Incremental MPC for Flexible Robot Manipulators

Tian Zheng
Zwischenbericht/Abschlussbericht Diplomarbeit/Studienarbeit
Supervisor: S. Supervisor

Institute of Automatic Control Engineering
Technical University of Munich

Outline

1. **Model**
2. **TDE**
3. **Incremental MPC**
4. **Simulation & Experiment**
5. **Possible Try**
6. **Timeline**

The dynamic model of the robot with compliant joints

Model

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{w}_l = \mathbf{\Gamma}$$

$$\mathbf{D}\ddot{\boldsymbol{\theta}} + \mathbf{w}_m + \mathbf{\Gamma} = \boldsymbol{\tau}$$

$$\mathbf{\Gamma} = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$$

Approximation of equations using Time-delayed Estimation

Two steps:

1. Separation

Introduce $\bar{\mathbf{M}}$ and $\bar{\mathbf{D}}$;
Rewrite the equation of motion into known and unknown parts

2. Approximation

$$(\mathbf{unknownpart})_{(t-L)} \cong (\mathbf{unknownpart})_{(t)}$$

with L is the delay time

Time-delayed Estimation

1. Introducing $\bar{M}$, we have

$$\bar{M}\cdot\ddot{q}+\underbrace{(M(q)-\bar{M})\ddot{q}+C(q,\dot{q})+G(q)+W_e}_{H_1}=K(\theta-q)$$

Assuming sampling period L is sufficiently small :

$$H_1 \approx H_1(t-L) = K(\theta_0-q_0)-\bar{M}\ddot{q}_0$$

$$\Rightarrow \bar{M}\ddot{q} + H_1(t-L) \approx K(\theta-q)$$

$\Rightarrow$ incremental system :

$$\ddot{q} = \ddot{q}_0 + \bar{M}^{-1}K(\Delta\theta-\Delta q)+\varepsilon$$

1. Introducing $\bar{D}$, we have

$$\bar{D}\cdot\ddot{\theta}+\underbrace{(D-\bar{D})\ddot{\theta}+W_m+T}_{H_2}=\tau$$

Assuming sampling period L is sufficiently small :

$$H_2 \approx H_2(t-L) = \tau_0-\bar{D}\cdot\ddot{\theta}_0$$

$$\Rightarrow \bar{D}\ddot{\theta} + H_2(t-L) \approx \tau$$

$\Rightarrow$ incremental system :

$$\ddot{\theta} = \ddot{\theta}_0 + \bar{D}^{-1}\Delta\tau + \varepsilon$$

Interim conclusion

Approximation based on TDE

$$\ddot{q} = \ddot{q}_0 + \bar{M}^{-1} K (\Delta\theta - \Delta q) + \varepsilon_q$$

$$\ddot{\theta} = \ddot{\theta}_0 + \bar{D}^{-1} \Delta\tau + \varepsilon_\theta$$

Linear system

1. Let $\varepsilon_{\mathbf{x}} = 0$ and $\varepsilon_{\mathbf{q}} = 0$
2. Change continuous to discrete-time form
3. Use Euler method

Linear system

Let $X(k) = \mathrm{col}(q(k), \dot{q}(k), \theta(k))$, then we have

$$X(k+1) = \begin{bmatrix} q(k+1) \\ \dot{q}(k+1) \\ \theta(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} I & T_s \cdot I & 0 \\ 0 & 2I - \bar{M}^{-1}KT_s^2 & \bar{M}^{-1}KT_s^2 \\ 0 & 0 & 2 \end{bmatrix}}_{A_1} \begin{bmatrix} q(k) \\ \dot{q}(k) \\ \theta(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}}_{A_2} \begin{bmatrix} q(k-1) \\ \dot{q}(k-1) \\ \theta(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \bar{D}^{-1}T_s \end{bmatrix}}_{B_1} \Delta\tau$$

$$\Rightarrow \bar{X}(k+1) = \begin{bmatrix} X(k+1) \\ X(k) \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}}_{A} \begin{bmatrix} X(k) \\ X(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 \\ 0 \end{bmatrix}}_{B} \Delta\tau$$

Incremental MPC 1. verison

Predicted joint dynamics error

$$\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right) := \dot{\tilde{\mathbf{q}}}_{k+j+1|k} + \mathbf{K}_{\mathrm{P}}\tilde{\mathbf{q}}_{k+j+1|k}$$

with $\tilde{\mathbf{q}} := \mathbf{q} - \mathbf{q}_{\mathrm{d}}$ tracking error; $\mathbf{K}_{\mathrm{P}} \succ 0$.

Cost function

$$\ell = \underbrace{\left\|\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right)\right\|_{\mathbf{Q}}^{2}}_{\text{predicted joint dynamics error}} + \underbrace{\left\|\Delta\boldsymbol{\tau}_{k+j|k}\right\|_{\mathbf{R}}^{2}}_{\text{control signal}}$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 1. version

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell\left(\mathbf{q}_{k+j+1|k}, \dot{\mathbf{q}}_{k+j+1|k}, \Delta\boldsymbol{\tau}_{k+j|k}\right)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

Optimization problem

rewrite into

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \Delta\bar{\tau}^T Q \Delta\bar{\tau} + \Delta\bar{\tau}^T L$$

s.t.

$$G_1 = C_1 \Delta\bar{\tau} + D_1 \leq 0$$

$$G_2 = C_2 \Delta\bar{\tau} + D_2 \leq 0$$

$$G_3 = C_3 \Delta\bar{\tau} + D_3 \leq 0$$

Selected mechanical model

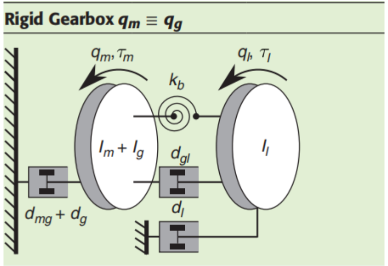

Fig1. Mechanical model

$$\mathbf{I}: \begin{bmatrix} I_m + I_g & 0 \\ 0 & I_l \end{bmatrix} \qquad \mathbf{D}: \begin{bmatrix} d_m + d_g + d_{gl} & -d_{gl} \\ -d_{gl} & d_l + d_{gl} \end{bmatrix}$$

$$\mathbf{q}: [q_m \ \ q_l]^T \qquad \mathbf{K}: \begin{bmatrix} k_b & -k_b \\ -k_b & k_b \end{bmatrix}$$

$$\mathbf{u}_q: [\tau_m \ \ \tau_l]^T \qquad \mathbf{B}_q: \begin{bmatrix} 0 & 0 & \frac{1}{I_g + I_m} & 0 \\ 0 & 0 & 0 & \frac{1}{I_l} \end{bmatrix}^T$$

$$\mathbf{A}_q: \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_b}{I_g + I_m} & \frac{k_b}{I_g + I_m} & -\frac{d_m + d_g + d_{gl}}{I_g + I_m} & \frac{d_{gl}}{I_g + I_m} \\ \frac{k_b}{I_l} & -\frac{k_b}{I_l} & \frac{d_{gl}}{I_l} & -\frac{d_l + d_{gl}}{I_l} \end{bmatrix}$$

Fig2. Dynamic terms

Incremental MPC 2. version

Predicted joint dynamics error

$$e(\bar{x}_{k+i|k}) = \underbrace{\begin{bmatrix} K_P & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\bar{K}} \bar{x}_{k+i|k} - \bar{x}_{ref}(k+i) \quad , \text{with } \bar{x}_{ref}(k+i) = \begin{bmatrix} q_{ref}(k+i) \\ \dot{q}_{ref}(k+i) \end{bmatrix}$$

Cost function

$$l(\bar{x}_{k+i|k}, \Delta u_{k+i|k}, k+i) = \| \bar{K}\bar{x}_{k+i|k} - \bar{x}_{ref}(k+i) \|_Q^2 + \| \Delta \tau_{k+i|k} \|_R^2$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 2. version

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell\left(x_{k+i|k} \;,\; \Delta u_{k+i|k} \;,\; k+i \right)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

result using function 1. & 2. version on Rigid Gearbox from Toolbox

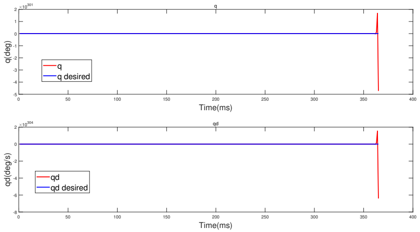

Fig3. Compare desired and is q and qd

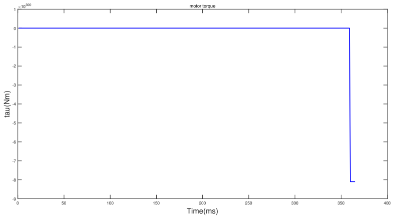

Fig4. motor Torque

Selected mechanical model

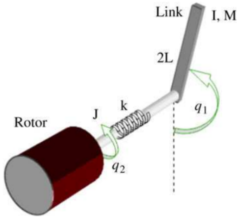

Fig5. Mechanical model [Ghahramani+ 2009]

$$\boldsymbol{I}(\boldsymbol{q}_1)\ddot{\boldsymbol{q}}_1 + \boldsymbol{C}(\boldsymbol{q}_1, \dot{\boldsymbol{q}}_1)\dot{\boldsymbol{q}}_1 + \boldsymbol{g}(\boldsymbol{q}_1) + \boldsymbol{K}(\boldsymbol{q}_1 - \boldsymbol{q}_2) = \boldsymbol{0}$$
$$J\ddot{\boldsymbol{q}}_2 - \boldsymbol{K}(\boldsymbol{q}_1 - \boldsymbol{q}_2) = \boldsymbol{u}$$

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{MgL}{I}\sin x_1 - \frac{k}{I}(x_1 - x_3) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{k}{J}(x_1 - x_3) + \frac{1}{J}u
\end{aligned}$$

Fig6. Dynamic

result using function 2. version on mechanical model in reference paper [Ghahramani+ 2009]

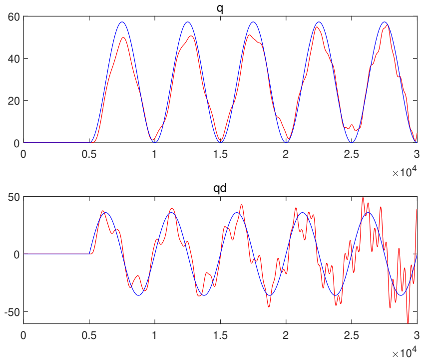

Fig7. Compare desired and is q and qd

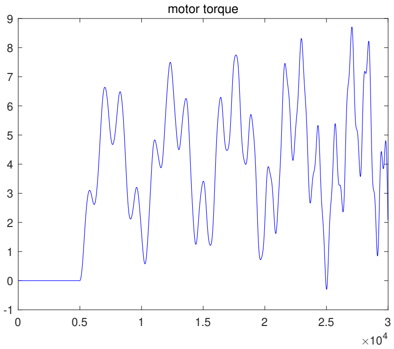

Fig8. motor Torque

Incremental MPC 3. version

Predicted joint dynamics error

$$e(\bar{x}_{k+i|k}) = \underbrace{\begin{bmatrix} K_P & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\bar{K}} \bar{x}_{k+i|k} - \bar{x}_{ref(k+i)} \quad \text{, with } \bar{x}_{ref(k+i)} = \begin{bmatrix} q_{ref(k+i)} \\ \dot{q}_{ref(k+i)} \end{bmatrix}$$

Cost function

$$\ell(\bar{x}_{k+i|k}, \Delta u_{k+i|k}, k+i) = \| \bar{K} \bar{x}_{k+i|k} - \bar{x}_{ref(k+i)} \|_Q^2 + \| \Delta \tau_{k+i|k} - \Delta \tau_{ref(k+i)} \|_R^2$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Cost function 3. version

① $$\ddot{q}(k+1) = (2-\bar{M}^{-1}KT_s^2)\,\dot{q}(k) + \bar{M}^{-1}KT_s^2\,\dot{\theta}(k) - \dot{q}(k-1)$$

$$\Rightarrow \dot{\theta}_d(k) = (\bar{M}^{-1}KT_s^2)^{-1}\left[\dot{q}_d(k+1) - (2-\bar{M}^{-1}KT_s^2)\,\dot{q}_d(k) + \dot{q}_d(k-1)\right]$$

② $$\dot{\theta}(k+1) = 2\dot{\theta}(k) - \dot{\theta}(k-1) + \bar{D}^{-1}T_s\,\Delta\tau(k)$$

$$\Rightarrow \Delta\tau_d(k) = (\bar{D}^{-1}T_s)^{-1}\left[\dot{\theta}_d(k+1) - 2\dot{\theta}_d(k) + \dot{\theta}_d(k-1)\right]$$

result using function 3. version on mechanical model in reference paper [Ghahramani+ 2009]

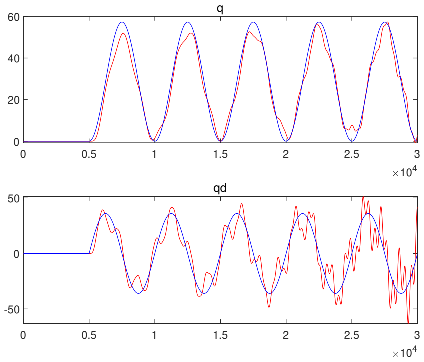

Fig9. Compare desired and is q and qd

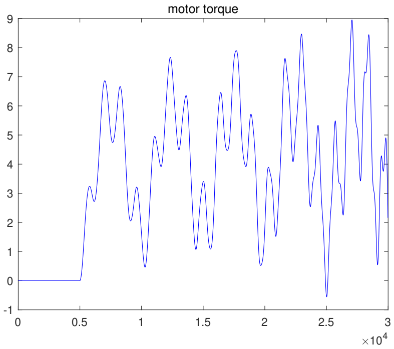

Fig10. motor Torque

Back to rigid gearbox in Toolbox

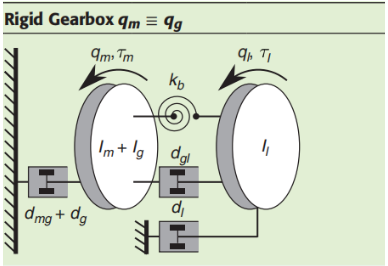

Fig11. Mechanical model

1. Gear transmission ratio included in the motor and gear inertial

2. Parameter highly affect performance: Similar performance using same paramter as in [Ghahramani+ 2009]

Short summary

Work by using IMPC to control the compliant joint with easy parameters
Similar result for using cost function 2. & 3. verison

Next step:
Add the desired motor position into cost function?

Timeline

- **Linear System formulation using TDE:** done
- **Incremental MPC: Cost function and constraints formulation:** still modifying
- **Simulation:** 01.Oktober ~20.November
  Integrate robot manipulator model into simulink
  Comparing the two solvers and different horizon (error and computation time)
- **Experiment:** 10.November ~10.December
  Comparing the two solvers and different horizon (error and computation time)
- **Possible Try:** 10.December ~30.December
  $\bar{M}$ and $\bar{D}$ online update

References

Nemat Ollah Ghahramani and Farzad Towhidkhah. **Constrained incremental predictive controller design for a flexible joint robot**. In: ISA transactions 48.3 (2009), pp. 321–326.