

Exercise : 2.1

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

$$\nabla f(x) = \begin{bmatrix} 200 \cdot (x_2 - x_1^2) \cdot (-2) + 2(1 - x_1) \cdot (-1) \\ 200 \cdot (x_2 - x_1^2) \end{bmatrix}$$

$$= \begin{bmatrix} -400x_2 + 400x_1^2 - 2 + 2x_1 \\ 200x_2 - 200x_1^2 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} 800x_1 + 2 & -400 \\ -400 & 200 \end{bmatrix}$$

when  $x^* = (1, 1)^T$ ,  $x_1 = 1$ ,  $x_2 = 1$

$$\nabla f(x^*) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\nabla^2 f(x^*) = \begin{bmatrix} 802 & -400 \\ -400 & 200 \end{bmatrix}$$

$$\det(\nabla^2 f(x^*)) = 2 > 0, \quad 802 > 0$$

$\therefore$  All determinants  $> 0$ .  $\therefore$  positive definite.

$\therefore x^*$  is local minimum.