MATLAB Programming NYCU Fall 2024 Assignment#2 due 2024/10/10

Topic: Normal distribution and Mahalanobis distance: You will

(1) Sample some 2-D points from a given normal distribution;
(2) Estimate the parameters of the normal distribution from the samples;
(3) Visualize the distribution using pseudo color;
(4) Draw contours that correspond to various Mahalanobis distances from the normal distribution.

Tasks:

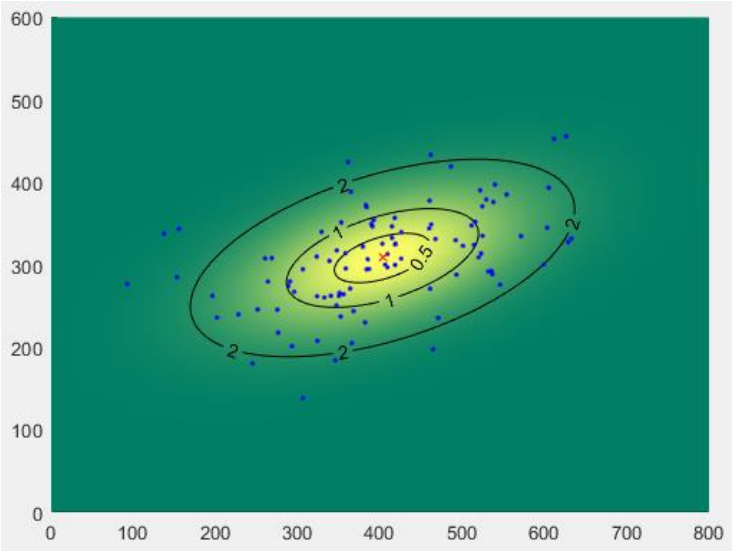
- To create the samples, start by using **`randn`** to get initial x and y coordinates of *N* points that have zero mean and unit standard deviation in both directions. Apply your desired rotation, scaling, and translation to these points to get the samples of a general normal distribution.
- A generalized normal distribution is given by

$$p(\boldsymbol{x}) = \frac{1}{\sqrt{(2\pi)^2 |\Sigma|}} \exp\left(-\frac{1}{2} (\boldsymbol{x}-\boldsymbol{\mu})^T \Sigma^{-1} (\boldsymbol{x}-\boldsymbol{\mu}) \right)$$

  All the vectors are column vectors. Each $\boldsymbol{x}$ here represents a 2-D point. Here $\boldsymbol{\mu}$ is the "mean" and Σ is the "covariance matrix". Compute the mean and covariance matrix of your samples using functions **`mean`** and **`cov`**, respectively. (Note: MATLAB uses row vectors for computing covariance.)
- Mahalanobis distance, defined as

$$\sqrt{(\boldsymbol{x}-\boldsymbol{\mu})^T \Sigma^{-1} (\boldsymbol{x}-\boldsymbol{\mu})} \ ,$$

  is a measure of the "distance" of a point $\boldsymbol{x}$ from a distribution. It reduces to Euclidean distance when the covariance matrix is an identity matrix.
- You need to create the pseudo-color visualization of the distribution $p(\boldsymbol{x})$ using **`imagesc`**. To do this, create a dense 2-D grid of points using **`meshgrid`**, then compute the Mahalanobis distances and $p(\boldsymbol{x})$ for all the grid points. Note: You should be able to do all the computations here without loops.
- Use **`contour`** to generate the contours of several Mahalanobis distances. Apply **`clabel`** to add labels to the contours; see MATLAB documentation for their usages.
- Plot the mean vector using a different symbol.
- Show the display of $p(\boldsymbol{x})$, Mahalanobis distance contours, the mean, and all the sample points all together as in the example shown here. Note: When you draw $p(\boldsymbol{x})$, you need to specify the value range in **`imagesc`**, or otherwise the value range will be affected by the call to **`contour`**.
- Make sure the direction of Y axis is upward, as shown in the example.
- (Parameters/attributes used for the example: Rotation of 20 degrees, scaling of 120 (x) and 50 (y), translation of [400 300], *N*=100, color space 'summer', contours at Mahalanobis distances of [0.5 1 2].)



Submission: Submit your code (m file) through E3. Name your file **`P2_#######.m`**, where the **`#######`** represents your student ID. There will be a three-day grace period after the due date, during which there will be a 10%/day deduction for your grade.

A "copy detection" will be applied to your submissions, and those found to have copied assignments will receive zero points for the assignment.

Your code should include sufficient comments. This will be part of the grade. Include your name and ID at the top of your code.

There will be demo session with the TAs (date/time to be announced later).