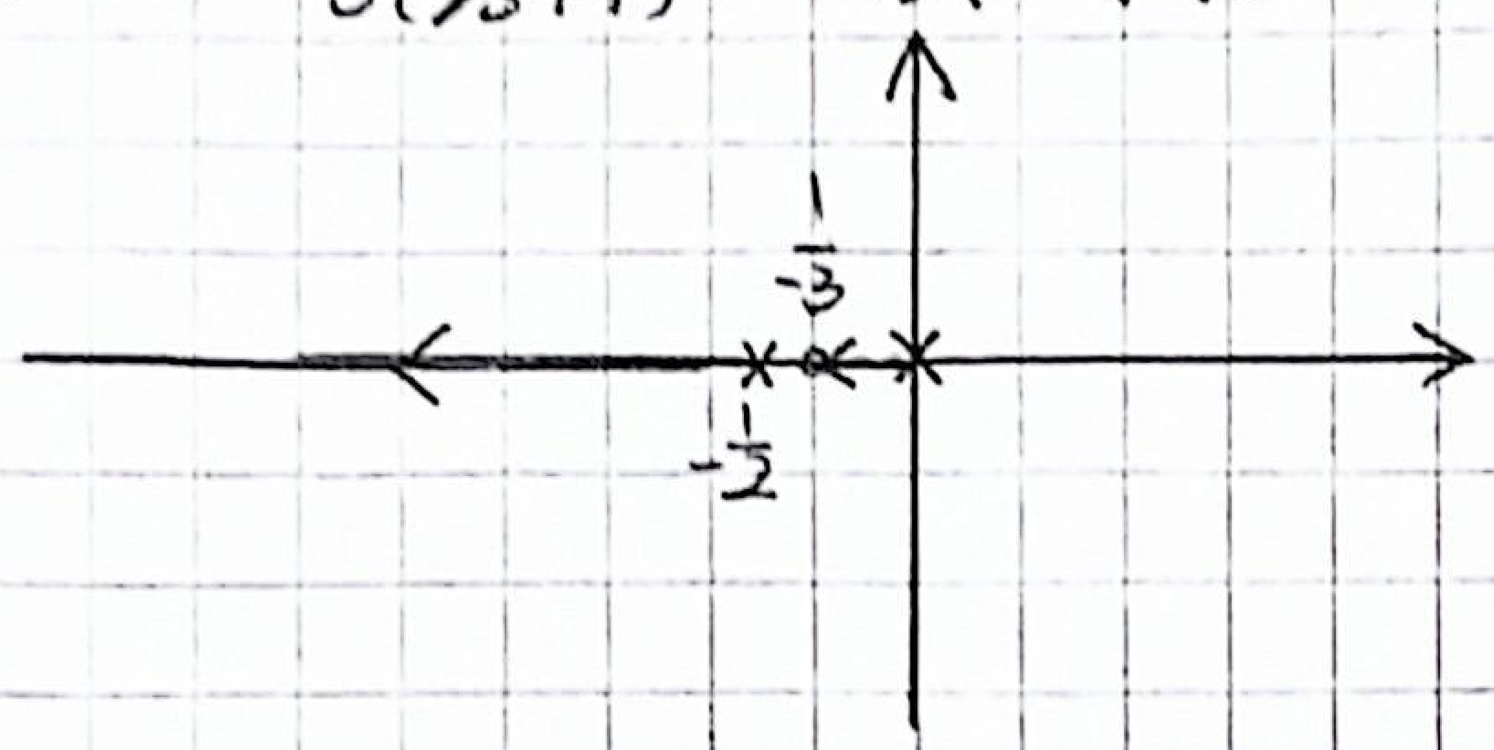
自控第五次作业 孙成 24.04.14

1. $G(s)=\dfrac{K(3s+1)}{s(2s+1)}$ 绘制根轨迹



$(-\infty,-\frac{1}{2}]\cup[-\frac{1}{3},0]$

[此题不写]

2. 单位负反馈传函如下

(1) $G(s)=\dfrac{K^*}{s(s+1)(s+10)}$ 产生纯虚根的开环增益

$$K_0=\frac{1}{10}K^* \quad D(s)=s(s+1)(s+10)+K^*=s^3+11s^2+10s+K^*.$$

解法① $D(j\omega)=0$ 即 $s=j\omega$ 是闭环极点

$$D(j\omega)=-j\omega^3-11\omega^2+10j\omega+K^*=j(10\omega-\omega^3)+(K^*-11\omega^2)$$

$$\begin{cases} j(10\omega-\omega^3)=0 \\ K^*-11\omega^2=0 \end{cases} \Rightarrow \begin{cases} \omega^2=10 \\ K^*=110 \end{cases} \quad K_0=\frac{K^*}{10}=11.$$

解法② 劳斯判据法. 虚轴上极点处于临界稳定.

s^3	1	10
s^2	11	K^*
s^1	$\frac{110-K^*}{11}$	
s^0	K^*	

$\Rightarrow$ 临界稳定 $K^*=110$ 或 $K^*=0$(舍).

$$K_0=\frac{K^*}{10}=11.$$

(2) $G(s)=\dfrac{K^*(s+z)}{s^2(s+10)(s+20)}$ 纯虚根为 $\pm j$ 的 z 值和 K^* 值

$$D(s)=s^4+30s^3+200s^2+K^*s+K^*z.$$

$$D(\pm j)=0 \Rightarrow (K^*z-199)+j(K^*-30)=0 \Rightarrow \begin{cases} K^*=30 \\ z=\frac{199}{30} \end{cases}$$

(3) 绘 $G(s)=\dfrac{K^*}{s(s+1)(s+3.5)(s+3+j2)(s+3-j2)}$ 的闭环根轨迹

$n=5,\ m=0.\ (n-m)=5$ 条分支

$p_1=0,\ p_2=-1,\ p_3=-3.5\quad p_{4,5}=-3\pm j2$

渐近线：$\theta=\frac{(2k+1)\pi}{n-m}=\pm\frac{\pi}{5},\ \pm\frac{3}{5}\pi,\ \pi$

$\sigma=\frac{\sum p_i-\sum z_i}{n-m}=-2.1$

分离点：$D(s)N'(s)-N(s)D'(s)=0 \rightarrow s^5+10.5s^4+43.5s^3+79.5s^2+40.5s$

$D(s)=s(s+1)(s+3.5)(s+3+j2)(s+3-j2) \quad N(s)=1$

$D'(s)=5s^4+42s^3+130.5s^2+159s+40.5 \quad N'(s)=0$

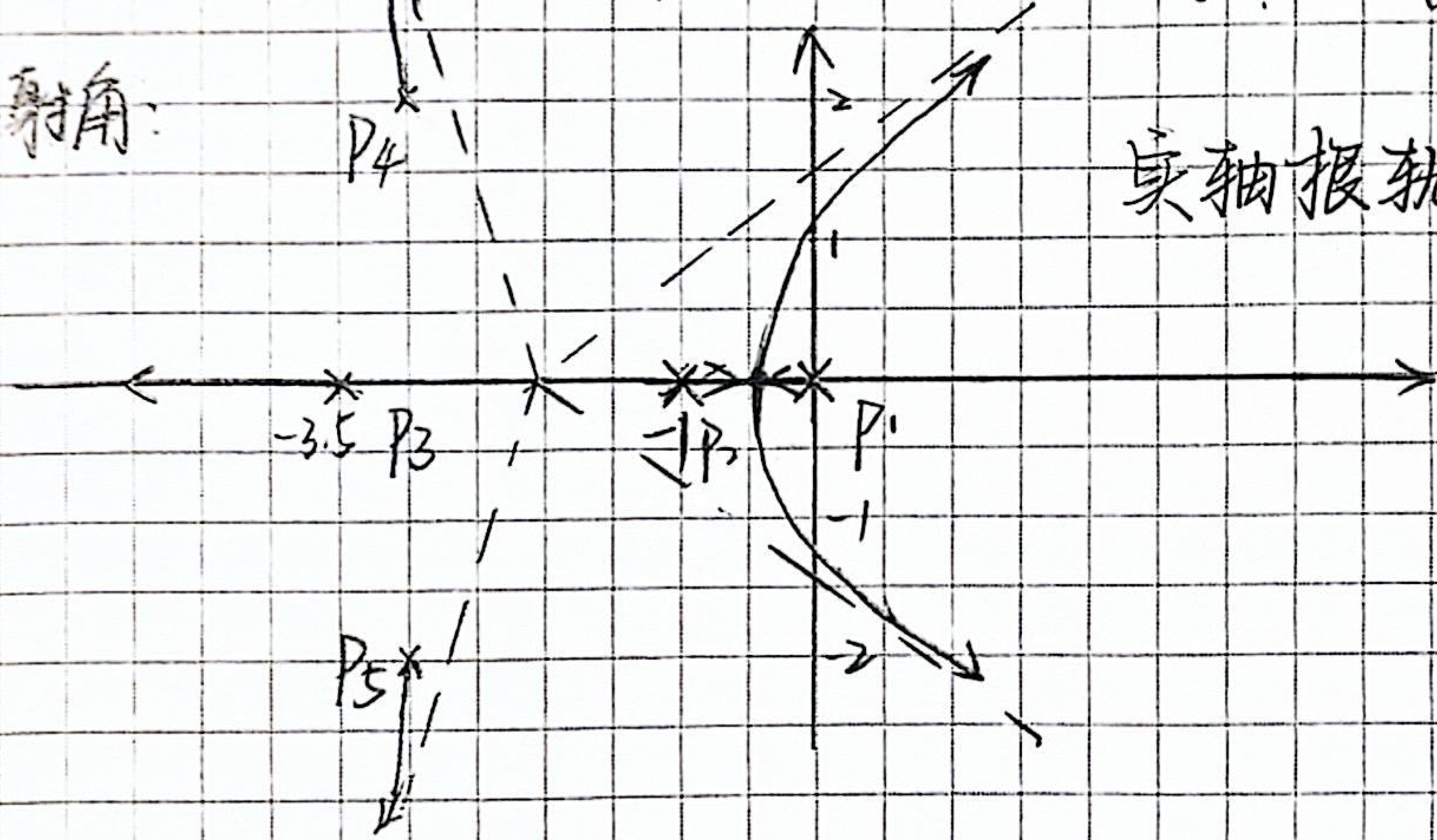
$d_1=-0.34$. $d_2=-2.6$（舍） $d_{3,4}=-2.72\pm j1.31$（舍）.

与虚轴交点：$D(j\omega)=0$.

$(j\omega)^5+10.5(j\omega)^4+43.5(j\omega)^3+79.5(j\omega)^2+40.5(j\omega)=0$

$$\begin{cases}10.5\omega^4-79.5\omega^2+K^*=0\\ \omega^5-43.5\omega^3+45.5=0\end{cases} \Rightarrow \begin{cases}\omega=\pm1.034, & K^*=73.04\\ \omega=\pm6.51\text{（舍）}\end{cases}$$

出射角：



实轴根轨迹：$(-\infty,-3.5)\cup[-1,0]$

$\theta_{p4}=180°-(90°+\arctan1.5)-135°-90°-\arctan4$

$=-92.73°$.

$\theta_{p5}=-92.73°$.

3. $G(s)=\dfrac{k^*(s+2)}{s(s+1)}$ 证明根轨迹以$(-2,0)$为圆心$\sqrt{2}$为半径的圆.

$z_1=-2$. $p_1=0$ $p_2=-1$

此题结论重要. 请牢记

$D(s)=s^2+(k^*+1)s+2k^*=0$

解得 $s_{1,2}=-\frac{1}{2}(k^*+1)\pm j\frac{1}{2}\sqrt{8k^*-(k^*+1)^2}$

即 $x=-\frac{1}{2}(k^*+1)$ $y=\frac{1}{2}\sqrt{8k^*-(k^*+1)^2}$

$\Downarrow$

$k^*=-(2x+1)$ 代入y可得

$4y^2=-8(2x+1)-4x^2 \Rightarrow x^2+4x+2+y^2=0$.

$(x+2)^2+y^2=2$.

即根轨迹以$(-2,0)$为圆心.$\sqrt{2}$为半径.

4. $G(s)=\dfrac{k^*}{s(s+4)(s^2+4s+20)}$ 绘制闭环根轨迹图

$n=4$ $m=0$. $(n-m)=4$条分支. $p_1=0$, $p_2=-4$, $p_{3,4}=-2\pm j4$.

渐近线 $\begin{cases}\theta=\dfrac{(2k+1)\pi}{4}=\pm\dfrac{\pi}{4},\pm\dfrac{3}{4}\pi. \\ \sigma=\dfrac{\sum p_i-\sum z_i}{4}=-2\end{cases}$ 实轴根轨迹$[-4,0]$

分离点. $D(s)=s(s+4)(s^2+4s+20) \rightarrow s^4+8s^3+36s^2+80s$ $N(s)=1$

$D'(s)=4s^3+24s^2+72s+80$ $N'(s)=0$

$D(s)N'(s)-N(s)D'(s)=0$.

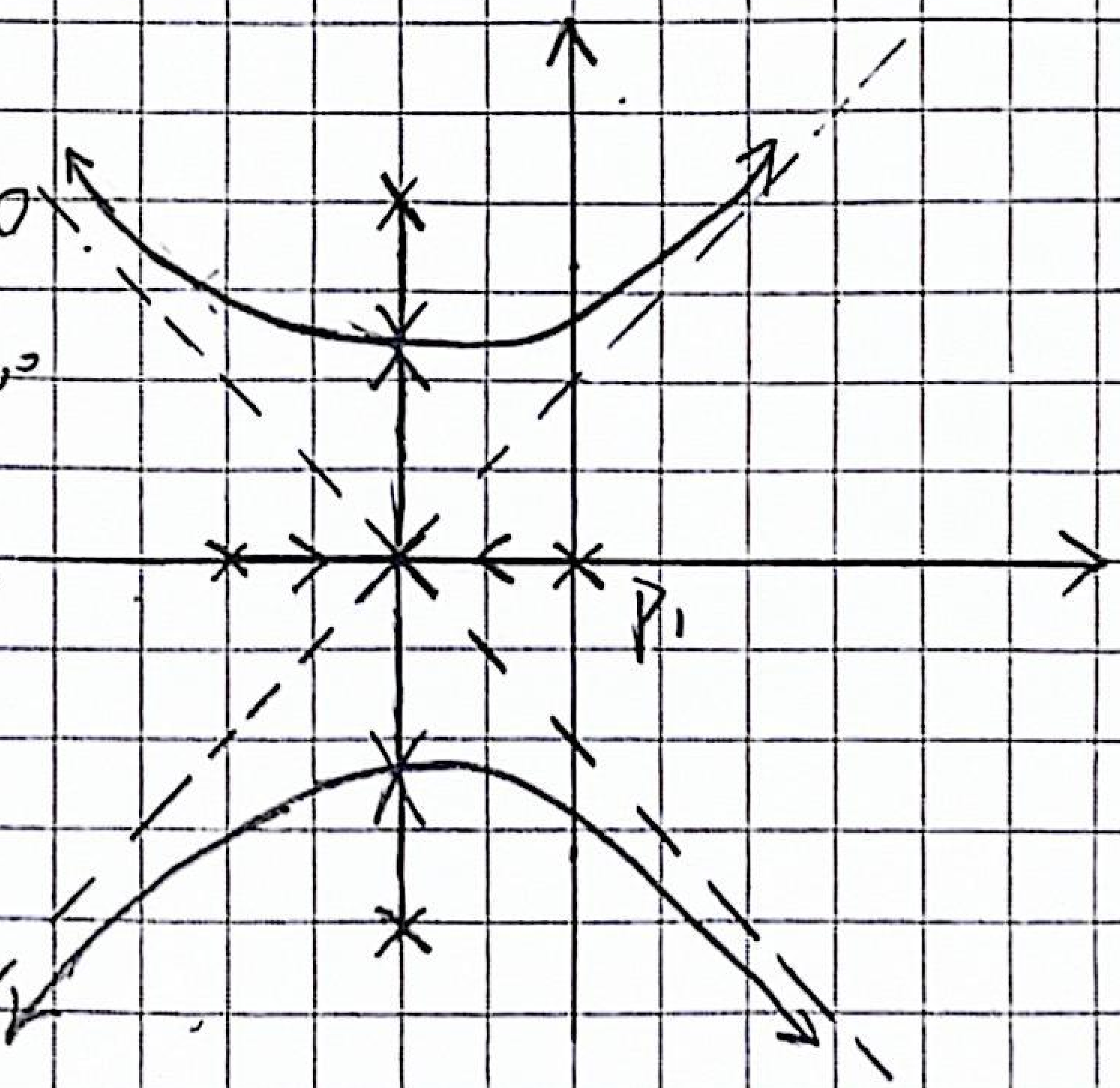
$\Rightarrow d=-2$. $d=-2\pm j\sqrt{6}$

与虚轴交点. $D(j\omega)=0$. $\Rightarrow (j\omega)^4+8(j\omega)^3+36(j\omega)^2+80j\omega+k^*=0$

$\begin{cases}\omega^4-36\omega^2+k^*=0 \\ j(80\omega-8\omega^3)=0\end{cases}$ $\begin{cases}\omega=\pm\sqrt{10} \\ k^*=260\end{cases}$

出射角: $0-(\theta_1+0+\varphi_3+\overset{0^\circ}{\varphi_4})=-180^\circ$ $\theta_1=180^\circ$

同理 易得 $\theta_2=0^\circ$ $\theta_3=-90^\circ$ $\theta_4=90^\circ$.



5. $G(s)=\frac{K^*(s+2)}{(s^2+4s+9)^2}$

$n=4 \quad m=1 \quad (n-m)=3$ 条分支.

$z_1=-2$. $\quad p_{1,2}=-2+j\sqrt{5} \quad p_{3,4}=-2-j\sqrt{5}$. 实轴根轨迹 $(-\infty,-2]$

渐近线: $\begin{cases} \theta=\frac{(2k+1)\pi}{n-m}=\pm\frac{\pi}{3},\pi \\ \sigma=\frac{\sum p_i-\sum z_i}{n-m}=-2 \end{cases}$ $\rightarrow s^4+8s^3+34s^2+72s+81$

分离点: $D(s)=(s^2+4s+9)^2 \qquad N(s)=(s+2)$.

$D'(s)=4s^3+24s^2+68s+72 \quad N'(s)=1$

$D(s)N'(s)-N(s)D'(s)=0$.

$d_{1,2}=-2\pm j\sqrt{5}$ (舍) $d_3=-0.71$ (舍) $d_4=-3.29$

与虚轴交点: $D(j\omega)=0 \Rightarrow (j\omega)^4+8(j\omega)^3+34(j\omega)^2+72j\omega+81+K^*j\omega+2K^*=0$.

$\begin{cases} \omega^4-34\omega^2+81+2K^*=0 \\ j(72\omega+K^*\omega-8\omega^3)=0 \end{cases} \rightarrow \begin{cases} K^*=96 \\ \omega=\pm\sqrt{21} \end{cases}$

注: 此题的分离点更推荐用以下方法.

$$\boxed{\sum\frac{1}{d-z}=\sum\frac{1}{d-p}}$$

$$\frac{2}{d+2+j\sqrt{5}}+\frac{2}{d+2-j\sqrt{5}}=\frac{1}{d+2}$$

$3d^2+12d+7=0$.

$d_1=-0.71$. $d_2=-3.29$.

初射角.

p_1: $90^\circ-(2\theta+90^\circ+90^\circ)=-180^\circ \quad \theta_1=45^\circ$

p_3: $90^\circ-(2\theta+90^\circ+90^\circ)=180^\circ \quad \theta_3=-135^\circ$

p_2: $90^\circ-(2\theta-90^\circ-90^\circ)=-180^\circ \quad \theta_2=-45^\circ$

p_4: $90^\circ-(2\theta-90^\circ-90^\circ)=180^\circ \quad \theta_4=135^\circ$