Replication and Robustness Analysis of Cologni & Manera (2008): 'Oil prices, inflation and interest rates in a structural cointegrated VAR model for the G-7 countries'.

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Abstract

The purpose of this project is to replicate and test the robustness of the econometric approach by Cologni & Manera (2008): "Oil prices, inflation and interest rates in a structural cointegrated VAR model for the G-7 countries". With specific focus on the United States, the replication results closely resembles the paper's macroeconomic dynamics. However, there are a few important distinctions: (i) the estimated impulse responses indicate that oil price shocks do not exert an as large negative effect on exchange rates for the U.S. and that the response stabilizes in the short-run; (ii) estimated structural coefficients reflect the significant short-run trade-off between oil prices and real GDP output; (iii) and the cointegration analysis indicates that interest rates have a significant indirect (spill-over) effect on excess output following sharp oil price increases.

1. Introduction

Substantial increases in the price of oil are well recognised to have significant effects on economic stability and macroeconomic policy. Moreover, the record-high oil prices registered from 2005 through 2008 fostered uncertainty about how the world economy would respond. To investigate the effect of oil price shocks on economic activity, Cologni & Manera (2008) considered a structural cointegrated VAR model for the G-7 countries. Specifically, the concise effects of an oil price shock on output, prices, and the reaction of monetary aggregates for sampled data from 1980q1 through 2003q4. [3]

With a specific focus on the United States (U.S.), the purpose of this project is to attempt to replicate the econometric results by Cologni & Manera (2008), and to further test the robustness of their approach and findings. In correspondence with the authors' methodology, the objectives are three-fold. Firstly, the effect of exogenous oil price shocks on macroeconomic variables is investigated

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Code and Data:
The code and data used in this project can be found at https://github.com/TianCater/Time_Series_Econometrics_Project.

[3] The authors selected this sample period to remove the spurious relationship between macroeconomic variables and the price of oil extant in the 1970s.

with a parsimonious econometric model. Secondly, in order to consider both long-run (cointegrated) restrictions and short-run (covariance) restrictions imposed by validated economic consensus, the empirical analysis involves a structural cointegrated vector autoregressive model (SVECM). Lastly, the estimated reduced-rank VECM is utilized to simulate impulse responses to an oil price shock. The simulation is to analyse the responses of the multi-level system following the oil price shock in the early 1990s, and its contribution to the economic tranquillity that followed. Moreover, the estimated SVECM allows measuring the direct effects of oil price innovations on monetary variables and corresponding spill-over effects.

To facilitate replication and robustness, I adopt Johansen & others (1992)'s method for modelling cointegration for the U.S. over the sample period 1880q1 to 2017q4, where the replication-sample results will continually be compared to Cologni & Manera (2008). The taxonomy is outlined as follows; the variables are pre-tested to conclude their order of integration, the unrestricted VAR in levels is then estimated and the adequacy of the model specification is considered. Next, the long-run co-integrating relationships are then estimated, and the long-run and short-run restrictions are imposed as necessary for identification. Finally, the resulting (reduced-rank) cointegrated VECM is estimated, impulse responses are simulated, and the economic dynamics of the model are considered. determine the specification of the long-run and short-run relationships as necessary for identification.

Notwithstanding some apparent differences, the results do resemble similar macroeconomic dynamics to that of Cologni & Manera (2008). The main economic findings in accordance with the authors are ; (i) an excess output function can be identified for the U.S., whereas most other countries are best described by a stationary money demand specification; (ii) The estimated structural VECM indicates that the U.S. oil prices have a significant effect on inflation, whereby interest rates indirectly rise to mitigate the impact of an inflationary shock; (iii) the evidence from the impulse responses suggests that oil price shocks exert an immediate, but the gradually-decaying effect on the prices.

The main economic results in this project differ in the following aspects: (i) the impulse responses indicate that oil price shocks do not exert an as large negative effect on exchange rates for the U.S. and that the response stabilizes in the short-run; (ii) estimated structural coefficients reflect the significant short-run trade-off between oil prices and real GDP output; (iii) the cointegration analysis indicates that interest rates have a significant indirect (spill-over) effect on excess output following sharp oil price increases.

The results will throughout be compared to the Cologni & Manera (2008)'s findings. The paper is laid out as follows. The econometric approach and identification requirements is outlined in section 2.The macroeconomic data collection strategy is discussed and compared to Cologni & Manera (2008)'s sample data in section 3. Section 4 is the detailed implementation of Johansen *et al.* (1992)'s methodology for modeling cointegration, with sections 4.1-4.2 conducting hypothesized tests for individual series and the adequacy of the estimated (unrestricted) VAR. Section 4.3 involves determining the number of cointegrating relationships, and considering specifications of the long-run and short-run relationships as necessary for identification. Lastly, the unrestricted VECM is estimated, impulse responses are simulated, and the economic content is intepreted in section **??**

2. The Econometric Approach

To replicate and test the robustness of the econometric methodology executed by Cologni & Manera (2008) I adopt Johansen *et al.* (1992)'s methodology for modeling cointegration. Before outlining the steps of this approach, the three model representations that will facilitate the method is provided. Firstly the reduced-form vector auto-regression model (VAR) with n time series and p legs is represented by:

$$\boldsymbol{Y}_t = \boldsymbol{\Phi D}_t + \boldsymbol{\Pi}_1 \boldsymbol{Y}_{t-1} + ... + \boldsymbol{\Phi}_p \boldsymbol{Y}_{t-p} + \boldsymbol{u}_t, \quad t = 1, ..., T, \tag{2.1}$$

and its corresponding vector error correction model (VECM) representation is specified as:

$$\Delta \boldsymbol{Y}_t = \boldsymbol{\Phi D}_t + \boldsymbol{\Pi Y}_{t-1} + \boldsymbol{\Gamma}_1 \boldsymbol{\Delta Y}_{t-1} + ... + \boldsymbol{\Gamma}_{p-1} \boldsymbol{\Delta Y}_{t-p+1} + \boldsymbol{u}_t, \quad t = 1, ..., T, \tag{2.2}$$

where $\boldsymbol{Y}_t$ is a $(n \times 1)$ vector of time series , $\boldsymbol{D}_t$ contains the deterministic terms (constant, trend, seasonal dummies etc.)[4], and the $(n \times n)$ matrix $\boldsymbol{\Pi}$ represents the long-run impact matrix which has reduced rank $r < n$. Since $\boldsymbol{\Pi}$ has rank r it can be decayed as the product $\boldsymbol{\Pi} = \boldsymbol{\alpha\beta'}$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $(n \times r)$ matrices with $rank(\boldsymbol{\alpha}) = rank(\boldsymbol{\beta}) = r$, and $\boldsymbol{\beta'}$ contains the cointegrating vectors in its r independent rows. $\boldsymbol{\Gamma}_i$, $i = 1, ..., p-1$ represents the short-run impact $(n \times n)$ matrices. The VAR parameters $\boldsymbol{\Pi}_i$ in (2.1) can be recovered from the VECM parameters $\boldsymbol{\Pi}$ and $\boldsymbol{\Gamma}_k$ in the following fashion:

$$\boldsymbol{\Pi}_1 = \boldsymbol{\Gamma}_1 + \boldsymbol{\Pi} + \boldsymbol{I}_n,$$
$$\boldsymbol{\Gamma}_k = \boldsymbol{\Gamma}_k - \boldsymbol{\Gamma}_{k-1}, \; k = 2, ..., p.$$

Finally, $\boldsymbol{u_t}$ is the $(n \times 1)$ white noise error vector that has a zero mean and a $(n \times n)$ invertible covariance matrix:

$$E[\boldsymbol{u}_t \boldsymbol{u}_t'] = \sum_u . \tag{2.3}$$

[4]The exact specification of the deterministic terms will be the first step of Johansen *et al.* (1992)'s approach

The following objective is to identify the structural shocks that dictates instructive responses by the system's n variables. The corresponding structural-form specification for systems (2.1) and (2.2) is:

$$\boldsymbol{A}\boldsymbol{Y}_t = \boldsymbol{\Lambda}_0\boldsymbol{D}_t + \boldsymbol{\Lambda}_1\boldsymbol{Y}_{t-1} + ... + \boldsymbol{\Lambda}_p\boldsymbol{Y}_{t-p} + \boldsymbol{\varepsilon}_t, \quad t = 1, ..., T, \tag{2.4}$$

where $\boldsymbol{\varepsilon}_t$ is the $(n \times 1)$ structural innovations vector with zero mean, unit variances, and a $(n \times n)$ covariance matrix with n non-zero elements:

$$E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] = \sum_{\varepsilon}. \tag{2.5}$$

The pre-multiplication of the primitive system (2.4) by $\boldsymbol{A}^{-1}$ produces the reduced-form system (2.1), noting that $\boldsymbol{\Phi} = \boldsymbol{A}^{-1}\boldsymbol{\Lambda}_0$,$\boldsymbol{\Pi}_1 = \boldsymbol{A}^{-1}\boldsymbol{\Lambda}_0$,$\ldots$,$\boldsymbol{\Pi}_p = \boldsymbol{A}^{-1}\boldsymbol{\Lambda}_p$ and, crucially, the relationship between the structural innovations ε_t and the residual error vector u_t is:

$$\boldsymbol{u}_t = \boldsymbol{A}^{-1}\boldsymbol{\varepsilon}_t = \boldsymbol{B}\boldsymbol{\varepsilon}_t. \tag{2.6}$$

Additionally, since the structural shocks are assumed to uncorrelated with unit variances ($\sum_\varepsilon = I_n$), (2.6) can be used to get:

$$\sum_u = E[\boldsymbol{u}_t\boldsymbol{u}_t'] = \boldsymbol{B}E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t']\boldsymbol{B'} = \boldsymbol{B}\sum_{\varepsilon}\boldsymbol{B'} = \boldsymbol{B}\boldsymbol{B'}. \tag{2.7}$$

The exact identification of the structural system (2.4) necessitates recovering the n^2 entries of $\boldsymbol{A}$, however, $\sum_u = \boldsymbol{B}\boldsymbol{B'}$ only identifies $(n^2 + n)/2$ unique parameters. Therefore, as stated by Lütkepohl (2005), an additional $(n^2 - n)/2$ restrictions are required for exact identification. Section **??**

3. Data 3

The variables used in the model are similar to Cologni & Manera (2008), with minor differences in its measurement due to data availability. The variables are reported in table 3.1 below. These variables include short-term interest rates (Federal Funds rate) (r_t), a monetary aggregate (M1) (m_t), the real gross domestic product (y_t), the Brent dated international oil price (U.S. dollars per barrel) (o_t), and the exchange rate expressed as the weighted average of the foreign exchange value of the U.S. dollar against a subset of the broad index currencies that circulate widely outside the U.S. (e_t)[5]. All variables are logarithmic transformed except for the interest rate (r_t).

The data is sampled quarterly for the period 1980q1 to 2017q4. Thereby including the sample period

[5] Cologni & Manera (2008) expresses the U.S. exchange rate as the ratio of the U.S. SDR rate to the average of the remaining G7 countries' SDR rates. However, due to the lack of availability of data, the exchange rate here is expressed as in Sims (1993).

of Cologni & Manera (2008) (1980q1 to 2003q4) on which the replication analysis will be conducted. Thereafter, it is compared to the results of the extended sample (1980q1 to 2017q4) that is extended further to consider if the results hold with more recent data.

Table 3.1: The Data.

United States- sample: 1980q1 to 2003q4, further extended to 2017q4	
Interest rates (r_t)	Effective Federal Funds rate - Percent per annum
Price Index (p_t)	Consumer Price Index
Gross Domestic Product (y_t)	Real Gross Domestic Product -billions of dollars, annual rate
Money Aggregate (m_t)	M1 - billions of U.S. Dollars
Exchange rates (e_t)	Nominal Advanced Foreign Economies U.S. Dollar Index
Crude Oil Prices (o_t)	Brent dated international average price - U.S. Dollars per barrel

[1] All data is retrieved from the Federal Reserve Bank of St.Louis: https://fred.stlouisfed.org/series. All series are gathered as seasonally adjusted except for the crude oil prices, which is manually adjusted for seasonality. The variables enter the model as logged.

4. Johansen's Methodology for Modeling Cointegration

The method proposed by Johansen *et al.* (1992) is applied to replicate and test the robustness of the econometric methodology executed by Cologni & Manera (2008). The steps are outlined as follows:

1. Pre-test the variables to conclude that they are (or may be) $I(1)$. (4.1)
2. Estimate the unrestricted VAR in levels and check the adequacy of the model specification. (4.2)
3. Select the specification of the deterministic component, determine the number of co-integrating relationships r, and determine the specification of the long-run and short-run relationships as necessary for identification. (4.3)
4. Estimate the resulting (reduced-rank) cointegrated VECM, interpret the economic dynamics of the model, and test further hypothesized restrictions. (4.4)

4.1. Pre-test the variables to conclude that they are (or may be) $I(1)$.

The time series of the macroeconomic variables are depicted in figure 4.1 below. Visually, all variables exhibit trending behavior. Augmented Dickey Fuller (ADF) tests are conducted for each variable to determine the order of integration to avoid the spurious regression concern. The results are reported in table 4.1. Since most series display trending behavior, the specification of the deterministic parts for each series includes a constant and a trend as done by Cologni & Manera (2008). The number of lags is determined by the Akaike Information Criteria (AIC), with the maximum number of lags assumed to be four as done by the authors.

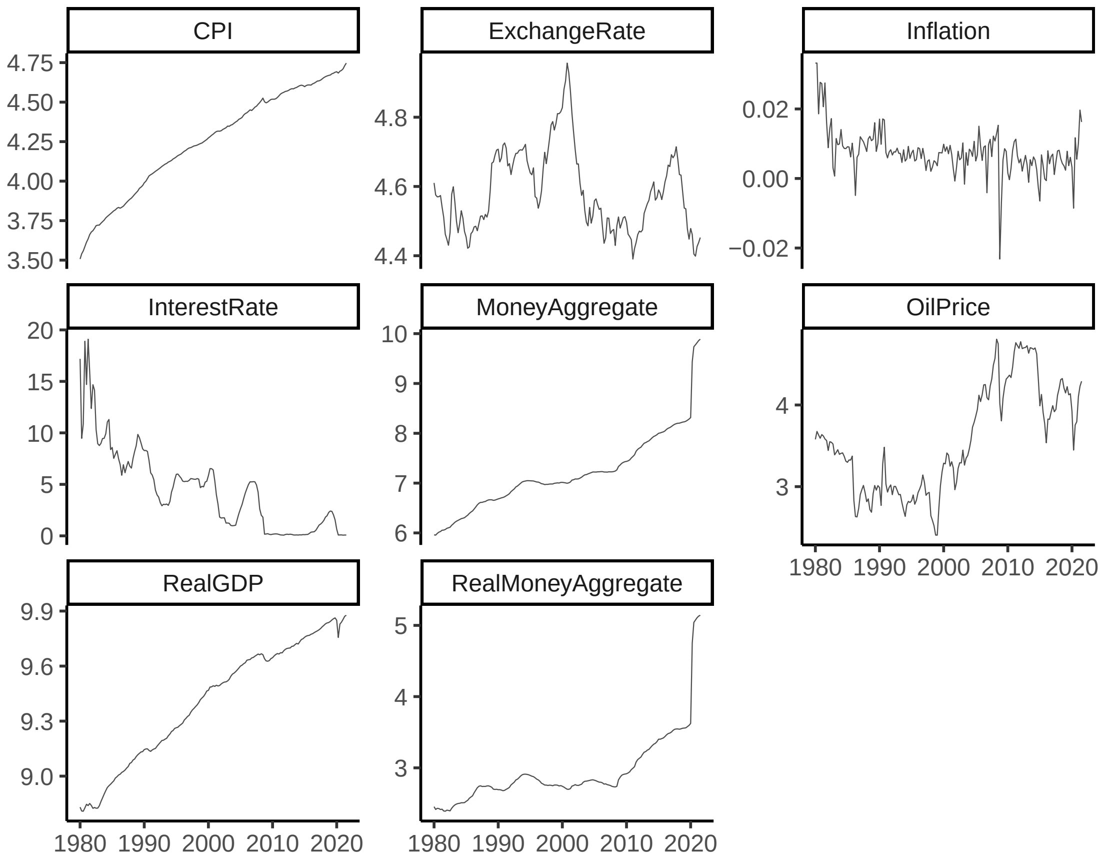


Figure 4.1: Time Series of the Macroeconomic Variables (Log-Transformed).

The results in table 4.1 suggests that interest rates (r_t), real GDP (y_t), and oil price (o_t) are $I(1)$, whereas the monetary aggregate (m_t) and the price level (p_t) seems to be $I(2)$. This is identical to the suggestions by Cologni & Manera (2008). To facilitate comparability, the system is transformed from $I(1)$ to $I(2)$ as done by Cologni & Manera (2008), by reviewing the inflation rate Δp_t and the real monetary aggregate ($m_t - p_t$). Failure to reject the null hypothesis of no unit root for the differenced inflation rate ($\Delta(\Delta p_t)$) and the differenced real monetary aggregate $\Delta(m_t - p_t)$ suggests that these series are $I(1)$, confirming the robustness of these transformations implemented by the authors.

Table 4.1: ADF Tests for Unit Roots and Order of Integration.

Variable	Deterministic terms	Lags	Test value	Critical values		
				1%	5%	10%
$\mathtt{r_t}$	constant, trend	2	–-2.73	−4.04	−3.45	−3.15
$\Delta\mathtt{r_t}$	constant	1	−10.04	−3.51	−2.89	−2.58
$\mathtt{p_t}$	constant, trend	4	−2.47	−4.04	−3.45	−3.15
$\Delta\mathtt{p_t}$	constant, trend	2	−2.5	−4.04	−3.45	−3.15
$\Delta(\Delta\mathtt{p_t})$	constant	1	−2.61	−2.6	−1.95	−1.61
$\mathtt{y_t}$	constant, trend	1	−2.31	−3.51	−2.89	−2.58
$\Delta\mathtt{y_t}$	constant	0	−2.8	−2.6	−1.95	−1.61
$\mathtt{e_t}$	constant, trend	3	−2.73	−4.04	−3.45	−3.15
$\Delta\mathtt{e_t}$	constant	1	−5.5	−3.51	−2.89	−2.58
$\mathtt{o_t}$	constant, trend	1	−2.65	−4.04	−3.45	−3.15
$\Delta\mathtt{o_t}$	constant	0	−8.15	−2.6	−1.95	−1.61
$\mathtt{m_t}$	constant, trend	2	−1.83	−4.04	−3.45	−3.15
$\Delta\mathtt{m_t}$	constant	1	−2.33	−3.51	−2.89	−2.58
$(\mathtt{m_t} - \mathtt{p_t})$	constant, trend	1	−1.80	−3.51	−2.89	−2.58
$\Delta(\mathtt{m_t} - \mathtt{p_t})$	constant	0	−3.33	−2.6	−1.95	−1.61

4.2. Estimate the unrestricted VAR in levels and check the adequacy of the model specification.

The order of lags of the unrestricted VAR(p) is selected based on the Akaike Information Criteria (AIC) reported in table 4.2. Similar to Cologni & Manera (2008), a lag order of two is selected. The unrestricted VAR(2) in levels is then estimated and the adequacy of its specification is investigated. Special attention is given to test that the errors are white noise. Since Johansen *et al.* (1992)'s method is a maximum-likelihood approach, the limiting distributions are derived assuming normal errors, however, specifications can be robust to deviations that permit some non-normality and heteroscedasticity under *i.i.d.* errors with finite variance. Critically, auto-correlated residuals, time-varying parameters, and structural breaks are unacceptable.

Table 4.2: Unrestricted VAR(p) Optimal Lag Length Criteria

	Number of lags (p)			
	1	2	3	4
AIC(n)	-41.77	-42.66	-42.50	-42.39
HQ(n)	-41.31	-41.80	-41.24	-40.73
SC(n)	-40.62	-40.52	-39.37	-38.28

Note:
Assumed maximum lag length of four in correspondence with Cologni & Manera (2008)

Table 4.3 shows the multivariate and uni-variate tests for normality (Jarque-Bera) and heteroskedasticity (ARCH). Despite evidence of non-normality in the uni-variate series' of the oil price o_t, the interest rate r_t, and the real monetary aggregate $(m_t - p_t)$,the null hypothesis of non-normally distributed residuals is rejected at the 5% level of significance for the multivariate VAR(2).

Table 4.3: Tests for Normality and Homoskedasticity for the unrestricted VAR(2).

	Jarque-Bera Test		ARCH Test	
		$H_0 : non-normal\ residuals$		$H_0 : Heteroskedasticity$
Variables	p-value[1]	Interpretation	p-value	Interpretation
$Multivariate$	0.51	Reject null of non-normality	0.42	Fail to reject null
y_t	0.67	Reject null of non-normality	0.603	Reject null
o_t	0.023	Fail to reject null	0.982	Reject null
r_t	0.00	Fail to reject null	0.193	Fail to reject null
Δp_t	0.804	Reject null of non-normality	0.43	Fail to reject null
e_t	0.77	Reject null of non-normality	0.678	Reject null
$(m_t - p_t)$	0.43	Fail to reject null	0.29	Fail to reject null

[1] Hypothesis are tested at the 5 percent significance level

The null hypothesis of heteroscedasticity (ARCH test) cannot be rejected at the 5% level of significance for the multivariate VAR(2), where the oil price o_t, the interest rate r_t, and the real monetary aggregate $(m_t - p_t)$ are each individually a source of this heteroscedasticity. The question becomes whether this level of heteroscedasticity still permits robust maximum-likelihood estimations. To answer this, more specification tests are conducted.

Both the Portmanteau (Asymptotic) and the Breusch-Godfrey LM tests reported in table 4.4 provide strong evidence that the residuals are not serially correlated. In addition, the (OLS) CUSUM tests and the empirical fluctuations plot for each series are given in figures .9 and .10 in Appendix A.1, providing significant evidence that the series have no structural breaks nor time-varying parameters.

Provided the strict requirements of no-serial-correlation, no-structural-breaks, and no-time-varying parameters are satisfied; the degree of homoscedasticity present within the unrestricted VAR(2) is assumed as acceptable.

Table 4.4: Tests for serial correlation unrestricted VAR(2)'s resduals.

Portmanteau Test (Asymptotic)		Breusch-Godfrey LM Test	
H_0 : *no serial correlation*		H_0 : *no serial correlation up to order p*	
p-value[1]	Interpretation	p-value	Interpretation
0.6539	Fail to reject the null	0.6317	Fail to reject the null

[1] Hypothesis are tested at the 5 percent significance level

4.3. Select the specification of the deterministic component, determine the number of cointegrating relationships r, and determine the specification of the long-run and short-run relationships as necessary for identification.

In accordance with Cologni & Manera (2008), the deterministic terms contained within $\boldsymbol{\Phi}_t \boldsymbol{D}_t = \boldsymbol{\mu}_t = \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 t$ (see 2.2) are assumed to contain a restricted trend to limit the trending nature of most of the series. The restricted VECM becomes:

$$\Delta \boldsymbol{Y}_t = \boldsymbol{\mu}_0 + \boldsymbol{\alpha}(\boldsymbol{\beta}' \boldsymbol{Y}_{t-1} + \boldsymbol{\rho}_1 t) + \boldsymbol{\Gamma}_1 \boldsymbol{\Delta} \boldsymbol{Y}_{t-1} + ... + \boldsymbol{\Gamma}_{p-1} \boldsymbol{\Delta} \boldsymbol{Y}_{t-p+1} + \boldsymbol{u}_t, \quad t = 1, ..., T, \tag{4.1}$$

where the series in $\boldsymbol{Y_t}$ are $I(1)$ with drift vector $\boldsymbol{\mu}_0$ and the cointegrating relations $\boldsymbol{\beta' Y_t}$ contain a linear trend term $\boldsymbol{\rho_1} t$.

The number of cointegrating relationships (r) contained within the $\boldsymbol{\beta}$ matrix is determined using Johansen (1995)'s LR approach. The Trace and Maximum-Eigenvalue test statistics are outlined in table 4.5 below. Similar to hypothesized results by Cologni & Manera (2008), both tests provide evidence of one co-integrating relationship at the 1% level of significance.

Table 4.5: Testing for the number of co-integrating relationships contained in β'.

		Trace Statistics[1]	0.05 Critical Value
H_o	H_1	*Trace test*	
$r=0$	$r=1$	144.21^{**}	114.90
$r\leq 1$	$r=2$	85.74	87.31
$r\leq 2$	$r=3$	42.24	62.99
$r\leq 3$	$r=4$	20.70	42.44
$r\leq 4$	$r=5$	10.56	25.32
H_o	H_1	*Max − eigenvalue test*	
$r=0$	$r=1$	58.48^{*}	43.97
$r\leq 1$	$r=2$	43.50^{***}	37.52
$r\leq 2$	$r=3$	21.53	31.46
$r\leq 3$	$r=4$	10.15	25.54
$r\leq 4$	$r=5$	7.25	18.96

[1] *,**,** represents rejection of the null hypothesis at the 1, 5, and 10 percent levels. The specification includes a constant and a trend.

4.3.1. Specification and identification of the long-run relationship

The next step is to estimate the co-integrating relationship ($r = 1$) of the restricted model. Cologni & Manera (2008) considers two long-run relationships as possible specifications for the co-integrating relationships. A standard money demand equation is the first long-run relationship considered:

$$m_t - p_t = \beta_o + \beta_1 y_t - \beta_2 r_t - \beta_3 \Delta p_t. \tag{4.2}$$

The second second long-run relationship equates excess output to the exchange rate, the inflation rate, the interest rate, and oil prices:

$$y_t - \beta_4 trend = \beta_5 + \beta_6 e_t - \beta_7 r_t - \beta_8 \Delta p_t - \beta_9 o_t. \tag{4.3}$$

Table 4.6 presents both the estimates of this paper and that of the authors. The authors found that the coefficient relating to the money demand specification is not statistically different from zero, and the LR test resulted in the rejection of null hypothesis of the conventional money demand function (4.2). Rather, the excess output function (4.3) is interpreted as the long run relationship for the United States.

Table 4.6: Estimated co-integrated relationships of the restricted system.

My Estimates [1]								
y_t	o_t	r_t	Δp_t	e_t	m_t	*constant*	*trend*	*LR test*
1	0.082	-1.693	-34.237	-	-	-4.070	-0.025	$\chi^2(2) =$
	(0.022)	(0.542)	(3.750)				(0.0009)	1.69[0.43]
Estimations by Cologni & Manera (2008)								
y_t	o_t	r_t	Δp_t	e_t	m_t	*constant*	*trend*	*LR test*
1	0.160	-	-26.019	0.218	-	-8.511	-0.009	$\chi^2(2) =$
	(0.036)		(3.191)	(0.059)			(0.0004)	2.44[0.36]

[1] Values in round brackets represents the parameter estimates standard errors, and in square brackets the p-values from the log-likelihood test for over-identifications.

Similarly, the LR test in this paper also rejects the null hypothesis of a conventional money demand function and adopts the excess output function as the specification for the long-run relationship. In particular, the data does not reject the existence of a negative long-run effect of oil prices on excess output. However, in contrast to Cologni & Manera (2008), the LR test for over-identification indicates that interest rates (r_t) has a significant negative effect on excess output, whereas exchange rates (e_t) has no effect. This minor difference in the identification of the long-run relationship produces an larger *p*-value [0.43] than estimated by the authors [0.33], resulting in a significantly lower-likelihood of rejecting the null of valid over-identifying restrictions.

4.3.2. Specification and identification of the short-run relationships

The model's short-run dynamics is described by adopting the identification strategy applied by Cologni & Manera (2008), that imposes a set of over-identifying restrictions on the coefficients of the $\boldsymbol{B}$ matrix in equation (2.2). The assumed short-run relationships are specified as:

$$u_m = b_{11}\varepsilon_m + b_{12}\varepsilon_r + b_{14}\varepsilon_{\Delta p}. \tag{4.4}$$

$$u_r = b_{22}\varepsilon_r + b_{23}\varepsilon_y + b_{24}\varepsilon_{\Delta p} + b_{25}\varepsilon_e + b_{26}\varepsilon_o. \tag{4.5}$$

$$u_y = b_{33}\varepsilon_y + b_{34}\varepsilon_{\Delta p} + b_{35}\varepsilon_e + b_{36}\varepsilon_o. \tag{4.6}$$

$$u_{\Delta p} = b_{44}\varepsilon_{\Delta p} + b_{45}\varepsilon_e + b_{46}\varepsilon_o. \tag{4.7}$$

$$u_e = b_{55}\varepsilon_e + b_{56}\varepsilon_o. \tag{4.8}$$

$$u_o = b_{66}\varepsilon_o, \tag{4.9}$$

where u_m, u_r, u_y, $u_{\Delta p}$, u_e, and u_o represents the reduced-form errors, whereas ε_m, ε_r, ε_y, $\varepsilon_{\Delta p}$, ε_e, and ε_o the structural shocks in money demand, money supply, real output, price setting, exchange rates, and oil prices.[6]

Specifications (4.4), (4.5), and (4.6) implies $b_{13} = b_{15} = b_{16} = 0$, $b_{21} = 0$, and $b_{31} = b_{32} = 0$, respectively. Where specifications (4.7), (4.8), and (4.9) equates to $b_{41} = b_{42} = b_{43} = 0$,$b_{51} = b_{52} = b_{53} = b_{54} = 0$, and $b_{61} = b_{62} = b_{63} = b_{64} = b_{65} = 0$, respectively.

4.4. Estimate the resulting (reduced-rank) cointegrated VECM, interpret the economic dynamics of the model, and test further hypothesized restrictions.

The set of short-run over-identification restrictions given by equations (4.4)-(4.9) are tested using the conventional likelihood-ratio (LR) statistic, after which the structural shocks are recovered by estimating the contemporaneous impact matrix $\boldsymbol{B}$. Lastly, impulse response analysis is conducted. The results are continuously compared with the findings by Cologni & Manera (2008).

The estimated coefficients for contemporaneous impact matrix B in the structural model with its corresponding LR statistic is given in table 2.7. The LR test statistic for the over-identifying short-run restrictions fails to reject the null hypothesis of valid restrictions at the 5% significance level.

[6] The economic theory applied to derive these short-run dynamics are omitted due to this paper's focus on the robustness of the econometric method. See Cologni & Manera (2008) for a detailed description.

Table 4.7: Estimated coefficients for contemporaneous impact matrix $\boldsymbol{B}$ in the structural model.

	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$	$b_{i,4}$	$b_{i,5}$	$b_{i,6}$
$b_{1,j}$	0.008*	-0.002***	-	-0.0022***	-	-
	(0.001)	(0.0008)		(0.001)		
$b_{2,j}$	-	0.957*	0.2375	0.3128**	0.0161	0.0352**
		(0.123)	(0.952)	(0.1141)	(0.1002)	(0.102)
$b_{3,j}$	-	-	0.0048*	0.001	0.0002*	-0.0001
			(0.0006)	(0.0006)	(0.0004)	(0.0005)
$b_{4,j}$	-	-	-	0.0026*	-0.0004	0.0014*
				(0.0003)	(0.0002)	(0.0003)
$b_{5,j}$	-	-	-	-	0.029*	0.0043
					(0.0034)	(0.0028)
$b_{6,j}$	-	-	-	-	-	0.126*
						(0.0176)
LR Test for overidentification: $\chi^2(3) = 3.27[0.19]$						

[1] b(i,j), i,j=1,..,6 are the estimated matrix B's parameters. Values in round brackets represents the parameter estimates standard errors, and in square brackets the p-values from the log-likelihood test for over-identifications.*,**,***, represents the 1,5, and 10 percent significance level of the coefficient.

The analysis of table 2.7 has similar results as Cologni & Manera (2008) in the following aspects: The short-run impact coefficient reflecting the effect of oil prices on the inflation rate ($b_{4,6}$) is positive and statistically significant at the 1% level. Also, there is no robust evidence that oil prices exerts an instantaneous effect on real GDP growth ($b_{3,6}$). Lastly, that a oil price shock has no contemporaneous impact on the exchange rates ($b_{5,6}$).

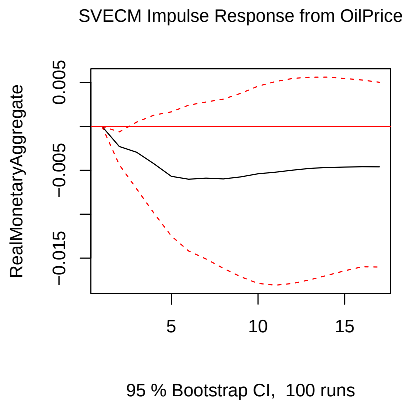
The empirical results differ from Cologni & Manera (2008) in the following ways: the estimates suggest that following a shock to the inflation rate, interest rates increase, being indicative of monetary tightening ($b_{2,4}$). Additionally, our estimations highlight the short-run negative trade-off between the inflation rate and GDP growth ($b_{3,4}$).

The estimated impulse responses following a one standard deviation shock to the oil price is depicted in figure 4.2, with the corresponding 95% confidence interval bandwidth provided.[7] Similar to Cologni

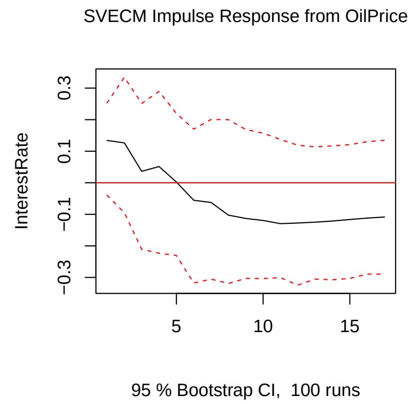
[7] As executed by Cologni & Manera (2008), the impulse response functions are computed by assuming 4 lags. The results are similar for the U.S. for lag lengths of 2,3, and 4.

& Manera (2008), an increase in the oil price tends to be followed by a rise in inflation. However, the impact is short lived, and progressively fades away towards its natural level. Moreover, the impulse responses are indicative that output is negatively affected by the sudden increase in oil prices, where Cologni & Manera (2008) explains that this is due to the U.S. being heavily dependent on foreign supplies of oil. However, in contrast to the authors findings, here the effects on output seems to be permanent and not temporary, being indicative of alternative specifications of the long-run relationships within the system.

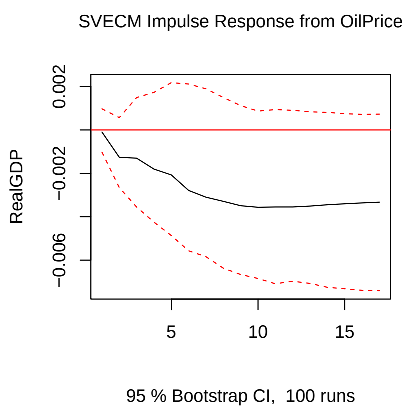
Another apparent distinction in the impulse responses is that innovations to the oil price does not exert an as large negative effect on exchange rates for the U.S. than the results by Cologni & Manera (2008) suggests.



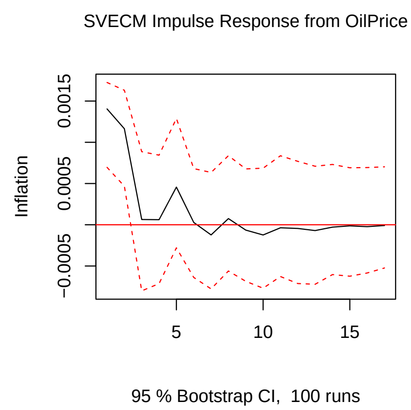
(a) (a)



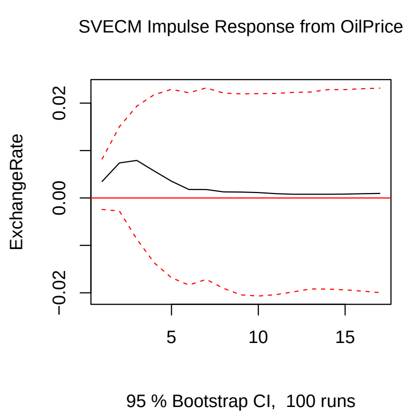
(b) (b)



(c) (c)



(d) (d)



(e) (e)

Figure 4.2: Impulse responses from an oil price shock

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10 Cologni, A. & Manera, M. 2008. Oil prices, inflation and interest rates in a structural cointegrated VAR model for the g-7 countries. *Energy economics*. 30(3):856–888.

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Appendix

A.1. Appendix A {-}

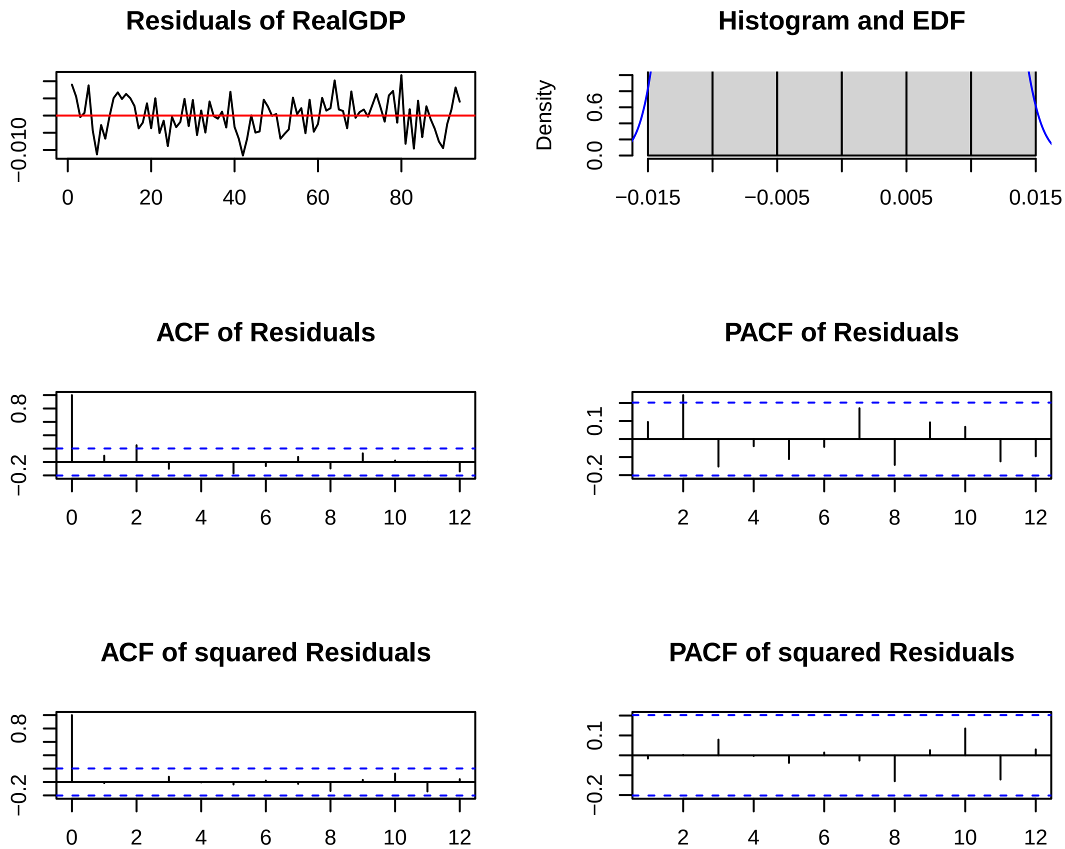

Figure .3: Diagnostics plot of VAR(2) for Real GDP

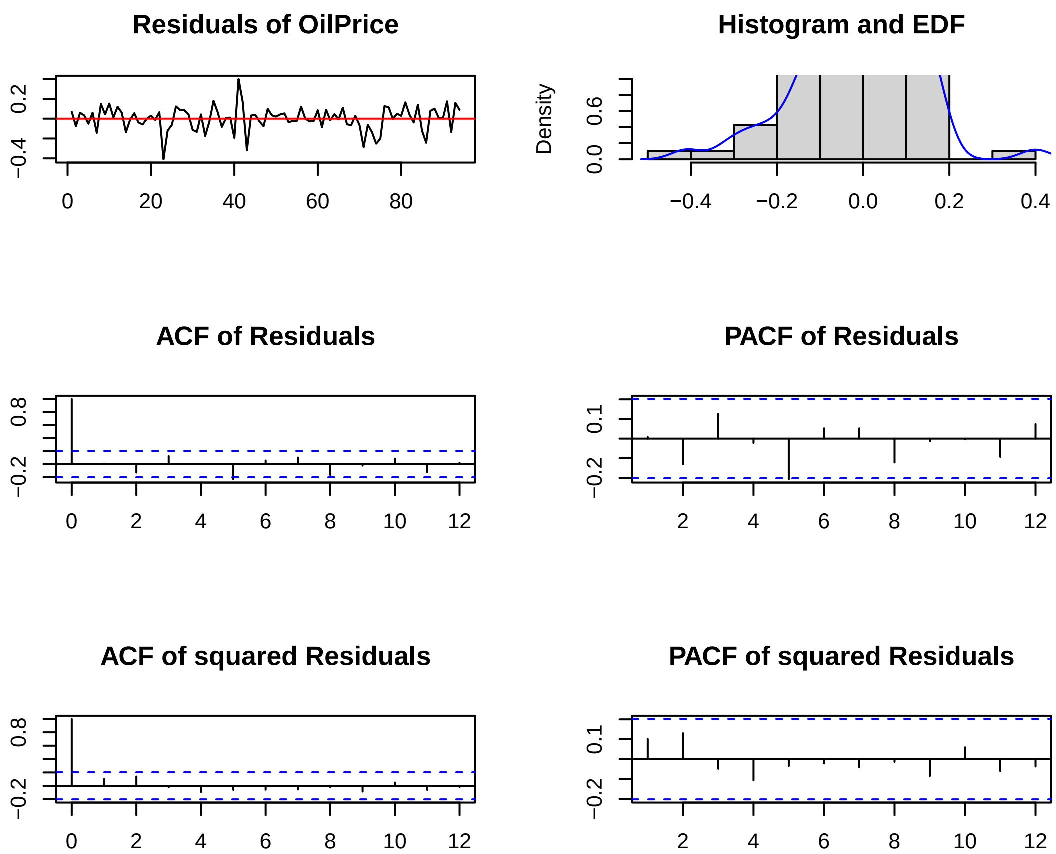

Figure .4: Diagnostics plot of VAR(2) for Oil Price

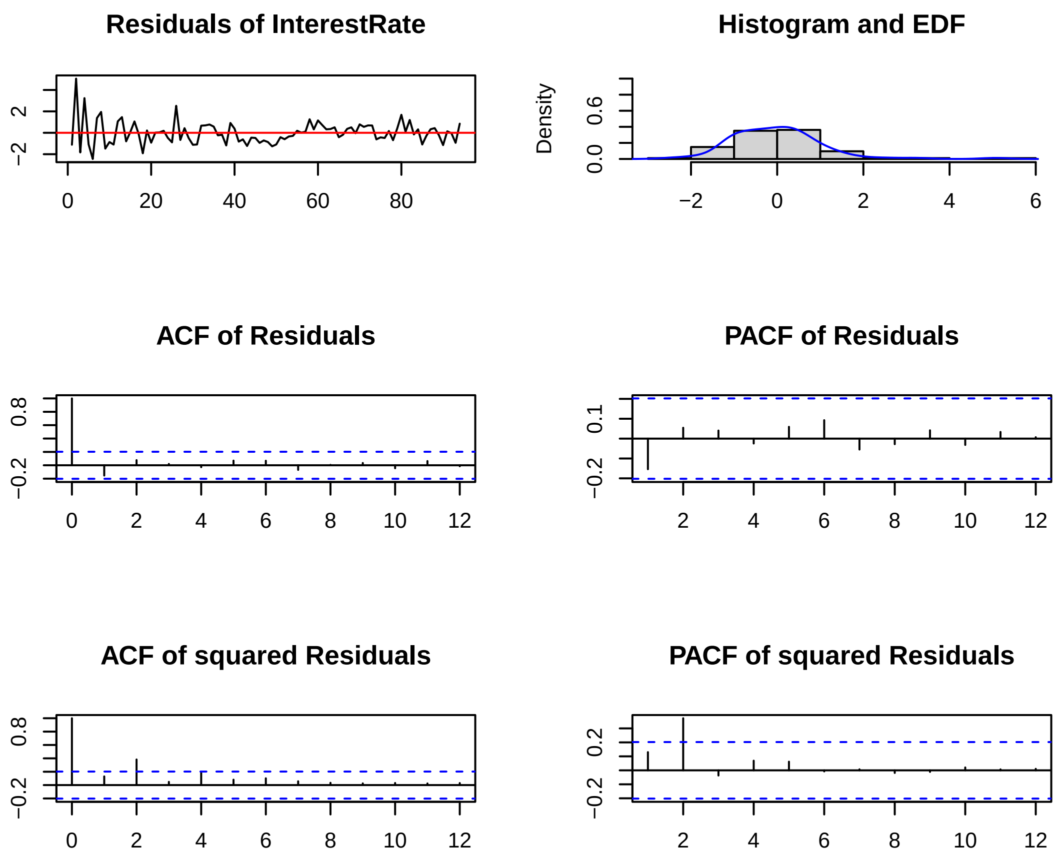

Figure .5: Diagnostics plot of VAR(2) for Interest Rate

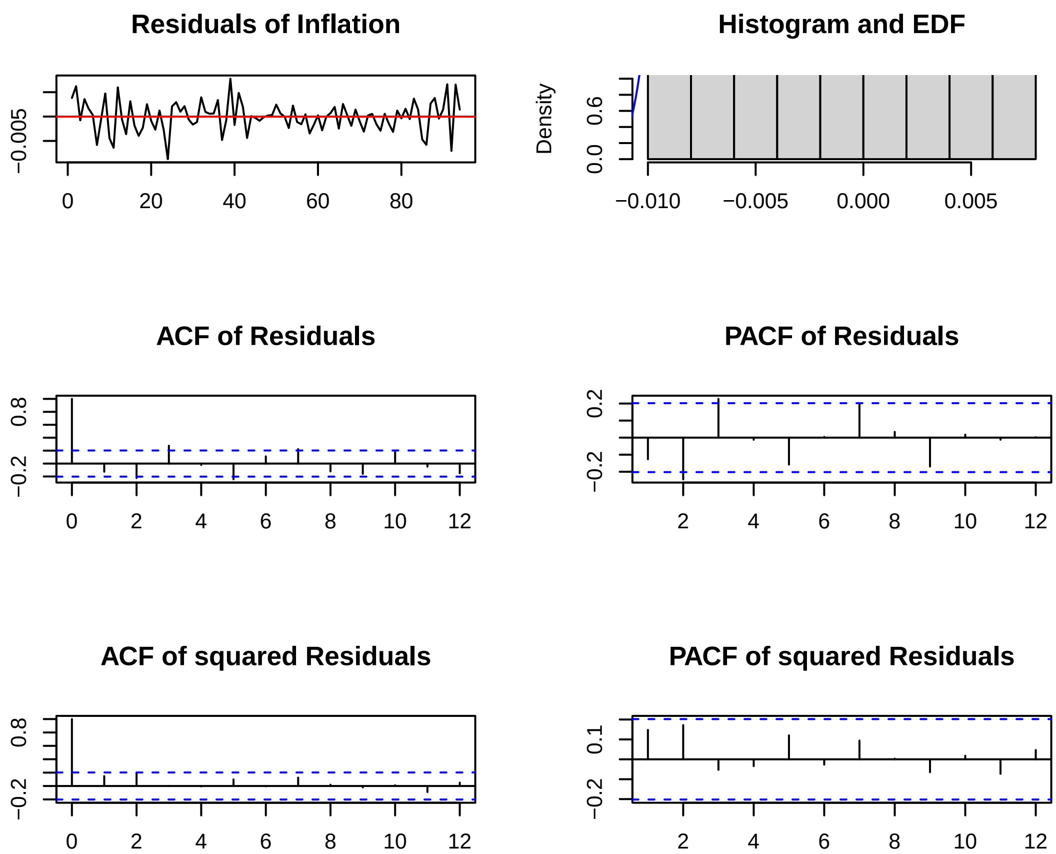

Figure .6: Diagnostics plot of VAR(2) for Inflation

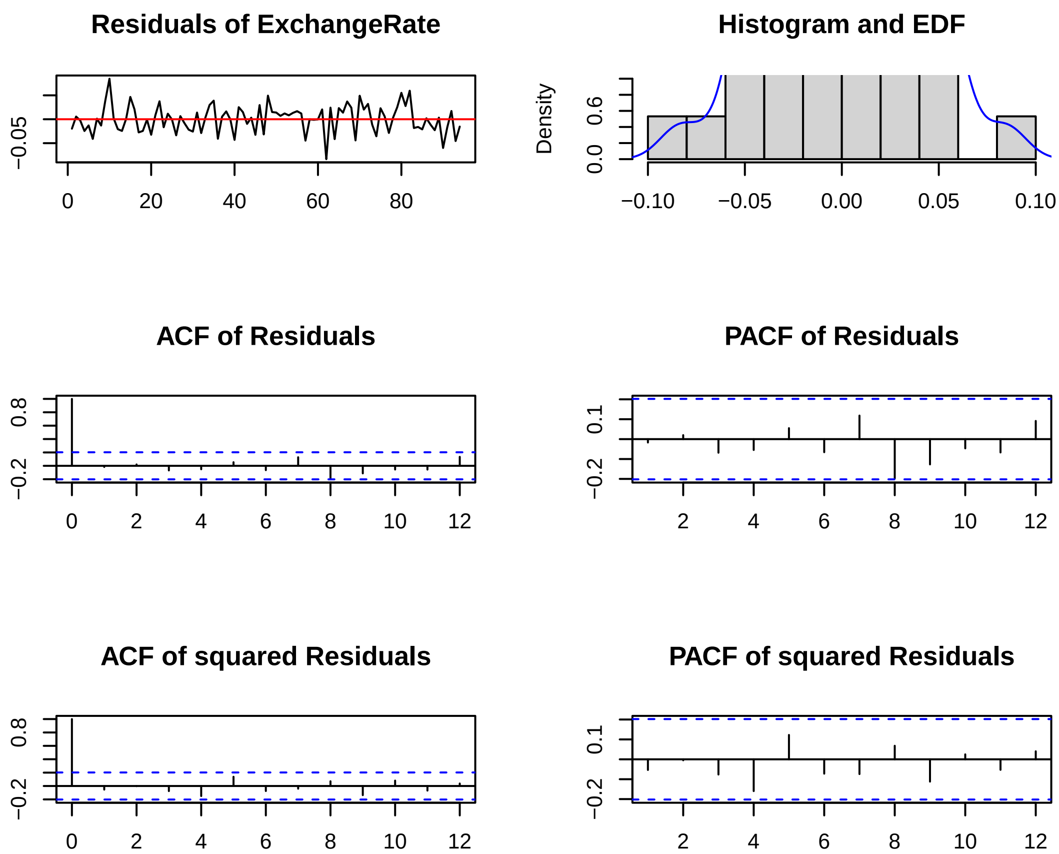

Figure .7: Diagnostics plot of VAR(2) for Exchange Rate

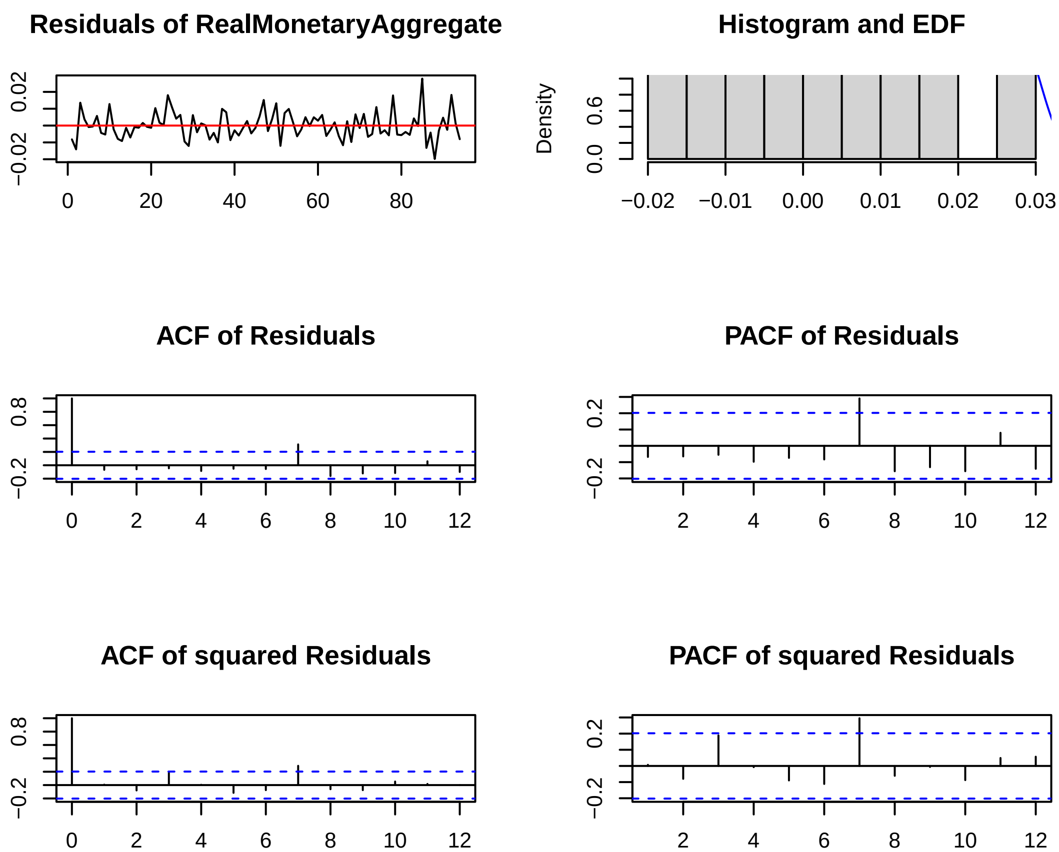

Figure .8: Diagnostics plot of VAR(2) for Real Money

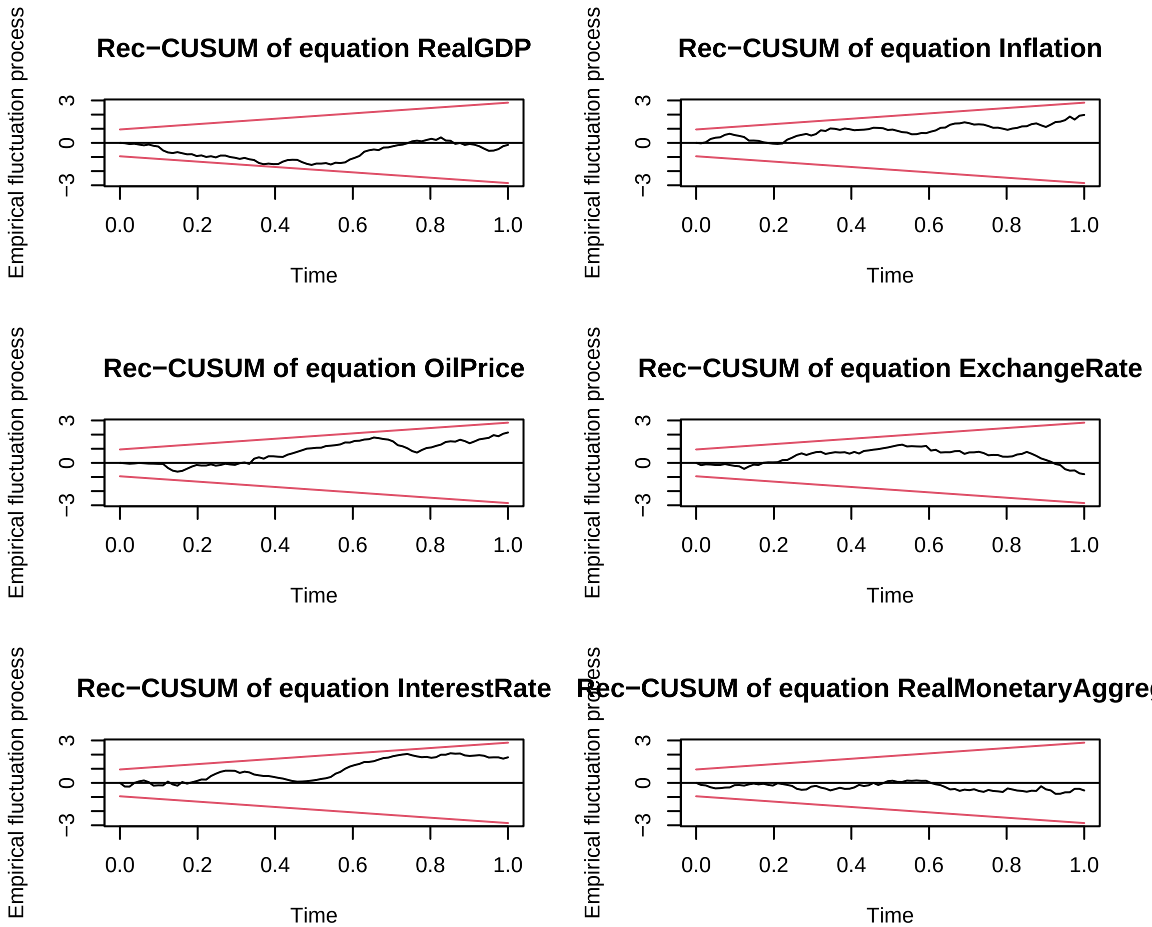

Figure .9: OLS-CUSUM test for VAR(2)

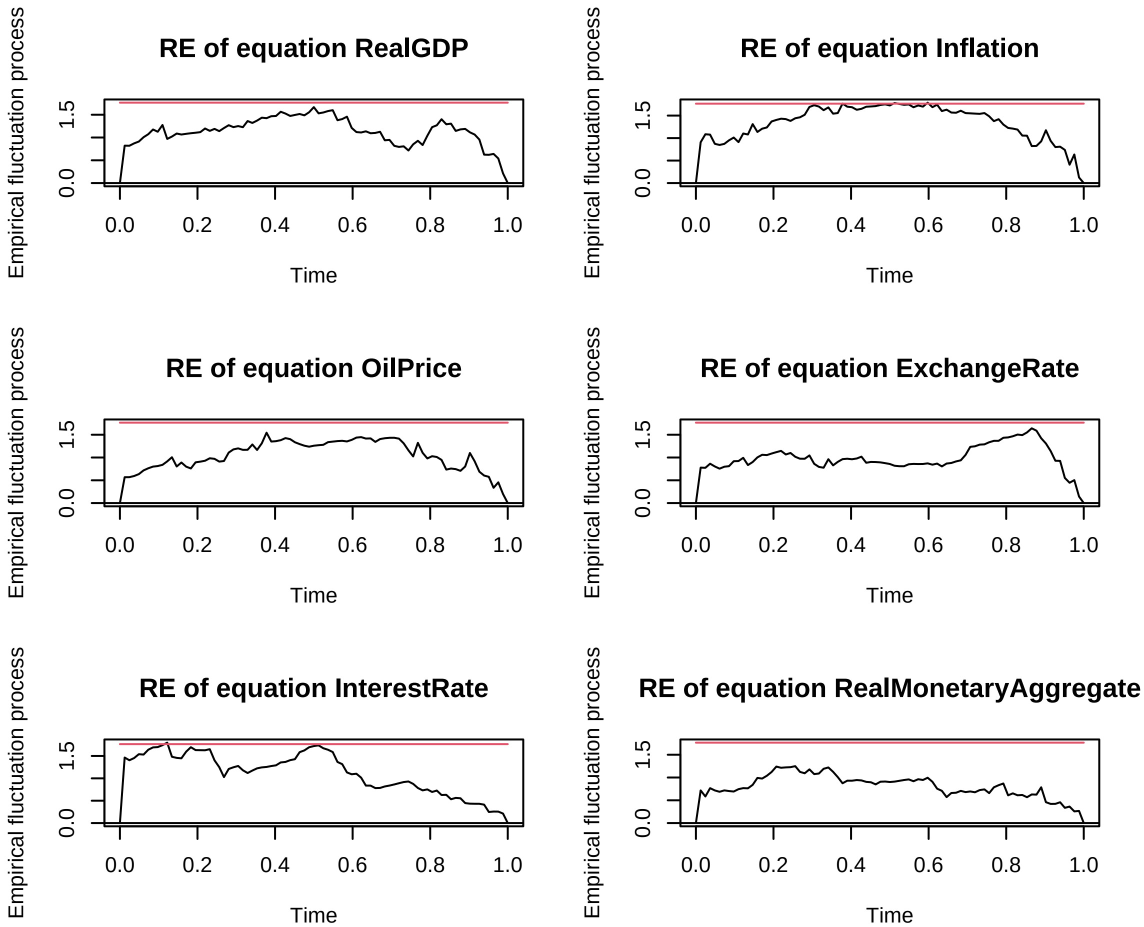

Figure .10: Empirical fluctuation plot for VAR(2)