Incremental MPC for Flexible Robot Manipulators

Tian Zheng
master thesis
Supervisor: Prof Jinoh Lee, Dr. Yongchao Wang

Institute of Automatic Control Engineering
Technical University of Munich

Outline

1. **Model**
2. **TDE**
3. **Incremental MPC**
4. **Simulation & Experiment**
5. **Possible Try**
6. **Timeline**

The dynamic model of the robot with compliant joints

Model

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{w}_l = \boldsymbol{\Gamma}$$

$$\mathbf{D}\ddot{\boldsymbol{\theta}} + \mathbf{w}_m + \boldsymbol{\Gamma} = \boldsymbol{\tau}$$

$$\boldsymbol{\Gamma} = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$$

Approximation of equations using Time-delayed Estimation

Two steps:

1. Separation

Introduce $\bar{\mathbf{M}}$ and $\bar{\mathbf{D}}$;
Rewrite the equation of motion into known and unknown parts

2. Approximation

$$(\mathbf{unknownpart})_{(t-L)} \cong (\mathbf{unknownpart})_{(t)}$$

with L is the delay time

Time-delayed Estimation

1. Introducing $\bar{M}$, we have

$$\bar{M}\cdot\ddot{q}+\underbrace{(M(q)-\bar{M})\ddot{q}+C(q,\dot{q})+G(q)+W_e}_{H_1}=K(\theta-q)$$

Assuming sampling period L is sufficiently small:

$$H_1\approx H_1(t-L)=K(\theta_0-q_0)-\bar{M}\ddot{q}_0$$

$$\Rightarrow \bar{M}\ddot{q}+H_1(t-L)\approx K(\theta-q)$$

$\Rightarrow$ incremental system:

$$\ddot{q}=\ddot{q}_0+\bar{M}^{-1}K(\Delta\theta-\Delta q)+\varepsilon$$

2. Introducing $\bar{D}$, we have

$$\bar{D}\cdot\ddot{\theta}+\underbrace{(D-\bar{D})\ddot{\theta}+W_m+\Gamma}_{H_2}=\tau$$

Assuming sampling period L is sufficiently small:

$$H_2\approx H_2(t-L)=\tau_0-\bar{D}\cdot\ddot{\theta}_0$$

$$\Rightarrow \bar{D}\ddot{\theta}+H_2(t-L)\approx\tau$$

$\Rightarrow$ incremental system:

$$\ddot{\theta}=\ddot{\theta}_0+\bar{D}^{-1}\Delta\tau+\varepsilon$$

Interim conclusion

Approximation based on TDE

$$\ddot{q} = \ddot{q}_0 + \bar{M}^{-1}K(\Delta\theta - \Delta q) + \varepsilon_q$$

$$\ddot{\theta} = \ddot{\theta}_0 + \bar{D}^{-1}\Delta\tau + \varepsilon_\theta$$

Linear system

1. Let $\varepsilon_{\mathbf{x}} = 0$ and $\varepsilon_{\mathbf{q}} = 0$

2. Change continuous to discrete-time form

3. Use Euler method

Linear system

Let $X(k) = \mathrm{col}\,(q(k), \dot{q}(k), \theta(k))$, then we have

$$X(k+1) = \begin{bmatrix} q(k+1) \\ \dot{q}(k+1) \\ \dot{\theta}(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} I & T_s \cdot I & 0 \\ 0 & 2I - \bar{M}^{-1} K T_s^2 & \bar{M}^{-1} K T_s^2 \\ 0 & 0 & 2 \end{bmatrix}}_{A_1} \begin{bmatrix} q(k) \\ \dot{q}(k) \\ \dot{\theta}(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}}_{A_2} \begin{bmatrix} q(k-1) \\ \dot{q}(k-1) \\ \dot{\theta}(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \bar{D}^{-1} T_s \end{bmatrix}}_{B_1} \Delta\tau$$

$$\Rightarrow \bar{X}(k+1) = \begin{bmatrix} X(k+1) \\ X(k) \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}}_{A} \begin{bmatrix} X(k) \\ X(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 \\ 0 \end{bmatrix}}_{B} \Delta\tau$$

Incremental MPC 1. verison

Predicted joint dynamics error

$$\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right) := \dot{\tilde{\mathbf{q}}}_{k+j+1|k} + \mathbf{K}_{\mathrm{P}}\tilde{\mathbf{q}}_{k+j+1|k}$$

with $\tilde{\mathbf{q}} := \mathbf{q} - \mathbf{q}_{\mathrm{d}}$ tracking error; $\mathbf{K}_{\mathrm{P}} \succ 0$.

Cost function

$$\ell = \underbrace{\left\|\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right)\right\|^2_{\mathbf{Q}}}_{\text{predicted joint dynamics error}} + \underbrace{\left\|\Delta\boldsymbol{\tau}_{k+j|k}\right\|^2_{\mathbf{R}}}_{\text{control signal}}$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 1. version

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell\left(\mathbf{q}_{k+j+1|k}, \dot{\mathbf{q}}_{k+j+1|k}, \Delta\boldsymbol{\tau}_{k+j|k}\right)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

Optimization problem

rewrite into

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \Delta\bar{\tau}^T Q \Delta\bar{\tau} + \Delta\bar{\tau}^T L$$

$$\text{s.t.}$$

$$G_1 = C_1 \Delta\bar{\tau} + D_1 \leq 0$$

$$G_2 = C_2 \Delta\bar{\tau} + D_2 \leq 0$$

$$G_3 = C_3 \Delta\bar{\tau} + D_3 \leq 0$$

Incremental MPC 2. Version

Predicted joint dynamics error

$$e(\bar{x}_{k+i|k}) = q_{k+i|k} - q_{d(k+i)}$$

Cost function

$$\ell(\bar{x}_{k+i|k}, \Delta u_{k+i|k}, k+i) = \| q_{k+i|k} - q_{d(k+i)} \|^2_{Q_1} + \| \dot{q}_{k+i|k} - \dot{q}_{d(k+i)} \|^2_{Q_2} + \| \Delta \tau_{k+i|k} \|^2_{R}$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 2. Version

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell\left(x_{k+i|k},\ \Delta u_{k+i|k},\ k+i \right)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

Recap

1. plant dynamics set as the complete one in toolbox by rewriting the equations instead of modify the block in toolbox
2. better tracking performance using 5th order polynomial reference than sinus wave reference:
no more significant fluctuate at the beginning
3. computing time and performance under different prediction horizon
4. find error in Simulink while testing step response

Progress No.1

debugged the error in Simulink:
wrong index of reference in Simulink block

Reference trajectory

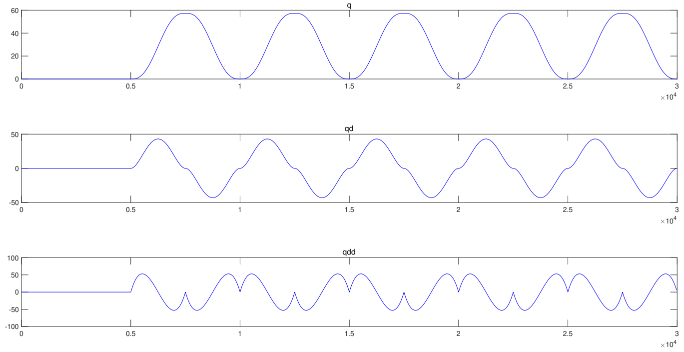
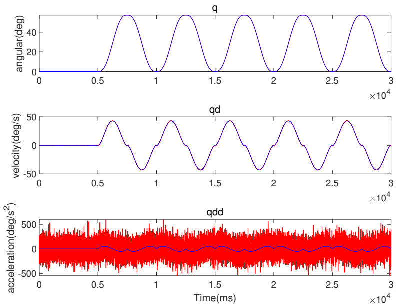
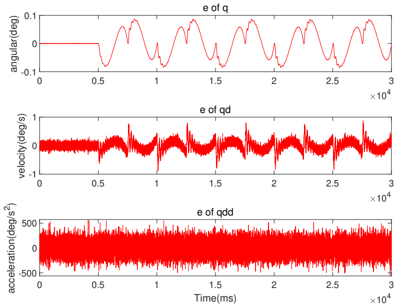


Fig3. reference trajectory

Performance under 5th order polynomial reference



World Map



Concrete and Constructions

Reference trajectory

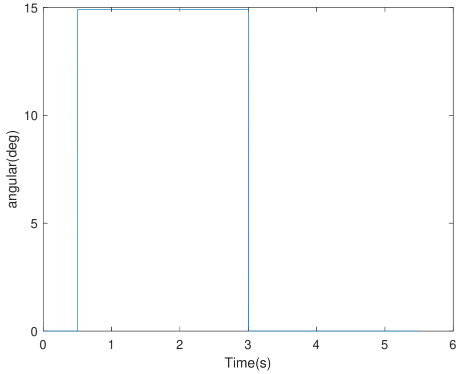
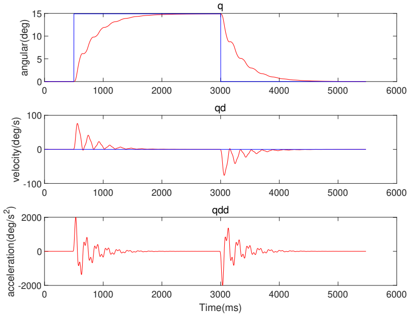
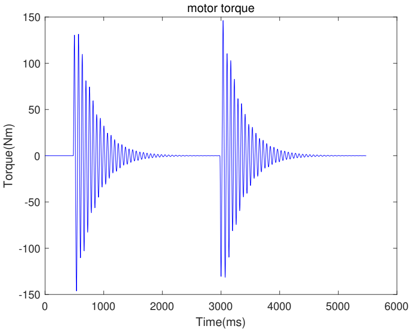


Fig3. reference trajectory

Performance under step reference



World Map



Concrete and Constructions

Progress No.2

performance test under constraints:
(5th order polynomial as reference)
5 different scenarios

scenario No.	Angular constraint	Velocity constraint	Input constraint
1	114.59 deg	114.59 deg/s	3 Nm
2	40 deg	114.59 deg/s	3 Nm
3	114.59 deg	30 deg/s	3 Nm
4	114.59 deg	114.59 deg/s	2.3 Nm
5	40 deg	114.59 deg/s	2.3 Nm

Scenario No.1: Angular constraint:114.59 deg;Velocity constraint:114.59 deg/s; Input constraint:3 Nm

trade-off between computation time and accuracy

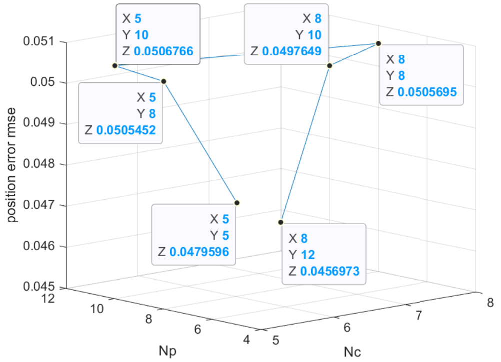


Fig3. max extended amount

Scenario No.1: Angular constraint:114.59 deg;Velocity constraint:114.59 deg/s; Input constraint:3 Nm

trade-off between computation time and accuracy

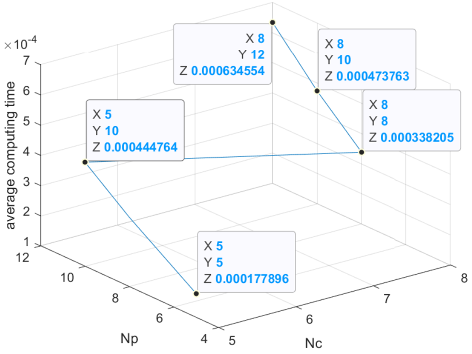
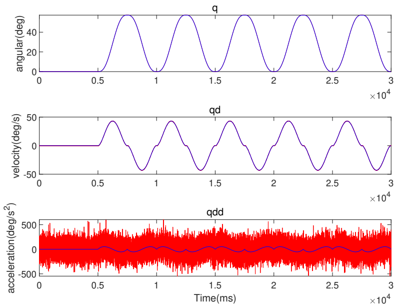


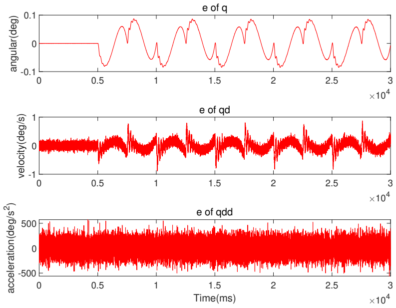
Fig3. max extended amount

Scenario No.1: Angular constraint:114.59 deg;Velocity constraint:114.59 deg/s; Input constraint:3 Nm

best performance under best horizon:



before



added x0 error to cost function

Scenario No.1: Angular constraint:114.59 deg;Velocity constraint:114.59 deg/s; Input constraint:3 Nm

best performance under best horizon:

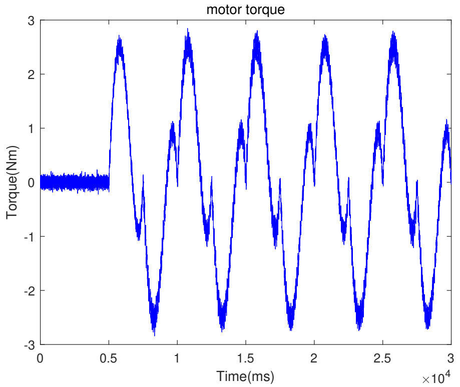


Fig3. max extended amount

Scenario No.2: Angular constraint:40 deg;Velocity constraint:114.59 deg/s; Input constraint:3 Nm

trade-off between computation time and accuracy

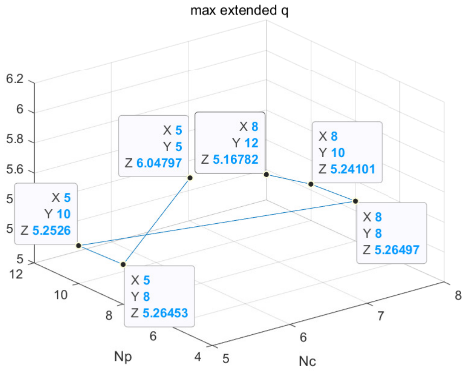


Fig3. max extended amount

Scenario No.2: Angular constraint:40 deg;Velocity constraint:114.59 deg/s; Input constraint:3 Nm

trade-off between computation time and accuracy

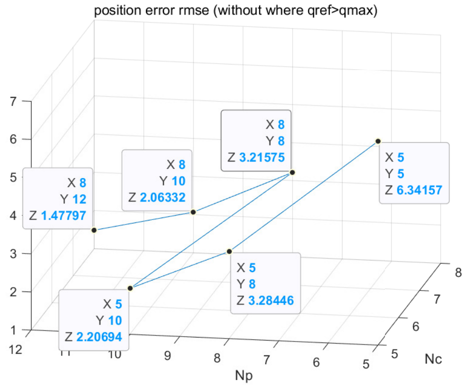


Fig3. max extended amount

Scenario No.2: Angular constraint:40 deg;Velocity constraint:114.59 deg/s; Input constraint:3 Nm

trade-off between computation time and accuracy

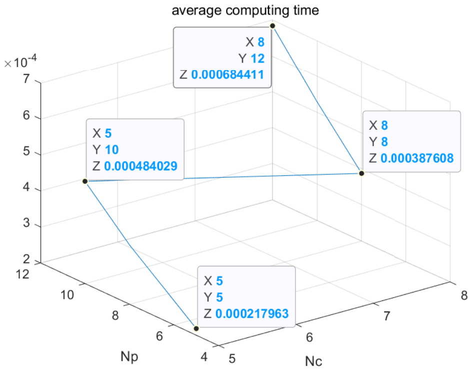
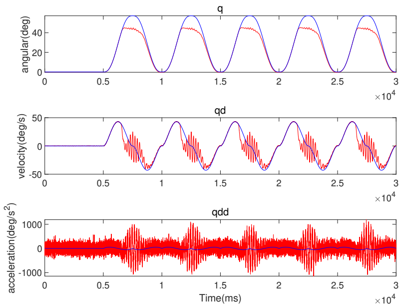


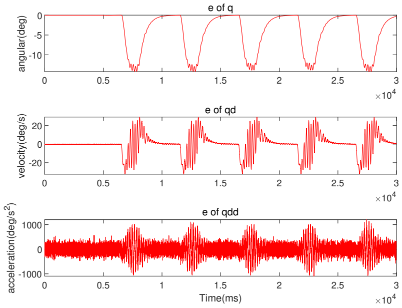
Fig3. max extended amount

Scenario No.2: Angular constraint:40 deg;Velocity constraint:114.59 deg/s; Input constraint:3 Nm

best performance under best horizon: Np=10;Nc=5



before



added x0 error to cost function

Scenario No.2: Angular constraint:40 deg;Velocity constraint:114.59 deg/s; Input constraint:3 Nm

best performance under best horizon: Np=10;Nc=5

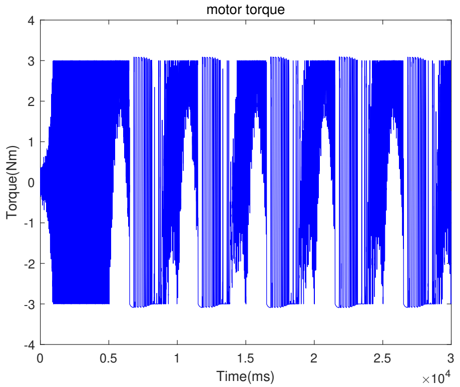


Fig3. max extended amount

Scenario No.3: Angular constraint:114.59 deg;Velocity constraint:30 deg/s; Input constraint:3 Nm

trade-off between computation time and accuracy

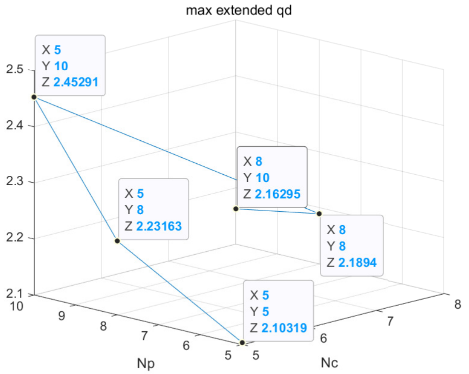


Fig3. max extended amount

Scenario No.3: Angular constraint:114.59 deg;Velocity constraint:30 deg/s; Input constraint:3 Nm

trade-off between computation time and accuracy

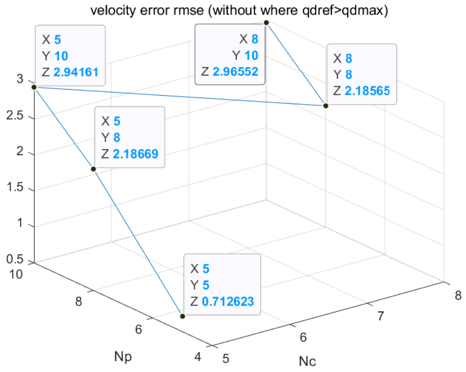


Fig3. max extended amount

Scenario No.3: Angular constraint:114.59 deg;Velocity constraint:30 deg/s; Input constraint:3 Nm

trade-off between computation time and accuracy

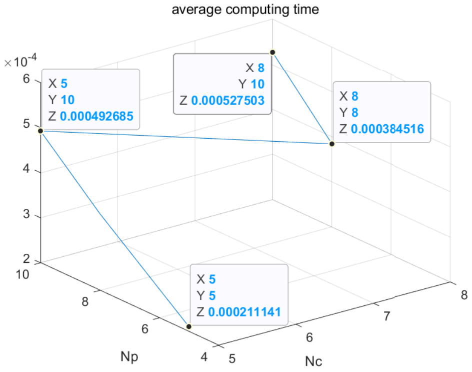
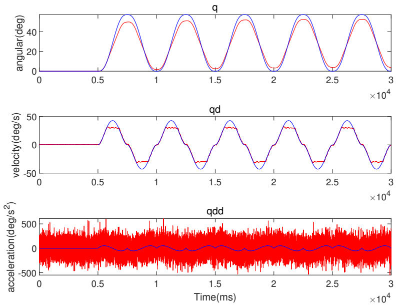


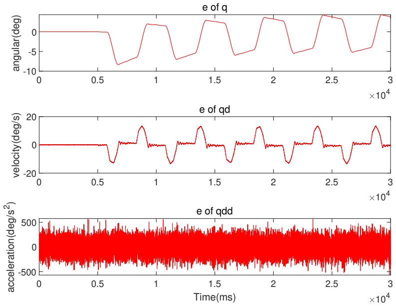
Fig3. max extended amount

Scenario No.3: Angular constraint:114.59 deg;Velocity constraint:30 deg/s; Input constraint:3 Nm

best performance under best horizon: Np=Nc=5



before



added x0 error to cost function

Scenario No.3: Angular constraint:114.59 deg;Velocity constraint:30 deg/s; Input constraint:3 Nm

best performance under best horizon: Np=Nc=5

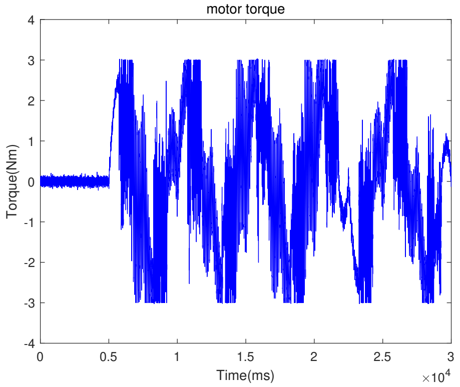
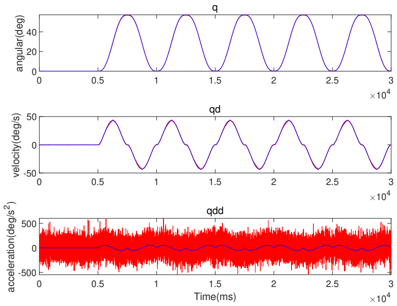


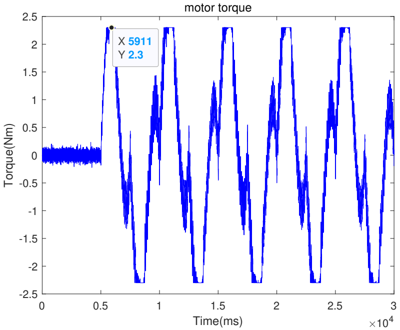
Fig3. max extended amount

Scenario No.4: Angular constraint:114.59 deg;Velocity constraint:114.59 deg/s; Input constraint:2.3 Nm

trade-off between computation time and accuracy:
Well performance under all kind of horizon combination mentioned in first two scenarios



before



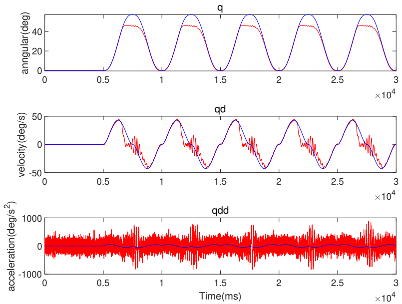
added x0 error to cost function

Scenario No.5: Angular constraint:40 deg;Velocity constraint:114.59 deg/s; Input constraint:2.3 Nm

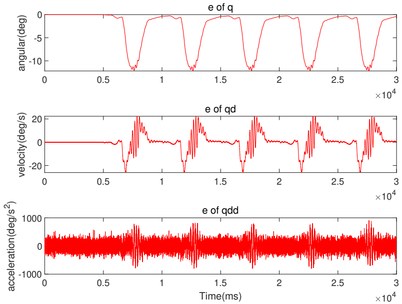
trade-off between computation time and accuracy:
Similar to the scenario No.2

Scenario No.5: Angular constraint:40 deg;Velocity constraint:114.59 deg/s; Input constraint:2.3 Nm

best performance under best horizon: Np10; Nc=5



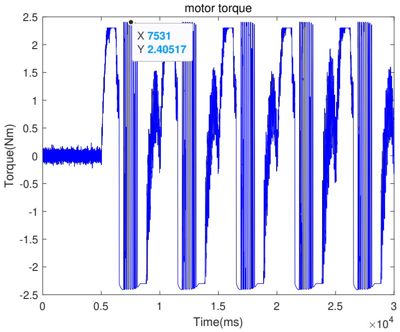
before



added x0 error to cost function

Scenario No.5: Angular constraint:40 deg;Velocity constraint:114.59 deg/s; Input constraint:2.3 Nm

best performance under best horizon: Np10; Nc=5



added x0 error to cost function

progress No.3

Nominal System : $\bar{x}(k+1) = \begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}}_{A} \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 \\ 0 \end{bmatrix}}_{B} \Delta\tau$

states in nominal system modified:

in case we have constraint on motor angular

change from $x = \begin{bmatrix} q \\ \dot{q} \\ \dot{\theta} \end{bmatrix}$ to $x = \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix}$

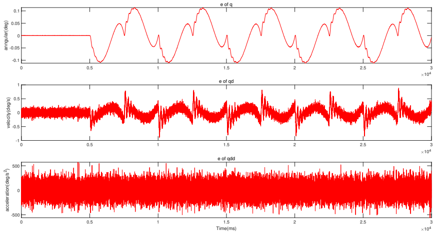
progress No.4

Cost function modified

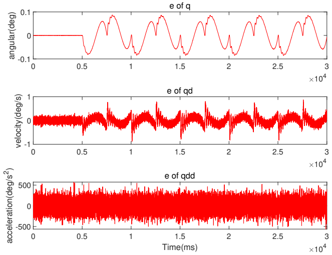
change from : $J = \left\| \begin{bmatrix} x_{k+1|k} \\ \vdots \\ x_{k+N_p|k} \end{bmatrix} - \begin{bmatrix} x_{ref}(k+1) \\ \vdots \\ x_{ref}(k+N_p) \end{bmatrix} \right\|_Q^2 + \left\| \begin{bmatrix} \Delta u_{k|k} \\ \vdots \\ \Delta u_{k+N_c-1|k} \end{bmatrix} \right\|_R^2$

to : $J = \left\| \begin{bmatrix} x_k \\ x_{k+1|k} \\ \vdots \\ x_{k+N_p|k} \end{bmatrix} - \begin{bmatrix} x_{ref}(k) \\ x_{ref}(k+1) \\ \vdots \\ x_{ref}(k+N_p) \end{bmatrix} \right\|_Q^2 + \left\| \begin{bmatrix} \Delta u_{k|k} \\ \vdots \\ \Delta u_{k+N_c-1|k} \end{bmatrix} \right\|_R^2$

progress No.4



before



added x0 error to cost function

progress No.5

tube based MPC: a type of robust MPC
stay inside of constraint while dynamic uncertainty exist

progress No.5

Nominal System : $\bar{x}(k+1) = A\,\bar{x}(k) + B\,\Delta\tau(k)$ $\big(\, z(k+1) = A\,z(k) + B\,v(k) \,\big)$

Real System : $\bar{x}(k+1) = A\,\bar{x}(k) + B\,\Delta\tau(k) + \underline{\underline{w}}(k)$

$w \triangleq$ Dynamic Uncertanty & External Disterbance .

---

we design $u(k) = v(k) + K\,(x(k) - z(k))$,

where $v(k)$ is the first optimal solution from the optimization problem regarding to $z(k)$.

$\Rightarrow$ if we define $e(k) = x(k) - z(k)$

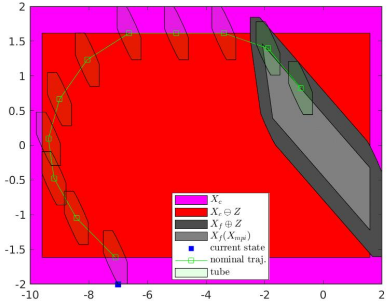
$\Rightarrow$ $e(k) = \sum_{i=1}^{k} A_k^{i-1}\, w(k-i) \subset \sum_{i=1}^{\infty} A_k^{\infty}\, \mathcal{W} \triangleq \mathcal{T}$, where $w(k) \in \mathcal{W}$

$\Rightarrow$ $z(k) \subset \mathcal{X} - \mathcal{T}$, $v(k) \subset \mathcal{U} - K\mathcal{T}$

progress No.5

tube based MPC: a type of robust MPC
stay inside of constraint while dynamic uncertainty exist

But stuck by selecting the K in
$u = v + K(x - z)$,
the K should let $T = \sum_{i=1}^{\infty} A_K^i W$ could
be calculated.



$\Rightarrow$ eigenvalue of $A_K = A + BK$ should stay small.

added x0 error to cost function

Short summary

- 1. survive from different constraints, but exceed under q and qd constraint
- 2. states in nominal system modified
- 3. Cost function modified
- 4. try to add Tube-based MPC to reach better performance, especially performance under constraints

Next step:

- 1. Learn by codes from XieJing to complete Tube-based MPC
- 2. keep try on with other solvers
- 3. Find colleague to do the experiment together.
  May we try on the fast-slow MPC algorithm to compare?

Timeline

- **Linear System formulation using TDE:** done
- **Incremental MPC: Cost function and constraints formulation:** still modifying
- **Simulation:** 01.Oktober ~20.November
  Integrate robot manipulator model into simulink
  Comparing the two solvers and different horizon (error and computation time)
- **Experiment:** 10.November ~10.December
  Comparing the two solvers and different horizon (error and computation time)
- **Possible Try:** 10.December ~30.December
  $\bar{M}$ and $\bar{D}$ online update

References