Statistics - Assignment 2

Announced: 2024/03/29
Due date: 2024/04/25

Finish this assignment and submit it on the E3 system, as a single PDF file (a file with context or photos should be good), by 2024/04/25 23:59. Please be as detailed as possible in your response (Mandarin or English). 可以打字或是手寫拍照貼到pdf上繳交。

Round off to the 3rd decimal place

1. **Reliability of a "one-shot" device.** A "one-shot" device can be used only once; after use, the device (e.g., a nuclear weapon, space shuttle, automobile air bag) is either destroyed or must be rebuilt. The destructive nature of a one-shot device makes repeated testing either impractical or too costly. Hence, the reliability of such a device must be determined with minimal testing. Consider a one-shot device that has some probability, p, of failure. Of course, the true value of p is unknown, so designers will specify a value of p that is the largest defective rate they are willing to accept. Designers will conduct n tests of the device and determine the success or failure of each test. If the number of observed failures, x, is less than or equal to some specified value, K, then the device is considered to have the desired failure rate. Consequently, the designers want to know the minimum sample size n needed so that observing K or fewer defectives in the sample will demonstrate that the true probability of failure for the one-shot device is no greater than p. (25 points)
    a. Suppose the desired failure rate for a one-shot device is p = .10. Also, suppose designers will conduct n = 20 tests of the device and conclude that the device is performing to specifications if K = 1 (i.e., if 1 or no failures are observed in the sample). Find P(x<= 1). (5 points)
    b. In reliability analysis, 1 - P(x<= K) is often called the level of confidence for concluding that the true failure rate is less than or equal to p. Find the level of confidence for the one-shot device described in part a. In your opinion, is this an acceptable level? Explain. (5 points)
    c. Demonstrate that the confidence level can be increased by either (1) increasing the sample size n or (2) decreasing the number K of failures allowed in the sample. (5 points)
    d. Typically, designers want a confidence level of .90, .95, or .99. Find the values of n and K to use so that the designers can conclude (with at least 95% confidence) that the failure rate for the one-shot device of **part a** is no greater than p = .10.（提示：先用**K=0(x=0)**算出一個**n**的初始值，寫出**P(K=1)=P(x<=1)=P(x=0)+P(x=1)**的計算式後，**n**慢慢往上加，可先**+5+5**的試，接近**0.05**後再改成**+1+1**，看加到多少**P**會最接近**0.05**）（**Please use K=0 and 1 in this quesiton**）(10 points)

2. A manufacturer produces safety jackets for competitive fencers. These jackets are rated by the minimum force, in newtons, that will allow a weapon to pierce the jacket. When this process is operating correctly, it produces jackets that have ratings with an average of 840 newtons and a standard deviation of 15 newtons. FIE, the international governing body for fencing, requires jackets to be rated at a minimum of 800 newtons. To check whether the process is operating correctly, a manager takes a sample of 50 jackets from the process, rates them, and calculates $\bar{x}$, the mean rating for jackets in the sample. She assumes that the standard deviation of the process is

fixed but is worried that the mean rating of the process may have changed. (20 points)

a. What is the sampling distribution of $\bar{x}$ (report distribution and parameters) if the process is still operating correctly? (fit ? distribution (and reason?)/ with mean=?, variance=?, please state if any of these are unknown) (5 points)

b. Suppose the manager's sample has a mean rating of 830 newtons. What is the probability of getting an $\bar{x}$ of 830 newtons or lower if the process is operating correctly? (5 points)

c. Given the manager's assumption that the standard deviation of the process is fixed, what does your answer to part b suggest about the current state of the process (i.e., does it appear that the mean jacket rating is still 840 newtons)? (5 points)

d. Now suppose that the mean of the process has not changed, but the standard deviation of the process has increased from 15 newtons to 45 newtons. What is the sampling distribution of $\bar{x}$ in this case? What is the probability of getting an x of 830 newtons or lower when $\bar{x}$ has this distribution? (5 points)

3. Researchers at the Terry College of Business at the University of Georgia sampled 344 business students and asked them this question: "Over the course of your lifetime, what is the maximum number of years you expect to work for any one employer?" The sample resulted in $\bar{x}$ = 19.1 years. Assume that the sample of students was randomly selected from the 6,000 undergraduate students at the Terry College and that σ = 6 years. (20 points)

   a. Describe the sampling distribution of the sample mean $\bar{x}$. (describe -> fit ? distribution (and reason?)/ with mean=?, variance=?, please state if any of these are unknown) (5 points)

   b. If the mean for the 6,000 undergraduate students is μ = 18.5 years, find $P(\bar{x} > 19.1)$. (5 points)

   c. If $P(\bar{x} > 19.1) = .5$, what is μ? (5 points)

   d. If $P(\bar{x} > 19.1) = .2$, is μ greater than or less than 19.1 years? Explain.(5 points)

4. The Advanced Subjects Test in the year 112 was held on July 12th and 13th, with Mark being one of the examinees. His goal was to be admitted into the Computer Science department @NYCU, hence he chose Mathematics A, Physics, and Chemistry as his subjects for the exam (15 points)
   a. In a mock exam held at school, Mark scored 75 in Physics. Assuming the Physics scores across the school follow a normal distribution, with a mean of 60 and a standard deviation of 10, what percentage of people did Mark outperform?
   b. On the day of the major exam, Mark found that none of the topics he was proficient in appeared on the Mathematics A test, and he couldn't solve the problems that were presented. In 3 multiple-choice questions, his calculated answers were not among the options. Fortunately, for each question, he was able to eliminate two options that were definitely incorrect. For the remaining two choices in each question, he decided to flip a fair coin to determine which answer to select. What is the probability that he answers all three questions correctly?
   c. Assuming that this year, the total number of candidates registering for the Chemistry subject is 10,000, and the probability of each person attending the exam is 0.8, what is the probability that the final number of attendees will be between 7,960 and 8,100? Please calculate using the normal distribution approximation method.

5. A soft-drink bottler purchases glass bottles from a vendor. The bottles are required to have an internal pressure of at least 150 pounds per square inch (psi). A prospective bottle vendor claims that its production process yields bottles with a mean internal pressure of 157 psi and a standard deviation of 3 psi. The bottler strikes an agreement with the vendor that permits the bottler to sample from the vendor's production process to verify the vendor's claim. The bottler randomly selects 40 bottles from the last 10,000 produced, measures the internal pressure of each, and finds the mean pressure for the sample to be 1.3 psi below the process mean cited by the vendor. (20 points)
    a. Assuming the vendor's claim to be true, what is the probability of obtaining a sample mean this far or farther below the process mean? What does your answer suggest about the validity of the vendor's claim? (5 points)
    b. If the process standard deviation were 3 psi as claimed by the vendor, but the mean were 156 psi and the sample mean remained the same with question a, would the observed sample result be more or less likely than in part a? What if the mean were 158 psi? (5 points)
    c. If the process mean were 157 psi as claimed, but the process standard deviation were 2 psi, would the sample result be more or less likely than in part a? What if instead the standard deviation were 6 psi? (10 points)