Incremental MPC for Flexible Robot Manipulators

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Outline

1. **Model**
2. **TDE**
3. **Incremental MPC**
4. **Simulation & Experiment**
5. **Possible Try**
6. **Timeline**

The dynamic model of the robot with compliant joints

Model

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{w}_l = \mathbf{\Gamma}$$

$$\mathbf{D}\ddot{\boldsymbol{\theta}} + \mathbf{w}_m + \mathbf{\Gamma} = \boldsymbol{\tau}$$

$$\mathbf{\Gamma} = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$$

Approximation of equations using Time-delayed Estimation

Two steps:

1. Separation

Introduce $\bar{\mathbf{M}}$ and $\bar{\mathbf{D}}$;
Rewrite the equation of motion into known and unknown parts

2. Approximation

$$(\mathbf{unknownpart})_{(t-L)} \cong (\mathbf{unknownpart})_{(t)}$$

with L is the delay time

Time-delayed Estimation

1. Introducing $\bar{M}$, we have

$$\bar{M}\cdot\ddot{q} + \underbrace{(M(q)-\bar{M})\ddot{q} + C(q,\dot{q}) + G(q) + W_g}_{H_1} = K(\theta - q)$$

Assuming sampling period L is sufficiently small:

$$H_1 \approx H_1(t-L) = K(\theta_0 - q_0) - \bar{M}\ddot{q}_0$$

$$\Rightarrow \bar{M}\ddot{q} + H_1(t-L) \approx K(\theta - q)$$

$$\Rightarrow \text{incremental system}: \quad \ddot{q} = \ddot{q}_0 + \bar{M}^{-1}K(\Delta\theta - \Delta q) + \varepsilon$$

1. Introducing $\bar{D}$, we have

$$\bar{D}\cdot\ddot{\theta} + \underbrace{(D-\bar{D})\ddot{\theta} + Nm + T}_{H_2} = \tau$$

Assuming sampling period L is sufficiently small:

$$H_2 \approx H_2(t-L) = \tau_0 - \bar{D}\cdot\ddot{\theta}_0$$

$$\Rightarrow \bar{D}\ddot{\theta} + H_2(t-L) \approx \tau$$

$$\Rightarrow \text{incremental system}: \quad \ddot{\theta} = \ddot{\theta}_0 + \bar{D}^{-1}\Delta\tau + \varepsilon$$

Interim conclusion

Approximation based on TDE

$$\ddot{q} = \ddot{q}_0 + \bar{M}^{-1} K (\Delta\theta - \Delta q) + \varepsilon_q$$

$$\ddot{\theta} = \ddot{\theta}_0 + \bar{D}^{-1} \Delta\tau + \varepsilon_\theta$$

Linear system

1. Let $\varepsilon_{\mathbf{x}} = 0$ and $\varepsilon_{\mathbf{q}} = 0$
2. Change continuous to discrete-time form
3. Use Euler method

Linear system

Let $X(k) = \mathrm{col}(q(k), \dot{q}(k), \theta(k))$, then we have

$$X(k+1) = \begin{bmatrix} q(k+1) \\ \dot{q}(k+1) \\ \theta(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} I & T_s \cdot I & 0 \\ 0 & 2I - \bar{M}^{-1} K T_s^2 & \bar{M}^{-1} K T_s^2 \\ 0 & 0 & 2 \end{bmatrix}}_{A_1} \begin{bmatrix} q(k) \\ \dot{q}(k) \\ \theta(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{A_2} \begin{bmatrix} q(k-1) \\ \dot{q}(k-1) \\ \theta(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \bar{D}^{-1} T_s \end{bmatrix}}_{B_1} \Delta\tau$$

$$\Rightarrow \bar{X}(k+1) = \begin{bmatrix} X(k+1) \\ X(k) \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}}_{A} \begin{bmatrix} X(k) \\ X(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 \\ 0 \end{bmatrix}}_{B} \Delta\tau$$

Incremental MPC 1. verison

Predicted joint dynamics error

$$\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right) := \dot{\tilde{\mathbf{q}}}_{k+j+1|k} + \mathbf{K}_{\mathrm{P}}\tilde{\mathbf{q}}_{k+j+1|k}$$

with $\tilde{\mathbf{q}} := \mathbf{q} - \mathbf{q}_{\mathrm{d}}$ tracking error; $\mathbf{K}_{\mathrm{P}} \succ 0$.

Cost function

$$\ell = \underbrace{\left\|\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right)\right\|^2_{\mathbf{Q}}}_{\text{predicted joint dynamics error}} + \underbrace{\left\|\Delta\boldsymbol{\tau}_{k+j|k}\right\|^2_{\mathbf{R}}}_{\text{control signal}}$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 1. version

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell\left(\mathbf{q}_{k+j+1|k}, \dot{\mathbf{q}}_{k+j+1|k}, \Delta\boldsymbol{\tau}_{k+j|k}\right)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

Optimization problem

rewrite into

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \Delta\bar{\tau}^T Q \Delta\bar{\tau} + \Delta\bar{\tau}^T L$$

s.t.

$$G_1 = C_1\Delta\bar{\tau} + D_1 \leq 0$$
$$G_2 = C_2\Delta\bar{\tau} + D_2 \leq 0$$
$$G_3 = C_3\Delta\bar{\tau} + D_3 \leq 0$$

Selected mechanical model

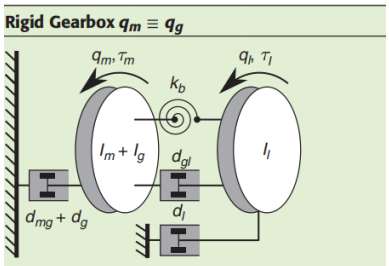

Fig1. Mechanical model

$$\mathbf{I}: \begin{bmatrix} I_m + I_g & 0 \\ 0 & I_l \end{bmatrix} \quad \mathbf{D}: \begin{bmatrix} d_m + d_g + d_{gl} & -d_{gl} \\ -d_{gl} & d_l + d_{gl} \end{bmatrix}$$

$$\mathbf{q}: [q_m \; q_l]^\mathrm{T} \quad \mathbf{K}: \begin{bmatrix} k_b & -k_b \\ -k_b & k_b \end{bmatrix}$$

$$\mathbf{u}_q: [\tau_m \; \tau_l]^\mathrm{T} \quad \mathbf{B}_q: \begin{bmatrix} 0 & 0 & \frac{1}{I_g + I_m} & 0 \\ 0 & 0 & 0 & \frac{1}{I_l} \end{bmatrix}^\mathrm{T}$$

$$\mathbf{A}_q: \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_b}{I_g + I_m} & \frac{k_b}{I_g + I_m} & -\frac{d_m + d_g + d_{gl}}{I_g + I_m} & \frac{d_{gl}}{I_g + I_m} \\ \frac{k_b}{I_l} & -\frac{k_b}{I_l} & \frac{d_{gl}}{I_l} & -\frac{d_l + d_{gl}}{I_l} \end{bmatrix}$$

Fig2. Dynamic terms

Incremental MPC 2. version

Predicted joint dynamics error

$$e(\bar{x}_{k+i|k}) = q_{k+i|k} - q_{d(k+i)}$$

Cost function

$$l(\bar{x}_{k+i|k}, \Delta u_{k+i|k}, k+i) = \|q_{k+i|k} - q_{d(k+i)}\|^2_{Q_1} + \|\dot{q}_{k+i|k} - \dot{q}_{d(k+i)}\|^2_{Q_2} + \|\Delta\tau_{k+i|k}\|^2_R$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 2. version

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} l(\chi_{k+i|k}, \Delta u_{k+i|k}, k+i)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

result using function 1. & 2. version on Rigid Gearbox from Toolbox using classic dynamic parameters

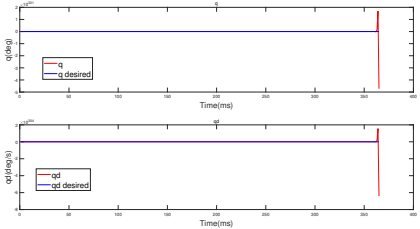

Fig3. Compare desired and is q and qd

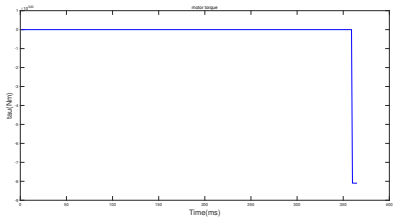

Fig4. motor Torque

used classic dynamic parameters

Actuator mass $m = 1kg$

Motor rotor inertia $I_m = 0.23Nm$

Gear inertia $I_g = 0.0717Nm$

Torsion bar inertia $I_l = 0.0001Nm$

Torsion bar stiffness $K_b = 6000Nm/rad$

Selected mechanical model

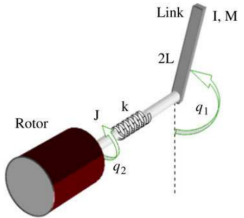

Fig5. Mechanical model [Ghahramani+ 2009]

$$\boldsymbol{I}(\boldsymbol{q}_1)\ddot{\boldsymbol{q}}_1 + \boldsymbol{C}(\boldsymbol{q}_1, \dot{\boldsymbol{q}}_1)\dot{\boldsymbol{q}}_1 + \boldsymbol{g}(\boldsymbol{q}_1) + \boldsymbol{K}(\boldsymbol{q}_1 - \boldsymbol{q}_2) = \boldsymbol{0}$$
$$\boldsymbol{J}\ddot{\boldsymbol{q}}_2 - \boldsymbol{K}(\boldsymbol{q}_1 - \boldsymbol{q}_2) = \boldsymbol{u}$$

where the n-dimensional vectors $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ represent the angles and the actuator angles, respectively, $\boldsymbol{J}$ is a diagonal m of actuator inertias reflected to the link side of the gears, a is an $n \times n$ diagonal matrix which stands for the joint stiffne single degree of freedom flexible joint manipulator introduc Spong [4] is considered as shown in Fig. 2.

The system dynamics are represented as follows:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{MgL}{I}\sin x_1 - \frac{k}{I}(x_1 - x_3)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{k}{J}(x_1 - x_3) + \frac{1}{J}u$$

Fig6. Dynamic

result using function 2. version on mechanical model in reference paper [Ghahramani+ 2009]

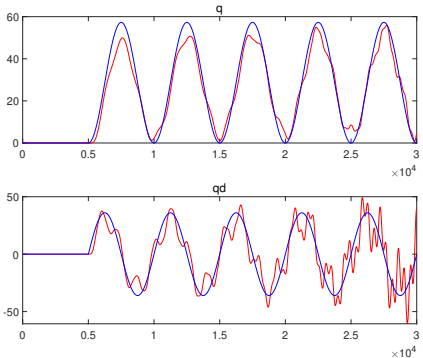

Fig7. Compare desired and is q and qd

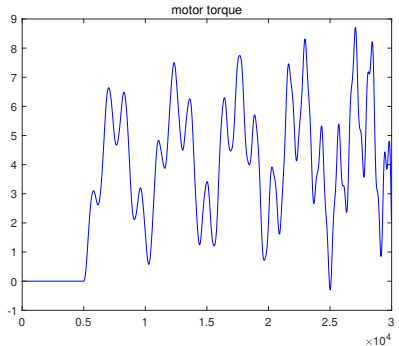

Fig8. motor Torque

used dynamic parameters

Actuator mass $m = 1kg$

Actuator length $l = 1m$

Moment inertia rotor $I_m + I_g = 1Nm$

Moment inertia of link $I_l = 1Nm$

Torsion bar stiffness $K_b = 100Nm/rad$

Incremental MPC 3. version

Predicted joint dynamics error

$$e(\bar{x}_{k+i|k}) = q_{k+i|k} - q_d(k+i)$$

Cost function

$$l(\bar{x}_{k+i|k}, \Delta u_{k+i|k}, k+i) = \| q_{k+i|k} - q_d(k+i) \|_{Q_1}^2 + \| \dot{q}_{k+i|k} - \dot{q}_d(k+i) \|_{Q_2}^2 + \| \Delta\tau_{k+i|k} - \Delta\tau_{ref}(k+i) \|_R^2$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

result using function 3. version on mechanical model in reference paper [Ghahramani+ 2009]

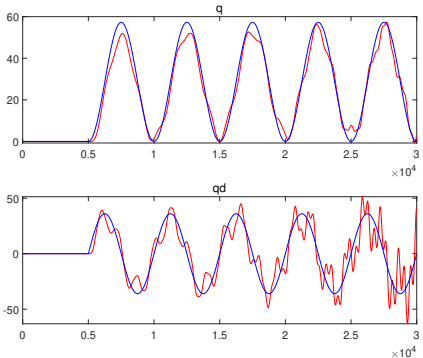

Fig9. Compare desired and is q and qd

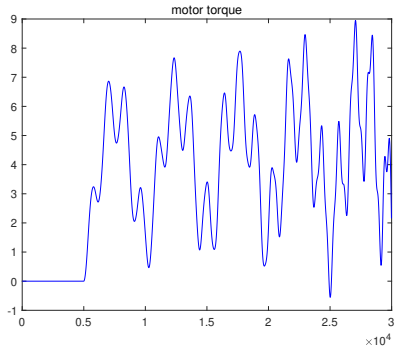

Fig10. motor Torque

Back to dynamic parameters

1. The experiment in [Ghahramani+ 2009] used the dynamic parameters to test step response
$\longrightarrow$ hard to track our desired trajectory because of small stiffness.
2. [Iskandar+ 2022] used the same manipulator with

Actuator mass $m = 1kg$

Actuator length $l = 1m$

Moment inertia rotor $I_m + I_g = 0.598Nm$

Moment inertia of link $I_l = 1Nm$

Torsion bar stiffness $K_b = 362Nm/rad$

result using function k^2. version on mechanical model in [Ghahramani+ 2009] using similar parameter as in [Iskandar+ 2022]

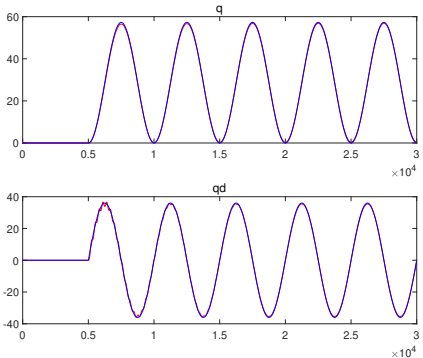

Fig11. Compare desired and is q and qd

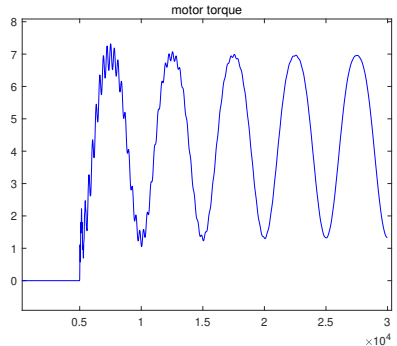

Fig12. motor Torque

result using function [illegible]. version on mechanical model in [Ghahramani+ 2009] using similar parameter as in [Iskandar+ 2022]

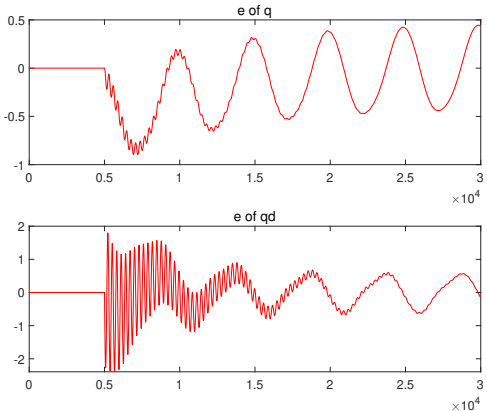

Fig13. Position and velocity error

result using function ^{2}. version on mechanical model in [Ghahramani+ 2009] using similar parameter as in [Iskandar+ 2022]

Np=Nc=10
change to Np=20; Nc=10

result using function ^{2}. version on mechanical model in [Ghahramani+ 2009] using similar parameter as in [Iskandar+ 2022]

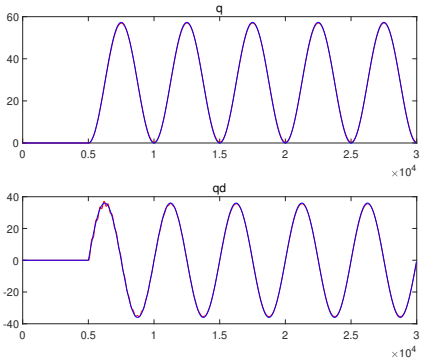

Fig14. Compare desired and is q and qd

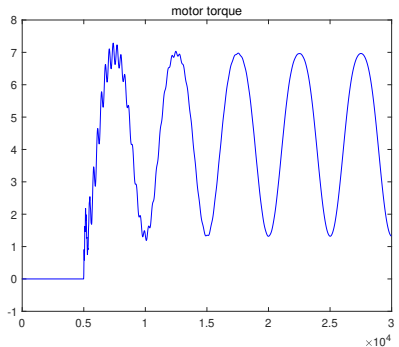

Fig15. motor Torque

result using function $\S$. version on mechanical model in [Ghahramani+ 2009] using similar parameter as in [Iskandar+ 2022]

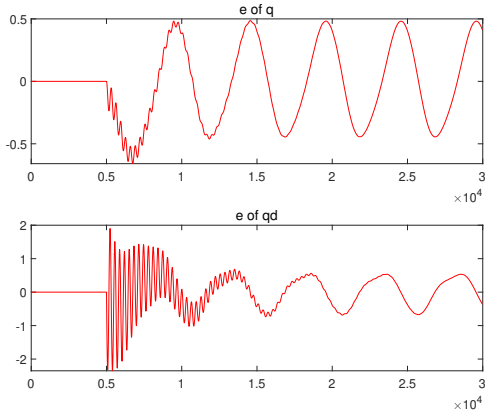

Fig16. Position and velocity error

compare Np=Nc=10 and Np=20;Nc=10

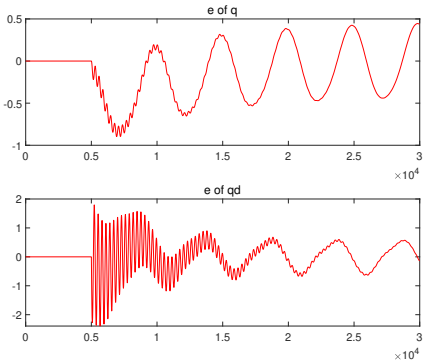

Fig17. Np=Nc=10

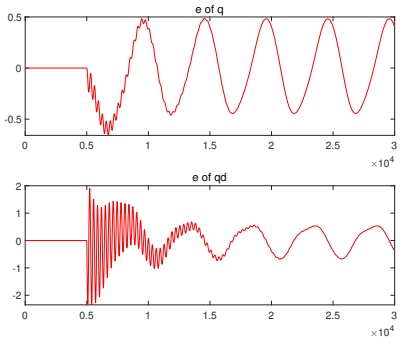

Fig18. Np=20;Nc=10

Find out

Unstable motor torque link torque in first period

Fast-Slow MPC from [Iskandar+ 2022] formulate the dynamic of fast part of link torque
fast part of link torque directly related to local vibrations in the joints
$\longrightarrow$constrain the motor torque with maximum feasible value

Short summary

Find feasible joint dynamic parameters

Less torque vibration using larger Np

Question:
Where to find complete dynamic parameters of our flexible joint robot

Next step:
Develop fast link torque dynamic to constrain it?

Timeline

- **Linear System formulation using TDE:** done
- **Incremental MPC: Cost function and constraints formulation:** still modifying
- **Simulation:** 01.Oktober ~20.November
  Integrate robot manipulator model into simulink
  Comparing the two solvers and different horizon (error and computation time)
- **Experiment:** 10.November ~10.December
  Comparing the two solvers and different horizon (error and computation time)
- **Possible Try:** 10.December ~30.December
  $\bar{M}$ and $\bar{D}$ online update

References

Nemat Ollah Ghahramani and Farzad Towhidkhah. **Constrained incremental predictive controller design for a flexible joint robot**. In: ISA transactions 48.3 (2009), pp. 321–326.

Maged Iskandar, Christiaan van Ommeren, Xuwei Wu, Alin Albu-Schaffer and Alexander Dietrich. **Model Predictive Control for Flexible Joint Robots**. In: arXiv preprint arXiv:2210.08084 (2022).