Replication of 'A Reconsideration of Money Growth Rules' by Belongia & Ireland (2020)

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Abstract

This project replicates the New Keynesian model composed by Belongia & Ireland (2020), estimated using Bayesian techniques, to reflect the recent post-financial-crisis episode of zero nominal interest rates in the US, and demonstrates the effects of substituting the Federal Reserve's traditional policy of managing interest rates with the alternative of money growth targeting. In addition, the sample is extended to the most recent data available to test the robustness of the findings. For both the benchmark and extended sample, counterfactual simulations illustrate similar findings to Belongia & Ireland (2020); a rule for modifying the money growth rate modestly and gradually in response to deviations in the output gap has performance levels comparable to the calibrated interest rate rule in stabilising inflation and output. Additionally, the impulse responses disclose that, under the same money growth rule, the US's post-2008 crisis economic recovery would have been accelerated due to a significantly shorter period of near-zero nominal interest rates. These findings insinuate that, with a binding zero lower bound constraint, policy rules for money growth can provide a simplistic method for monetary policy to reduce the uneasiness of monetary policy efficacy.

1. Introduction

In the wake of the 2008 Global Financial Crisis (GFC), the Federal Reserve (Fed) implemented the accustomed expansionary monetary policy strategy of interest rate targeting to spur economic growth and curb rising inflation. This strategy involved the Fed lowering the target range for the Federal Funds Rate, the average interest rate banks pay for overnight borrowing in the federal funds market, to ultimately transmit into reduced long-run market rates and increased aggregate demand. However, the efficacy of managing interest rates post-crisis was constrained by the Zero Lower Bound (ZLB). Three waves of unconventional Quantitative Easing were executed as an unorthodox alternative strategy to stabilise output inflation.

To this extent, Belongia & Ireland (2020) retrospectively investigates the efficacy of money growth targeting as the Fed's monetary policy strategy under the restrictive ZLB in the US, specifically

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during the crisis-induced global recession. With a similar *modus operandi*, this project conducts Bayesian estimation to replicate the model by Belongia & Ireland (2020) and performs counterfactual simulation analysis with alternative monetary policy rules, namely an interest rate rule, a (flexible) money supply rule, and a constant money supply rule. In addition, Bayesian estimation techniques are conducted on an extended sample, to give an indication of the robustness of the authors' findings.

For both the benchmark and extended sample the results are similar to that of Belongia & Ireland (2020); impulse response simulations, estimated over the sample data from 1983 up to 2019 (and extended to the end of 2021) on the US economy, show that a (flexible) money growth rule, whereby deviations in the output gap are counteracted with modest and gradual changes to the money growth rate, has efficacy levels comparable to the estimated interest rate rule in stabilising output and inflation. Moreover, under a constant money growth rule, output and inflation illustrate substantially more volatility, consistent with the findings by Ireland (2000), Collard & Dellas (2005), and Gali (2015).

Crucially, the findings of this project support Belongia & Ireland (2020)'s counterfactual simulations, disclosing that the US economy would have recovered faster from the crisis-induced recession under the same money growth rule and a significantly shorter period of ground-level interest rates. Additionally, the simulations presented here show that a money growth rule has satisfactory performance in both the good and the bad times. These findings iterate that the Fed, and other monetary authorities, should consider money growth targeting as a strategy during periods whereby the ZLB diminishes the historical and conventional method of interest rate targeting.

The practice of strictly targeting the Federal Funds rate has been the principal monetary policy strategy since the early 1990s, with economic academia also portraying the Fed's policy as one of interest rate management. At the forefront of understanding interest rate control in theory and practice, Taylor (1993) presented the renowned policy rate rule (the Taylor rule), which accurately describes the Fed's adjustments to the Federal Funds rate target in response to deviations in the output gap and inflation during the period 1987 to 1992. Moreover, variants of Taylor (1993)'s rule are considered the workhorse structural representation of monetary policy in New Keynesian economics[2].

Ireland (2003), Collard & Dellas (2005), Piazzesi, Rogers & Schneider (2019), and Rupert & Šustek (2019) also illustrate the preference for interest rate management is modern New Keynesian modelling, specifically during successive periods of money demand shocks[3]. Under these circumstances, the evidence reflects that excess volatility in output and inflation is generated when monetary policies impose a constant money growth rate rather than implement interest rate targeting. In these stochastic IS-LM model economies, the added output and price volatility are insulated from the impact of

[2]Accredited examples of such variants are given by Woodford (2012), Gali (2015) and Walsh (2010)

[3]These Taylor rule type specifications are modern variants of the classical contribution by Poole (1970), where the variants allow for a dynamic aggregate price level. Therefore, the Taylor rule becomes a natural benchmark for structuring monetary policy in economic models

demand shocks under policies of interest rate management. Additionally, by construction, the widely accepted forward-looking specifications of the New Keynesian Phillips curve imply that the sole policy instrument available to monetary authorities in their pursuit to stabilise output and inflation is the power to affect the current and expected future dynamics of the short-term nominal interest rate. Therefore, understandably, the Fed has held the longstanding stance that the workhorse monetary policy strategy is that of interest rate management.

However, the recent past of interest rates gravitating near-zero has forced monetary authorities globally to find alternative price-and-growth-stabilization tactics. During the periods 2009 to 2015, the Fed's interest rate targeting strategy was constrained by the ZLB, and three rounds of unconventional open market operations in the form of Quantitative Easing were conducted, where interest payments on bank reserves and reverse repurchase agreements for financial institutions are among some other actions taken by the Fed. Therefore, in this pursuit of discovering alternative stabilising strategies that Belongia & Ireland (2020) investigates the targeting of the money stock as a potential monetary policy strategy. As in Belongia & Ireland (2018), this project finds simulated evidence favouring the adoption of money growth management by monetary authorities.

2. The Log-Linearized Model

$$\hat{c}_t = \hat{y}_t \tag{2.1}$$

$$(z-\beta\lambda)(z-\gamma)\hat{\lambda}_t = \gamma z\hat{y}_{t-1} - (z^2+\beta\gamma^2)\hat{y}_t + \beta\gamma z E_t\hat{y}_{t+1} + (z-\beta\gamma\rho_a)(z-\gamma)\hat{a}_t - \gamma z\hat{z}_t \tag{2.2}$$

$$\hat{\lambda}_t = \hat{r}_t + E_t\hat{\lambda}_{t+1} - E_t\hat{\pi}_{t+1} \tag{2.3}$$

$$\gamma z\hat{q}_{t-1} - (z^2+\beta\gamma^2)\hat{q}_t + \beta\gamma z E_t\hat{q}_{t+1} + \beta\gamma(z-\gamma)(1-\rho_a)\hat{a}_t - \gamma z\hat{z}_t = 0 \tag{2.4}$$

$$\hat{x}_t = \hat{y}_t - \hat{q}_t \tag{2.5}$$

$$(1+\beta\alpha)\hat{\pi}_t = \alpha\hat{\pi}_{t-1} + \beta E_t\hat{\pi}_{t+1} - \psi\hat{\lambda}_t + \psi\hat{a}_t + \hat{e}_t \tag{2.6}$$

$$\hat{r}_t = \rho_r\hat{r}_{t-1} + \rho_\pi\hat{\pi}_{t-1} + \rho_x\hat{x}_{t-1} + \varepsilon_{rt} \tag{2.7}$$

$$\delta_r(r-1)\hat{\lambda}_t - \delta_r(r-1)\hat{a}_t - \hat{u}_t + \phi\hat{z}_t = \phi\hat{m}_{t-1} - [\phi(1+\beta)+1]\,\hat{m}_t + \beta\phi\hat{m}_{t+1} - \delta_r\hat{r}_t \tag{2.8}$$

$$\hat{\mu}_t = \hat{z}_t + \hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t \tag{2.9}$$

$$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \tag{2.10}$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{at} \tag{2.11}$$

$$\hat{z}_t = \varepsilon_{zt} \tag{2.12}$$

$$\hat{u}_t = \rho_u\hat{u}_{t-1} + \varepsilon_{ut} \tag{2.13}$$

$$\hat{e}_t = \rho_e\hat{e}_{t-1} + \varepsilon_{et} \tag{2.14}$$

The final model is a fully specified four-equation New Keynesian(NK) DSGE model that consists of an IS relation , a AD relation, a NK Phillips curve, and a monetary policy function. The monetary policy function is adapted under three distinct specifications, namely, a modified Taylor (1993) rule, a zero interest rate rule, and a money growth rule. Excluding the monetary policy function and its specifications, the final model is derived as:

$$\hat{x}_t = E_t\hat{x}_{t+1} - (\hat{r}_t - E_t\hat{\pi}_{t+1}) + (1-\rho_a)\hat{a}_t, \tag{2.15}$$

$$\hat{m}_t = \delta_r(r-1)\hat{y}_t - \delta_r\hat{r}_t + \hat{u}_t, \tag{2.16}$$

$$(1+\beta\alpha)\hat{\pi}_t = \alpha\hat{\pi}_{t-1} + \beta E_t\hat{\pi}_{t+1} - \psi\hat{\lambda}_t + \psi\hat{a}_t + \hat{e}_t. \tag{2.17}$$

Equation (2.15) represents the model's NK IS relationship, formulated by combining equations (2.2) - (2.5), that relates the real interest rate $\hat{r}_t - E_t\hat{\pi}_{t+1}$ to deviations in the output gap $\hat{x}_t$. Here, the special case of zero habit formation in consumption ($\gamma = 0$) is applied to simplify the relation (2.15) is purely-forward looking.

Again, in adopting the special case of zero habit formation in consumption and adjustment costs for real balances in the money demand relationship (2.8), so that $\gamma = 0$ and $\phi_m = 0$, yields the final NK AD relationship (2.16). In this specification, δ_r is the interest semi-elasticity of money demand and u_t represents a shock to money demand. Importantly, for relatively low levels of the steady state nominal interest rate, the coefficient on $\hat{y}_t$ will be small, and therefore the demand for real money balances depends more heavily on Z_t, the permanent component of income, than on the transitory component of income, captured by $\hat{y}_t$.

The NK Phillips curve is given by equation 2.17, and includes a backward-looking component $\alpha\hat{\pi}_{t-1}$ when $\alpha > 0$, such that the price stickiness of individual goods are indexed to past inflation. The following subsections completes the model under the different specifications for the monetary policy function.

2.1. Monetary Policy Specification 1: The Taylor Rule

The first specification for the monetary policy function is the modified Taylor (1993) rule given by equation 2.7, re-copied here:

$$\hat{r}_t = \rho_r\hat{r}_{t-1} + \rho_\pi\hat{\pi}_{t-1} + \rho_x\hat{x}_{t-1} + \varepsilon_{rt}, \tag{2.18}$$

where ρ_r is the interest rate smoothing parameter, with ρ_π and ρ_x representing the severity of monetary policy's response to deviations in inflation and the output gap, respectively.

2.2. Monetary Policy Specification 2: The Flexible Money Growth Rule

As an alternative monetary policy instrument, when monetary authorities are unable to lower the policy rate due to the ZLB, Belongia & Ireland (2020) specifies a money growth rule as:

$$\hat{\mu}_t = \rho_{mm}\hat{\mu}_{t-1} + \rho_{m\pi}\hat{\pi}_{t-1} + \rho_{mx}\hat{x}_{t-1}, \tag{2.19}$$

where ρ_{mm} represents the money growth smoothing parameter, with $\rho_{m\pi}$ and ρ_{mx} the severity of monetary policy's money growth response to deviations in inflation and the output gap ,respectively.In the case that $\rho_{m\pi} < 0$ and $\rho_{mx} < 0$, the money growth rate rule gives the monetary authority the ability to actively stabilize inflation and the output gap in response to exogenous shocks that stuns the economy. In addition, if $\rho_{mm} > 0$, the money growth rule (2.19) dictates a gradual response in money growth following deviations in inflation and the output gap in a similar fashion that interest rate smoothing ρ_r does in the Taylor (1993) rule given by (2.18).

2.3. Monetary Policy Specification 3: The Constant Money Growth Rule

In the case that $\rho_{mm} = \rho_{m\pi} = \rho_{mx} = 0$, (2.19) is narrowed to the constant money growth rule advocated for by Friedman (1968), and further studied in Ireland (2000), Collard & Dellas (2005), and Gali (2015). As will be motivated in section 3 below, the flexible money growth rate (2.19) is replaced by the constant money growth rate:

$$\hat{\mu}_t = 1.0136. \tag{2.20}$$

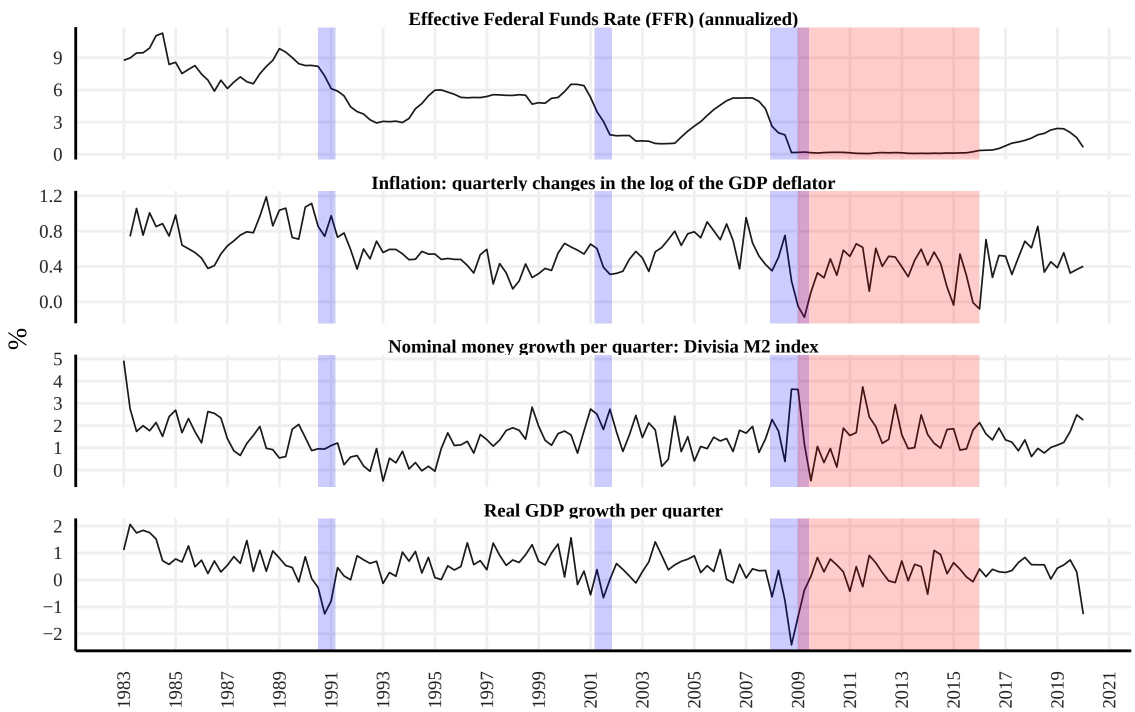
3. Data and Calibration of Steady State

All data used to estimate the model are quarterly, with the benchmark sample running in tandem to Belongia & Ireland (2020) from 1983:1 through 2019:1. Additionally, the extended sample runs from 1983:1 through 2020:4. The estimation method treats four of the model's variables as observable, gathered using US macroeconomic time series data drawn from the Federal Reserve Bank of St. Louis' FRED database, and is depicted in Figure 3.1 below.

The short-term nominal interest rate $\hat{r}_t$ is measured by the effective federal funds rate, scaled by 400 to transform the data to quarterly rates as in the model. Output growth ($\hat{g}_t$) is measured as the quarterly changes in the per capita natural log of real GDP.[4] Inflation $\hat{\pi}_t$ is measured as the quarterly changes in the log of the GDP deflator. Lastly, the quarterly changes in the Divisia M2 index of money, scaled to per capita terms, measures the nominal money growth rate $\hat{\mu}_t$.

During the periods from 2009:1 through 2015:4 the FED held short-term nominal interest rates close to zero, following Belongia & Ireland (2020), the Taylor (1993) specification (2.18) is replaced by the following zero interest rate condition in the estimated model:

$$\hat{r}_t = -\ln(r). \tag{3.1}$$



All data is sourced from the Federal Reserve Bank of St. Louis' FRED database. The blue shaded areas reflect economic recessions as defined by the NBER, whereas the red shaded is the period when the FED was constrained by the ZLB.

Figure 3.1: Observed Time Series Used for Estimation.

The model is estimated in its log-linearized form as stated by equation (2.1)-(2.14) above. Therefore,

[4] Population is measured as ages 16 and over of the non-institutional civilian population.

pre-estimation, the calibration strategy treats the steady state variables of inflation (π), nominal interest rate (r), and output growth ($g = z$) as exogenously observable. These values for both the benchmark and extended sample is given in Table 3.1, and is calculated as the mean values of the respective sample data. However, for the steady state nominal interest rate (r), I apply the strategy used by Ireland (2011), and is calculated as $r = \frac{(z\pi)}{\beta}$. Finally, the adopted discount factor (β) of 0.9987, marginally smaller than assumed by Belongia & Ireland (2020), is as in the models presented in Ireland (2011), Gali (2015), and Rupert & Šustek (2019).

Table 3.1: Steady state values calculated using data.

Parameter		Benchmark sample	Extented sample
Steady state inflation	π	1.00978	1.00997
Steady state nominal interest rate	r	1.00617	1.00658
Steady state output growth	$g = z$	1.00039	1.00042
Discount factor parameter	β	0.9987	0.9987

Importantly, when simulating (for the benchmark sample) and estimating (for the extended sample) the structural model under the flexible money growth rule (2.19), I calibrate the parameters as in Belongia & Ireland (2020). That is, the parameters for money growth smoothing ($\rho_{mm} = 1$), and the severity of the policy's response to deviations in inflation ($\rho_{m\pi} = 0$) and the output gap ($\rho_{mx} = -0.125$). Money growth smoothing (ρ_{mm}) is normalised, combined with the negative policy response output gap deviations (ρ_{mx}), permitting the monetary authority to actively stabilise the output gap in response to exogenous shocks that maim the economy similarly to interest rate smoothing (ρ_r) role in the Taylor (1993) rule (2.19). However, it is assumed that policy doesn't actively respond to deviations in inflation as $\rho_{m\pi} = 0$.

The constant money growth specification for monetary policy (2.20) implies the case where $\rho_{mm} = \rho_{m\pi} = \rho_{mx} = 0$, and the flexible money growth rule (2.19) is transformed into the constant money growth rule advocated for by Friedman (1968). In this specification, the (gross) constant money growth rate is set to equal to 1.0136 based on the estimated findings by Ireland (2011) and justified by the similar implementations by Beck & Wieland (2008) and Gali (2015)[5].

[5]See Castelnuovo (2012) and Tarassow (2019) for an in-depth analysis of the intricacies surrounding estimating constant money growth values for the US.

4. Priors and Bayesian Estimation

The priors for the remaining 16 structural parameters are set to match that of Belongia & Ireland (2020). The priors and posterior estimates under the Taylor (1993) rule (2.18) for both the benchmark and extended samples are given in Table 4.1, and is plotted in more detail in Figures A.1 in Appendix A and B.1 in Appendix B for the benchmark sample and extended sample, respectively. Additionally, under the same priors, the posterior estimates for the extended sample under the flexible money growth rule (2.19) is given in Table 4.2 and is plotted in more detail C.1 in Appendix C.

Table 4.1: Priors and Metropolis-Hastings Estimated Posterior Distributions for Structural Parameters: Taylor rule for both benchmark and extended samples.

		Prior			**Posterior** *Benchmark Sample*		**Posterior** *Extended Sample*	
Parameter		Distribution	Mean	Stdev.	Mean	Stdev.	Mean	Stdev.
Habit formation	γ	Beta	0.500	0.2000	0.638	0.0243	0.9624	0.0073
Price indexation	α	Beta	0.500	0.2000	0.244	0.0770	0.1543	0.1481
Phillips-Curve slope	ψ	Gamma	0.100	0.0300	0.023	0.0044	0.0070	0.0018
Money-demand semi-elasticity	δ_r	Gamma	15.000	5.0000	10.092	1.8756	5.7916	1.853
Money-demand adjustment cost	ϕ	Gamma	10.000	10.0000	41.273	14.3314	9.7462	6.9789
Interest rate smoothing	ρ_r	Beta	0.750	0.1000	0.904	0.0160	0.9497	0.0182
Policy response to inflation	ρ_π	Gamma	0.400	0.1000	0.252	0.0660	0.341	0.0641
Policy response to output gap	ρ_x	Gamma	0.200	0.1000	0.395	0.0460	0.2725	0.0985
Preference shock persistence	ρ_a	Beta	0.750	0.1000	0.936	0.0101	0.4687	0.0093
Money-demand shock persistence	ρ_u	Beta	0.750	0.1000	0.968	0.0088	0.9479	0.0216
Cost-push shock persistence	ρ_e	Beta	0.500	0.1000	0.394	0.0151	0.5788	0.1010
Monetary policy innovation	ε_r	Inverse Gamma	0.003	0.0160	0.005	0.0003	0.003	0.0001
Preference innovation	ε_a	Inverse Gamma	0.013	0.0660	0.196	0.0240	0.0364	0.0049
Productivity innovation	ε_z	Inverse Gamma	0.013	0.0660	0.010	0.0016	0.0062	0.0007
Money demand innovation	ε_u	Inverse Gamma	0.013	0.0660	0.085	0.0185	0.0285	0.0131
Cost-push innovation	ε_e	Inverse Gamma	0.003	0.0160	0.001	0.0001	0.0011	0.0002

Note: Prior distributions for the standard deviations σ_i, $i = a, z, u, e, r$, are those induced by assuming that the associated variance σ_i^2 has the inverse chi-squared distribution with scale parameter 0.012 for $i = a, z, u$ or 0.00252 for $i = e, r$ and 4 degrees of freedom.

Table 4.2: Priors and Metropolis-Hastings Estimated Posterior Distributions for Structural Parameters: Flexible money growth rule for extended sample.

		Prior			Posterior	
Parameter		Distribution	Mean	Stdev.	Mean	Stdev.
Habit formation	γ	Beta	0.500	0.2000	0.0093	0.0060
Price indexation	α	Beta	0.500	0.2000	0.2070	0.1109
Phillips-Curve slope	ψ	Gamma	0.100	0.0300	0.0095	0.0019
Money-demand semi-elasticity	δ_r	Gamma	15.000	5.0000	62.1706	4.3367
Money-demand adjustment cost	ϕ	Gamma	10.000	10.0000	0.1615	0.1753
Preference shock persistence	ρ_a	Beta	0.750	0.1000	0.3902	0.0638
Money-demand shock persistence	ρ_u	Beta	0.750	0.1000	0.9497	0.0127
Cost-push shock persistence	ρ_e	Beta	0.500	0.1000	0.4494	0.0801
Preference innovation	ε_a	Inverse Gamma	0.013	0.0660	0.0931	0.0063
Productivity innovation	ε_z	Inverse Gamma	0.013	0.0660	0.1062	0.0059
Money demand innovation	ε_u	Inverse Gamma	0.013	0.0660	0.1176	0.0090
Cost-push innovation	ε_e	Inverse Gamma	0.003	0.0160	0.0012	0.0002

Note: Prior distributions for the standard deviations σ_i, $i = a, z, u, e$, are those induced by assuming that the associated variance σ_i^2 has the inverse chi-squared distribution with scale parameter 0.012 for $i = a, z, u$ or 0.00252 for $i = e$ and 4 degrees of freedom.

4.1. Blanchard-Khan Stability Conditions

The assumption that macroeconomic models assume perfect foresight has been widely critiqued. However, the assumption has been made possible due to improvement of simulation algorithms. For the model to have a unique solution, the Blanchard-Khan conditions have to be met. These conditions are easy to check, in terms of eigenvalues computed at the steady state of the model. This unique solution for the model is determined if and only if the number of unstable eigenvalues is equal to the number of non-predetermined variables. The Blanchard-Khan condition for the DSGE model of this project is satisfied as there are five eigenvalues larger than one in the modulus for five forward looking variables in the model.

The estimations mode check plots for the benchmark sample Taylor (1993) rule (2.18), the extended sample Taylor (1993) rule (2.19), and flexible money growth rule (2.19) is depicted in Figure A.2 in Appendix A, Figure B.2 in Appendix B, and Figure C.2 in Appendix C, respectively. The differences in the shape between the likelihood kernel and the posterior likelihood indicate the role of the prior in influencing the curvature of the likelihood function. Ideally, the estimated mode should be at the maximum of the posterior likelihood. These plots further confirms that the Blanchard-Khan conditions have been met for all three estimations.

4.2. Monte Carlo Markov Chain (MCMC) Convergence Diagnostics

For all three estimation specifications the number of Metropolis-Hastings Chains is set to two, corresponding to 25 000 draws, with the respective acceptance ratio's given in Table 4.3 below. Since the DSGE model specified has more than five parameters, we would expect to see an acceptance ratio of around 23 percent according to Bedard (2008). This is clearly satisfied for all three estimations.

In addition, the estimated Brooks & Gelman (1998) MCMC multivariate and univariate convergence diagnostic plots for the benchmark sample Taylor (1993) rule (2.18), the extended sample Taylor (1993) rule (2.19), and flexible money growth rule (2.19) is depicted in Figures A.3 and B.4 in Appendix A, Figures B.3 and B.4 in Appendix B, and Figures C.3 and C.4 in Appendix C, respectively. For each estimation procedure, the chains for the multivariate and univariate convergence diagnostic plots converges and stabilises horizontally, and are clearly satisfactory.

Table 4.3: MCMC Acceptance Ratios.

	Benchmark sample	**Extended sample**	
	Taylor rule (2.18)	Taylor rule (2.18)	Flexible money growth rule (2.19)
Chain 1	24.012 %	23.912 %	26.748 %
Chain 2	23.774 %	21.048 %	26.896 %

5. Analysis of Results- Impulse Responses

The benchmark sample impulse response function comparissons for the estimated Taylor (1993) rule (2.18), and the calibrated-to-match these structural parameter estimates flexible money growth rule (2.20) and constant money growth rule (2.20) following a one-standard-deviation preference, productivity, money demand, and cost-push shock are illustrated by Figure 5.1. With some minor differences, the results are strikingly similar to that reported by Belongia & Ireland (2020).

Importantly, the extended sample impulse response functions are provided in Figure B.5 in Appendix B, however, yields similar dynamics in response to the above-mentioned shocks than the benchmark sample. Therefore, I only analyse the impulse response in Figure 5.1 below.

5.1. Comparisson to flexible money growth rule (2.19)

The benchmark sample impulse response function comparissons for the estimated Taylor (1993) rule (2.18), and the calibrated-to-match these structural parameter estimates flexible money growth rule

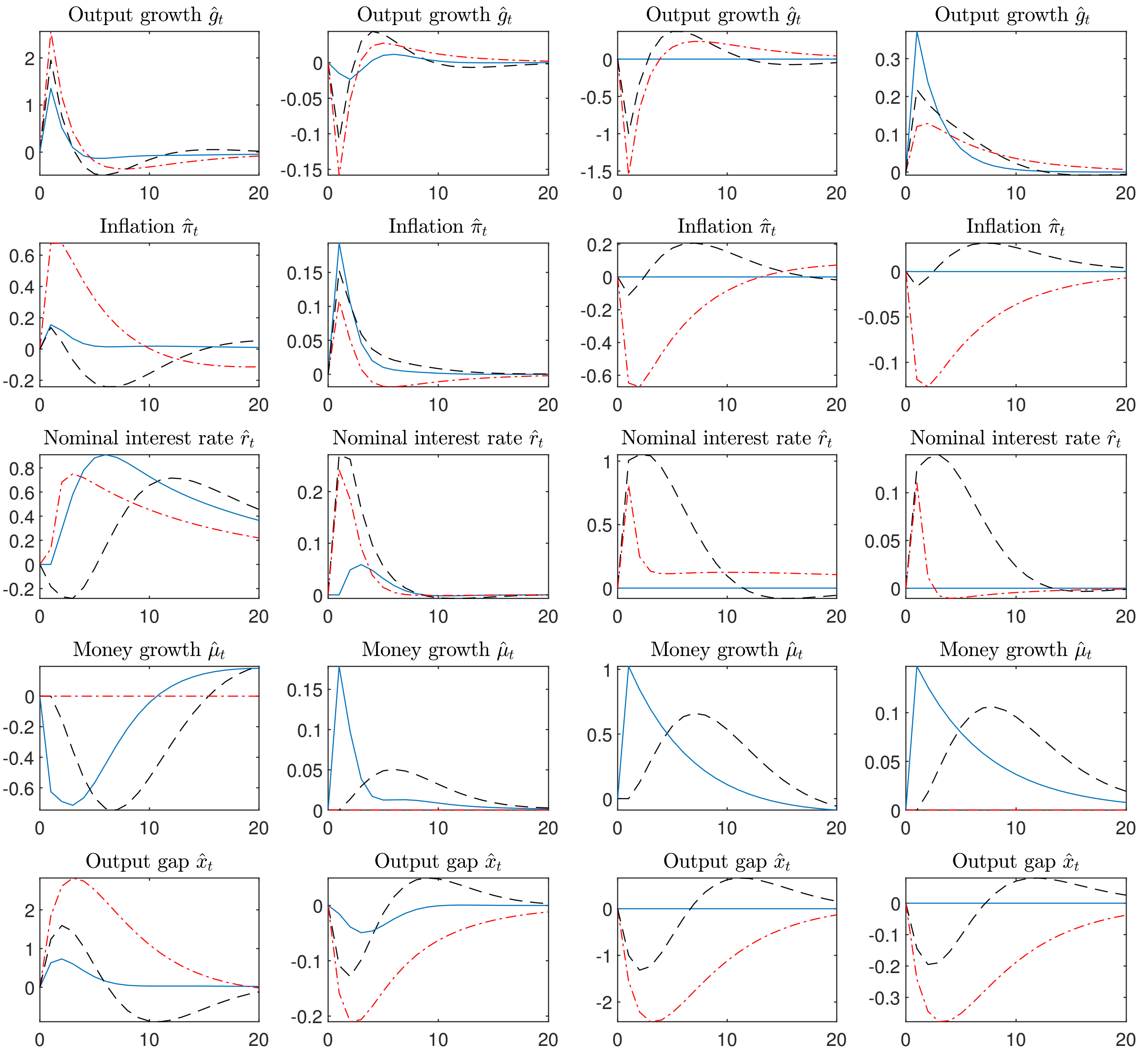
(2.20) and constant money growth rule (2.20) following a one-standard-deviation preference, productivity, money demand, and cost-push shock are illustrated by Figure 5.1. With some minor differences, the results are strikingly similar to that reported by Belongia & Ireland (2020).

In response to an expansionary preference shock depicted in column (a) of Figure 5.1, both the Taylor rule and the flexible money growth rule results in monetary tightening through increased interest rates and decreased money supply growth. Under both rules, the monetary tightening effectively eradicates the inflationary effects induced by the preference shock and assists in stabilising output growth and the output gap.

The impulse response functions illustrated in column (d) of Figure 5.1 are consistent with the findings by Ireland (2003); following a productivity shock under sticky prices, the Taylor rule specification for monetary policy necessitates an increase in money growth to instigate the systematic increase in output growth while holding the output gap unvaried. Similarly, in response to such an accommodating productivity shock, the flexible money growth rule dictates a monetary expansion that grants a more efficient output recovery, minimising the severity of inflation's response.

Moreover, column (c) of Figure 5.1 shows that by maintaining a fixed short-term nominal rate, the Taylor rule cushions output growth, inflation, and the output gap by wholly accommodating a money demand shock[6]. However, this ideal is not present under the flexible money growth rule. The nominal interest rate instantly jumps to a higher level as the money demand shock hits the model economy. Nevertheless, the flexible money growth rule still encompasses a persistent increase in money growth, which accommodates the increase in the demand for money balances. Finally, the impulse responses following a cost-push shock depicted in (b) of Figure 5.1 show that the flexible money growth rule has near-identical dynamics to the Taylor rule.

[6]This result corroborates the conclusions in the classic Keynesian analysis by Poole (1970).



(a) **PREFERENCE SHOCK** (b) **COST PUSH SHOCK** (c) **MONEY DEMAND SHOCK** (d) **PRODUCTIVITY SHOCK**

Figure 5.1: Impulse responses to the indicated shock under the benchmark sample. Each column shows the percentage-point response of the indicated variable to a one-standard-deviation σ_i,for $i = a_t, z_t, u_t, e_t$, under the estimated Taylor rule (2.18), the flexible money growth rule (2.19),and the constant money growth rule (2.20).

5.2. Comparisson to constant money growth rule (2.20)

The constant money growth rule (2.20) represents the case where $\rho_{mm} = \rho_{m\pi} = \rho_{mx} = 0$ is applied to the flexible money growth rule (2.20). Additionally, as motivated in section 3, the (gross) constant money growth rate $\hat{\mu}_t$ is set equal to 1.0136 as in Ireland (2011). The estimated prior and posterior distributions under the constant money growth rule by Ireland (2000), Collard & Dellas (2005), Gali (2015), and Belongia & Ireland (2020) indicate that macroeconomic volatility would have been severely escalated if the Fed had followed a constant money growth rule as specified here. Specifically, Belongia & Ireland (2020) finds that median estimates of the standard deviations of output growth and inflation are more than 50 per cent larger under this constant money growth rule than under the conventional Taylor rule. The impulse response functions under this constant money growth rule, given in Figure 5.1, supports these authors' findings, and the analysis is discussed below.

In general, the efficacy of the constant money growth rule, simulated in Figure 5.1, performs rather poorly, especially relative to both the estimated Taylor and flexible money growth rules. The constant money growth rule fails to exhibit the monetary tightening induced by both the Taylor and flexible money growth rules. Therefore, output growth, inflation, and the output gap under constant money growth illustrate substantially more volatility in response to a preference shock column (a) of Figure 5.1.

Similarly, column (d) of Figure 5.1 shows that in response to an accommodative productivity shock, the constant money growth rule is absent of the gradual increase in money growth that facilitates an efficient recovery in the economy evident in both the Taylor rule and the flexible money growth rule. As a result, even though the output gap and inflation are more volatile, output growth notably exhibits more stability in the case of a constant money growth rule in response to a productivity shock.

In stark contrast to the Taylor and flexible money growth rule, column (c) of Figure 5.1 shows that the constant money growth rule permits money demand shocks to exacerbate macroeconomic volatility.

Lastly, some positives can be drawn from column (b) of Figure 5.1, illustrating that the constant money growth rule performs moderately better than the Taylor and flexible money growth rules in stabilising inflation in response to a cost-push shock. However, this inflation stability comes at the cost of permitting substantially more significant output growth and output gap variability.

The literature consensus indicates that QE lowers longer-term interest rates through a central bank balance sheet re-balancing channel on the term premium, which in turn generates expansionary effects on output and inflation (Borio & Zabai, 2018; and Carlson, D'Amico, Fuentes-Albero, Schlusche & Wood, 2020). Therefore, assuming some combination of a flexible or constant money growth rule and a rule for central bank open market operations is undoubtedly attractive for future studies; however, it is outside the scope of this project [7].

[7] See Gertler & Karadi (2011), Gertler & Karadi (2018), and Bernanke (2020) for possible QE rules that can be combined with money growth rules for monetary policy.

6. Analysis of Results- Estimated Smoothed Variables

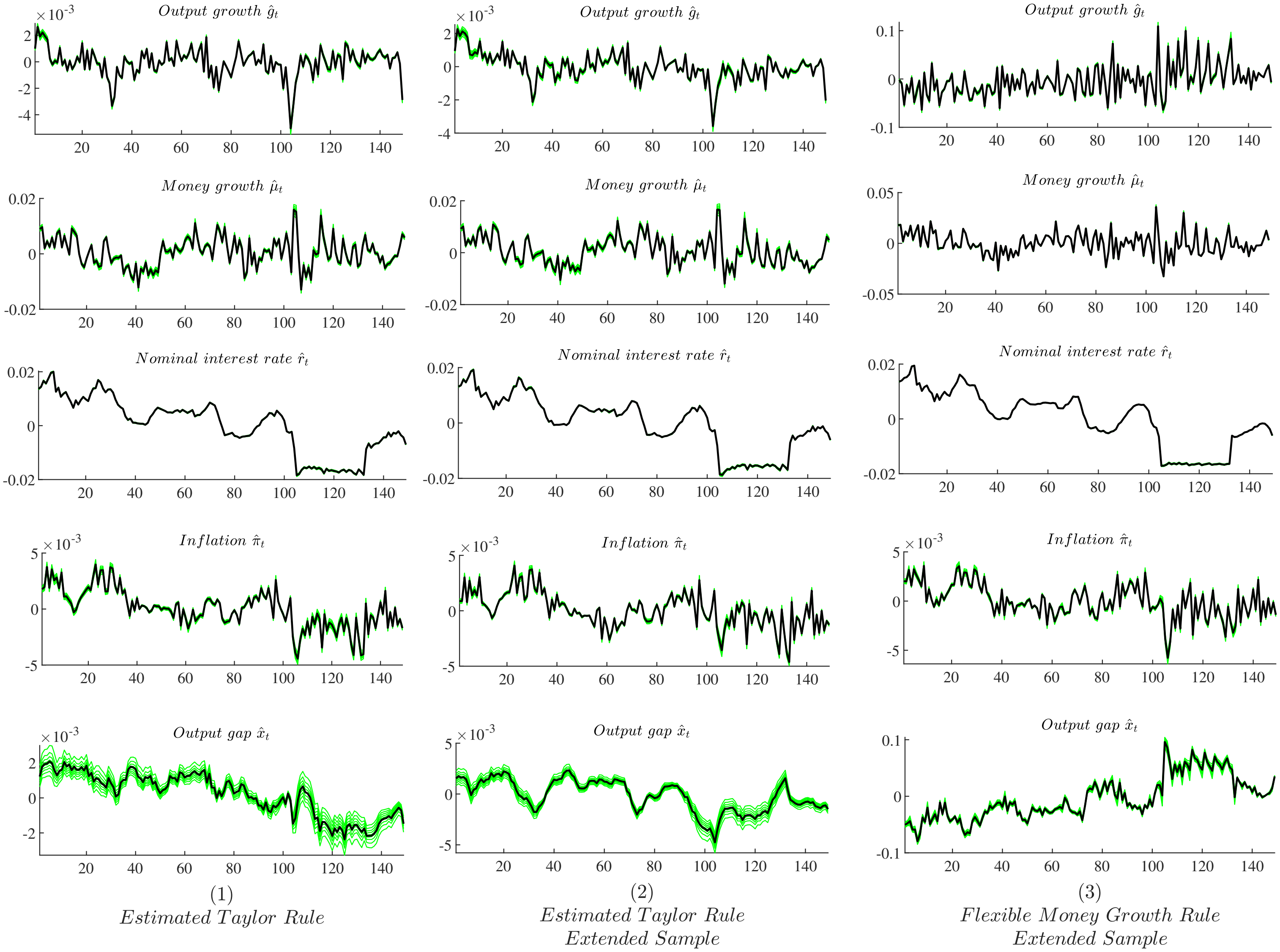


Figure 6.1: Smoothed variables – The black line depicts the estimate of the smoothed variable ("best guess for the observed variable given all observations"), derived from the Kalman smoother at the posterior mean (Bayesian estimation). The green lines represent the deciles of the one step ahead forecast distribution.

The Bayesian estimates of the smoothed variables ("best guess for the observed variable given all observations"), derived from the Kalman smoother at the posterior mean are shown in Figure 6.1 for the benchmark sample Taylor rule, and the extended sample Taylor rule and flexible money growth rule in columns (1), (2), and (3), respectively. Once the sample has been extended, the Taylor rule performs similar than under the benchmark sample, with the exception of exhibiting significantly less volatility in the output gap.

However, the main findings of Belongia & Ireland (2020) are still apparent when the sample is extended; The flexible money growth rule could have caused the US economy to regain its strength quicker following the the 2008-9 recession, with a significantly shorter period of extraordinarily low interest rates.

7. Conclusion

This project replicates model by Belongia & Ireland (2020) using Bayesian estimation techniques and performs simulation analysis with alternative monetary policy rules, namely an estimated interest rate rule, a (flexible) money supply rule, and a constant money supply rule. In addition, the sample is extended, to verify the robustness of the authors' findings.

The estimated results are similar to that of Belongia & Ireland (2020); impulse response simulations, estimated using sample data from 1983:1 up to 2019:1 on the US economy, show that a (flexible) money growth rule, whereby deviations in the output gap are counteracted with modest and gradual changes to the money growth rate, has efficacy levels comparable to the estimated interest rate rule in stabilising output and inflation. Moreover, under a constant money growth rule, output and inflation illustrate substantially more volatility, consistent with the findings by Ireland (2000), Collard & Dellas (2005), and Gali (2015).

Crucially, the findings of this project support Belongia & Ireland (2020)'s counterfactual simulations, disclosing that the US economy would have recovered faster from the crisis-induced recession under the same money growth rule and a significantly shorter period of ground-level interest rates. Additionally, the simulations presented here show that a money growth rule has satisfactory performance in both the good and the bad times. These findings iterate that the Fed, and other monetary authorities, should consider money growth targeting as a strategy during periods whereby the ZLB diminishes the historical and conventional method of interest rate targeting.

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Appendix A. Bayesian Esimation Results- Taylor Rule (2.18), Benchmark Sample

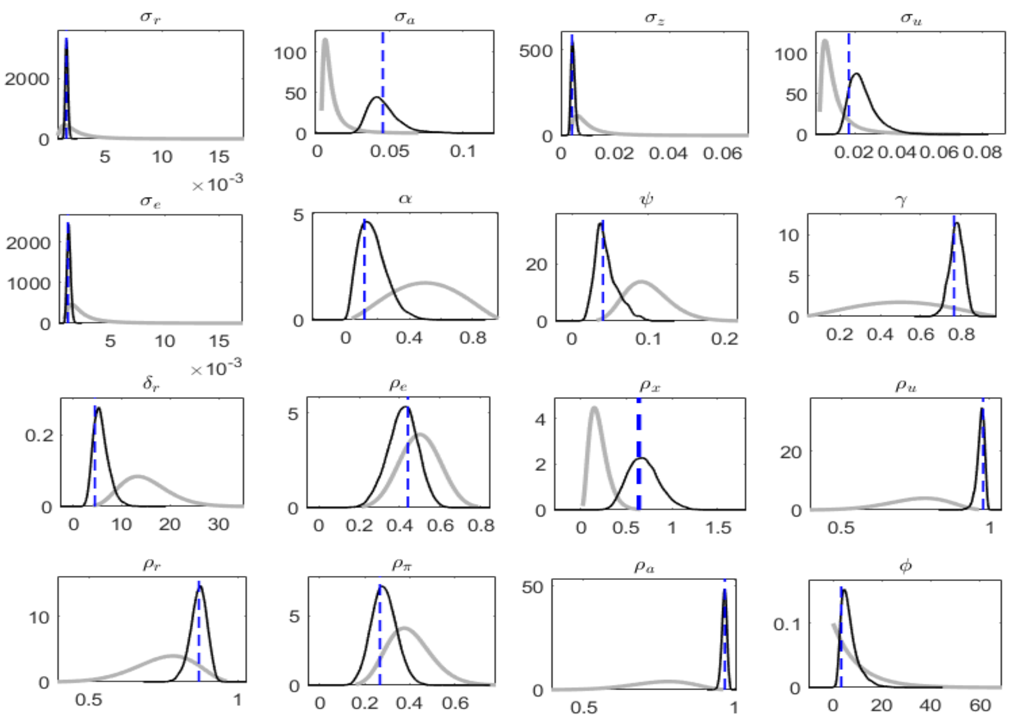

Figure A.1: Estimated posterior distributions (black solid line) for the benchmark sample under the Taylor rule. The grey line shows the prior density and the black line the density of the posterior distribution. The blue horizontal line indicates the posterior mode.

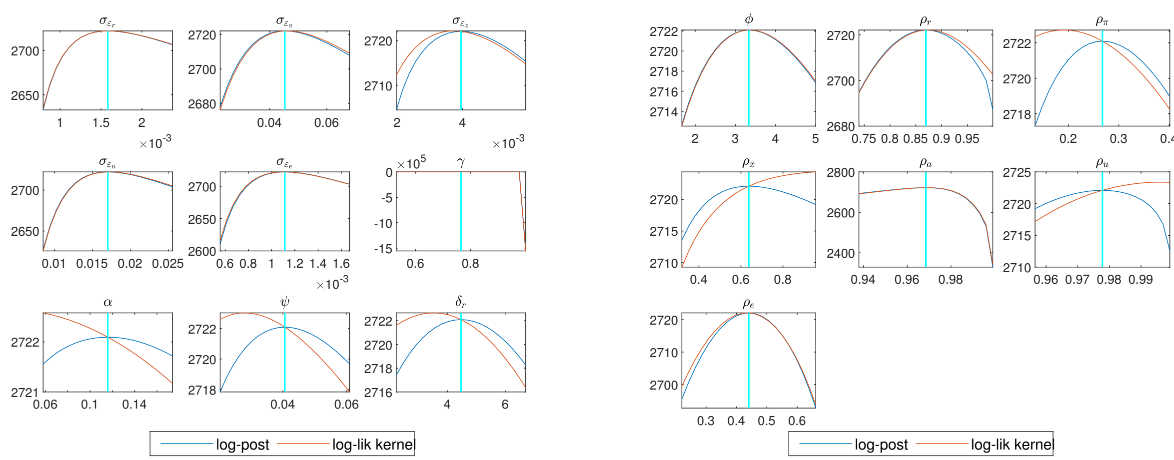

Figure A.2: Estimated structural parameters mode check plots for benchmark sample under Taylor rule. The difference in the shapes of the likelihood kernel (red line) and the posterior likelihood (blue line) indicates the role of the prior in influencing the curvature of the likelihood function.

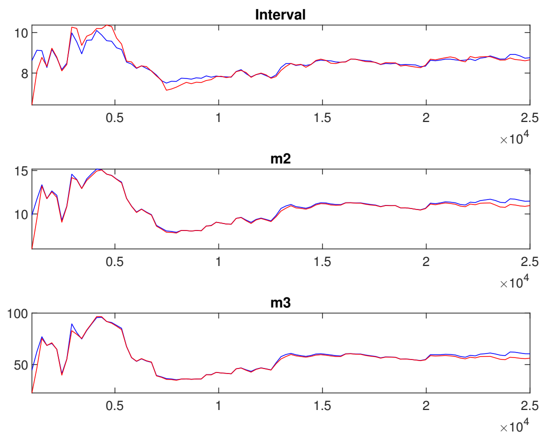

Figure A.3: MCMC multivariate diagnostics of structural parameters for benchmark sample under the estimated Taylor rule [@brooks1998]. The first plot shows the convergence diagnostics for the 80 per cent interval. The second and third plots shows the estimates of the second and third central moments (m2 and m3), respectively. The red line shows the 80 per cent quantile range based on the 25 0000 pooled draws from all sequences and the blue line shows the mean interval range based on the draws of the individual sequences.

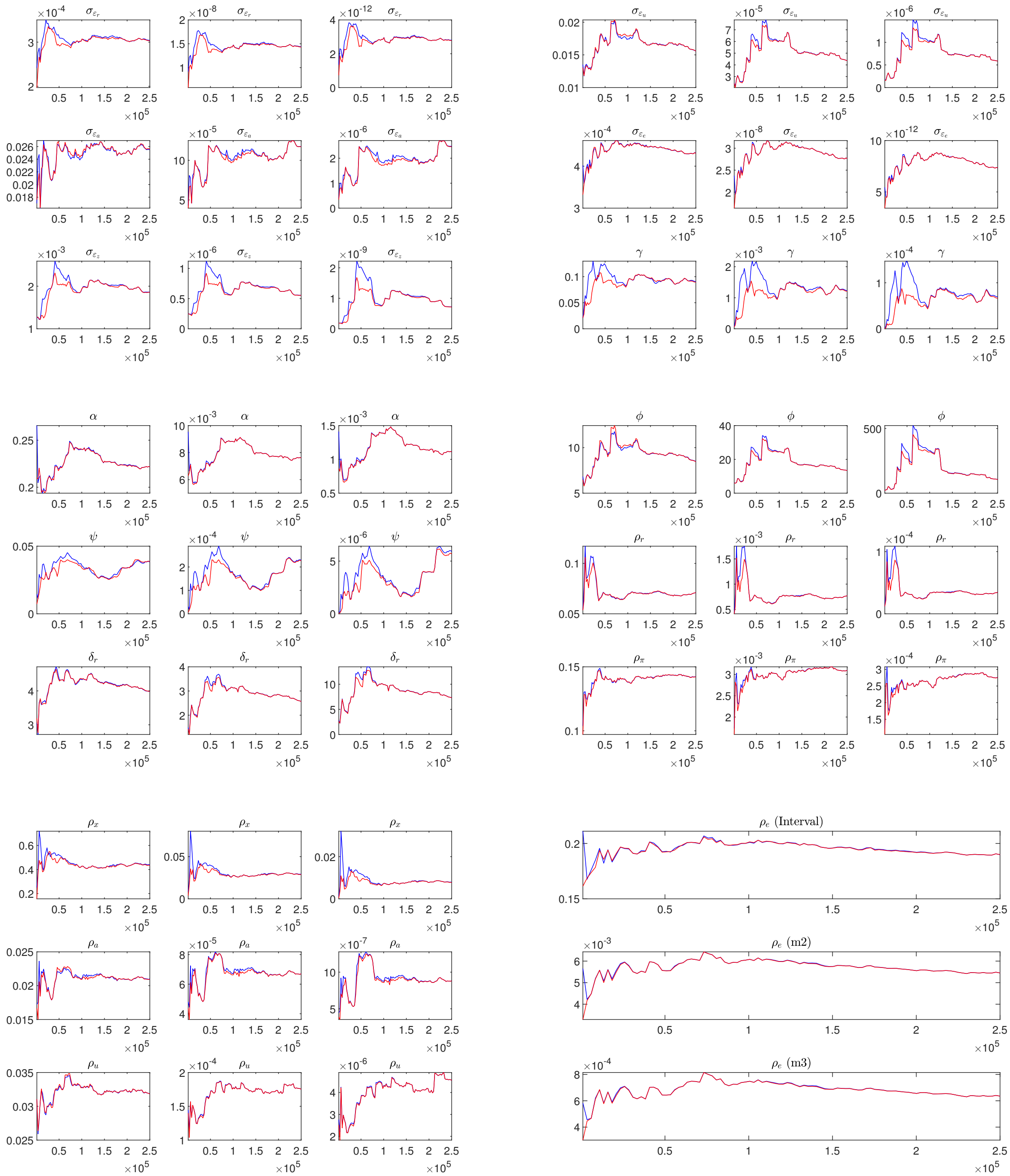

Figure A.4: MCMC univariate diagnostics of structural parameters for extended sample under estimated the Taylor rule (2.18) [@brooks1998]. For each parameter, the first column shows the convergence diagnostics for the 80 per cent interval. The second and third columns shows the estimates of the second and third central moments (m2 and m3), respectively. The red line shows the 80 per cent quantile range based on the 25 000 pooled draws from all sequences and the blue line shows the mean interval range based on the draws of the individual sequences.

Appendix B. Bayesian Esimation Results- Taylor Rule (2.18) , Extended Sample

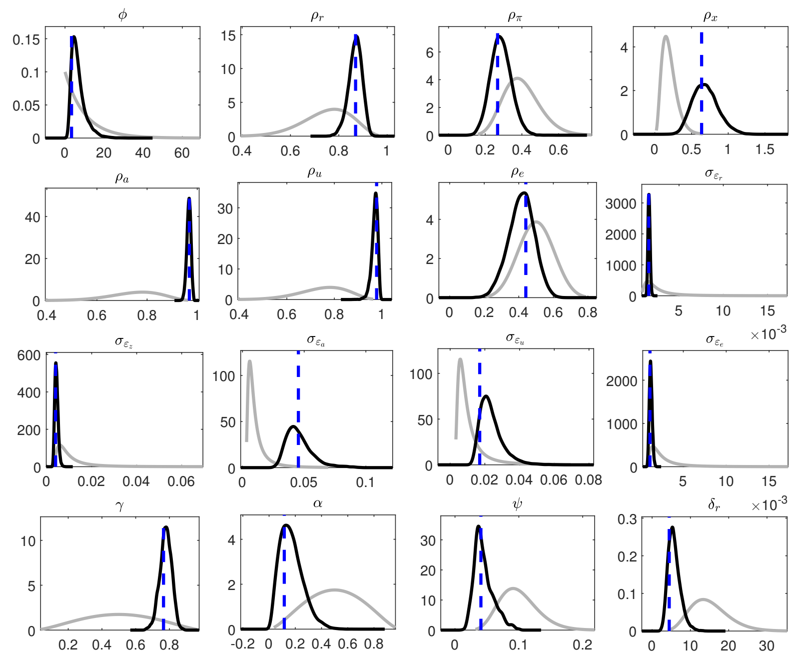

Figure B.1: Estimated posterior distributions (black solid line) for the extended sample under the Taylor rule. The grey line shows the prior density and the black line the density of the posterior distribution. The blue horizontal line indicates the posterior mode.

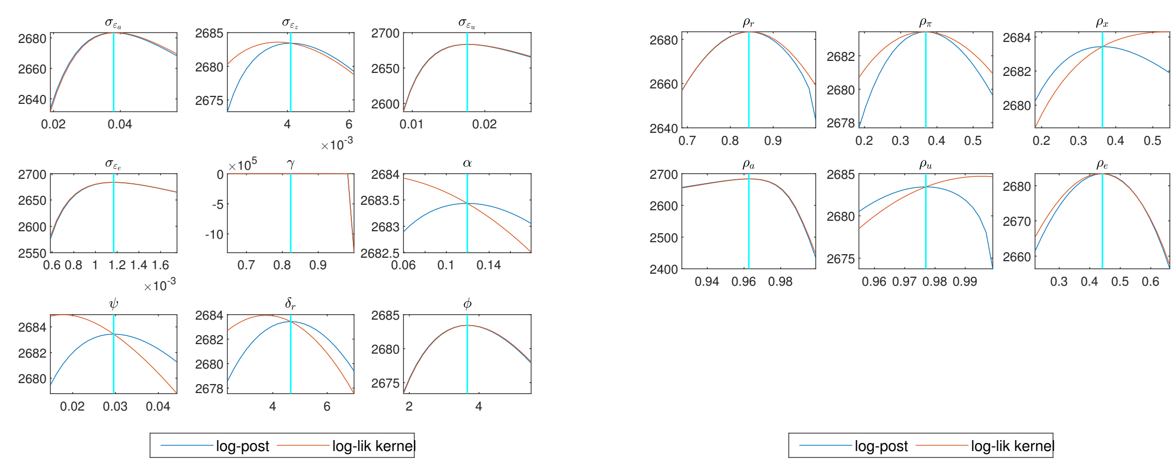

Figure B.2: Estimated structural parameters mode check plots for extended sample under the Taylor rule (2.18). The difference in the shapes of the likelihood kernel (red line) and the posterior likelihood (blue line) indicates the role of the prior in influencing the curvature of the likelihood function.

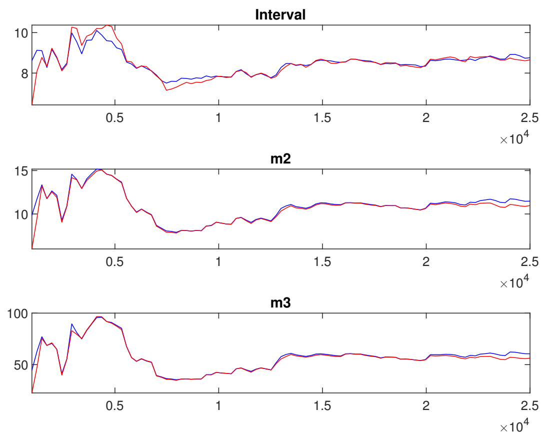

Figure B.3: MCMC multivariate diagnostics of structural parameters for extended sample under the Taylor rule (2.18) [@brooks1998]. The first plot shows the convergence diagnostics for the 80 per cent interval. The second and third plots shows the estimates of the second and third central moments (m2 and m3), respectively. The red line shows the 80 per cent quantile range based on the 25 000 pooled draws from all sequences and the blue line shows the mean interval range based on the draws of the individual sequences.

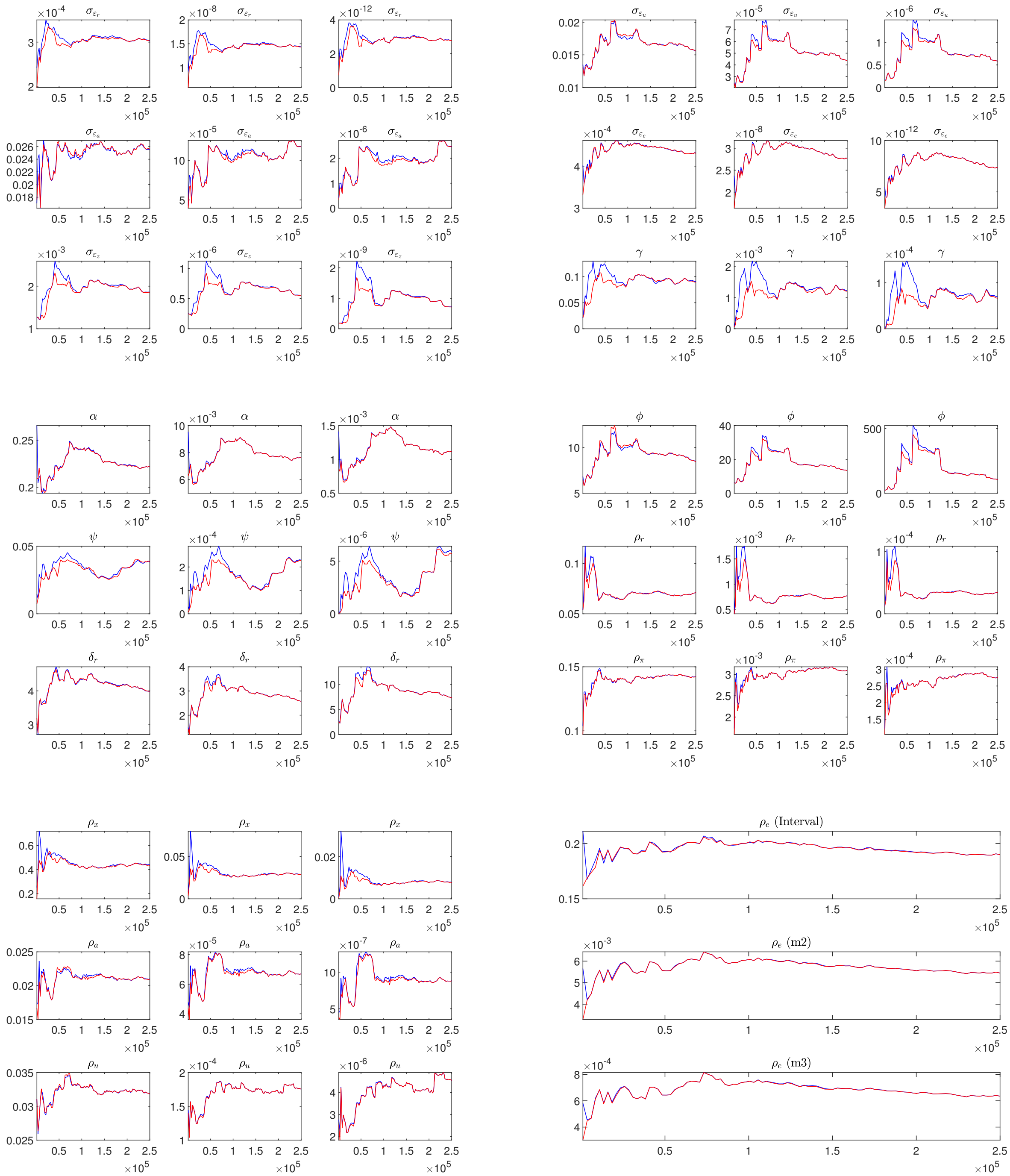

Figure B.4: MCMC univariate diagnostics of structural parameters for extended sample under the estimated Taylor rule [@brooks1998]. For each parameter, the first column shows the convergence diagnostics for the 80 per cent interval. The second and third columns shows the estimates of the second and third central moments (m2 and m3), respectively. The red line shows the 80 per cent quantile range based on the pooled draws from all sequences and the blue line shows the mean interval range based on the 25 000 draws of the individual sequences.

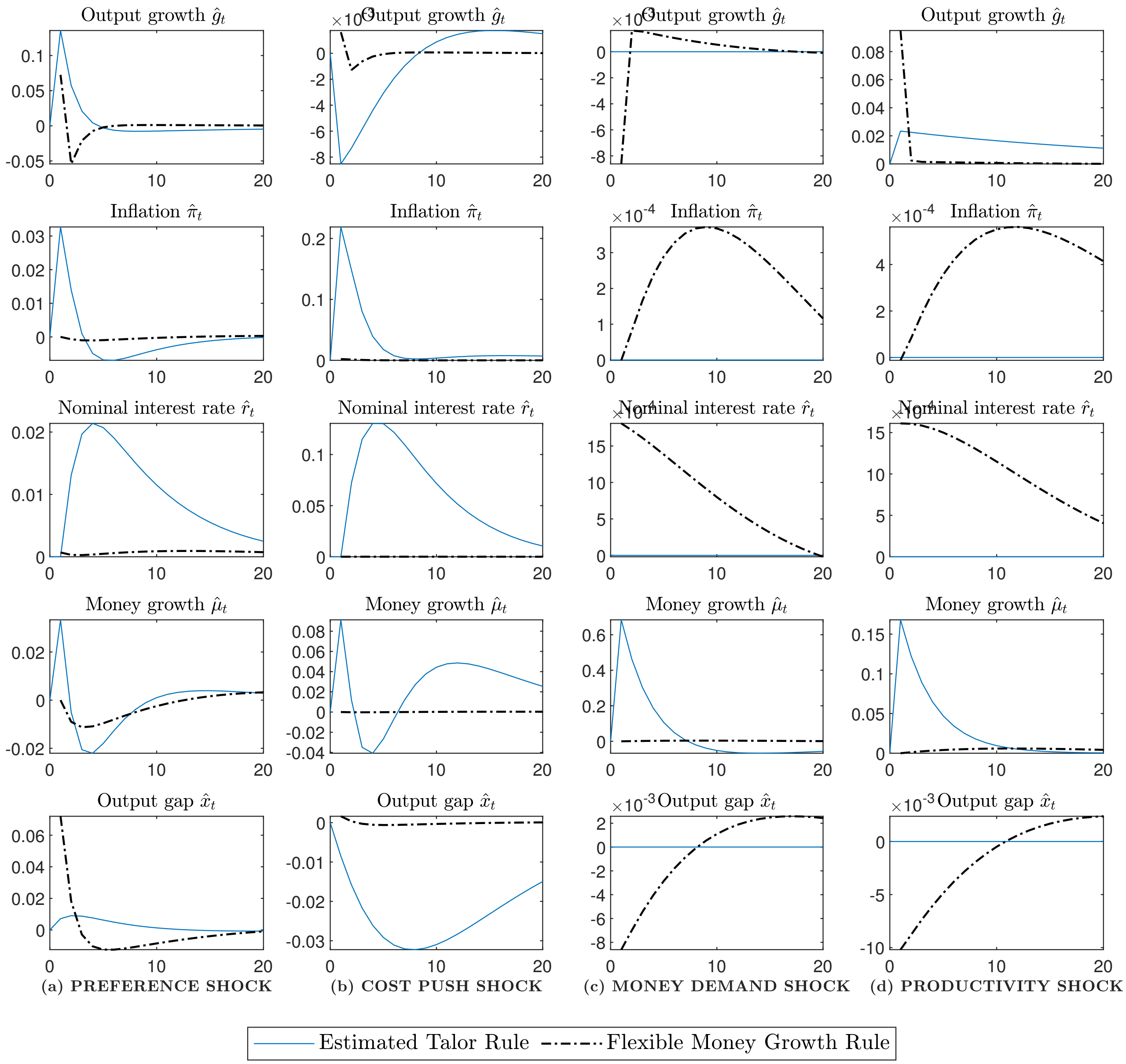

Figure B.5: Impulse responses to the indicated shock under the extended sample. Each column shows the percentage-point response of the indicated variable to a one-standard-deviation σ_i,for $i = a_t, z_t, u_t, e_t$, under the estimated taylor rule (2.18) and the flexible money growth rule (2.19).

Appendix C. Bayesian Esimation Results- Flexible Money Growth Rule (2.19), Extended Sample

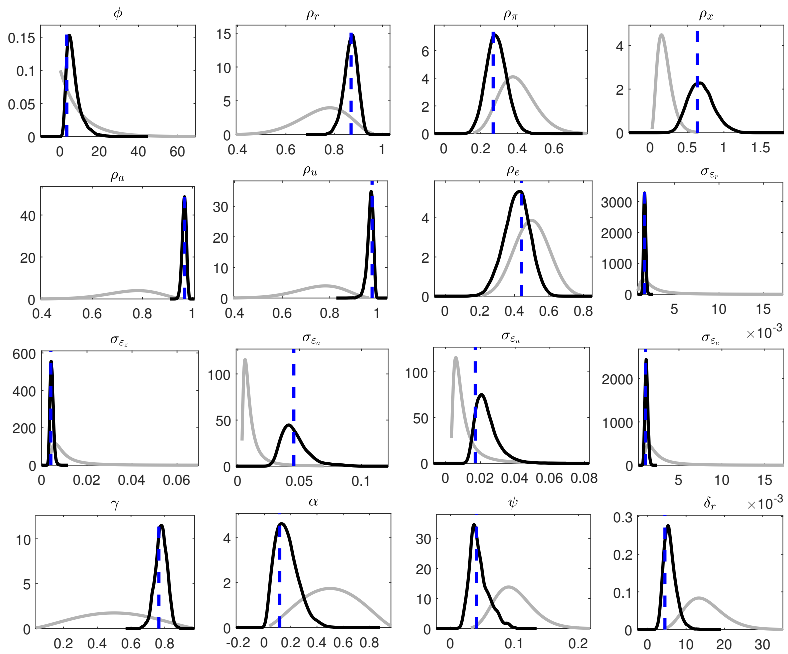

Figure C.1: Estimated posterior distributions (black solid line) for the extended sample under the flexible money growth rule (2.19). The grey line shows the prior density and the black line the density of the posterior distribution. The blue horizontal line indicates the posterior mode.

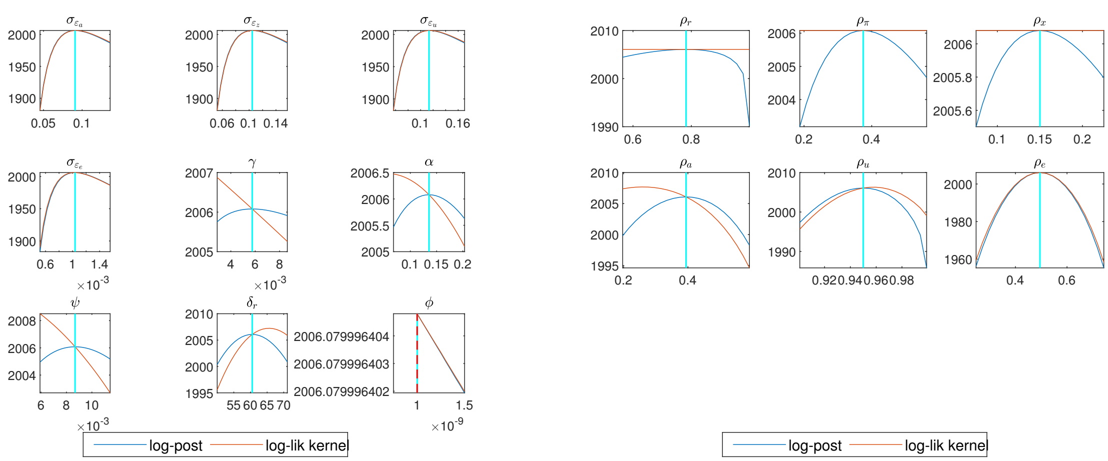

Figure C.2: Estimated structural parameters mode check plots for extended sample under flexible money growth rule (2.19). The difference in the shapes of the likelihood kernel (red line) and the posterior likelihood (blue line) indicates the role of the prior in influencing the curvature of the likelihood function.

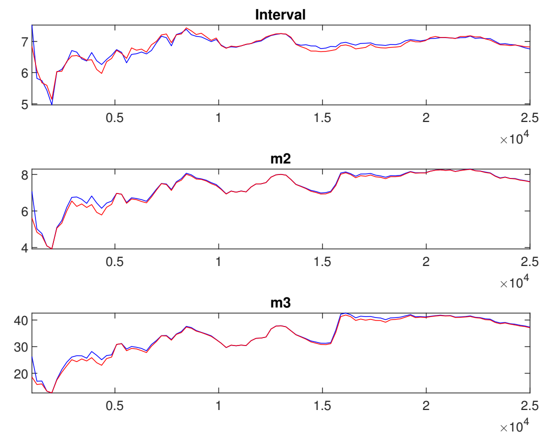

Figure C.3: MCMC multivariate diagnostics of structural parameters for extended sample under the flexible money growth rule (2.19) [@brooks1998]. The first plot shows the convergence diagnostics for the 80 per cent interval. The second and third plots shows the estimates of the second and third central moments (m2 and m3), respectively. The red line shows the 80 per cent quantile range based on the 25 0000 pooled draws from all sequences and the blue line shows the mean interval range based on the draws of the individual sequences.

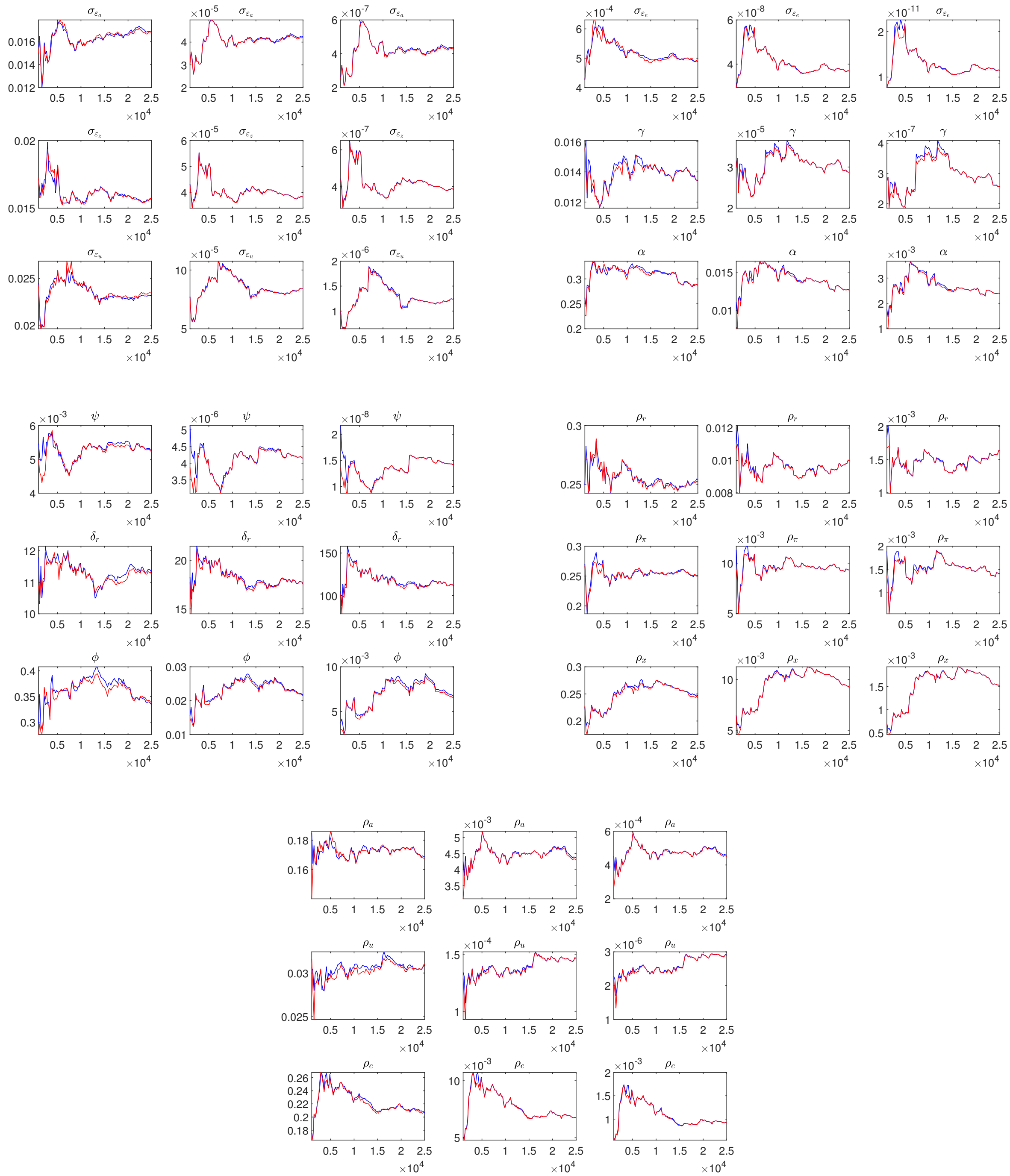

Figure C.4: MCMC univariate diagnostics of structural parameters for extended sample under the flexible money growth rule (2.19) [@brooks1998]. For each parameter, the first column shows the convergence diagnostics for the 80 per cent interval. The second and third columns shows the estimates of the second and third central moments (m2 and m3), respectively. The red line shows the 80 per cent quantile range based on the pooled draws from all sequences and the blue line shows the mean interval range based on 25 000 the draws of the individual sequences.