Assignment 2

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1. a. $P(x<=1)=P(X=0)+P(X=1)$.

By the Binomial Probability Distribution table, we have $P(X=0)+P(X=1)\fallingdotseq 0.122+0.270 \fallingdotseq 0.392$.

b. In a. $P(X<=K)=P(X<=1)=0.392$, the level of confidence $=1-P(X<=1)=\underline{0.608}$.

This might not be an acceptable level because this means that there's approximately a 60.8% chance that the true failure rate is not greater than 0.10, which is not high enough to convince people, we often want the value to be 0.90, 0.95 or 0.99, ... etc.

c. (1) Increase the sample size n, compared to a.

If we make the sample size $n=25$, then $P(X<=1)=P(X=0)+P(X=1)\fallingdotseq 0.072+0.199=0.2$

The level of confidence $=1-P(X<=1)=0.729$; larger than 0.608.

(2) Decrease the number K of failures allowed, compared to a.

If we make $K=0$, then $P(X=0)=0.122$. $1-P(X=0)=0.878$, larger than 0.608.

So both increasing the sample size or decreasing the number K can increase the confidence leve

d. For $K=0$, $P(X<=0)=P(X=0)$, $0.9^n>0.05$, $n\log 0.9>\log 0.05$, $n>28.433$, $\underline{n=29}$, so $n=29$ is require

For $K=1$, $P(X<=1)=P(X=0)+P(X=1)=0.9^n+0.9^{n-1}\times 0.1$,

when $n=29$, $P(X<=1)\fallingdotseq 0.0523$,

when $n=30$, $P(X<=1)\fallingdotseq 0.0471$, so $\underline{n=30 \text{ is required.}}$

2. a. By Central Limited Theorem, the distribution $\bar{x}$ $\overset{\text{approximately}}{\wedge}$ fits the normal distribution since $n=50>30$, with

$\text{Mean}=\mu_{\bar{x}}=\underline{\mu=840}$, and

$SD=\underline{\sigma_{\bar{x}}}=\frac{\sigma}{\sqrt{n}}=\frac{15}{\sqrt{50}}\approx\underline{2.121}$.

b. The probability would be $P(Z\leq\frac{830-840}{2.121})\fallingdotseq P(Z\leq-4.714)\fallingdotseq \underline{0.5}-0.5=0$.

c. Since the probability to have a mean of 830 or smaller is extremely small if the true mean is 840, we tend to believe that the true mean is not 840, but some value less than 840.

d. If $\sigma=45N$, then the SD $\sigma_{\bar{x}}=\frac{45}{\sqrt{50}}\fallingdotseq 6.364$.

The probability would be $P(Z\leq\frac{830-840}{6.364})\fallingdotseq P(Z\leq-1.571)\fallingdotseq \underline{0.058}$. #

3. a. Since the sample size is $344 > 30$, by Central Limit Theorem, we know that the sampling distribution of $\bar{x}$ fits the normal distribution with SD $= \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{344}} \fallingdotseq 0.324$, and the mean is unknown.

b. $P(\bar{x}>19.1) = P(Z> \frac{19.1-18.5}{0.324}) \fallingdotseq P(Z>1.852) \fallingdotseq 0.032$.

c. $P(\bar{x}>19.1) = 0.5$, then since $P(Z>0)=0.5$ and $0 = \frac{19.1-\mu}{\sigma_{\bar{x}}}$, $\mu = 19.1$.

d. If $P(\bar{x}>19.1)=0.2$, then $P(Z>\frac{19.1-\mu}{0.324}) = 0.2$.

So, because if $\mu \geq 19.1$, then $P(\bar{x}>19.1) \geq 0.5$, so $\mu < 19.1$. ※

4. a. Since it follows the normal distribution with $\mu = 60$ and $\sigma = 10$, $P(Z \geq \frac{75-60}{10}) = P(Z \geq 1.5) \fallingdotseq 0.067$, so $P(Z<1.5) \fallingdotseq 0.933$, he outperforms 93.3% of people.

b. The probability would be $(\frac{1}{2})^3 = \frac{1}{8} = 12.5\%$.

c. First we calculate the mean $= np = 10000 \times 0.8 = 8000$.

Then $Z_1 = (7960-8000)/40 = -1$,

$Z_2 = (8100-8000)/40 = 2.5$

So, $P(-1 \leq Z \leq 2.5) = 1 - 0.158655 - 0.006210 \fallingdotseq 0.835135 \fallingdotseq 83.514\%$

5. a. By Central Limit Theorem, we can know that assuming the vendor's claim to be true, we have mean $\mu_{\bar{x}} = 157$ psi, and SD, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{40}} \fallingdotseq 0.474$.

Then the probability of mean $= 157 - 1.3 = 155.7$ would be $P(Z \leq \frac{155.7-157}{0.474}) = P(Z \leq -2.743) \fallingdotseq 0.003$.

Since this probability is too small, we tend to believe that the population mean is below 157.

b. If $\mu = 156$, $\sigma = 3$, then the probability $= P(Z \leq \frac{155.7-156}{0.474}) \fallingdotseq P(Z \leq -0.633) \fallingdotseq 26.435\%$

If $\mu = 158$, $\sigma = 3$, then the probability $= P(Z \leq \frac{155.7-158}{0.474}) \fallingdotseq P(Z \leq -4.852) \fallingdotseq 0$.

So the observed sample result would be more likely for $\mu = 156$ than part a, and would be less likely for $\mu = 158$ than part a.

c. If $\mu = 157$, $\sigma = 2$, then the probability $= P(Z \leq \frac{155.7-157}{0.316}) \fallingdotseq P(Z \leq -4.114) \fallingdotseq 0$.

If $\mu = 157$, $\sigma = 6$, then the probability $= P(Z \leq \frac{155.7-157}{0.948}) \fallingdotseq P(Z \leq -1.371) \fallingdotseq 0.085$.

So the sample result would be less likely for $\sigma = 2$ than part a, and more likely for $\sigma = 6$ than part