Física Aplicada à Computação – Lic. Engenharia Informática / IPBeja
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Tema: Epidemiologia da Malária

Bibliografia: Danby, J. M. A. (1997), “Computer Modeling: From Sports To Spaceflight….From Order To Chaos”, Willmann-Bell, Richmond USA.

7.9 The Epidemiology of Malaria

A description of this model can be found in the text by N.T.J.Bailey[6]. There are two species, humans and mosquitos, denoted by variables with subscripts 1 and 2. Let

n_i be the total population at any time t,
y_i be the number of infected individuals,
f_i be the proportion of infected individuals who are also infectious,
g_i be the recovery rate,
m_i be the birthrate,
k_i be the death rate, where $i = 1, 2$.

Let the rate at which mosquitos bite people be b_2; then in time dt, y_2 infected mosquitos make $b_2 f_2 y_2 dt$ infectious bites. The numbers of susceptible humans is $n_1 - y_1$, so the proportion of susceptible humans is $(n_1 - y_1)/n_1$. So the number of new infections during the time dt is

$$\frac{b_2 f_2 y_2 (n_1 - y_1) dt}{n_1}.$$

So, taking into account the recovery and death rates,

$$\frac{dy_1}{dt} = \frac{b_2 f_2 y_2 (n_1 - y_1)}{n_1} - (g_1 + k_1) y_1. \tag{7.9.1}$$

Also, in time dt, $n_2 - y_2$ susceptible mosquitos make $b_2(n_2 - y_2)$ bites. The proportion of humans bitten that is infectious is $f_1 y_1 / n_1$. So

$$\frac{dy_2}{dt} = \frac{b_2 f_1 y_1 (n_2 - y_2)}{n_1} - (g_2 + k_2) y_2. \tag{7.9.2}$$

It is assumed that all newborn babies are uninfected, but are immediately susceptible.

For simplification, suppose that the birth and death rates are the same, i.e., $m_i = k_i$. Suppose also that for humans k_1 is much smaller than g_1, while the opposite is true for mosquitos. Let

$$Y_1 = y_1/n_1, \ Y_2 = y_2/n_2, \ n = n_2/n_1, \tag{7.9.3}$$

where n is constant, under our hypotheses. Then

$$\left.\begin{aligned} \frac{dY_1}{dt} &= b_2 f_2 n Y_2 (1 - Y_1) - (g_1 + k_1) Y_1, \\ \frac{dY_2}{dt} &= b_2 f_1 Y_1 (1 - Y_2) - (g_2 + k_2) Y_2. \end{aligned}\right] \tag{7.9.4}$$

The simplified equations are

$$\left.\begin{aligned} \frac{dY_1}{dt} &= b_2 f_2 Y_2 n (1 - Y_1) - g_1 Y_1, \\ \frac{dY_2}{dt} &= b_2 f_1 Y_1 (1 - Y_2) - m_2 Y_2. \end{aligned}\right] \tag{7.9.5}$$

Confirm that there are two equilibria. (Set the derivatives to zero; divide each equation by the product $Y_1 Y_2$, then solve the linear equations for $1/Y_1$ and $1/Y_2$.) The one at the origin will not be called "trivial" for this project, because it is just the equilibrium that we would like to see occur. A theoretical result is that the equilibrium at the origin is stable provided that

$$P = \frac{n b_2^2 f_1 f_2}{g_1 m_2} < 1. \tag{7.9.6}$$

Can you confirm this numerically?

For your project, start by choosing simple values for the parameters so that the origin is stable. Can you find the other equilibrium? You may find it helpful to plot solutions in the $Y_1 - Y_2$ plane. Now increase b_2 (or another parameter, such as n) so that P becomes larger than one; the change should be made gradually, with the case $P = 1$ included. What happens when the origin becomes unstable? Investigate the *hypothesis* that the solutions starting close to the origin move toward the other equilibrium, which is stable.

The simplifying assumptions just made can be dispensed with at the cost of two further equations. Suppose the populations n_1 and n_2 vary logistically. Then we can write

$$\left.\begin{aligned} \frac{dn_1}{dt} &= m_1 n_1 (P_1 - n_1) - k_1 y_1, \\ \frac{dn_2}{dt} &= m_2 n_2 (P_2 - n_2) - k_2 y_2, \end{aligned}\right] \tag{7.9.7}$$

where deaths due to malaria have been included. P_1 and P_2 are the maximum sustainable populations of humans and mosquitos. If we let $y_3 = n_1$ and $y_4 = n_2$, the complete system of equations can be written

$$\left.\begin{aligned} \frac{dy_1}{dt} &= \frac{b_2 f_2 y_2 (y_3 - y_1)}{y_3} - (g_1 + k_1) y_1, \\ \frac{dy_2}{dt} &= \frac{b_2 f_1 y_1 (y_4 - y_2)}{y_3} - (g_2 + k_2) y_2, \\ \frac{dy_3}{dt} &= m_1 y_3 (P_1 - y_3) - k_1 y_1, \\ \frac{dy_4}{dt} &= m_2 y_4 (P_2 - y_4) - k_2 y_2. \end{aligned}\right] \tag{7.9.8}$$

Finally, experiment with periodic variation of infectiousness by allowing b_2 to vary periodically.

7.10 The Spread of Gonorrhea

An excellent discussion of this model and some of its variations is given in the text by Braun [17]. There you will find, with more detail than you may wish, a discussion of the parameters and their dependence on the personal preferences of the groups involved. Gonorrhea has a short incubation period of 3 to 7 days, which will be neglected in this model. The disease does not confer immunity: immediately after recovery, an individual is as susceptible as ever — assuming of course that he or she hasn't learned any better.

The two species in this model are males and females. Suppose that there are in a community, at time t,

$m(t)$ active and promiscuous males, of which $x(t)$ are infected, and
$f(t)$ active and promiscuous females, of which $y(t)$ are infected.

We assume that male infectives are cured at the rate $a_1 x$, and females, at the rate $a_2 y$. a_1 is larger than a_2 since the male symptoms develop rapidly and are painful; females can be asymptomatic and so are infectious for much longer periods of time. We assume that the rate of increase of x is proportional to the product of y, the number of infected females, and $(m - x)$, the number of uninfected males, with a similar property for y. Then we have the following model (which includes only heterosexual contacts):

$$\left.\begin{aligned} \frac{dx}{dt} &= -a_1 x + b_1 y (m - x), \\ \frac{dy}{dt} &= -a_2 y + b_2 x (f - y). \end{aligned}\right] \tag{7.10.1}$$