Incremental MPC for Flexible Robot Manipulators

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Outline

1. Model
2. TDE
3. Incremental MPC
4. Simulation & Experiment
5. Possible Try
6. Timeline

The dynamic model of the robot with compliant joints

Model

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{w}_l = \mathbf{\Gamma}$$

$$\mathbf{D}\ddot{\boldsymbol{\theta}} + \mathbf{w}_m + \mathbf{\Gamma} = \boldsymbol{\tau}$$

$$\mathbf{\Gamma} = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$$

Approximation of equations using Time-delayed Estimation

Two steps:

1. Separation

Introduce $\bar{\mathbf{M}}$ and $\bar{\mathbf{D}}$;
Rewrite the equation of motion into known and unknown parts

2. Approximation

$$(\mathbf{unknownpart})_{(t-L)} \cong (\mathbf{unknownpart})_{(t)}$$

with L is the delay time

Time-delayed Estimation

1. Introducing $\bar{M}$, we have

$$\bar{M}\cdot\ddot{q} + \underbrace{(M(q)-\bar{M})\ddot{q} + C(q,\dot{q}) + G(q) + w_e}_{H_1} = K(\theta - q)$$

Assuming sampling period L is sufficiently small :

$$H_1 \approx H_1(t-L) = K(\theta_0 - q_0) - \bar{M}\ddot{q}_0$$

$$\Rightarrow \bar{M}\ddot{q} + H_1(t-L) \approx K(\theta - q)$$

$\Rightarrow$ incremental system :

$$\ddot{q} = \ddot{q}_0 + \bar{M}^{-1}K(\Delta\theta - \Delta q) + \varepsilon$$

1. Introducing $\bar{D}$, we have

$$\bar{D}\cdot\ddot{\theta} + \underbrace{(D-\bar{D})\ddot{\theta} + w_m + T}_{H_2} = \tau$$

Assuming sampling period L is sufficiently small :

$$H_2 \approx H_2(t-L) = \tau_0 - \bar{D}\cdot\ddot{\theta}_0$$

$$\Rightarrow \bar{D}\ddot{\theta} + H_2(t-L) \approx \tau$$

$\Rightarrow$ incremental system :

$$\ddot{\theta} = \ddot{\theta}_0 + \bar{D}^{-1}\Delta\tau + \varepsilon$$

Interim conclusion

Approximation based on TDE

$$\ddot{q} = \ddot{q}_0 + \bar{M}^{-1}K(\Delta\theta - \Delta q) + \varepsilon_q$$

$$\ddot{\theta} = \ddot{\theta}_0 + \bar{D}^{-1}\Delta\tau + \varepsilon_\theta$$

Linear system

1. Let $\varepsilon_{\mathbf{x}} = 0$ and $\varepsilon_{\mathbf{q}} = 0$

2. Change continuous to discrete-time form

3. Use Euler method

Linear system

Let $x(k) = \mathrm{col}(q(k), \dot{q}(k), \theta(k))$, then we have

$$x(k+1) = \begin{bmatrix} q(k+1) \\ \dot{q}(k+1) \\ \theta(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} I & T_s \cdot I & 0 \\ 0 & 2I - \bar{M}^{-1}KT_s^2 & \bar{M}^{-1}KT_s^2 \\ 0 & 0 & 2 \end{bmatrix}}_{A_1} \begin{bmatrix} q(k) \\ \dot{q}(k) \\ \theta(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}}_{A_2} \begin{bmatrix} q(k-1) \\ \dot{q}(k-1) \\ \theta(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \bar{D}^{-1}T_s \end{bmatrix}}_{B_1} \Delta\tau$$

$$\Rightarrow \bar{x}(k+1) = \begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}}_{A} \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 \\ 0 \end{bmatrix}}_{B} \Delta\tau$$

Incremental MPC 1. verison

Predicted joint dynamics error

$$\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right) := \dot{\tilde{\mathbf{q}}}_{k+j+1|k} + \mathbf{K}_{\mathrm{P}}\tilde{\mathbf{q}}_{k+j+1|k}$$

with $\tilde{\mathbf{q}} := \mathbf{q} - \mathbf{q}_{\mathrm{d}}$ tracking error; $\mathbf{K}_{\mathrm{P}} \succ 0$.

Cost function

$$\ell = \underbrace{\left\|\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right)\right\|_{\mathbf{Q}}^{2}}_{\text{predicted joint dynamics error}} + \underbrace{\left\|\Delta\boldsymbol{\tau}_{k+j|k}\right\|_{\mathbf{R}}^{2}}_{\text{control signal}}$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 1. version

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell\left(\mathbf{q}_{k+j+1|k}, \dot{\mathbf{q}}_{k+j+1|k}, \Delta\boldsymbol{\tau}_{k+j|k}\right)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

Optimization problem

rewrite into

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \Delta\bar{\tau}^T Q \Delta\bar{\tau} + \Delta\bar{\tau}^T L$$

$$\text{s.t.}$$

$$G_1 = C_1 \Delta\bar{\tau} + D_1 \leq 0$$

$$G_2 = C_2 \Delta\bar{\tau} + D_2 \leq 0$$

$$G_3 = C_3 \Delta\bar{\tau} + D_3 \leq 0$$

Reference signal

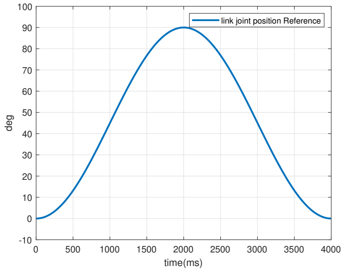

Fig1. Link joint position reference

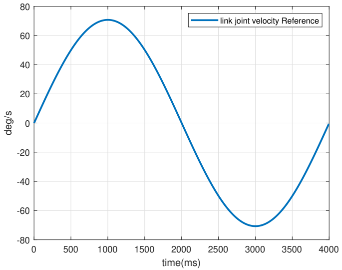

Fig2. Link joint velocity reference

Selected mechanical model

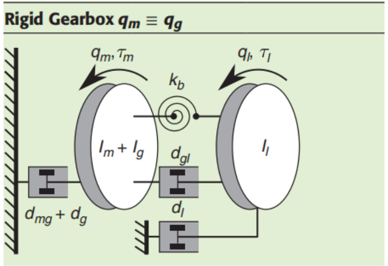

Fig3. Mechanical model

$$\mathbf{I}: \begin{bmatrix} I_m + I_g & 0 \\ 0 & I_l \end{bmatrix} \quad \mathbf{D}: \begin{bmatrix} d_m + d_g + d_{gl} & -d_{gl} \\ -d_{gl} & d_l + d_{gl} \end{bmatrix}$$

$$\mathbf{q}: [q_m \;\; q_l]^{\mathrm{T}} \quad \mathbf{K}: \begin{bmatrix} k_b & -k_b \\ -k_b & k_b \end{bmatrix}$$

$$\mathbf{u}_q: [\tau_m \;\; \tau_l]^{\mathrm{T}} \quad \mathbf{B}_q: \begin{bmatrix} 0 & 0 & \frac{1}{I_g + I_m} & 0 \\ 0 & 0 & 0 & \frac{1}{I_l} \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{A}_q: \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_b}{I_g + I_m} & \frac{k_b}{I_g + I_m} & -\frac{d_m + d_g + d_{gl}}{I_g + I_m} & \frac{d_{gl}}{I_g + I_m} \\ \frac{k_b}{I_l} & -\frac{k_b}{I_l} & \frac{d_{gl}}{I_l} & -\frac{d_l + d_{gl}}{I_l} \end{bmatrix}$$

Fig4. Dynamic terms

result: Controller not working

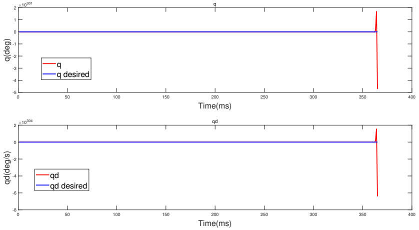

Fig5. Compare desired and is q and qd

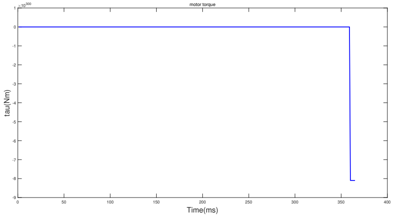

Fig6. motor Torque

Incremental MPC 2. version

Predicted joint dynamics error

$$e(\bar{x}_{k+i|k}) = \underbrace{\begin{bmatrix} 1 & 0 & 0 & & 0 & \\ 0 & 1 & 0 & & & \\ & & & 1 & 0 & 0 \\ & 0 & & 0 & 1 & 0 \end{bmatrix}}_{\bar{K}} \bar{x}_{k+i|k} - \bar{x}_{ref(k+i)} \quad , \text{with } \bar{x}_{ref(k+i)} = \begin{bmatrix} q_{ref}(k+i) \\ \dot{q}_{ref}(k+i) \\ q_{ref}(k+i-1) \\ \dot{q}_{ref}(k+i-1) \end{bmatrix}$$

Cost function

$$\ell(\bar{x}_{k+i|k}, \Delta u_{k+i|k}, k+i) = \| \bar{K} \bar{x}_{k+i|k} - \bar{x}_{ref(k+i)} \|_Q^2 + \| \Delta \tau_{k+i|k} \|_R^2$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 2. version

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell(\chi_{k+i|k}, \Delta u_{k+i|k}, k+i)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

cost function 2. version

same result as using cost function 1. version

Implement IMPC on rigid manipulator

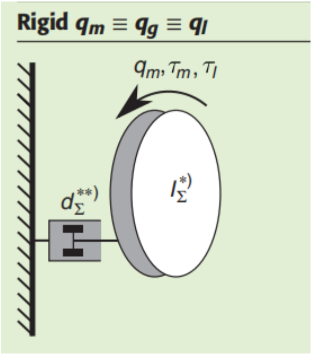

Fig7. Rigid echanical model

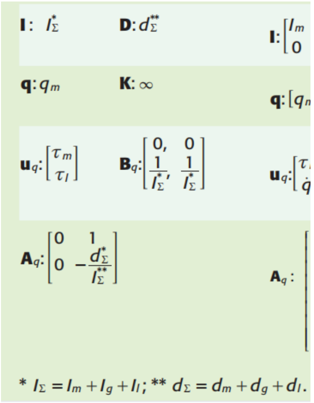

Fig8. Dynamic terms

Implement IMPC on rigid manipulator

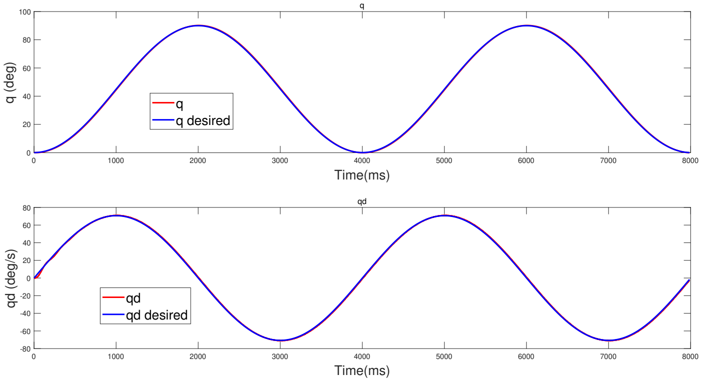

Fig9. Compare desired and is q and qd

Implement IMPC on rigid manipulator

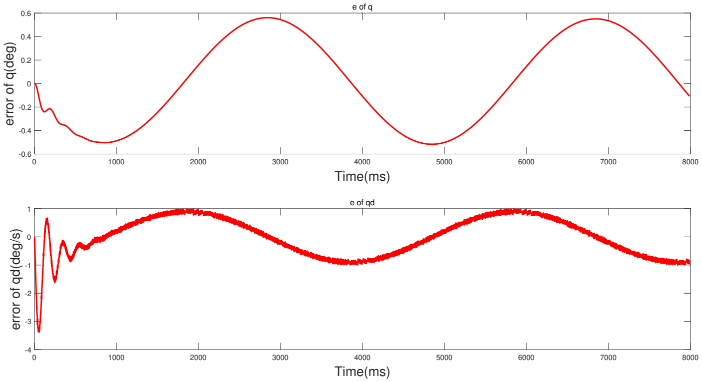

Fig10. position and velocity error

Implement IMPC on rigid manipulator

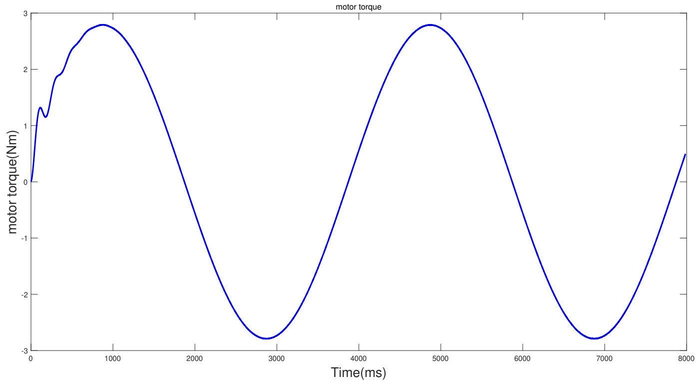

Fig11. motor torque

Incremental MPC 3. version

Predicted joint dynamics error

$$e(\bar{x}_{k+i|k}) = \underbrace{\begin{bmatrix} \begin{smallmatrix}1&0&0\\0&1&0\end{smallmatrix} & 0 \\ 0 & \begin{smallmatrix}1&0&0\\0&1&0\end{smallmatrix} \end{bmatrix}}_{\bar{K}} \bar{x}_{k+i|k} - \bar{x}_{ref(k+i)} \quad , \text{with} \quad \bar{x}_{ref(k+i)} = \begin{bmatrix} q_{ref}(k+i) \\ \dot{q}_{ref}(k+i) \\ q_{ref}(k+i-1) \\ \dot{q}_{ref}(k+i-1) \end{bmatrix}$$

Cost function

$$\ell(\bar{x}_{k+i|k}, \Delta u_{k+i|k}, k+i) = \| \bar{K}\bar{x}_{k+i|k} - \bar{x}_{ref(k+i)} \|_Q^2 + \| \Delta\tau_{k+i|k} - \Delta\tau_{ref(k+i)} \|_R^2$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Incremental MPC 3. version

How to get $\Delta\tau_{ref}$?

1. Set $x(k) = \begin{bmatrix} q(k) \\ \dot{q}(k) \\ \tau(k) \end{bmatrix}$, $x_{ref}(k) = \begin{bmatrix} q_{ref}(k) \\ \dot{q}_{ref}(k) \\ \tau_{ref}(k) \end{bmatrix}$.

2. $\tau_{ref}(k) = \bar{M}\ddot{q}(k) + H_1(k-1)$

$$= \bar{M}\ddot{q}(k) + \tau_{ref}(k-1) - \bar{M}\ddot{q}(k-1)$$

3. $\bar{x}(k+1) = \begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} = A\,\bar{x}(k) + B\,\Delta\tau(k) \Rightarrow \Delta\tau_{ref}(k) = (B^TB)^{-1}B^T\left(\bar{x}_{ref}(k+1) - A\,\bar{x}_{ref}(k)\right)$

Short summary

Work fine by using IMPC to control the rigid joint
Failed by using IMPC to control the compliant joint

Tried:

1. Modify TDE formulation
2. Modify cost function formulation to 2. verison

Next step:
Try the cost function 3. version?

More parameters

Parameter	Value
$\bar{M}$	I_l
$\bar{D}$	$I_m + I_g$
K	k_b
Weighting matrix of dynamics error Q	
Weighting matrix of control signals R	

Timeline

- **Linear System formulation using TDE:** done
- **Incremental MPC: Cost function and constraints formulation:** still modifying
- **Simulation:** 01.Oktober ~20.November
  Integrate robot manipulator model into simulink
  Comparing the two solvers and different horizon (error and computation time)
- **Experiment:** 10.November ~10.December
  Comparing the two solvers and different horizon (error and computation time)
- **Possible Try:** 10.December ~30.December
  $\bar{M}$ and $\bar{D}$ online update

References