HW4

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1. a. The population of interest is senior human resources executives at US companies.

   b. The population parameter of interest p is the proportion of all senior human resources executives at U.S companies who believe that their hiring managers are interviewing too many people to find qualified candidates for the job.

   Here $\hat{p} = \frac{211}{502} \fallingdotseq 0.420$.

   c. We can calculate $n\hat{p}$ and $n(1-\hat{p})$.

   $n\hat{p} \approx 502 \times 0.420 = 210.84 \geq 15$

   $n(1-\hat{p}) \approx 502 \times 0.580 = 291.16 \geq 15$.

   So both $n\hat{p}$ and $n(1-\hat{p}) \geq 15$, the sample size is large enough to provide reliable estimate of p

   d. The confidence interval is $\left[\hat{p} - Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$.

   We use a confidence level of 98%, so $Z_{\alpha/2} = Z_{0.01} \fallingdotseq 233$.

   The interval is $\left[0.420 - 2.33\sqrt{\frac{0.42 \times 0.58}{502}},\ 0.420 + 2.33\sqrt{\frac{0.42 \times 0.58}{502}}\right] \fallingdotseq [0.351, 0.453]$.

   e. Since $Z_{\alpha/2}$ is lower, the confidence interval would become narrower.

2. a. The point estimate $\hat{p} = 23/244 \fallingdotseq 0.094$.

   b. Calculate $n\hat{p}$ and $n(1-\hat{p})$,

   $n\hat{p} \fallingdotseq 244 \times 0.094 = 22.936 \geq 15$

   $n(1-\hat{p}) \fallingdotseq 244 \times 0.906 = 221.064 \geq 15$.

   So both $n\hat{p}$ and $n(1-\hat{p}) \geq 15$, the sample size is large enough to use the normal approximation.

   C. The confidence interval is $\left[\hat{p} - Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$

   With the confidence level of 95%, $Z_{\alpha/2} = Z_{0.025} \fallingdotseq 1.96$.

   the interval is $\left[0.094 - 1.96\sqrt{\frac{0.094 \times 0.906}{244}},\ 0.094 + 1.96\sqrt{\frac{0.094 \times 0.906}{244}}\right] \fallingdotseq [0.057, 0.131]$.

   d. The point estimate is the sampled chance that an ice cream bar consumer refuse to purchase ice cream 6 months after the outbreak because of the potential of food poisoning.

   There is 95% chance that the chance consumers refuse to purchase ice cream 6 months after the outbreak because of the potential of food poisoning in the whole population is within this interval

3. We can apply t-test, with the interval of the true mean be

$[\bar{x}-t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}, \bar{x}+t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}]$. since we use 99% confidence and 9 blocks per hour, we have $t_{\alpha/2,n-1}=t_{0.005,8}\doteqdot 3.355$, so the interval is $[985.6-3.355\cdot\frac{22.9}{\sqrt{9}}, 985.6+3.355\cdot\frac{22.9}{\sqrt{9}}]$.

$\doteqdot[959.990, 1011.210]$

Since 1000 psi is within this 99% confidence interval, the process can keep going.

4. a. Given the standard deviation is 0.001 inch, assume that the distribution is normal distribution, then $\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}=\frac{0.001}{\sqrt{25}}=0.0002$.

The probability is then $Pr(\frac{-0.0001}{0.0002}\leq\frac{\bar{x}-\mu}{0.0002}\leq\frac{0.0001}{0.0002})=Pr(-0.5\leq Z\leq 0.5)=Pr(Z\leq 0.5)-Pr(Z\leq -0.5)$

$\doteqdot 0.691-0.308=\underline{0.383}$.

b. If the distribution is extremely skewed, the approximation would be inaccurate. also, the CLT would not apply because the sample size $=25<30$

5. a. With 99% confidence interval, we have $\alpha=0.01$, $Z_{\alpha/2}\doteqdot 2.575$,

The confidence interval is then $[\bar{x}-2.575\cdot\frac{S}{\sqrt{n}}, \bar{x}+2.575\cdot\frac{S}{\sqrt{n}}]=[1.13-2.575\cdot\frac{2.21}{\sqrt{72}}, 1.13+2.575\cdot\frac{2.21}{\sqrt{72}}]$

$\doteqdot\underline{[0.459, 1.801]}$.

Thus, there's 99% chance that the actual average number of pecks each chicken took of the blue string over a specified time is between 0.459 and 1.801.

b. Since $\mu=7.5$, the 99% confidence interval is $[0.4594, 1.801]$, there is a strong evidence that chickens are more apt to peak at the white string than the blue string because μ is far from this interval.

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6.
a. $H_0: \mu = 1050.$

$H_1: \mu > 1050.$

b. With the 12 data we have $\bar{x} \doteqdot 2509.431$, $S \doteqdot 2149.263$.

With $\alpha = 0.05$, We have $t_{\alpha, n-1} \doteqdot 1.796$, the lower bound is $2509.431 - 1.796 \cdot \frac{2149.263}{\sqrt{12}} \doteqdot 1395.123$

Thus, $\mu_{\bar{x}} = 1395.123 > 1050$, there is a strong evidence that the frequency program would be profitable for the company if adopted worldwide.

7. Here we let $H_0: \mu = 1220$, $H_1: \mu < 1220$.

With the 10 data, we have $\bar{x} = 989.8$, $s \doteqdot 160.676$.

Assume that $\alpha = 0.05$, we have $t_{\alpha, n-1} \doteqdot 1.833$, then $\bar{x} + t_{\alpha, n-1} \cdot \frac{s}{\sqrt{n}} \doteqdot 989.8 + 1.833 \cdot \frac{160.676}{\sqrt{10}} \doteqdot 1082.935$.

Since $1082.935 < 1220$, H_0 is rejected, there is a strong evidence that the peak-hour pricing succeeded in reducing the average number of vehicles attempting to use the Lincoln Tunnel during the peak rush hour. *