GAME2016
Mathematical Foundation of Game Design and Animation

Lecture 3

Matrices

Dr. Paolo Mengoni
pmengoni@hkbu.edu.hk
Senior Lecturer @HKBU Department of Interactive Media

Agenda

- Basic properties and operations of matrices
  - Mathematical perspective.
  - Geometrical perspective.


matrix
vector
transformation
matrices
column
vector
row
multiplication
linear
example
basis
algebra
element
coordinate
transformations
multiply
right
product
equations
elements
rows
result
space
book
columns
numbers
important
two
know
original
one
use
used
left
also
identity
scalar
dimension
array
notation
like
square
diagonal
multiplied
standard
programming
geometric
operations
following
resulting
chapter
visualize
order
Matrix
basic
written
system
game
place
many
first
lines
number
using
distinction
words
give
just
key
cube
corresponding
transform
represented
mathematical
may
Matrices
look
perform
formed
defined
lets
especially
previous
combination
allowed
concerning
general
Now
systems
derive
explains
get
mathematical

Matrix: An Algebraic Definition

Definitions

- Algebraic definition of a matrix: a table of scalars in square brackets.

- Matrix *dimension* is the width and height of the table, $w \times h$.

- Typically, we use dimensions 2 x 2 for 2D work, and 3 x 3 for 3D work.

- We'll find a use for 4 x 4 matrices also.

Matrix Components

- Entries are numbered by row and column
- eg. m_{ij} is the entry in row i, column j.
- Start numbering at 1, not 0.

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

Square Matrices

- Same number as rows as columns.
- Entries m_{ii} are called the *diagonal* entries. The others are called *nondiagonal* entries

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

Diagonal Matrices

- A diagonal matrix is a square matrix whose nondiagonal elements are zero.

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

The Identity Matrix

- The identity matrix of dimension n, denoted $\mathbf{I}_n$, is the n x n matrix with 1s on the diagonal and 0s elsewhere.

$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Vectors as Matrices

- A row vector is a 1 x *n* matrix.
- A column vector is an *n* x 1 matrix.
- They were pretty much interchangeable in the lecture on Vectors.
- They're not once you start treating them as matrices.

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

Transpose of a Matrix

- The transpose of an *r* x *c* matrix **M** is a *c* x *r* matrix called $\mathbf{M}^T$.
- Take every row and rewrite it as a column.
  - Equivalently, flip about the diagonal

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

- Transpose is its own inverse: $(\mathbf{M}^T)^T = \mathbf{M}$ for all matrices **M**.
- $\mathbf{D}^T = \mathbf{D}$ for all diagonal matrices **D** (including the identity matrix **I**).

Transpose of a Vector

- If **v** is a row vector, $\mathbf{v}^T$ is a column vector and vice-versa

$$\begin{bmatrix} x & y & z \end{bmatrix}^T = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix}^T = \begin{bmatrix} x & y & z \end{bmatrix}$$

Multiplying By a Scalar

- Can multiply a matrix by a scalar.
  - Result is a matrix of the same dimension.
- To multiply a matrix by a scalar, multiply each component by the scalar.

$$k\mathbf{M} = k\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & m_{43} \end{bmatrix} = \begin{bmatrix} km_{11} & km_{12} & km_{13} \\ km_{21} & km_{22} & km_{23} \\ km_{31} & km_{32} & km_{33} \\ km_{41} & km_{42} & km_{43} \end{bmatrix}$$

Matrix Addition

- Two matrices must have an equal number of rows and columns to be added.
- The sum of two matrices **A** and **B** will be a matrix which has the same number of rows and columns as **A** and **B**.
- The sum of **A** and **B**, denoted **A** + **B**, is computed by adding corresponding elements of **A** and **B.**
- Matrix subtraction follows the same rules

$$\mathbf{A}+\mathbf{B} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{bmatrix}$$

Matrix Addition Examples

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 0 \\ 7 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 7 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 \\ 12 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 7 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 \\ 1+7 & 0+5 \\ 1+2 & 2+1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 8 & 5 \\ 3 & 3 \end{bmatrix}$$

Matrix Multiplication

- Multiplying an *r* x *n* matrix **A** by an *n* x *c* matrix **B** gives an *r* x *c* result **AB**.

$$\underset{\substack{r\times n\\ 4\times 2}}{\overset{\mathbf{A}}{\begin{bmatrix} ? & ? \\ ? & ? \\ ? & ? \\ ? & ? \end{bmatrix}}}
\underset{\substack{n\times c\\ 2\times 5}}{\overset{\mathbf{B}}{\begin{bmatrix} ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \end{bmatrix}}}
=
\underset{\substack{r\times c\\ 4\times 5}}{\overset{\mathbf{AB}}{\begin{bmatrix} ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \end{bmatrix}}}$$

Multiplication: Result

- Multiply an $r \times n$ matrix **A** by an $n \times c$ matrix **B** to give an $r \times c$ result **C** = **AB**.
- Then **C** = $[c_{ij}]$, where c_{ij} is the dot product of the i^{th} row of **A** with the j^{th} column of **B**.
- That is:

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}.$$

Example

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} \end{bmatrix} =$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \end{bmatrix}$$

$$c_{24} = a_{21}b_{14} + a_{22}b_{24}$$

Another Way of Looking at It

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix} \quad \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} \end{bmatrix}$$

$$c_{43} = a_{41}b_{13} + a_{42}b_{23}$$

2 x 2 Case

$$\mathbf{AB} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

2 x 2 Example

$$\mathbf{A} = \begin{bmatrix} -3 & 0 \\ 5 & 1/2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -7 & 2 \\ 4 & 6 \end{bmatrix}$$

$$\begin{aligned} \mathbf{AB} &= \begin{bmatrix} -3 & 0 \\ 5 & 1/2 \end{bmatrix} \begin{bmatrix} -7 & 2 \\ 4 & 6 \end{bmatrix} \\ &= \begin{bmatrix} (-3)(-7) + (0)(4) & (-3)(2) + (0)(6) \\ (5)(-7) + (1/2)(4) & (5)(2) + (1/2)(6) \end{bmatrix} \\ &= \begin{bmatrix} 21 & -6 \\ -33 & 13 \end{bmatrix} \end{aligned}$$

3 x 3 Case

$$
\mathbf{AB} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}
$$

$$
= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}
$$

3 x 3 Example

$$\mathbf{A} = \begin{bmatrix} 1 & -5 & 3 \\ 0 & -2 & 6 \\ 7 & 2 & -4 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -8 & 6 & 1 \\ 7 & 0 & -3 \\ 2 & 4 & 5 \end{bmatrix}$$

$$\begin{aligned} \mathbf{AB} &= \begin{bmatrix} 1 & -5 & 3 \\ 0 & -2 & 6 \\ 7 & 2 & -4 \end{bmatrix} \begin{bmatrix} -8 & 6 & 1 \\ 7 & 0 & -3 \\ 2 & 4 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 1\cdot(-8) + (-5)\cdot 7 + 3\cdot 2 & 1\cdot 6 + (-5)\cdot 0 + 3\cdot 4 & 1\cdot 1 + (-5)\cdot(-3) + 3\cdot 5 \\ 0\cdot(-8) + (-2)\cdot 7 + 6\cdot 2 & 0\cdot 6 + (-2)\cdot 0 + 6\cdot 4 & 0\cdot 1 + (-2)\cdot(-3) + 6\cdot 5 \\ 7\cdot(-8) + 2\cdot 7 + (-4)\cdot 2 & 7\cdot 6 + 2\cdot 0 + (-4)\cdot 4 & 7\cdot 1 + 2\cdot(-3) + (-4)\cdot 5 \end{bmatrix} \\ &= \begin{bmatrix} -37 & 18 & 31 \\ -2 & 24 & 36 \\ -50 & 26 & -19 \end{bmatrix} \end{aligned}$$

Identity Matrix

- Recall that the identity matrix **I** (or $\mathbf{I}_n$) is a diagonal matrix whose diagonal entries are all 1.

- Now that we've seen the definition of matrix multiplication, we can say that **IM** = **MI** = **M** for all matrices **M** (dimensions appropriate)

$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix Multiplication Facts

- Not commutative: in general $\mathbf{AB} \neq \mathbf{BA}$.
- Associative:

$$(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$$

- Associates with scalar multiplication:

$$k(\mathbf{AB}) = (k\mathbf{A})\mathbf{B} = \mathbf{A}(k\mathbf{B})$$

- $(\mathbf{AB})^\mathrm{T} = \mathbf{B}^\mathrm{T}\mathbf{A}^\mathrm{T}$
- $(\mathbf{M}_1\mathbf{M}_2\mathbf{M}_3 \ldots \mathbf{M}_n)^\mathrm{T} = \mathbf{M}_n^\mathrm{T} \ldots \mathbf{M}_3^\mathrm{T}\mathbf{M}_2^\mathrm{T}\mathbf{M}_1^\mathrm{T}$
  - Very important for GDA!

Row Vector Times Matrix Multiplication

- Can multiply a row vector times a matrix

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} =$$

$$\begin{bmatrix} xm_{11} + ym_{21} + zm_{31} & xm_{12} + ym_{22} + zm_{32} & xm_{13} + ym_{23} + zm_{33} \end{bmatrix}$$

Matrix Times Column Vector Multiplication

- Can multiply a matrix times a column vector.

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} xm_{11} + ym_{12} + zm_{13} \\ xm_{21} + ym_{22} + zm_{23} \\ xm_{31} + ym_{32} + zm_{33} \end{bmatrix}$$

Row vs. Column Vectors

- Row vs. column vector matters now. Here's why:
- Let $\mathbf{v}$ be a row vector, $\mathbf{M}$ a matrix.
  - $\mathbf{vM}$ is legal, $\mathbf{Mv}$ is undefined
  - $\mathbf{Mv}^T$ is legal, $\mathbf{v}^T\mathbf{M}$ is undefined
- DirectX uses row vectors.
- OpenGL uses column vectors.

Vector-Matrix Multiplication Facts 1

- Associates with vector multiplication.
  - Let **v** be a row vector:

$$\mathbf{v}(\mathbf{AB}) = (\mathbf{vA})\mathbf{B}$$

  - Let **v** be a column vector:

$$(\mathbf{AB})\mathbf{v} = \mathbf{A}(\mathbf{Bv})$$

Vector-Matrix Multiplication Facts 2

- Vector-matrix multiplication distributes over vector addition:

$$(\mathbf{v} + \mathbf{w})\mathbf{M} = \mathbf{vM} + \mathbf{wM}$$

- That was for row vectors **v**, **w**. Similarly for column vectors.

Matrix – a Geometric Interpretation

Matrices and Geometry

- A square matrix can perform any *linear transformation*.

- What’s that?
  - Preserves straight lines
  - Preserves parallel lines.
  - No translation: the axes do not move.

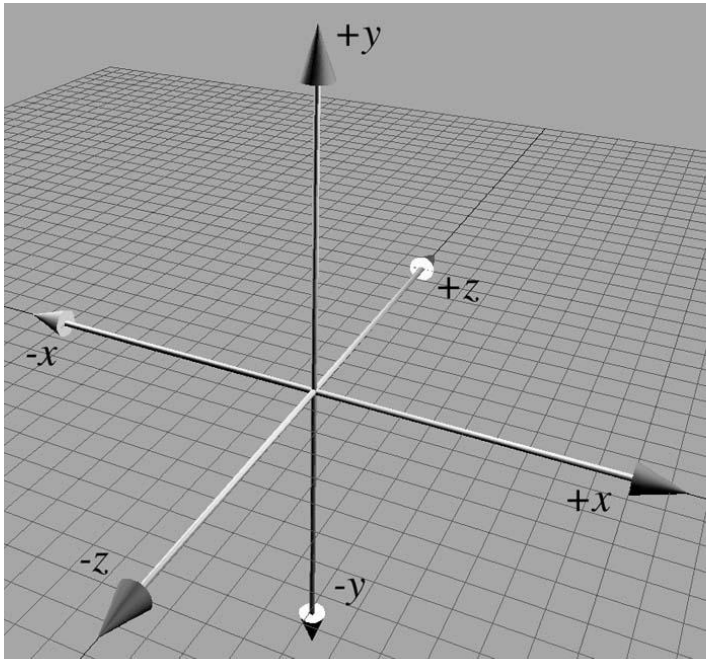
Linear Transformations

- Rotation
- Scaling
- Orthographic projection
- Reflection
- Shearing
- More about these in the next chapter.

A Movie Quote

- "Unfortunately, no-one can be told what The Matrix is – you have to see it for yourself."

- Actually, it's all about *basis vectors*.
  - Roughly speaking the vectors that represent each axis



Axial Displacements

Can rewrite any vector $\mathbf{v} = [x\ y\ z]$ as a sum of *axial displacements*.

$$V = [x\ y\ z]$$
$$= [x\ 0\ 0] + [0\ y\ 0] + [0\ 0\ z]$$
$$= x\,[1\ 0\ 0] + y\,[0\ 1\ 0] + z\,[0\ 0\ 1]$$

Basis Vectors

- Define three unit vectors along the axes:

$$\mathbf{p} = [1\ 0\ 0]$$
$$\mathbf{q} = [0\ 1\ 0]$$
$$\mathbf{r} = [0\ 0\ 1]$$

- Then we can rewrite the axial displacement equation as

$$\mathbf{v} = x\mathbf{p} + y\mathbf{q} + z\mathbf{r}$$

- **p**, **q**, **r** are known as *basis vectors*

Arbitrary Basis Vectors

- Can use any three linearly independent vectors

$$\mathbf{p} = [p_x \; p_y \; p_z]$$
$$\mathbf{q} = [q_x \; q_y \; q_z]$$
$$\mathbf{r} = [r_x \; r_y \; r_z]$$

- *Linearly independent* means that there do <u>not</u> exist scalars *a,b,c* such that:

$$a\mathbf{p} + b\mathbf{q} + c\mathbf{r} = \mathbf{0}$$

Orthonormal Basis Vectors

- Best to use an orthonormal basis
- Orthonormal means unit vectors that are pairwise orthogonal:

$$\mathbf{p} \cdot \mathbf{q} = \mathbf{q} \cdot \mathbf{r} = \mathbf{r} \cdot \mathbf{p} = 0$$

- Otherwise, things can get weird.

Matrix From Basis Vectors

Construct a matrix **M** using **p**, **q**, **r** as the rows of the matrix:

$$\mathbf{M} = \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \\ \mathbf{r} \end{bmatrix} = \begin{bmatrix} \mathbf{p}_x & \mathbf{p}_y & \mathbf{p}_z \\ \mathbf{q}_x & \mathbf{q}_y & \mathbf{q}_z \\ \mathbf{r}_x & \mathbf{r}_y & \mathbf{r}_z \end{bmatrix}$$

What Does This Matrix Do?

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} \mathbf{p}_x & \mathbf{p}_y & \mathbf{p}_z \\ \mathbf{q}_x & \mathbf{q}_y & \mathbf{q}_z \\ \mathbf{r}_x & \mathbf{r}_y & \mathbf{r}_z \end{bmatrix}$$

$$= \begin{bmatrix} x\mathbf{p}_x + y\mathbf{q}_x + z\mathbf{r}_x & x\mathbf{p}_y + y\mathbf{q}_y + z\mathbf{r}_y & x\mathbf{p}_z + y\mathbf{q}_z + z\mathbf{r}_z \end{bmatrix}$$

$$= x\mathbf{p} + y\mathbf{q} + z\mathbf{r}$$

Transformation by a Matrix

- If we interpret the rows of a matrix as the basis vectors of a coordinate space, then multiplication by the matrix performs a coordinate space transformation.

- If **aM** = **b**, we say that vector **a** is *transformed* by the matrix **M** into vector **b**.

Conversely

- See what **M** does to the original basis vectors [1 0 0], [0 1 0], [0 0 1].

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \begin{bmatrix} m_{21} & m_{22} & m_{23} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \begin{bmatrix} m_{31} & m_{32} & m_{33} \end{bmatrix}$$

Visualize The Matrix

- Each row of a matrix is a basis vector after transformation.
- Given an arbitrary matrix, visualize the transformation by its effect on the standard basis vectors – the rows of the matrix.
- Given an arbitrary linear transformation, create the matrix by visualizing what it does to the standard basis vectors and using that for the rows of the matrix.

2D Matrix Example

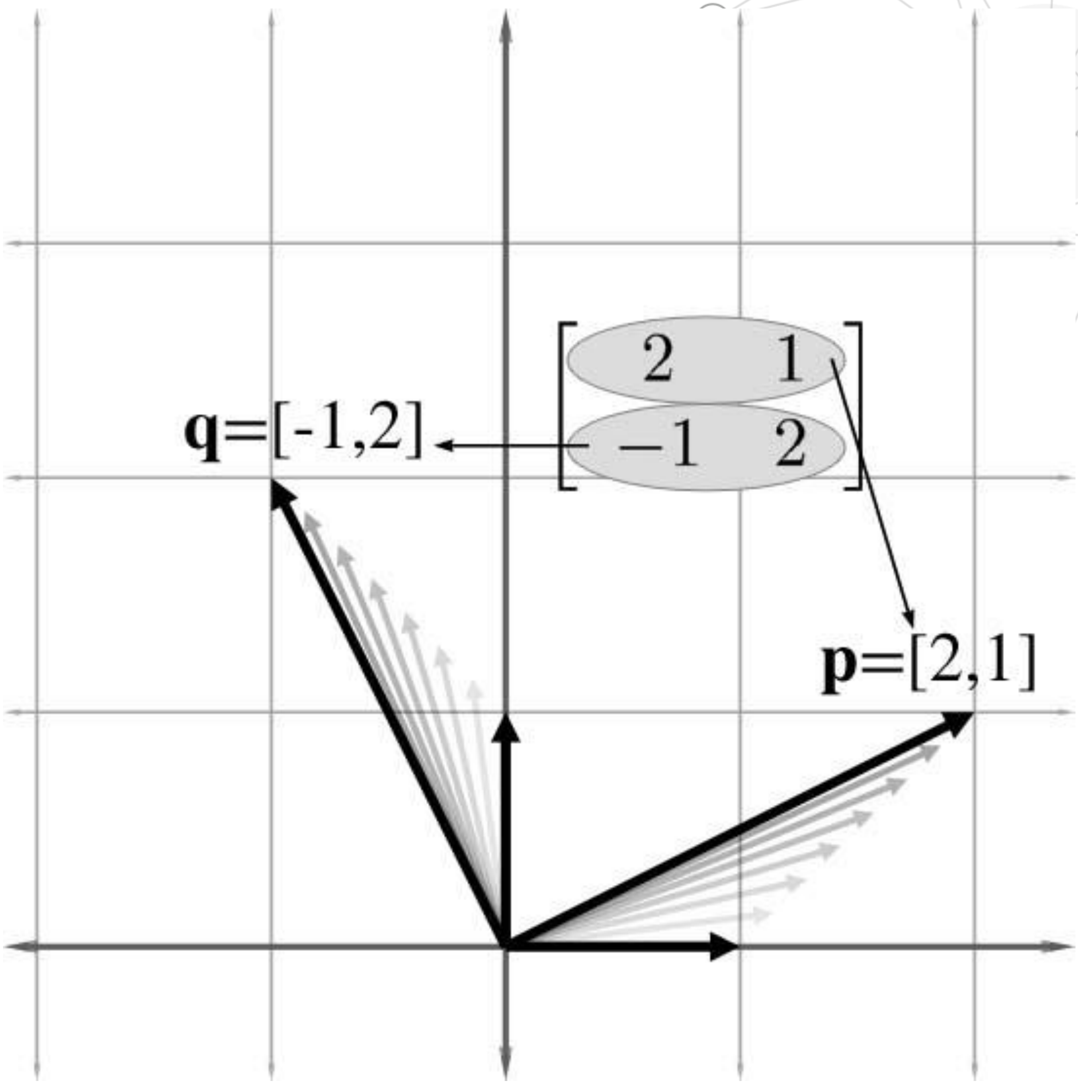
- What does the following 2D matrix do?

$$\mathbf{M} = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$$

- Extract the basis vectors (the rows of **M**)

p = [2 1]

q = [-1,2]



$\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$
q=[-1,2]
p=[2,1]

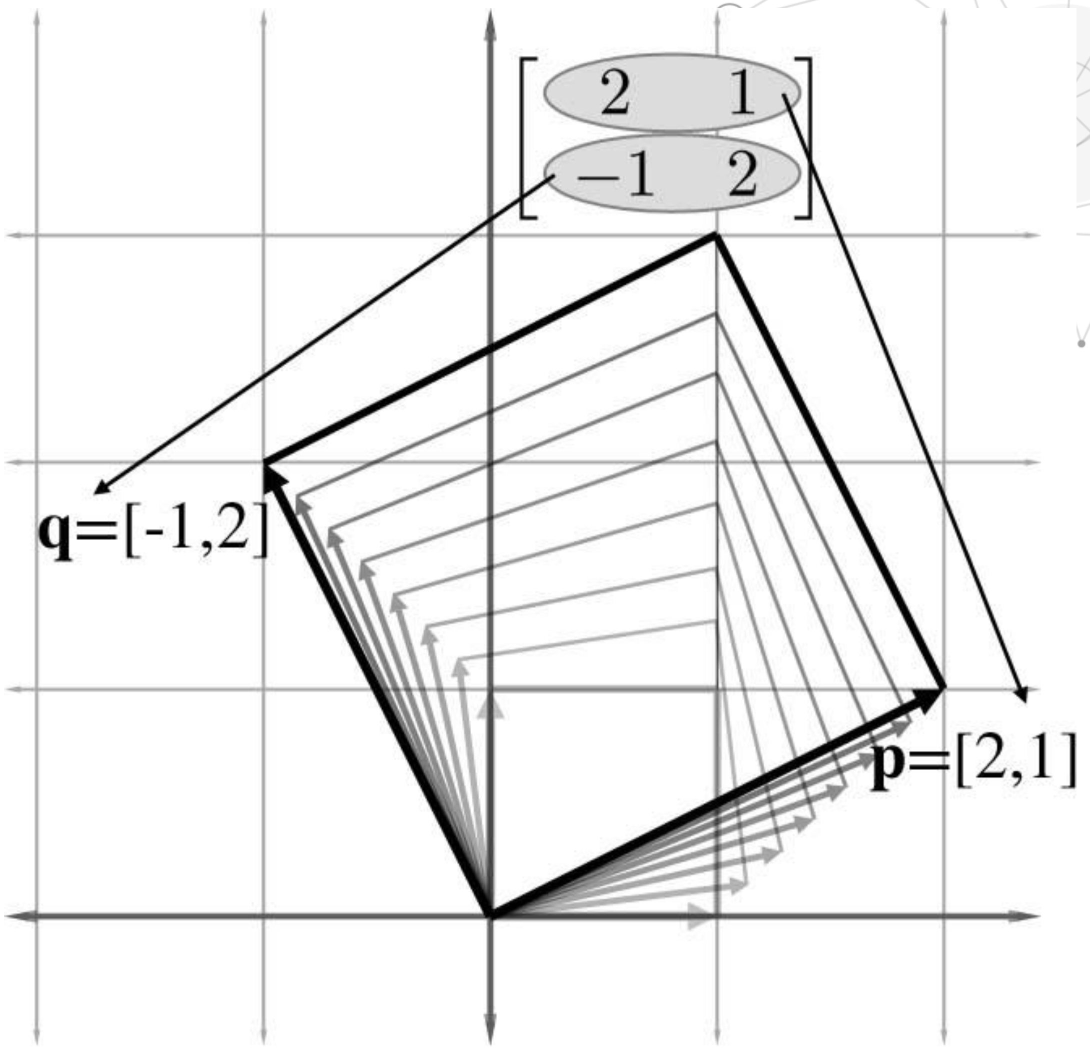
What's the Transformation?

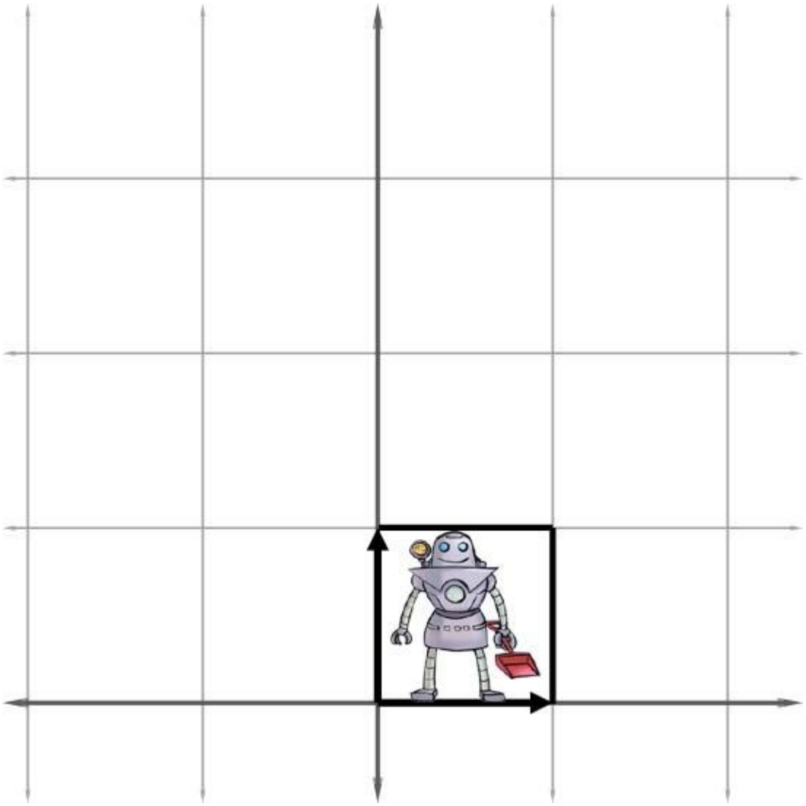
- It moves the unit axes [1, 0] and [0, 1] to the new axes.

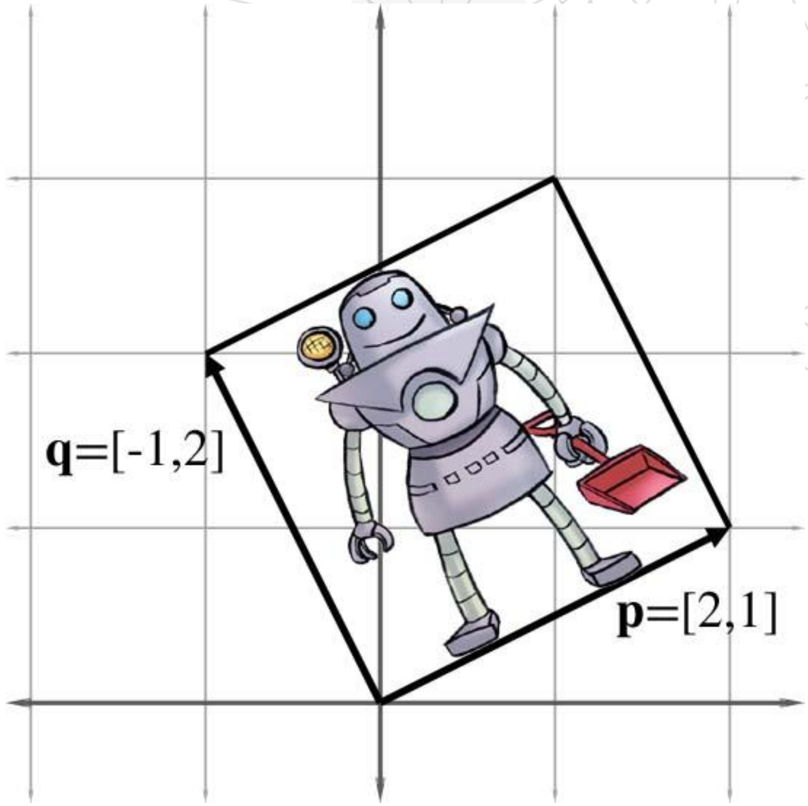
- It does the same thing to all vectors.

- Visualize a box being transformed from one coordinate system to the other.

- This is called a *skew box*.


$\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$
q=[-1,2]
p=[2,1]





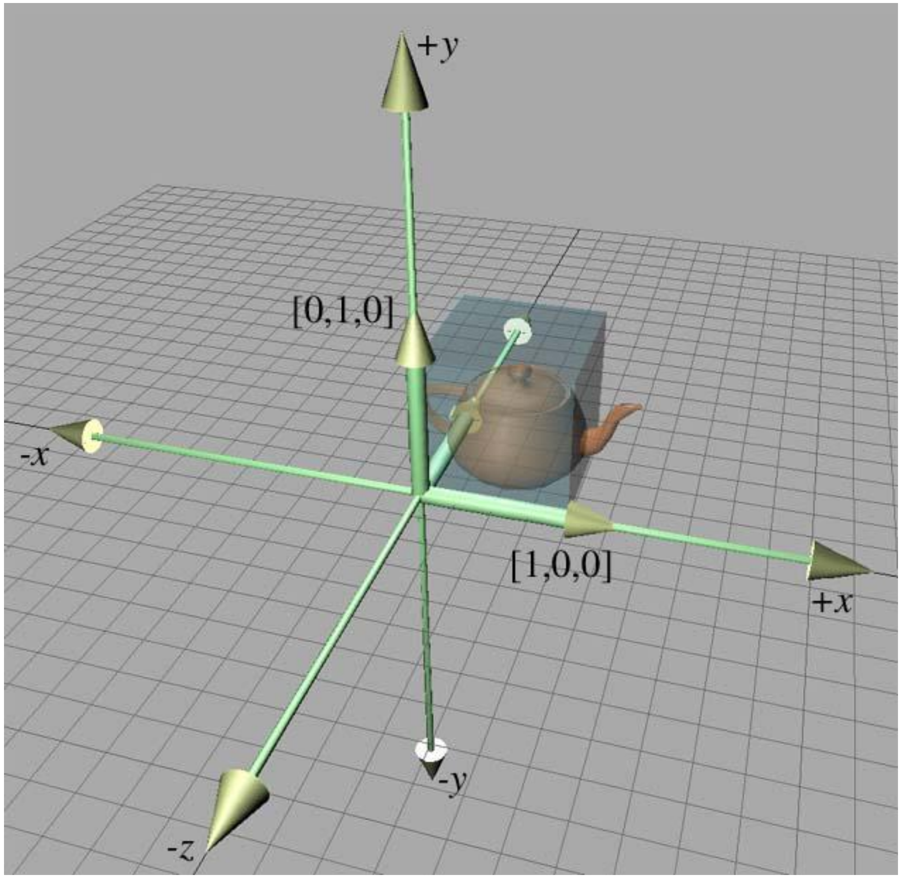
Before

After

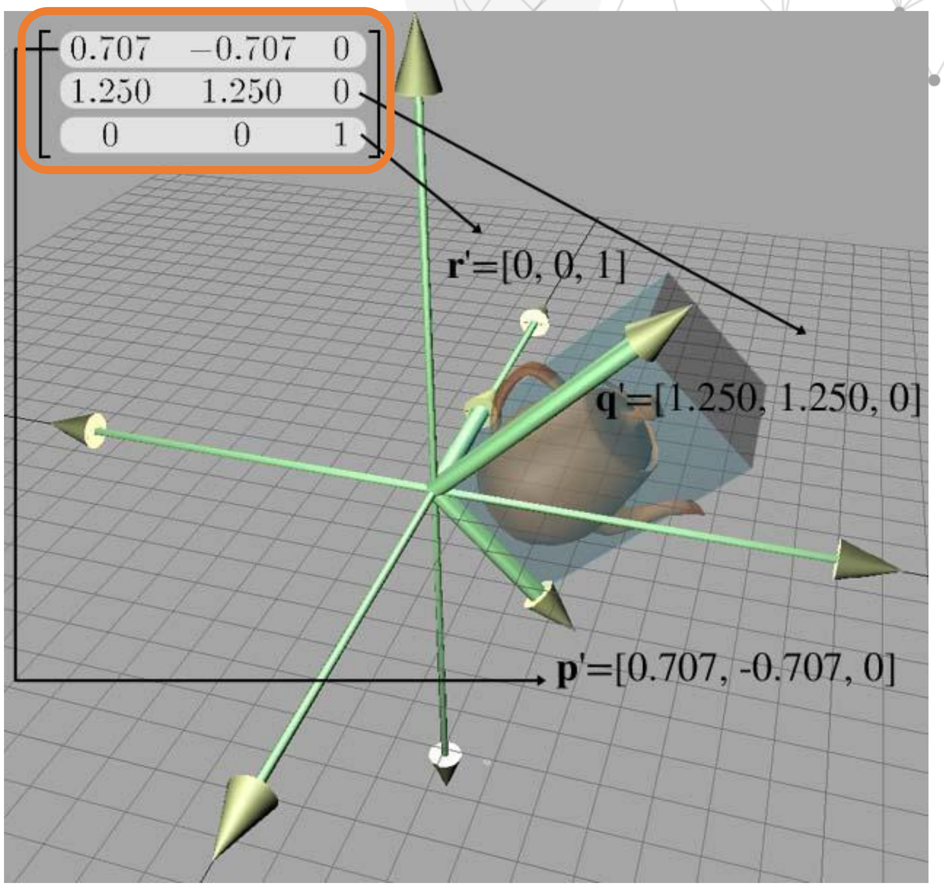
So What Does It Do?

- Rotates objects counterclockwise by a small amount.
- Scales them up by a factor of two.

3D Transformation Example



Before



After

What's the Matrix?

- Get rows of matrix from new basis vectors.

$$\begin{bmatrix} 0.707 & -0.707 & 0 \\ 1.250 & 1.250 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, what does it do?

- Rotates objects clockwise by 45°.
- Scales them up along the *y* axis.

Constructing & Deconstructing Matrices

- By interpreting the rows of a matrix as basis vectors, we have a tool for deconstructing a matrix.
- But we also have a tool for constructing one! Given a desired transformation (e.g. rotation, scale, etc.), we can derive a matrix which represents that transformation.
- All we have to do is figure out what the transformation does to basis vectors, and then place those transformed basis vectors into the rows of a matrix.
- We'll use this tool repeatedly to derive the matrices to perform the linear basic transformations such as rotation, scale, shear, and reflection that we mentioned earlier.