1. (a)

$$\times(-0.59)\quad \left[\begin{array}{ccc|c} 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.30 & 1.03 \\ 2.68 & 3.04 & -1.48 & -0.53 \end{array}\right]$$

$$\Rightarrow \times(-1.07)\quad \left[\begin{array}{ccc|c} 2.51 & 1.48 & 4.53 & 0.05 \\ 0 & 0.06 & -3.97 & 1.00 \\ 2.68 & 3.04 & -1.48 & -0.53 \end{array}\right]$$

$$\Rightarrow \times(-24.3)\quad \left[\begin{array}{ccc|c} 2.51 & 1.48 & 4.53 & 0.05 \\ 0 & 0.06 & -3.97 & 1.00 \\ 0 & 1.46 & -6.33 & -0.58 \end{array}\right]$$

$-1.25-2.36$

$$\Rightarrow \left[\begin{array}{ccc|c} 2.51 & 1.48 & 4.53 & 0.05 \\ 0 & 0.06 & -3.97 & 1.00 \\ 0 & 0 & 90.14 & -24.88 \end{array}\right]$$

By backsubstitution. $z \fallingdotseq -0.276$, $y \fallingdotseq -1.595$, $x \fallingdotseq 1.458$

(b)

$$\left[\begin{array}{ccc|c} 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.30 & 1.03 \\ 2.68 & 3.04 & -1.48 & -0.53 \end{array}\right]$$

-0.024

$$\begin{array}{l} \\ \times(-0.94) \\ \times(-0.55) \end{array} \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.53 \\ 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.30 & 1.03 \end{array}\right]$$

$$\Rightarrow \times(-0.54)\quad \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.53 \\ 0 & -1.38 & 5.92 & 0.55 \\ 0 & -0.74 & -0.49 & 1.32 \end{array}\right]$$

$0.40996-4.82144$

$$\Rightarrow \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.53 \\ 0 & -1.38 & 5.92 & 0.55 \\ 0 & 0 & -3.69 & 1.02 \end{array}\right]$$

By back-substitution: $z \fallingdotseq -0.276$, $y \fallingdotseq -1.583$, $x \fallingdotseq 1.445$.

(c)

$$\left[\begin{array}{ccc|c} 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.30 & 1.03 \\ 2.68 & 3.04 & -1.48 & -0.53 \end{array}\right]$$

$$\Rightarrow \begin{array}{l} \\ \times(-0.93) \\ \times(-0.55) \end{array} \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.53 \\ 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.30 & 1.03 \end{array}\right]$$

$$\Rightarrow \times(-0.55)\quad \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.53 \\ 0 & -1.34 & 5.90 & 0.54 \\ 0 & -0.74 & -0.48 & 1.32 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2.68 & 3.04 & -1.48 & -0.53 \\ 0 & -1.34 & 5.90 & 0.54 \\ 0 & 0 & -3.72 & 1.02 \end{array}\right]$$

By back-substitution: $z \fallingdotseq -0.274$, $y \fallingdotseq -1.609$, $x \fallingdotseq 1.476$.

(d)

(a): $\begin{bmatrix} 0.0487 \\ 1.03329 \\ -0.53288 \end{bmatrix}$, |total difference| = 0.00747.

(b): $\begin{bmatrix} 0.03383 \\ 1.02521 \\ -0.53124 \end{bmatrix}$, |total difference| = 0.0222.

(c): $\begin{bmatrix} 0.08227 \\ 1.04431 \\ -0.53016 \end{bmatrix}$, |total difference| = 0.04669. $\Rightarrow$ I don't know why but (a) matches the best.

2.

(a) The code is attached in the file.

(b) $X = \begin{bmatrix} 46.3415 \\ 85.3659 \\ 95.1220 \\ 95.1220 \\ 85.3659 \\ 46.3415 \end{bmatrix}$.

(c) For operational count, In the reduction phase, there are n-1 rows for which there are 2 multiplies and 2 subtracts, which yields a total of 4(n-1) operations.

In the back substitution phase, there is one division in row n. For rows n-1 to row 1, there is on multiplication, subtraction and division, so there are 3(n-1)+1 operations for this phase.

$\Rightarrow 4(n-1)+3(n-1)+1 = 7n-6$.

$\Rightarrow$ There are $\underline{7n-6}$. operations needed.

(a)

Jacobi Method:

$$x^{(k+1)} = -D^{-1} * (L+U) * x^{(k)} + D^{-1}b.$$

$\Rightarrow$ We could get $\underline{x = \begin{bmatrix} -8.9892 \\ -9.4844 \\ 10.0509 \end{bmatrix}}$ after 234 times of iteration, no row exchange need since A is already diagonal dominant

the code is attached in the file.

(b)

Gauss-Seidel Method:

$$x^{(k+1)} = -(L+D)^{-1} * U * x^{(k)} + (LD)^{-1}b.$$

$\Rightarrow$ We could get $\underline{x = \begin{bmatrix} -8.9893 \\ -9.4844 \\ 10.0510 \end{bmatrix}}$ after 112 times of iteration, no row exchange needed.

The code is attached in the file.

$$(D+\omega L)x = \omega b - [\omega U + (\omega-1)D]x$$

$$x^{(k+1)} = (D+\omega L)^{-1}(\omega b - [\omega U + (\omega-1)D]x^{(k)})$$

$$x^{(k+1)} = -(\omega L+D)^{-1}[\omega U + (\omega-1)D]x^{(k)} + (\omega L+D)^{-1}\omega b.$$

$$x^{(k+1)} = -(\omega L+D)^{-1}[\omega(U+D) - D]x^{(k)} + (\omega L+D)^{-1}\omega b$$

After several tests, I found that it can have minimum times of iteration at $\underline{\omega \doteq 1.461, \text{ with only } 22 \text{ times needed}}$

The code is attached in the file.

(a). Condition Number = 10^{20}.

(b). Condition Number = 1.

(c). Condition Number = 1.

(d). Singular. Condition Number = ∞.

$\Rightarrow$ (a)(d) are ill-conditioned.

(b)(c) are well-conditioned.

The code to compute conditional numbers is attached in the file.