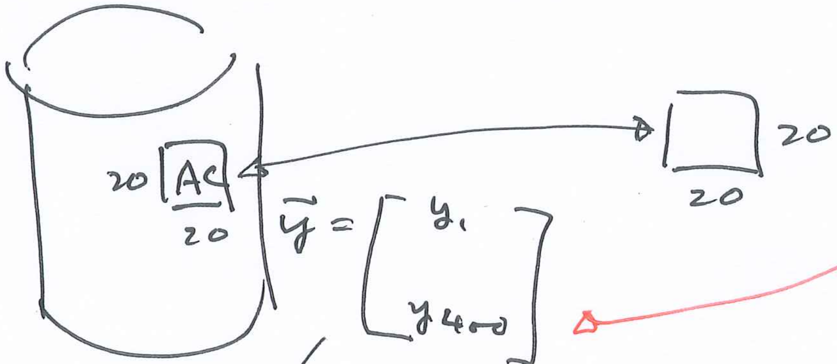
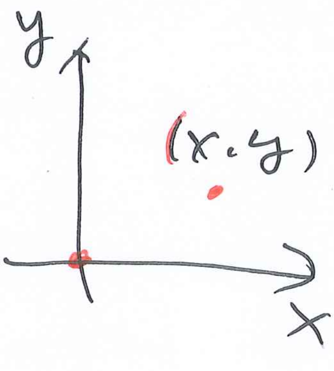
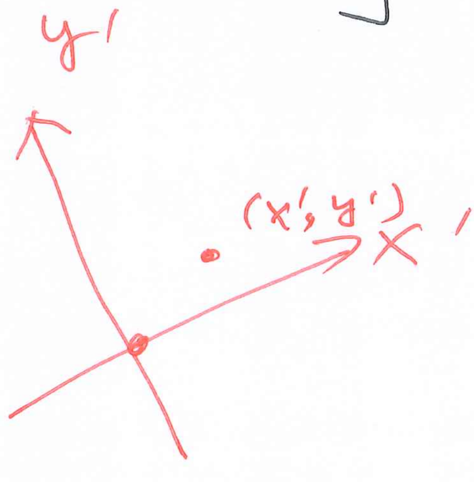


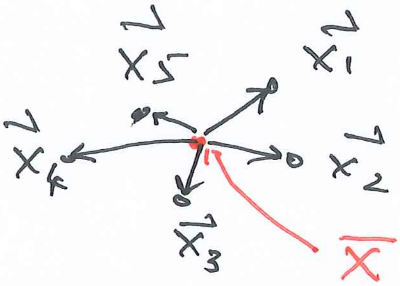

$$\vec{y} = \begin{bmatrix} y_1 \\ \\ y_{400} \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x_1 \\ \\ x_{400} \end{bmatrix}$$

$$D = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots + (x_{400} - y_{400})^2}$$

$$\vec{y}\,' = \begin{bmatrix} y_1 \\ \\ y_{20} \end{bmatrix} \qquad \vec{x}\,' = \begin{bmatrix} x_1 \\ \\ x_{20} \end{bmatrix}$$



$$\underset{2\times 1}{\vec{x}_1 = \begin{bmatrix}1\\1\end{bmatrix}} \qquad \bar{x} = \begin{bmatrix}2\\2\end{bmatrix}$$

$$\vec{x}_2 = \begin{bmatrix}2\\2\end{bmatrix}$$

$$\vec{x}_3 = \begin{bmatrix}3\\3\end{bmatrix}$$

---

$$\overleftrightarrow{S} = \underbrace{(\vec{x}_1-\bar{x})(\vec{x}_1-\bar{x})^T}_{2\times 2} + \overbrace{(\vec{x}_2-\bar{x})(\vec{x}_2-\bar{x})^T}^{2\times 2} + \underbrace{(\vec{x}_3-\bar{x})(\vec{x}_3-\bar{x})^T}_{2\times 2}$$

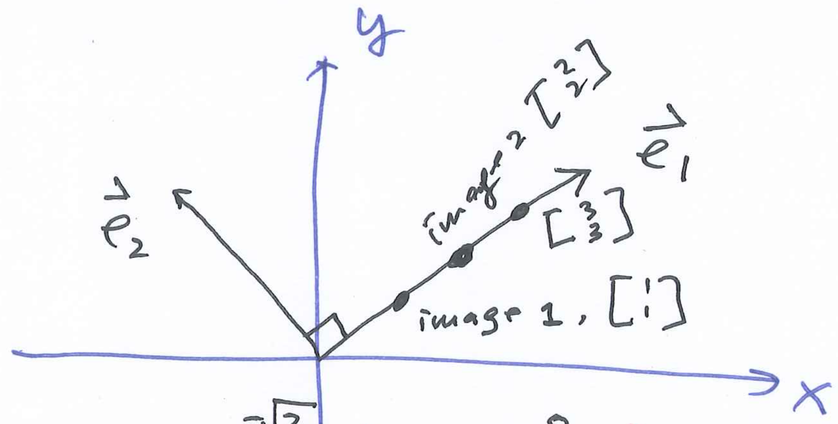
$$= \begin{bmatrix}-1\\-1\end{bmatrix}\begin{bmatrix}-1 & -1\end{bmatrix} + \dots$$

$$= \underset{2\times 2}{\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}} + \dots\dots = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$$

---

eigenvectors $\vec{e}_1 = \begin{bmatrix}0.7071\\0.7071\end{bmatrix}$ ($\frac{1}{\sqrt{2}}$) $\quad \vec{e}_2 = \begin{bmatrix}-0.7071\\0.7071\end{bmatrix}$

eigenvalues $\lambda_1 = 4 \quad \lambda_2 = 0$




$$\vec{x}_1 = \bar{x} + \overset{-\sqrt{2}}{\boxed{g_{11}}}\,\vec{e}_1 + \overset{0}{\boxed{g_{12}}}\,\vec{e}_2$$

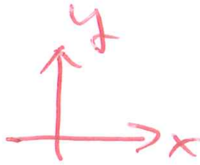
$$g_{11} = (\vec{x}_1-\bar{x})\cdot\vec{e}_1$$

$$= \begin{bmatrix}-1\\-1\end{bmatrix}\cdot\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} = -\sqrt{2}$$

$$g_{12} = (\vec{x}_1-\bar{x})\cdot\vec{e}_2$$

$$= \begin{bmatrix}-1\\-1\end{bmatrix}\cdot\begin{bmatrix}-\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} = 0$$

old representation




$1^{st}$ image $\begin{bmatrix}1\\1\end{bmatrix}$ ; $\vec{x}_1 = (1)\begin{bmatrix}1\\0\end{bmatrix} + (1)\begin{bmatrix}0\\1\end{bmatrix}$

$2^{nd}$ image $\begin{bmatrix}2\\2\end{bmatrix}$ ; $\vec{x}_2 = (2)\begin{bmatrix}1\\0\end{bmatrix} + (2)\begin{bmatrix}0\\1\end{bmatrix}$

$3^{rd}$ image $\begin{bmatrix}3\\3\end{bmatrix}$ ; $\vec{x}_3 = (3)\begin{bmatrix}1\\0\end{bmatrix} + (3)\begin{bmatrix}0\\1\end{bmatrix}$

Old coordinate frame Base $= \left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \right\}$ Basis

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new representation

eigenface

$1^{st}$ image $-\sqrt{2}$ ; $\vec{x}_1 = \bar{x} + \overset{g_{11}}{(-\sqrt{2})}\begin{bmatrix}0.7071\\0.7071\end{bmatrix} + \overset{g_{12}}{(0)}\begin{bmatrix}-0.7071\\0.7071\end{bmatrix}$

$2^{nd}$ image $0$ ; $\vec{x}_2 = \bar{x} + \overset{g_{21}}{(0)}\begin{bmatrix}0.7071\\0.7071\end{bmatrix} + \overset{g_{22}}{(0)}\begin{bmatrix}-0.7071\\0.7071\end{bmatrix}$

$3^{rd}$ image $\sqrt{2}$ ; $\vec{x}_3 = \bar{x} + \overset{g_{31}}{(\sqrt{2})}\begin{bmatrix}0.7071\\0.7071\end{bmatrix} + \overset{g_{32}}{(0)}\begin{bmatrix}-0.7071\\0.7071\end{bmatrix}$

New coordinate frame Base $= \left\{ \begin{bmatrix}0.7071\\0.7071\end{bmatrix}, \begin{bmatrix}-0.7071\\0.7071\end{bmatrix} \right\}$ Basis

$\vec{x}_1 \rightarrow -\sqrt{2}$

$\vec{x}_2 \rightarrow 0$

$\vec{x}_3 \rightarrow \sqrt{2}$