

Incremental MPC for Flexible Robot Manipulators

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Outline

1. Model
2. TDE
3. Incremental MPC
4. Simulation & Experiment
5. Possible Try
6. Timeline

The dynamic model of the robot with compliant joints

Model

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{w}_l = \mathbf{\Gamma}$$

$$\mathbf{D}\ddot{\boldsymbol{\theta}} + \mathbf{w}_m + \mathbf{\Gamma} = \boldsymbol{\tau}$$

$$\mathbf{\Gamma} = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$$

Approximation of equations using Time-delayed Estimation

Two steps:

1. Separation

Introduce $\bar{\mathbf{M}}$ and $\bar{\mathbf{D}}$;
Rewrite the equation of motion into known and unknown parts

2. Approximation

$$(\text{unknownpart})_{(t-L)} \cong (\text{unknownpart})_{(t)}$$

with L is the delay time

Time-delayed Estimation

1. Introducing \bar{M} , we have

$$\bar{M} \cdot \ddot{q} + \underbrace{(M(q) - \bar{M}) \ddot{q} + C(q, \dot{q}) + G(q) + W_L}_{H_1} = K(\theta - q)$$

Assuming sampling period L is sufficiently small:

$$H_1 \approx H_1(t-L) = K(\theta_0 - q_0) - \bar{M} \ddot{q}_0$$

$$\Rightarrow \bar{M} \ddot{q} + H_1(t-L) \approx K(\theta - q)$$

\Rightarrow incremental system:

$$\ddot{q} = \ddot{q}_0 + \bar{M} K (\Delta \theta - \Delta q) + \varepsilon$$

1. Introducing \bar{D} , we have

$$\bar{D} \cdot \ddot{\theta} + \underbrace{(D - \bar{D}) \ddot{\theta} + W_m + T}_{H_2} = \tau$$

Assuming sampling period L is sufficiently small:

$$H_2 \approx H_2(t-L) = \tau_0 - \bar{D} \cdot \ddot{\theta}_0$$

$$\Rightarrow \bar{D} \ddot{\theta} + H_2(t-L) \approx \tau$$

\Rightarrow incremental system:

$$\ddot{\theta} = \ddot{\theta}_0 + \bar{D}^{-1} \Delta \tau + \varepsilon$$

Interim conclusion

Approximation based on TDE

$$\ddot{\mathbf{q}} = \ddot{\mathbf{q}}_0 + \bar{\mathbf{M}}\mathbf{K}(\Delta\boldsymbol{\theta} - \Delta\mathbf{q}) + \boldsymbol{\varepsilon}_q$$

$$\ddot{\boldsymbol{\theta}} = \ddot{\boldsymbol{\theta}}_0 + \bar{\mathbf{D}}^{-1}\Delta\boldsymbol{\tau} + \boldsymbol{\varepsilon}_\theta$$

Linear system

1. Let $\varepsilon_x = 0$ and $\varepsilon_q = 0$
2. Change continuous to discrete-time form
3. Use Euler method

Linear system

Let $\chi(k) = \text{col}(q(k), \dot{q}(k), \theta(k))$, then we have

$$\chi(k+1) = \begin{bmatrix} q(k+1) \\ \dot{q}(k+1) \\ \theta(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} I & T_s \cdot I & 0 \\ 0 & 2I - \bar{M}^{-1} K T_s^2 & \bar{M}^{-1} K T_s^2 \\ 0 & 0 & 2 \end{bmatrix}}_{A_1} \begin{bmatrix} q(k) \\ \dot{q}(k) \\ \theta(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}}_{A_2} \begin{bmatrix} q(k-1) \\ \dot{q}(k-1) \\ \theta(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \bar{D}^{-1} T_s \end{bmatrix}}_{B_1} \Delta \tau$$

$$\Rightarrow \bar{\chi}(k+1) = \begin{bmatrix} \chi(k+1) \\ \chi(k) \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}}_A \begin{bmatrix} \chi(k) \\ \chi(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 \\ 0 \end{bmatrix}}_B \Delta \tau$$

Incremental MPC 1. version

Predicted joint dynamics error

$$\mathbf{e}(\vec{\mathbf{x}}_{k+j+1|k}) := \dot{\tilde{\mathbf{q}}}_{k+j+1|k} + \mathbf{K}_P \tilde{\mathbf{q}}_{k+j+1|k}$$

with $\tilde{\mathbf{q}} := \mathbf{q} - \mathbf{q}_d$ tracking error; $\mathbf{K}_P \succ 0$.

Cost function

$$\ell = \underbrace{\|\mathbf{e}(\vec{\mathbf{x}}_{k+j+1|k})\|_{\mathbf{Q}}^2}_{\text{predicted joint dynamics error}} + \underbrace{\|\Delta \boldsymbol{\tau}_{k+j|k}\|_{\mathbf{R}}^2}_{\text{control signal}}$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 1. version

$$\Delta\bar{\tau}^* = \arg \min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell(\mathbf{q}_{k+j+1|k}, \dot{\mathbf{q}}_{k+j+1|k}, \Delta\boldsymbol{\tau}_{k+j|k})$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^j \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

Optimization problem

rewrite into

$$\Delta\bar{\tau}^* = \arg \min_{\Delta\bar{\tau}} \Delta\bar{\tau}^T Q \Delta\bar{\tau} + \Delta\bar{\tau}^T L$$

s.t.

$$G_1 = C_1 \Delta\bar{\tau} + D_1 \leq 0$$

$$G_2 = C_2 \Delta\bar{\tau} + D_2 \leq 0$$

$$G_3 = C_3 \Delta\bar{\tau} + D_3 \leq 0$$

Reference signal

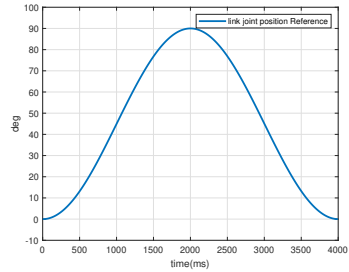


Fig1. Link joint position reference

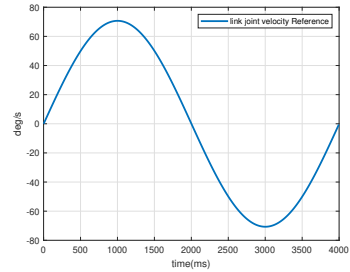


Fig2. Link joint velocity reference

Selected mechanical model

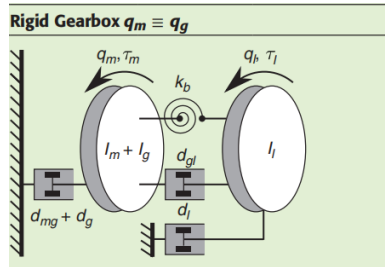


Fig3. Mechanical model

$$\mathbf{I} : \begin{bmatrix} I_m + I_g & 0 \\ 0 & I_l \end{bmatrix} \quad \mathbf{D} : \begin{bmatrix} d_m + d_g + d_{gl} & -d_{gl} \\ -d_{gl} & d_l + d_{gl} \end{bmatrix}$$

$$\mathbf{q} : [q_m \ q_l]^T \quad \mathbf{K} : \begin{bmatrix} k_b & -k_b \\ -k_b & k_b \end{bmatrix}$$

$$\mathbf{u}_q : [\tau_m \ \tau_l]^T \quad \mathbf{B}_q : \begin{bmatrix} 0 & 0 & \frac{1}{I_g + I_m} & 0 \\ 0 & 0 & 0 & \frac{1}{I_l} \end{bmatrix}^T$$

$$\mathbf{A}_q : \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_b}{I_g + I_m} & \frac{k_b}{I_g + I_m} & -\frac{d_m + d_g + d_{gl}}{I_g + I_m} & \frac{d_{gl}}{I_g + I_m} \\ \frac{k_b}{I_l} & -\frac{k_b}{I_l} & \frac{d_{gl}}{I_l} & -\frac{d_l + d_{gl}}{I_l} \end{bmatrix}$$

Fig4. Dynamic terms

result: Controller not working

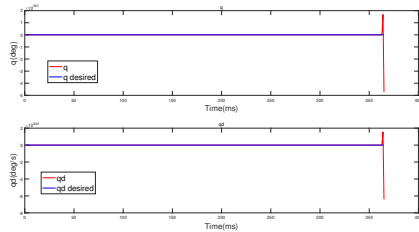


Fig5. Compare desired and is q and qd

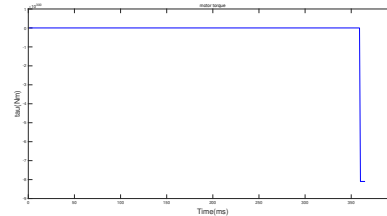


Fig6. motor Torque

Incremental MPC 2. version

Predicted joint dynamics error

$$e(\bar{x}_{k+i|k}) = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\bar{K}} \bar{x}_{k+i|k} - \bar{x}_{\text{ref}}(k+i) \quad , \text{ with } \bar{x}_{\text{ref}}(k+i) = \begin{bmatrix} q_{\text{ref}}(k+i) \\ \dot{q}_{\text{ref}}(k+i) \\ q_{\text{ref}}(k+i-1) \\ \dot{q}_{\text{ref}}(k+i-1) \end{bmatrix}$$

Cost function

$$\ell(\bar{x}_{k+i|k}, \Delta u_{k+i|k}, k+i) = \| \bar{K} \bar{x}_{k+i|k} - \bar{x}_{\text{ref}}(k+i) \|_Q^2 + \| \Delta z_{k+i|k} \|_R^2$$

with $Q, R \succ 0$.

Optimization problem 2. version

$$\Delta \bar{\tau}^* = \arg \min_{\Delta \bar{\tau}} \sum_{j=0}^{N-1} \ell(\chi_{k+j|k}, \Delta u_{k+j|k}, k+j)$$

s.t.

$$\vec{x}_{k+j+1|k} = \mathbf{A} \vec{x}_{k+j|k} + \mathbf{B} \Delta \tau_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\tau_{\min} \leq \tau_0 + \sum_{s=0}^j \Delta \tau_{k+s|k} \leq \tau_{\max}$$

cost function 2. version

same result as using cost function 1. version

Implement IMPC on rigid manipulator

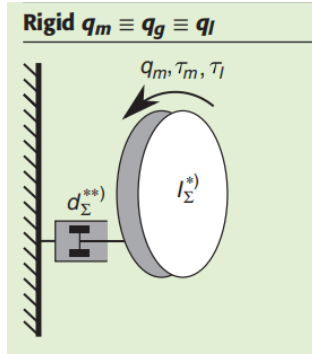


Fig7. Rigid echanical model

$\mathbf{I}: I_\Sigma^*$	$\mathbf{D}: d_\Sigma^{**}$	$\mathbf{l}: \begin{bmatrix} l_m \\ 0 \end{bmatrix}$
$\mathbf{q}: q_m$	$\mathbf{K}: \infty$	$\mathbf{q}: [q_n]$
$\mathbf{u}_q: \begin{bmatrix} \tau_m \\ \tau_l \end{bmatrix}$	$\mathbf{B}_q: \begin{bmatrix} 0, & 0 \\ \frac{1}{I_\Sigma^*}, & \frac{1}{I_\Sigma^*} \end{bmatrix}$	$\mathbf{u}_q: \begin{bmatrix} \tau_m \\ \dot{q} \end{bmatrix}$
$\mathbf{A}_q: \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d_\Sigma^{**}}{I_\Sigma^*} \end{bmatrix}$		$\mathbf{A}_q: \begin{bmatrix} \dots \\ \dots \end{bmatrix}$

* $I_\Sigma = I_m + I_g + I_l$; ** $d_\Sigma = d_m + d_g + d_l$.

Fig8. Dynamic terms

Implement IMPC on rigid manipulator

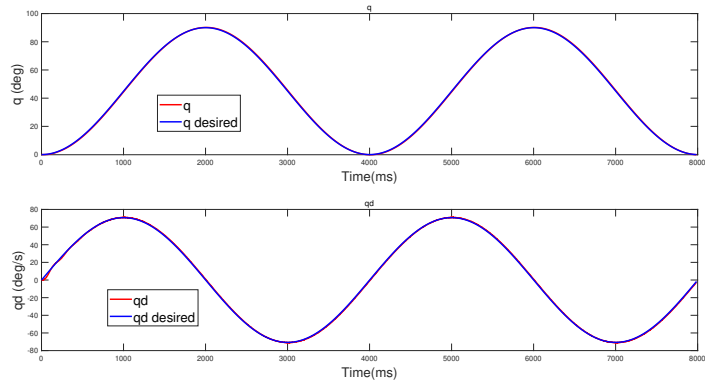


Fig9. Compare desired and is q and q_d

Implement IMPC on rigid manipulator

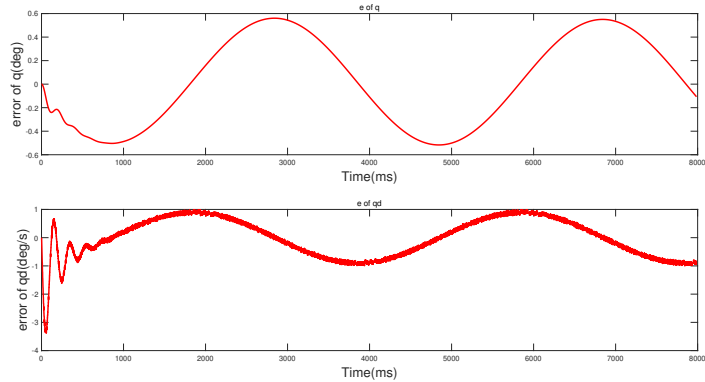


Fig10. position and velocity error

Implement IMPC on rigid manipulator

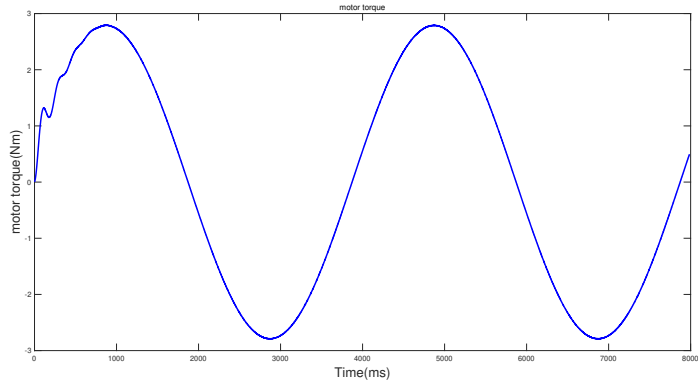


Fig11. motor torque

Incremental MPC 3. version

Predicted joint dynamics error

$$e(\bar{\chi}_{k+i|k}) = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\bar{K}} \bar{\chi}_{k+i|k} - \bar{\chi}_{\text{ref}}(k+i) \quad , \text{ with } \bar{\chi}_{\text{ref}}(k+i) = \begin{bmatrix} q_{\text{ref}}(k+i) \\ \dot{q}_{\text{ref}}(k+i) \\ q_{\text{ref}}(k+i-1) \\ \dot{q}_{\text{ref}}(k+i-1) \end{bmatrix}$$

Cost function

$$\ell(\bar{\chi}_{k+i|k}, \Delta u_{k+i|k}, k+i) = \| \bar{K} \bar{\chi}_{k+i|k} - \bar{\chi}_{\text{ref}}(k+i) \|_Q^2 + \| \Delta z_{k+i|k} - \Delta z_{\text{ref}}(k+i) \|_R^2$$

with $Q, R \succ 0$.

Incremental MPC 3. version

How to get $\Delta \tau_{\text{ref}}$?

1. Set $\chi(k) = \begin{bmatrix} q(k) \\ \dot{q}(k) \\ p(k) \end{bmatrix}$, $\chi_{\text{ref}}(k) = \begin{bmatrix} q_{\text{ref}}(k) \\ \dot{q}_{\text{ref}}(k) \\ p_{\text{ref}}(k) \end{bmatrix}$.

2.
$$\begin{aligned} P_{\text{ref}}(k) &= \bar{M} \ddot{q}(k) + H_1(k-1) \\ &= \bar{M} \ddot{q}(k) + P_{\text{ref}}(k-1) - \bar{M} \ddot{q}(k-1) \end{aligned}$$

3.
$$\bar{\chi}(k+1) = \begin{bmatrix} \chi(k+1) \\ \chi(k) \end{bmatrix} = A \bar{\chi}(k) + B \Delta \tau(k) \Rightarrow \Delta \tau_{\text{ref}}(k) = (B^T B)^{-1} B^T (\bar{\chi}_{\text{ref}}(k+1) - A \bar{\chi}_{\text{ref}}(k))$$

Short summary

Work fine by using IMPC to control the rigid joint
Failed by using IMPC to control the compliant joint

Tried:

1. Modify TDE formulation
2. Modify cost function formulation to 2. version

Next step:

Try the cost function 3. version?

More parameters

Parameter	Value
\bar{M}	I_l
\bar{D}	$I_m + I_g$
K	k_b
Weighting matrix of dynamics error Q	
Weighting matrix of control signals R	

Timeline

- **Linear System formulation using TDE:** done
- **Incremental MPC: Cost function and constraints formulation:** still modifying
- **Simulation:** 01.Oktober ~20.November
Integrate robot manipulator model into simulink
Comparing the two solvers and different horizon (error and computation time)
- **Experiment:** 10.November ~10.December
Comparing the two solvers and different horizon (error and computation time)
- **Possible Try:** 10.December ~30.December
 \bar{M} and \bar{D} online update

References