Incremental MPC for Flexible Robot Manipulators

Tian Zheng
Zwischenbericht/Abschlussbericht Diplomarbeit/Studienarbeit
Supervisor: S. Supervisor

Institute of Automatic Control Engineering
Technical University of Munich

Outline

The dynamic model of the robot with compliant joints

Model

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{w}_l = \mathbf{\Gamma}$$

$$\mathbf{D}\ddot{\boldsymbol{\theta}} + \mathbf{w}_m + \mathbf{\Gamma} = \boldsymbol{\tau}$$

$$\mathbf{\Gamma} = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$$

Approximation of equations using Time-delayed Estimation

Two steps:

1. Separation

Introduce $\bar{\mathbf{M}}$ and $\bar{\mathbf{D}}$;
Rewrite the equation of motion into known and unknown parts

2. Approximation

$$(\mathbf{unknownpart})_{(t-L)} \cong (\mathbf{unknownpart})_{(t)}$$

with L is the delay time

Time-delayed Estimation

1. Introducing $\bar{M}$, we have

$$\bar{M}\cdot\ddot{q}+\underbrace{(M(q)-\bar{M})\ddot{q}+C(q,\dot{q})+G(q)+W_e}_{H_1}=K(\theta-q)$$

Assuming sampling period L is sufficiently small :

$$H_1 \approx H_1(t-L) = K(\theta_0-q_0)-\bar{M}\ddot{q}_0$$

$$\Rightarrow \bar{M}\ddot{q}+H_1(t-L)\approx K(\theta-q)$$

$\Rightarrow$ incremental system :

$$\ddot{q}=\ddot{q}_0+\bar{M}^{-1}K(\Delta\theta-\Delta q)+\varepsilon$$

1. Introducing $\bar{D}$, we have

$$\bar{D}\cdot\ddot{\theta}+\underbrace{(D-\bar{D})\ddot{\theta}+W_m+T}_{H_2}=\tau$$

Assuming sampling period L is sufficiently small :

$$H_2\approx H_2(t-L)=\tau_0-\bar{D}\cdot\ddot{\theta}_0$$

$$\Rightarrow \bar{D}\ddot{\theta}+H_2(t-L)\approx\tau$$

$\Rightarrow$ incremental system :

$$\ddot{\theta}=\ddot{\theta}_0+\bar{D}^{-1}\Delta\tau+\varepsilon$$

Interim conclusion

Approximation based on TDE

$$\ddot{q} = \ddot{q}_0 + \bar{M}^{-1}K(\Delta\theta - \Delta q) + \varepsilon_q$$

$$\ddot{\theta} = \ddot{\theta}_0 + \bar{D}^{-1}\Delta\tau + \varepsilon_\theta$$

Linear system

1. Let $\varepsilon_{\mathbf{x}} = 0$ and $\varepsilon_{\mathbf{q}} = 0$

2. Change continuous to discrete-time form

3. Use Euler method

Linear system

Let $X(k) = \mathrm{col}(q(k), \dot{q}(k), \theta(k))$, then we have

$$X(k+1) = \begin{bmatrix} q(k+1) \\ \dot{q}(k+1) \\ \theta(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} I & T_s \cdot I & 0 \\ 0 & 2I - \bar{M}^{-1}KT_s^2 & \bar{M}^{-1}KT_s^2 \\ 0 & 0 & 2 \end{bmatrix}}_{A_1} \begin{bmatrix} q(k) \\ \dot{q}(k) \\ \theta(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}}_{A_2} \begin{bmatrix} q(k-1) \\ \dot{q}(k-1) \\ \theta(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \bar{D}^{-1}T_s \end{bmatrix}}_{B_1} \Delta\tau$$

$$\Rightarrow \bar{X}(k+1) = \begin{bmatrix} X(k+1) \\ X(k) \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}}_{A} \begin{bmatrix} X(k) \\ X(k-1) \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 \\ 0 \end{bmatrix}}_{B} \Delta\tau$$

Incremental MPC 1. verison

Predicted joint dynamics error

$$\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right) := \dot{\tilde{\mathbf{q}}}_{k+j+1|k} + \mathbf{K}_{\mathrm{P}}\tilde{\mathbf{q}}_{k+j+1|k}$$

with $\tilde{\mathrm{q}} := \mathbf{q} - \mathbf{q}_{\mathrm{d}}$ tracking error; $\mathbf{K}_{\mathrm{P}} \succ 0$.

Cost function

$$\ell = \underbrace{\left\|\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right)\right\|_{\mathbf{Q}}^2}_{\text{predicted joint dynamics error}} + \underbrace{\left\|\Delta\boldsymbol{\tau}_{k+j|k}\right\|_{\mathbf{R}}^2}_{\text{control signal}}$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 1. version

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell\left(\mathbf{q}_{k+j+1|k}, \dot{\mathbf{q}}_{k+j+1|k}, \Delta\boldsymbol{\tau}_{k+j|k}\right)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

Optimization problem

rewrite into

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \Delta\bar{\tau}^T Q \Delta\bar{\tau} + \Delta\bar{\tau}^T L$$

s.t.

$$G_1 = C_1\Delta\bar{\tau} + D_1 \leq 0$$

$$G_2 = C_2\Delta\bar{\tau} + D_2 \leq 0$$

$$G_3 = C_3\Delta\bar{\tau} + D_3 \leq 0$$

Incremental MPC 2. version

Predicted joint dynamics error

$$e(\bar{x}_{k+i|k}) = q_{k+i|k} - q_{d(k+i)}$$

Cost function

$$\ell(\bar{x}_{k+i|k}, \Delta u_{k+i|k}, k+i) = \| q_{k+i|k} - q_{d(k+i)} \|^2_{Q_1} + \| \dot{q}_{k+i|k} - \dot{q}_{d(k+i)} \|^2_{Q_2} + \| \Delta \tau_{k+i|k} \|^2_{R}$$

with $\mathbf{Q}, \mathbf{R} \succ 0$.

Optimization problem 2. version

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell(x_{k+i|k}, \Delta u_{k+i|k}, k+i)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

Recap

1. better tracking performance using simplified plant dynamics than using complete plant dynamics in Compliant Joint Toolbox
2. fluctuate at the beginning because bad select of reference trajectory (acceleration at t_0 not equal to 0)
3. no noises or damping added into plant yet
4. still using classic qpOASES solver

change No.1

change plant dynamics into the complete one in toolbox by rewriting the equations instead of modify the block inn toolbox

Selected mechanical model

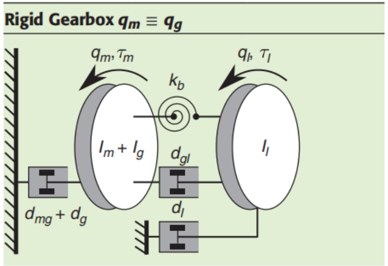


Fig1. Mechanical model

$\mathbf{I}: \begin{bmatrix} I_m + I_g & 0 \\ 0 & I_l \end{bmatrix}$ $\quad \mathbf{D}: \begin{bmatrix} d_m + d_g + d_{gl} & -d_{gl} \\ -d_{gl} & d_l + d_{gl} \end{bmatrix}$

$\mathbf{q}: [q_m \;\; q_l]^T$ $\quad \mathbf{K}: \begin{bmatrix} k_b & -k_b \\ -k_b & k_b \end{bmatrix}$

$\mathbf{u}_q: [\tau_m \;\; \tau_l]^T$ $\quad \mathbf{B}_q: \begin{bmatrix} 0 & 0 & \frac{1}{I_g + I_m} & 0 \\ 0 & 0 & 0 & \frac{1}{I_l} \end{bmatrix}^T$

$$\mathbf{A}_q: \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_b}{I_g + I_m} & \frac{k_b}{I_g + I_m} & -\frac{d_m + d_g + d_{gl}}{I_g + I_m} & \frac{d_{gl}}{I_g + I_m} \\ \frac{k_b}{I_l} & -\frac{k_b}{I_l} & \frac{d_{gl}}{I_l} & -\frac{d_l + d_{gl}}{I_l} \end{bmatrix}$$

Fig2. Dynamic terms

change No.2

change of reference trajectory: change from sinus trajectory into 5th order polynomial

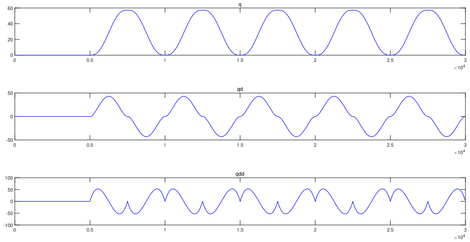


Fig3. reference trajectory

change No.3

Used dynamic parameters:

Motor rotor plus gear inertia [Nm] $I_m + I_g = 0.598$

Torsion bar inertia [Nm] $I_l = 1$

Torsion bar stiffness [Nm/rad] $K_b = 362$

Damping and noises added:

Motor Damping plus Gearbox damping [Nms/rad] $d_m + d_g = 2.2036$

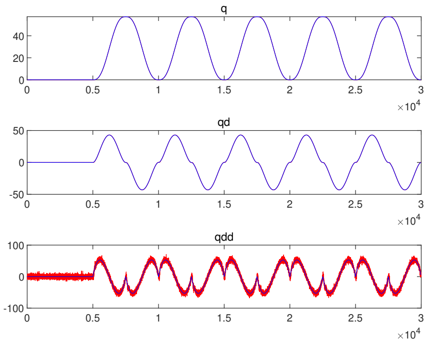
Torsion bar damping [Nms/rad] $d_l = 1$

Torsion bar internal damping [Nms/rad] $d_gl = 1.0000$

added input noises $N(0, \sqrt{var_u})$ $var_u = 1e - 10$

added output noises $N(0, \sqrt{var_y})$: $var_y = 1e - 15$ ();

performance



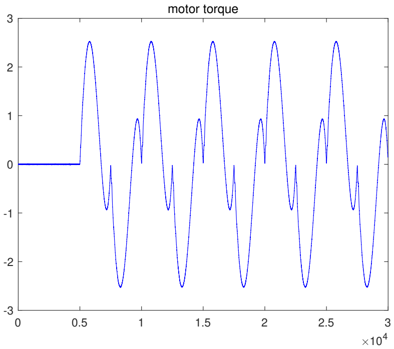


Fig4. Compare desired and is q and qd

Fig5. motor Torque

performance

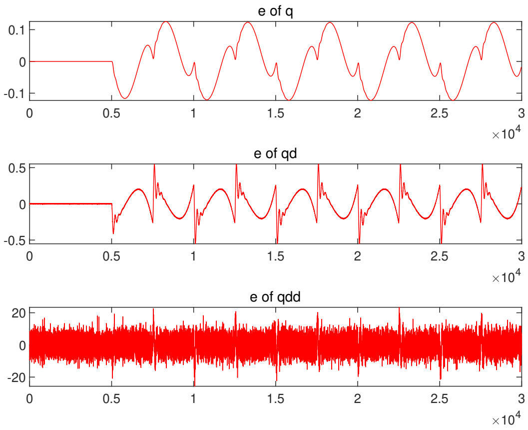


Fig6. error

Performance compare using different Horizon

Np=20; Nc=10
Np=20; Nc=20
Np=30; Nc=10
Np=30; Nc=30

Computing time compare using different Horizon

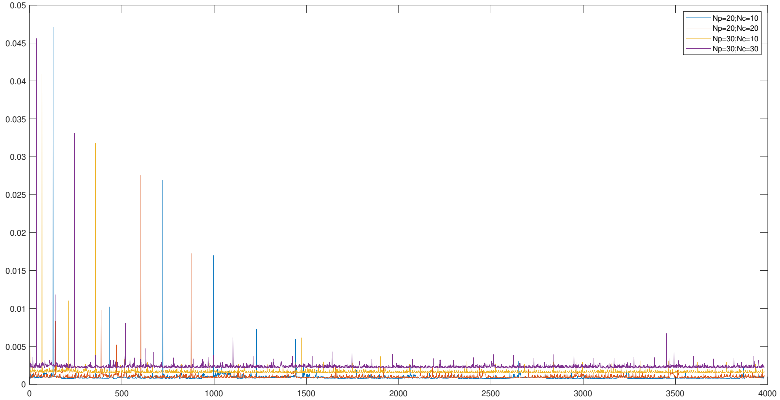


Fig7. Computing time using different Horizon

Performance compare using different Horizon

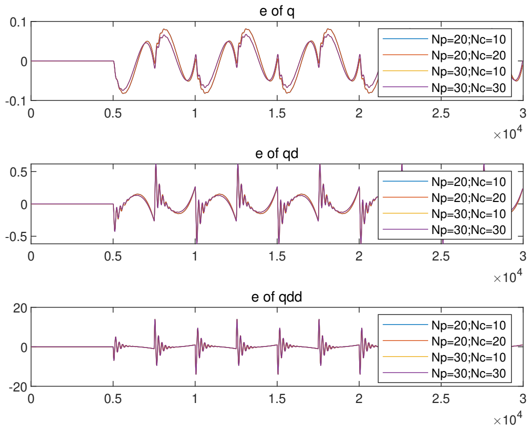
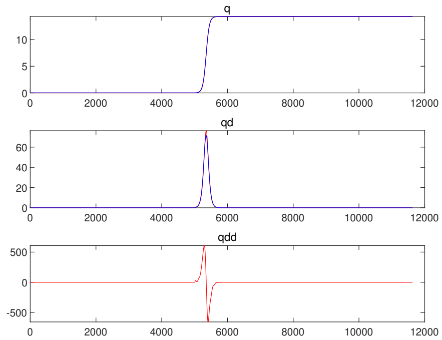


Fig8. Performance using different Horizon

step response (smooth step)

Q1 = 0; Q2 = 1000



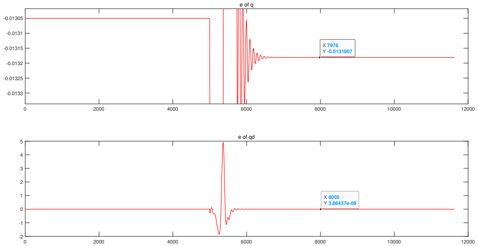


Fig9. Compare desired and is q and qd

Fig10. error

step response (smooth step)

Q1 = 10000; Q2 = 1000

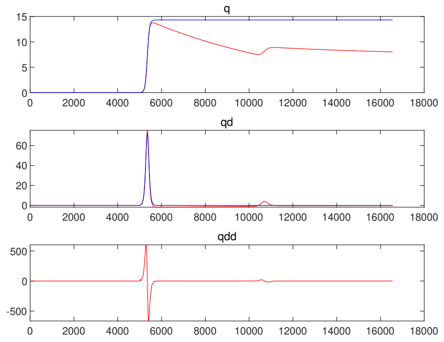


Fig11. Compare desired and is q and qd

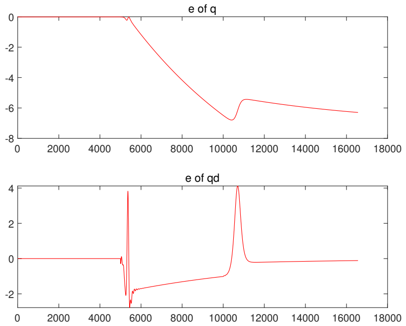


Fig12. error

Find out

maybe error formulation or coding regarding position error

Short summary

change to complete plant dynamics with damping and noises

check step response

Next step:
keep try on with other solvers

Timeline

- **Linear System formulation using TDE:** done
- **Incremental MPC: Cost function and constraints formulation:** still modifying
- **Simulation:** 01.Oktober ~20.November
  Integrate robot manipulator model into simulink
  Comparing the two solvers and different horizon (error and computation time)
- **Experiment:** 10.November ~10.December
  Comparing the two solvers and different horizon (error and computation time)
- **Possible Try:** 10.December ~30.December
  $\bar{M}$ and $\bar{D}$ online update

References