Incremental MPC for Flexible Robot Manipulators

Tian Zheng
Zwischenbericht/Abschlussbericht Diplomarbeit/Studienarbeit
Supervisor: S. Supervisor

Institute of Automatic Control Engineering
Technical University of Munich

Outline

1. **Model**
2. **TDE**
3. **Incremental MPC**
4. **Simulation & Experiment**
5. **Possible Try**
6. **Timeline**

The dynamic model of the robot with compliant joints

Model

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{w}_l = \mathbf{\Gamma}$$

$$\mathbf{D}\ddot{\boldsymbol{\theta}} + \mathbf{w}_m + \mathbf{\Gamma} = \boldsymbol{\tau}$$

$$\mathbf{\Gamma} = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$$

Approximation of equations using Time-delayed Estimation

Two steps:

1. Separation

Introduce $\bar{\mathbf{M}}$ and $\bar{\mathbf{D}}$;
Rewrite the equation of motion into known and unknown parts

2. Approximation

$$(\mathbf{unknownpart})_{(t-L)} \cong (\mathbf{unknownpart})_{(t)}$$

with L is the delay time

Time-delayed Estimation

Introducing $\bar{\mathbf{M}}$, we have

$$\bar{\mathbf{M}}\ddot{\mathbf{q}} + \underbrace{(\mathbf{M}(\mathbf{q}) - \bar{\mathbf{M}})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{w}_l + \mathbf{K}\mathbf{q}}_{\mathbf{H}_1} = \mathbf{K}\boldsymbol{\theta}$$

Assuming the sampling period L is sufficiently small, we obtain

$$\mathbf{H}_1 \approx (\mathbf{H}_1)_{(t-L)} := \mathbf{K}\boldsymbol{\theta}_0 - \bar{\mathbf{M}}\ddot{\mathbf{q}}_0$$

Then, the following incremental system is obtained:

$$\ddot{\mathbf{q}} = \ddot{\mathbf{q}}_0 + \bar{\mathbf{M}}^{-1}\mathbf{K}\Delta\boldsymbol{\theta} + \boldsymbol{\epsilon}$$

where $\Delta\boldsymbol{\theta} := \boldsymbol{\theta} - \boldsymbol{\theta}_0$, $\boldsymbol{\epsilon}$ is the TDE error.

Introducing $\bar{\mathbf{D}}$, we have

$$\bar{\mathbf{D}}\ddot{\boldsymbol{\theta}} + \underbrace{(\mathbf{D} - \bar{\mathbf{D}})\ddot{\boldsymbol{\theta}} + \mathbf{w}_m + \boldsymbol{\Gamma}}_{\mathbf{H}_2} = \boldsymbol{\tau}$$

Similarly,

$$\mathbf{H}_2 \approx (\mathbf{H}_2)_{(t-L)} := \boldsymbol{\tau}_0 - \bar{\mathbf{D}}\ddot{\boldsymbol{\theta}}_0$$

Finally,

$$\ddot{\boldsymbol{\theta}} = \ddot{\boldsymbol{\theta}}_0 + \bar{\mathbf{D}}^{-1}\Delta\boldsymbol{\tau} + \boldsymbol{\varepsilon}$$

Interim conclusion

Approximation based on TDE

$$\ddot{\mathbf{q}} = \ddot{\mathbf{q}}_{(t-L)} + \bar{\mathbf{M}}^{-1}\mathbf{K}\Delta\boldsymbol{\theta} + \boldsymbol{\varepsilon}_{\mathbf{q}}$$

$$\ddot{\theta} = \ddot{\theta}_{(t-L)} + \bar{\mathbf{D}}^{-1}\Delta\boldsymbol{\tau} + \boldsymbol{\varepsilon}_{\boldsymbol{\theta}}$$

Linear system

1. Let $\boldsymbol{\varepsilon}_{\mathbf{x}} = 0$ and $\boldsymbol{\varepsilon}_{\mathbf{q}} = 0$
2. Change continuous to discrete-time form
3. Use Euler method

Linear system

Let $\mathbf{X}(k) := \mathrm{col}(\mathbf{q}(k), \dot{\mathbf{q}}(k), \dot{\boldsymbol{\theta}}(k))$, then we have

$$\underbrace{\begin{bmatrix} \mathbf{q}(k+1) \\ \dot{\mathbf{q}}(k+1) \\ \dot{\boldsymbol{\theta}}(k+1) \end{bmatrix}}_{\mathbf{X}(k+1)} = \underbrace{\begin{bmatrix} \mathbf{I} & T_s\mathbf{I} & \mathbf{O} \\ \mathbf{O} & 2\mathbf{I} & \bar{\mathbf{M}}^{-1}\mathbf{K}T_s^2 \\ \mathbf{O} & \mathbf{O} & 2\mathbf{I} \end{bmatrix}}_{\mathbf{A}_1} \underbrace{\begin{bmatrix} \mathbf{q}(k) \\ \dot{\mathbf{q}}(k) \\ \dot{\boldsymbol{\theta}}(k) \end{bmatrix}}_{\mathbf{X}(k)} + \underbrace{\begin{bmatrix} \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & -\mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & -\mathbf{I} \end{bmatrix}}_{\mathbf{A}_2} \underbrace{\begin{bmatrix} \mathbf{q}(k-1) \\ \dot{\mathbf{q}}(k-1) \\ \dot{\boldsymbol{\theta}}(k-1) \end{bmatrix}}_{\mathbf{X}(k-1)} + \underbrace{\begin{bmatrix} \mathbf{O} \\ \mathbf{O} \\ \bar{\mathbf{D}}^{-1}T_s \end{bmatrix}}_{\mathbf{B}_1} \Delta\boldsymbol{\tau}$$

Finally, let $\bar{\mathbf{X}}(k) := \mathrm{col}(\mathbf{X}(k), \mathbf{X}(k-1))$, we obtain

$$\underbrace{\begin{bmatrix} \mathbf{X}(k+1) \\ \mathbf{X}(k) \end{bmatrix}}_{\bar{\mathbf{X}}(k+1)} = \underbrace{\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{O} & \mathbf{I} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{X}(k) \\ \mathbf{X}(k-1) \end{bmatrix}}_{\bar{\mathbf{X}}(k)} + \underbrace{\begin{bmatrix} \mathbf{B}_1 \\ \mathbf{O} \end{bmatrix}}_{\mathbf{B}} \Delta\boldsymbol{\tau}$$

Incremental MPC

Predicted joint dynamics error

$$\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right) := \dot{\tilde{\mathbf{q}}}_{k+j+1|k} + \mathbf{K}_{\mathrm{P}}\tilde{\mathbf{q}}_{k+j+1|k}$$

with $\tilde{\mathbf{q}} := \mathbf{q} - \mathbf{q}_{\mathrm{d}}$ tracking error; $\mathbf{K}_{\mathrm{P}} \succ 0$.

Cost function

$$\ell = \underbrace{\left\|\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right)\right\|_{\mathbf{Q}}^{2}}_{\text{predicted joint dynamics error}} + \underbrace{\left\|\Delta\boldsymbol{\tau}_{k+j|k}\right\|_{\mathbf{R}}^{2}}_{\text{control signal}}$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell\left(\mathbf{q}_{k+j+1|k}, \dot{\mathbf{q}}_{k+j+1|k}, \Delta\boldsymbol{\tau}_{k+j|k}\right)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

Optimization problem

rewrite into

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \Delta\bar{\tau}^T Q \Delta\bar{\tau} + \Delta\bar{\tau}^T L$$

$$\text{s.t.}$$

$$G_1 = C_1\Delta\bar{\tau} + D_1 \leq 0$$

$$G_2 = C_2\Delta\bar{\tau} + D_2 \leq 0$$

$$G_3 = C_3\Delta\bar{\tau} + D_3 \leq 0$$

More parameters

Parameter	Value	Unit
$\bar{M}$	?	-
$\bar{D}$	?	-
Weighting matrix of dynamics error Q	?	-
Weighting matrix of control signals R	?	-
Limitation of joint position $q_{\min}, q_{\max}$	$-?,+?$	deg
Limitation of joint velocity $\dot{q}_{\min}, \dot{q}_{\max}$	$-?,+?$	deg/s
Limitation of torque $\tau_{\min}, \tau_{\max}$	$-?,+?$	Nm

solver

QPOASES vs QPSWIFT

$\bar{M}$ and $\bar{D}$ online update

$\bar{M}$ and $\bar{D}$ online update using recursive least square(RLS)

Timeline

- **Linear System formulation using TDE:** done
- **Incremental MPC: Cost function and constraints formulation:** done
- **Simulation:** 01.Oktober ~01.November
  Integrate robot manipulator model into simulink
  Comparing the two solvers and different horizon (error and computation time)
- **Experiment:** 20.Oktober ~20.November
  Comparing the two solvers and different horizon (error and computation time)
- **Possible Try:** 20.November ~10.December
  $\bar{M}$ and $\bar{D}$ online update

References