This is a test!

$$X - \mu = LF + \epsilon$$

- Covariance ($Cov$) and correlation ($corr$) are related parameters that indicate the extent to which two random variables **co-vary**
- Correlation is the covariance of standardized variables.

$$Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

$$corr(X, Y) = \frac{Cov(X, Y)}{sd(X)sd(Y)}$$

$$\begin{aligned}
X_1 &= l_{11}F_1 + l_{12}F_2 + \cdots + l_{1m}F_m + \epsilon_1 \\
X_2 &= l_{21}F_1 + l_{22}F_2 + \cdots + l_{2m}F_m + \epsilon_2 \\
&\vdots \\
X_p &= l_{p1}F_1 + l_{p2}F_2 + \cdots + l_{pm}F_m + \epsilon_p
\end{aligned}$$

$$\underset{(p\times 1)}{\boldsymbol{X}} = \underset{(p\times m)}{\boldsymbol{L}} \times \underset{(m\times 1)}{\boldsymbol{F}} + \underset{(p\times 1)}{\boldsymbol{\epsilon}}$$

$\boldsymbol{L}$: loading of factors with $l_{ij}$ represents association between $X_i$ and hidden factor $F_j$.

$\boldsymbol{F}$: $m$ unknown factors with variance 1

$\boldsymbol{\epsilon}$: specific errors, where $\epsilon_i$ represents random error + $X_i$ specific factors.

PCA

$$
\begin{aligned}
C_1 &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1p}x_p \\
C_2 &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2p}x_p \\
&\cdots\cdots \\
C_p &= a_{p1}x_1 + a_{p2}x_2 + \cdots + a_{pp}x_p
\end{aligned}
$$

$$
\begin{aligned}
x_1 &= a_{11}C_1 + a_{21}C_2 + \cdots + a_{k1}C_k + \cdots + a_{p1}C_p \\
x_2 &= a_{12}C_1 + a_{22}C_2 + \cdots + a_{k2}C_k + \cdots + a_{p2}C_p \\
&\cdots\cdots \\
x_p &= a_{1p}C_1 + a_{2p}C_2 + \cdots + a_{kp}C_k + \cdots + a_{pp}C_p
\end{aligned}
$$

$$x_1 = a_{11}C_1 + a_{21}C_2 + \cdots + a_{k1}C_k + \delta_1$$

$$x_2 = a_{12}C_1 + a_{22}C_2 + \cdots + a_{k2}C_k + \delta_2$$

$$\cdots\cdots$$

$$x_p = a_{1p}C_1 + a_{2p}C_2 + \cdots + a_{kp}C_k + \delta_p$$

$$Var(C_1) \geq Var(C_2) \geq \cdots \geq Var(C_k)$$

$$x_1 = a_{11}\sqrt{Var(C_1)}\frac{C_1}{\sqrt{Var(C_1)}} + a_{21}\sqrt{Var(C_2)}\frac{C_2}{\sqrt{Var(C_2)}} + \cdots + a_{k1}\sqrt{Var(C_k)}\frac{C_k}{\sqrt{Var(C_k)}} + \delta_1$$

$$x_1 = l_{11}F_1 + l_{12}F_2 + \cdots + l_{1k}F_k + \delta_1$$

$$x_2 = l_{21}F_1 + l_{22}F_2 + \cdots + l_{2k}F_k + \delta_2$$

$$\cdots\cdots$$

$$x_p = l_{p1}F_1 + l_{p2}F_2 + \cdots + l_{pk}F_k + \delta_p$$

$$F_j = \frac{C_j}{\sqrt{Var(C_j)}}$$

$$Var(F_1) = Var(F_2) = \cdots = Var(F_p) = 1$$

$l_{ij} = a_{ij}\sqrt{Var(C_j)}$

$Var(X_p) = Var(l_{p1}F_1 + l_{p2}F_2 + \cdots + l_{pk}F_k) + Var(\delta_p)$

$L_{ij}$ is the factor laoding of the $X_i$ on $F_j$

Higher $|L_{ij}|$, stronger association between $X_i$ and $F_j$

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