	84.94	7.1	82.7	–
9	0.12	133.425	2.1	80.8	–
10	0.18	82.507	17.7	64.517	–
11	0.1	46.07	0.85	49.4	–

12	0.09	60.097	0.64	49.1	–
13	0.0693	44.094	–	–	–
14	0.0577	58.12	–	–	–
15	0.05	58.12	–	–	–
16	0.2	137.4	–	–	–
17	0.26	120.9	–	–	–
18	0.55	170.9	–	–	–
19	0.06	62.501	–	–	–
20	0.1	84.94	–	–	–
21	0.12	133.425	–	–	–
22	0.18	82.507	–	–	–
23	0.1	46.07	–	–	–
24	0.09	60.097	–	–	–

Table B.2. *Data set for benchmark function G20*

The G21 Function

$$\begin{aligned} & f(x) = x_1 \\ & g_1(x) = -x_1 + 35x_2^{0.6} + 35x_3^{0.6} \leq 0 \\ & h_1(x) = -300x_3 + 7500x_5 - 7500x_6 - 25x_4x_5 + 25x_4x_6 + x_3x_4 = 0 \\ & h_2(x) = 100x_2 + 155.365x_4 + 2500x_7 - x_2x_4 - 25x_4x_7 - 15536.5 = 0 \\ & h_3(x) = -x_5 + \ln(-x_4 + 900) = 0 \\ & h_4(x) = -x_6 + \ln(x_4 + 300) = 0 \\ & h_5(x) = -x_7 + \ln(-2x_4 + 700) = 0 \end{aligned} \qquad [B.22]$$

where $0 \leq x_1 \leq 1000$, $0 \leq x_2, x_3 \leq 40$, $100 \leq x_4 \leq 300$, $6.3 \leq x_5 \leq 6.7, 5.9 \leq x_6 \leq 6.4$, and $4.5 \leq x_7 \leq 6.25$. The optimum solution is $f\left(x^*\right) = 193.724510070035$.

The G22 Function

$$
\begin{aligned}
& f(x) = x_1 \\
& g_1(x) = -x_1 + x_2^{0.6} + x_3^{0.6} + x_4^{0.6} \le 0 \\
& h_1(x) = x_5 - 100000x_8 + 1\times 10^7 = 0 \\
& h_2(x) = x_6 + 100000x_8 - 100000x_9 = 0 \\
& h_3(x) = x_7 + 100000x_9 - 5\times 10^7 = 0 \\
& h_4(x) = x_5 + 100000x_{10} - 3.3\times 10^7 = 0 \\
& h_5(x) = x_6 + 100000x_{11} - 4.4\times 10^7 = 0 \\
& h_6(x) = x_7 + 100000x_{12} - 6.6\times 10^7 = 0 \\
& h_7(x) = x_5 - 120x_2x_{13} = 0 \\
& h_8(x) = x_6 - 80x_3x_{14} = 0 \\
& h_9(x) = x_7 - 40x_4x_{15} = 0 \\
& h_{10}(x) = x_8 - x_{11} + x_{16} = 0 \\
& h_{11}(x) = x_9 - x_{12} + x_{17} = 0 \\
& h_{12}(x) = -x_{18} + \ln(x_{10} - 100) = 0 \\
& h_{13}(x) = -x_{19} + \ln(-x_8 + 300) = 0 \\
& h_{14}(x) = -x_{20} + \ln(x_{16}) = 0 \\
& h_{15}(x) = -x_{21} + \ln(-x_9 + 400) = 0 \\
& h_{16}(x) = -x_{22} + \ln(x_{17}) = 0 \\
& h_{17}(x) = -x_8 - x_{10} + x_{13}x_{18} - x_{13}x_{19} + 400 = 0 \\
& h_{18}(x) = x_8 - x_9 - x_{11} + x_{14}x_{20} - x_{14}x_{21} + 400 = 0 \\
& h_{19}(x) = x_9 - x_{12} - 4.60517x_{15} + x_{15}x_{22} + 100 = 0
\end{aligned}
\qquad \text{[B.23]}
$$

where $0 \le x_1 \le 20000$, $0 \le x_2, x_3, x_4 \le 1\times 10^6$, $0 \le x_5, x_6, x_7 \le 4\times 10^7$, $100 \le x_8 \le 290.99$, $100 \le x_9 \le 399.99$, $100.01 \le x_{10} \le 300$, $100 \le x_{11} \le 400$, $100 \le x_{12} \le 600$, $0 \le x_{13}, x_{14}, x_{15} \le 500$, $0.01 \le x_{16} \le 300$, $0.01 \le x_{17} \le 400$ and $-4.7 \le x_{18}, x_{19}, x_{20}, x_{21}, x_{22} \le 6.25$. The optimum solution is $f(x^*) = 236.430975504001$.

The G23 Function

$$
\begin{aligned}
& f(x) = -9x_5 - 15x_8 + 6x_1 + 16x_2 + 10(x_6 + x_7) \\
& g_1(x) = x_9x_3 + 0.02x_6 - 0.025x_5 \le 0 \\
& g_2(x) = x_9x_4 + 0.02x_7 - 0.015x_8 \le 0 \\
& h_1(x) = x_1 + x_2 - x_3 - x_4 = 0 \\
& h_2(x) = 0.03x_1 + 0.01x_2 - x_9(x_3 + x_4) = 0 \\
& h_3(x) = x_3 + x_6 - x_5 = 0 \\
& h_4(x) = x_4 + x_7 - x_8 = 0
\end{aligned} \tag{B.24}
$$

where $0 \le x_1, x_2, x_6 \le 300$, $0 \le x_3, x_5, x_7 \le 100$, $0 \le x_4, x_8 \le 200$ and $0.01 \le x_9 \le 0.03$. The optimum solution is $f(x^*) = -400.055099999999584$.

The G24 Function

$$
\begin{aligned}
& f(x) = -x_1 - x_2 \\
& g_1(x) = -2x_1^4 + 8x_1^3 - 8x_1^2 + x_2 - 2 \le 0 \\
& g_2(x) = -4x_1^4 + 32x_1^3 - 88x_1^2 + 96x_1 + x_2 - 36 \le 0
\end{aligned} \tag{B.25}
$$

where $0 \le x_1 \le 3$ and $0 \le x_2 \le 4$. The optimum solution is $f(x^*) =$ -5.50801327159536.

The C01 Function

$$
\begin{aligned}
& f(x) = -\left| \frac{\sum_{i=1}^{n} \cos^4(z_i) - 2\prod_{i=1}^{n} \cos^2(z_i)}{\sqrt{\sum_{i=1}^{n} iz^2}} \right| \\
& g_1(x) = 0.75 - \prod_{i=1}^{n} z_i \le 0 \\
& g_2(x) = \sum_{i=1}^{n} z_i - 0.75D \le 0 \\
& \quad x_i \in [0, 10]
\end{aligned} \tag{B.26}
$$

where $z_i = x_i - o_i$ for $i \in [1, n]$. In this function and the functions below, we use o_i to refer to a random offset and M to refer to a random rotation matrix.

The C02 Function

$$
\begin{aligned}
f(x) &= \max_i (z_i) \\
g_1(x) &= 10 - \frac{1}{n}\sum_{i=1}^{n}[z_i^{\,2} - 10\cos(2\pi z_i) + 10] \le 0 \\
g_2(x) &= \frac{1}{n}\sum_{i=1}^{n}[z_i^{\,2} - 10\cos(2\pi z_i) + 10] - 15 \le 0 \\
h(x) &= \frac{1}{n}\sum_{i=1}^{n}[y_i^{\,2} - 10\cos(2\pi y_i) + 10] - 20 = 0 \\
x_i &\in [-5.12, 5.12]
\end{aligned}
\qquad [B.27]
$$

where $z_i = x_i - o_i$ and $y_i = z_i - 0.5$ for $i \in [1, n]$.

The C03 Function

$$
\begin{aligned}
f(x) &= \sum_{i=1}^{n-1}\left[100(z_i^{\,2} - z_{i+1})^2 + (z_i^{\,2} - 1)^2\right] \\
h(x) &= \sum_{i=1}^{n-1}(z_i - z_{i+1})^2 = 0 \\
x_i &\in [-1000, 1000]
\end{aligned}
\qquad [B.28]
$$

where $z_i = x_i - o_i$ for $i \in [1, n]$.

The C04 Function

$$
\begin{aligned}
f(x) &= \max_i (z_i) \\
h_1(x) &= \frac{1}{n}\sum_{i=1}^{n}(z_i \cos(\sqrt{|z_i|}) = 0 \\
h_2(x) &= \sum_{i=1}^{n/2-1}(z_i - z_{i+1})^2 = 0 \\
h_3(x) &= \sum_{i=n/2+1}^{n}(z_i^{\,2} - z_{i+1})^2 = 0 \\
h_4(x) &= \sum_{i=1}^{n} z_i = 0 \\
x_i &\in [-50, 50]
\end{aligned}
\qquad [B.29]
$$

where $z_i = x_i - o_i$ for $i \in [1, n]$.

The C05 Function

$$
\begin{aligned}
f(x) &= \max_i (z_i) \\
h_1(x) &= \frac{1}{n}\sum_{i=1}^{n}(-z_i \sin(\sqrt{|z_i|}) = 0 \\
h_2(x) &= \frac{1}{n}\sum_{i=1}^{n}(-z_i \cos(\sqrt{|z_i|}) = 0 \\
x_i &\in [-600, 600]
\end{aligned}
\quad [B.30]
$$

where $z_i = x_i - o_i$ for $i \in [1, n]$.

The C06 Function

$$
\begin{aligned}
f(x) &= \max_i (z_i) \\
y_i &= (z_i + 483.6106156535)M - 483.6106156535 \\
h_1(x) &= \frac{1}{n}\sum_{i=1}^{n}(-y_i \sin(\sqrt{|y_i|}) = 0 \\
h_2(x) &= \frac{1}{n}\sum_{i=1}^{n}(-y_i \cos(0.5\sqrt{|y_i|}) = 0 \\
x_i &\in [-600, 600]
\end{aligned}
\quad [B.31]
$$

where $z_i = x_i - o_i$ for $i \in [1, n]$.

The C07 Function

$$
\begin{aligned}
f(x) &= \sum_{i=1}^{n-1}\left[100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2\right] \\
g(x) &= 0.5 - \exp(-0.1\sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^2}) \\
&\quad - 3\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(0.1 y_i) + \exp(1) \le 0 \\
x_i &\in [-140, 140]
\end{aligned}
\quad [B.32]
$$

where $y_i = x_i - o_i$ and $z_i = x_i + 1 - o_i$ for $i \in [1, n]$.

The C08 Function

$$f(x)=\sum_{i=1}^{n-1}\left[100(z_i^{\,2}-z_{i+1})^2+(z_i-1)^2\right]$$
$$g(x)=0.5-\exp(-0.1\sqrt{\frac{1}{n}\sum_{i=1}^{n}y_i^{\,2}})$$
$$-3\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(0.1y_i)+\exp(1)\le 0 \qquad [B.33]$$
$$x_i\in[-140,140]$$

where $y_i=(x_i-o_i)M$ and $z_i=x_i+1-o_i$ for $i\in[1,n]$.

The C09 Function

$$f(x)=\sum_{i=1}^{n-1}\left[100(z_i^{\,2}-z_{i+1})^2+(z_i-1)^2\right]$$
$$h(x)=\sum_{i=1}^{n}(y\sin\sqrt{|y_i|})=0 \qquad [B.34]$$
$$x_i\in[-500,500]$$

where $y_i=x_i-o_i$ and $z_i=x_i+1-o_i$ for $i\in[1,n]$.

The C10 Function

$$f(x)=\sum_{i=1}^{n-1}\left[100(z_i^{\,2}-z_{i+1})^2+(z_i-1)^2\right]$$
$$h(x)=\sum_{i=1}^{n}(y_i\sin\sqrt{|y_i|})=0 \qquad [B.35]$$
$$x_i\in[-500,500]$$

where $y_i=(x_i-o_i)M$ and $z_i=x_i+1-o_i$ for $i\in[1,n]$.

The C11 Function

$$f(x) = \frac{1}{n}\sum_{i=1}^{n}\left[-z_i \cos\left(2\sqrt{|z_i|}\right)\right]$$
$$h(x) = \sum_{i}^{n-1}(100(y_i - y_{i+1})^2 + (y_i - 1)^2) = 0 \qquad [B.36]$$
$$x_i \in [-100, 100]$$

where $y_i = (x_i - o_i)M$ and $z_i = x_i + 1 - o_i$ for $i \in [1, n]$.

The C12 Function

$$f(x) = \sum_{i=1}^{n} z_i \sin\sqrt{|z_i|}$$
$$h(x) = \sum_{i=1}^{n}(z_i^2 - z_{i+1})^2 = 0 \qquad [B.37]$$
$$g(x) = \sum_{i=1}^{n}(z_i - 100\cos(0.1 z_i) + 10) \le 0$$
$$x_i \in [-1000, 1000]$$

where $z_i = x_i - o_i$ for $i \in [1, n]$.

The C13 Function

$$f(x) = \frac{1}{n}\sum_{i=1}^{n}\left[-z_i \sin\sqrt{|z_i|}\right] \qquad [B.38]$$
$$g_1(x) = -50 + \frac{1}{100n}\sum_{i}^{n} z_i^2 \le 0$$

$$g_2(x) = \frac{50}{n}\sum_{i=1}^{n}\sin(\frac{1}{50}\pi z_i) \le 0$$

$$g_3(x) = 75 - 50\left[\sum_{i=1}^{n}\frac{z_i^{\ 2}}{4000} - \prod_{i=1}^{n}\cos(\frac{z_i}{\sqrt{i}}) + 1\right] \le 0$$

$$x_i \in [-500, 500]$$

where $z_i = x_i - o_i$ for $i \in [1, n]$.

The C14 Function

$$f(x) = \sum_{i=1}^{n-1}\left[100(z_i^{\ 2} - z_{i+1})^2 + (z_i - 1)^2\right]$$

$$g_1(x) = \sum_{i=1}^{n}(-y_i \cos\sqrt{|y_i|}) - n \le 0$$

$$g_2(x) = \sum_{i=1}^{n}(y_i \cos\sqrt{|y_i|}) - n \le 0 \qquad \text{[B.39]}$$

$$g_3(x) = \sum_{i=1}^{n}(y_i \sin\sqrt{|y_i|}) - 10n \le 0$$

$$x_i \in [-1000, 1000]$$

where $y_i = x_i - o_i$ and $z_i = x_i + 1 - o_i$ for $i \in [1, n]$.

The C15 Function

$$f(x) = \sum_{i=1}^{n-1}\left[100(z_i^{\ 2} - z_{i+1})^2 + (z_i - 1)^2\right]$$

$$g_1(x) = \sum_{i=1}^{n}(-y_i \cos\sqrt{|y_i|}) - n \le 0$$

$$g_2(x) = \sum_{i=1}^{n}(y_i \cos\sqrt{|y_i|}) - n \le 0 \qquad \text{[B.40]}$$

$$g_3(x) = \sum_{i=1}^{n}(y_i \sin\sqrt{|y_i|}) - 10n \le 0$$

$$x_i \in [-1000, 1000]$$

where $y_i = (x_i - o_i)M$ and $z_i = x_i + 1 - o_i$ for $i \in [1, n]$.

The C16 Function

$$
\begin{aligned}
& f(x) = \sum_{i=1}^{n} \frac{z_i^{\,2}}{4000} - \prod_{i=1}^{n} \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1 \\
& g_1(x) = \sum_{i=1}^{n} [z_i^{\,2} - 100\cos(\pi z_i) + 10] \le 0 \\
& g_2(x) = \prod_{i=1}^{n} z_i \le 0 \\
& h_1(x) = \sum_{i=1}^{n} (z_i \sin\sqrt{|z_i|}) = 0 \\
& h_2(x) = \sum_{i=1}^{n} (-z_i \sin\sqrt{|z_i|}) = 0 \\
& \quad x_i \in [-10, 10]
\end{aligned}
\qquad [\text{B.41}]
$$

where $z_i = x_i - o_i$ for $i \in [1, n]$.

The C17 Function

$$
\begin{aligned}
& f(x) = \sum_{i=1}^{n-1} (z_i - z_{i+1})^2 \\
& g_1(x) = \prod_{i=1}^{n} z_i \le 0 \\
& g_2(x) = \sum_{i=1}^{n} z_i \le 0 \\
& h(x) = \sum_{i=1}^{n} z_i \sin(4\sqrt{|z_i|}) = 0 \\
& \quad x_i \in [-10, 10]
\end{aligned}
\qquad [\text{B.42}]
$$

where $z_i = x_i - o_i$ for $i \in [1, n]$.

The C18 Function

$$
\begin{aligned}
& f(x) = \sum_{i=1}^{n-1} (z_i - z_{i+1})^2 \\
& g(x) = \frac{1}{n} \sum_{i=1}^{n} (-z_i \sin\sqrt{|z_i|}) \le 0
\end{aligned}
\qquad [\text{B.43}]
$$

$$h(x) = \frac{1}{n}\sum_{i=1}^{n}(z_i \sin\sqrt{|z_i|}) = 0$$
$$x_i \in [-50, 50]$$

where $z_i = x_i - o_i$ for $i \in [1, n]$.