Incremental MPC for Flexible Robot Manipulators

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Outline

1. **Model**

2. **TDE**

3. **Incremental MPC**

4. **Simulation & Experiment**

5. **Possible Try**

6. **Timeline**

The dynamic model of the robot with compliant joints

Model

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{w}_l = \mathbf{\Gamma}$$

$$\mathbf{D}\ddot{\boldsymbol{\theta}} + \mathbf{w}_m + \mathbf{\Gamma} = \boldsymbol{\tau}$$

$$\mathbf{\Gamma} = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$$

Approximation of equations using Time-delayed Estimation

Two steps:

1. Separation

Introduce $\bar{\mathbf{M}}$ and $\bar{\mathbf{D}}$;
Rewrite the equation of motion into known and unknown parts

2. Approximation

$$(\mathbf{unknownpart})_{(t-L)} \cong (\mathbf{unknownpart})_{(t)}$$

with L is the delay time

Time-delayed Estimation

Introducing $\bar{\mathbf{M}}$, we have

$$\bar{\mathbf{M}}\ddot{\mathbf{q}} + \underbrace{(\mathbf{M}(\mathbf{q}) - \bar{\mathbf{M}})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{w}_l + \mathbf{K}\mathbf{q}}_{\mathbf{H}_1} = \mathbf{K}\boldsymbol{\theta}$$

Assuming the sampling period L is sufficiently small, we obtain

$$\mathbf{H}_1 \approx (\mathbf{H}_1)_{(t-L)} := \mathbf{K}\boldsymbol{\theta}_0 - \bar{\mathbf{M}}\ddot{\mathbf{q}}_0$$

Then, the following incremental system is obtained:

$$\ddot{\mathbf{q}} = \ddot{\mathbf{q}}_0 + \bar{\mathbf{M}}^{-1}\mathbf{K}\Delta\boldsymbol{\theta} + \boldsymbol{\epsilon}$$

where $\Delta\boldsymbol{\theta} := \boldsymbol{\theta} - \boldsymbol{\theta}_0$, $\boldsymbol{\epsilon}$ is the TDE error.

Introducing $\bar{\mathbf{D}}$, we have

$$\bar{\mathbf{D}}\ddot{\boldsymbol{\theta}} + \underbrace{(\mathbf{D} - \bar{\mathbf{D}})\ddot{\boldsymbol{\theta}} + \mathbf{w}_m + \boldsymbol{\Gamma}}_{\mathbf{H}_2} = \boldsymbol{\tau}$$

Similarly,

$$\mathbf{H}_2 \approx (\mathbf{H}_2)_{(t-L)} := \boldsymbol{\tau}_0 - \bar{\mathbf{D}}\ddot{\boldsymbol{\theta}}_0$$

Finally,

$$\ddot{\boldsymbol{\theta}} = \ddot{\boldsymbol{\theta}}_0 + \bar{\mathbf{D}}^{-1}\Delta\boldsymbol{\tau} + \boldsymbol{\varepsilon}$$

Interim conclusion

Approximation based on TDE

$$\ddot{\mathbf{q}} = \ddot{\mathbf{q}}_{(t-L)} + \bar{\mathbf{M}}^{-1}\mathbf{K}\Delta\boldsymbol{\theta} + \boldsymbol{\varepsilon}_{\mathbf{q}}$$

$$\ddot{\theta} = \ddot{\theta}_{(t-L)} + \bar{\mathbf{D}}^{-1}\Delta\boldsymbol{\tau} + \boldsymbol{\varepsilon}_{\boldsymbol{\theta}}$$

Linear system

1. Let $\varepsilon_{\mathbf{x}} = 0$ and $\varepsilon_{\mathbf{q}} = 0$

2. Change continuous to discrete-time form

3. Use Euler method

Linear system

Let $\mathbf{X}(k) := \mathrm{col}(\mathbf{q}(k), \dot{\mathbf{q}}(k), \dot{\boldsymbol{\theta}}(k))$, then we have

$$\underbrace{\begin{bmatrix} \mathbf{q}(k+1) \\ \dot{\mathbf{q}}(k+1) \\ \dot{\boldsymbol{\theta}}(k+1) \end{bmatrix}}_{\mathbf{X}(k+1)} = \underbrace{\begin{bmatrix} \mathbf{I} & T_s\mathbf{I} & \mathbf{O} \\ \mathbf{O} & 2\mathbf{I} & \bar{\mathbf{M}}^{-1}\mathbf{K}T_s^2 \\ \mathbf{O} & \mathbf{O} & 2\mathbf{I} \end{bmatrix}}_{\mathbf{A}_1} \underbrace{\begin{bmatrix} \mathbf{q}(k) \\ \dot{\mathbf{q}}(k) \\ \dot{\boldsymbol{\theta}}(k) \end{bmatrix}}_{\mathbf{X}(k)} + \underbrace{\begin{bmatrix} \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & -\mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & -\mathbf{I} \end{bmatrix}}_{\mathbf{A}_2} \underbrace{\begin{bmatrix} \mathbf{q}(k-1) \\ \dot{\mathbf{q}}(k-1) \\ \dot{\boldsymbol{\theta}}(k-1) \end{bmatrix}}_{\mathbf{X}(k-1)} + \underbrace{\begin{bmatrix} \mathbf{O} \\ \mathbf{O} \\ \bar{\mathbf{D}}^{-1}T_s \end{bmatrix}}_{\mathbf{B}_1} \Delta\boldsymbol{\tau}$$

Finally, let $\bar{\mathbf{X}}(k) := \mathrm{col}(\mathbf{X}(k), \mathbf{X}(k-1))$, we obtain

$$\underbrace{\begin{bmatrix} \mathbf{X}(k+1) \\ \mathbf{X}(k) \end{bmatrix}}_{\bar{\mathbf{X}}(k+1)} = \underbrace{\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{O} & \mathbf{I} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{X}(k) \\ \mathbf{X}(k-1) \end{bmatrix}}_{\bar{\mathbf{X}}(k)} + \underbrace{\begin{bmatrix} \mathbf{B}_1 \\ \mathbf{O} \end{bmatrix}}_{\mathbf{B}} \Delta\boldsymbol{\tau}$$

Incremental MPC

Predicted joint dynamics error

$$\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right) := \dot{\tilde{\mathbf{q}}}_{k+j+1|k} + \mathbf{K}_{\mathrm{P}}\tilde{\mathbf{q}}_{k+j+1|k}$$

with $\tilde{\mathbf{q}} := \mathbf{q} - \mathbf{q}_{\mathrm{d}}$ tracking error; $\mathbf{K}_{\mathrm{P}} \succ 0$.

Cost function

$$\ell = \underbrace{\left\|\mathbf{e}\left(\vec{\mathbf{x}}_{k+j+1|k}\right)\right\|_{\mathbf{Q}}^{2}}_{\text{predicted joint dynamics error}} + \underbrace{\left\|\Delta\boldsymbol{\tau}_{k+j|k}\right\|_{\mathbf{R}}^{2}}_{\text{control signal}}$$

with $\mathbf{Q}, \mathbf{R} \succ 0.$

Optimization problem

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \sum_{j=0}^{N-1} \ell\left(\mathbf{q}_{k+j+1|k}, \dot{\mathbf{q}}_{k+j+1|k}, \Delta\boldsymbol{\tau}_{k+j|k}\right)$$

s.t.

$$\vec{\mathbf{x}}_{k+j+1|k} = \mathbf{A}\vec{\mathbf{x}}_{k+j|k} + \mathbf{B}\Delta\boldsymbol{\tau}_{k+j|k}$$

$$\mathbf{q}_{\min} \leq \mathbf{q}_{k+j+1|k} \leq \mathbf{q}_{\max}$$

$$\dot{\mathbf{q}}_{\min} \leq \dot{\mathbf{q}}_{k+j+1|k} \leq \dot{\mathbf{q}}_{\max}$$

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau}_0 + \sum_{s=0}^{j} \Delta\boldsymbol{\tau}_{k+s|k} \leq \boldsymbol{\tau}_{\max}$$

Optimization problem

rewrite into

$$\Delta\bar{\tau}^* = \arg\min_{\Delta\bar{\tau}} \Delta\bar{\tau}^T Q \Delta\bar{\tau} + \Delta\bar{\tau}^T L$$

$$\text{s.t.}$$

$$G_1 = C_1 \Delta\bar{\tau} + D_1 \leq 0$$

$$G_2 = C_2 \Delta\bar{\tau} + D_2 \leq 0$$

$$G_3 = C_3 \Delta\bar{\tau} + D_3 \leq 0$$

Reference signal

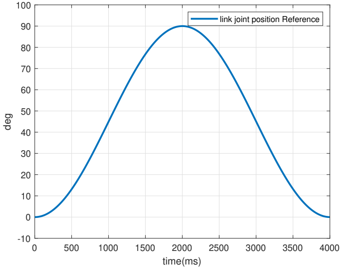

Fig1. Link joint position reference

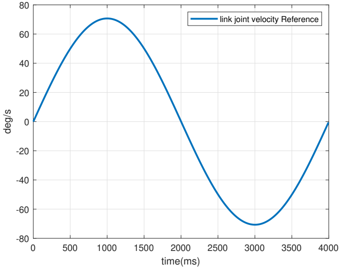

Fig2. Link joint velocity reference

Selected mechanical model

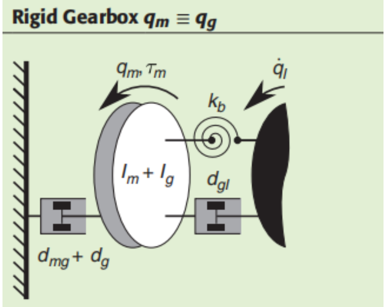

Fig3. Mechanical model

I: $I_m + I_g$ **D**: $d_m + d_g + d_{gl}$

q: $[q_m \;\; q_l]^T$ **K**: k_b

$\mathbf{u}_q$: $\begin{bmatrix} \tau_m \\ \dot{q}_l \end{bmatrix}$ $\mathbf{B}_q$: $\begin{bmatrix} 0, & 0, & \frac{1}{I_m + I_g} \\ 0, & 1, & \frac{d_{gl}}{I_m + I_g} \end{bmatrix}^T$

$$\mathbf{A}_q: \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -\frac{k_b}{I_m + I_g} & \frac{k_b}{I_m + I_g} & -\frac{d_m + d_g + d_{gl}}{I_m + I_g} \end{bmatrix}$$

Fig4. Dynamic terms

Selected mechanical model

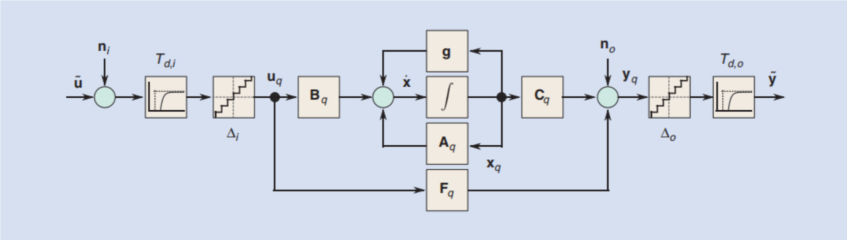

Fig5. The model structure for the mechanical subsystem

result: Controller not working

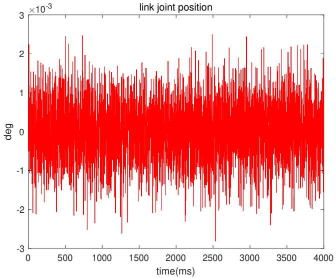

Fig6. Link joint position result

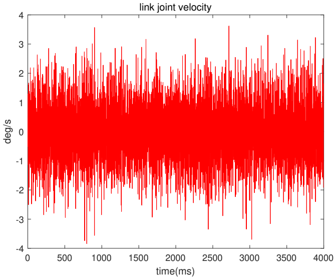

Fig7. Link joint velocity result

Compare

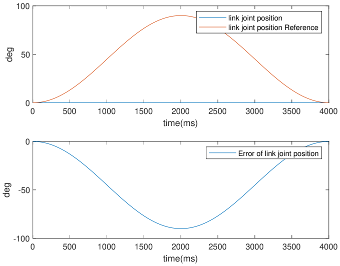

Fig8. Link joint position error

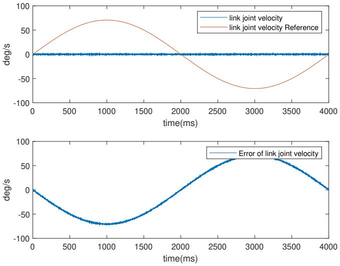

Fig9. Link joint velocity error

Working with simple controller (torque controller)

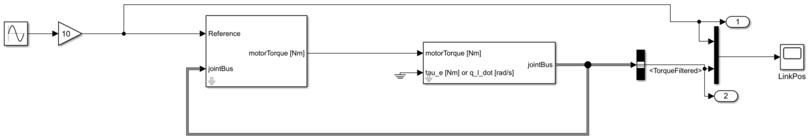

Fig10. Torque controller

Working with simple controller (torque controller)

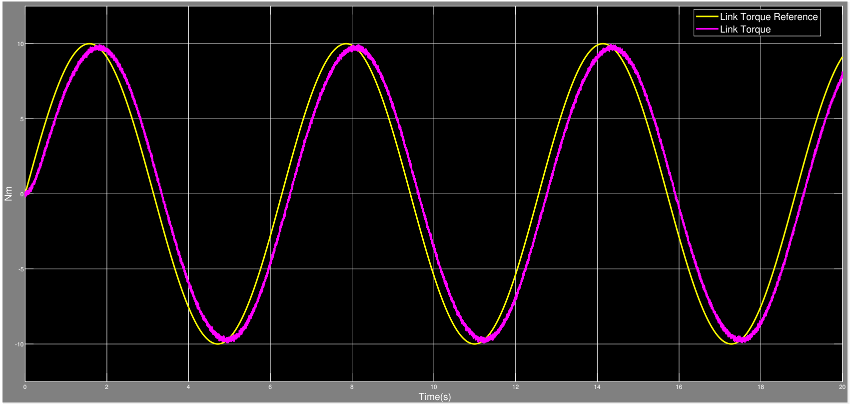

Fig11. Performance using torque controller

Short summary

Work fine by using simple PD controller to control the selected compliant joint:
⇒ Correct use of the "Compliant Joint Toolbox"
⇒ 1.Error in implementation of IMPC controller
2.Wrong setting of parameters($\bar{M}$, $\bar{D}$)

Next step:
Find the way to calculate $\bar{M}$ and $\bar{D}$ at initial gesture from the mechanical model dynamic

More parameters

Parameter	Value	Unit
$\bar{M}$	?	-
$\bar{D}$	?	-
Weighting matrix of dynamics error Q	?	-
Weighting matrix of control signals R	?	-
Limitation of joint position $q_{\text{min}}, q_{\text{max}}$	$-?,+?$	deg
Limitation of joint velocity $\dot{q}_{\text{min}}, \dot{q}_{\text{max}}$	$-?,+?$	deg/s
Limitation of torque $\tau_{\text{min}}, \tau_{\text{max}}$	$-?,+?$	Nm

Timeline

- **Linear System formulation using TDE:** done
- **Incremental MPC: Cost function and constraints formulation:** done
- **Simulation:** 01.Oktober ˜01.November
  Integrate robot manipulator model into simulink
  Comparing the two solvers and different horizon (error and computation time)
- **Experiment:** 20.Oktober ˜20.November
  Comparing the two solvers and different horizon (error and computation time)
- **Possible Try:** 20.November ˜10.December
  $\bar{M}$ and $\bar{D}$ online update

References