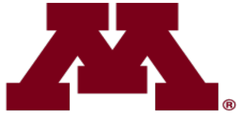
Rare Region Effects on the Ising-nematic Quantum Phase Transition

Tianbai Cui, Rafael Fernandes



UNIVERSITY OF MINNESOTA

What is Ising-nematic phase?

Isotropic

total melting

Rotation

Nematic

partial melting

Translation & Rotation

Smectic

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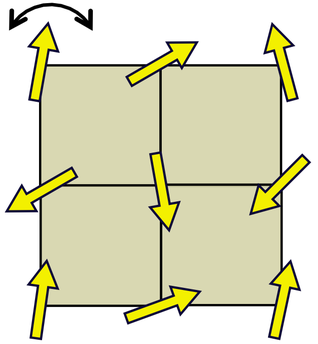

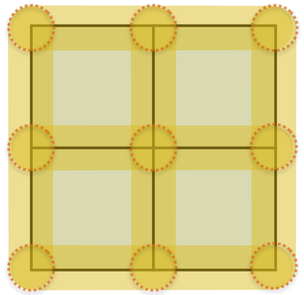

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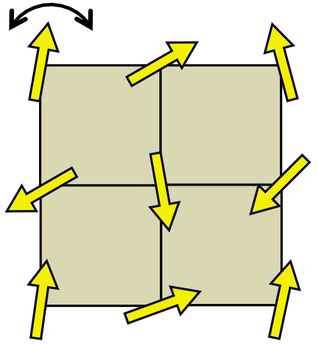

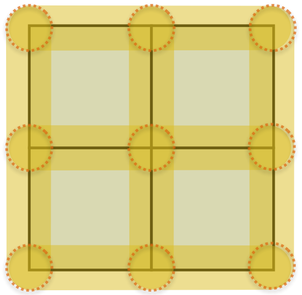

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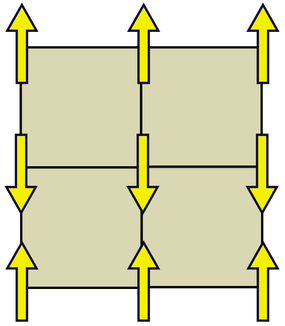

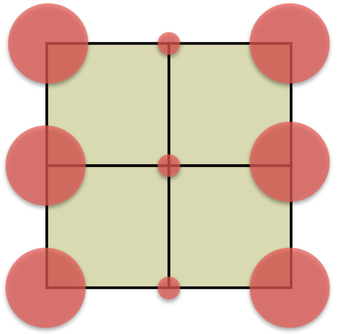

C_4 & Translation

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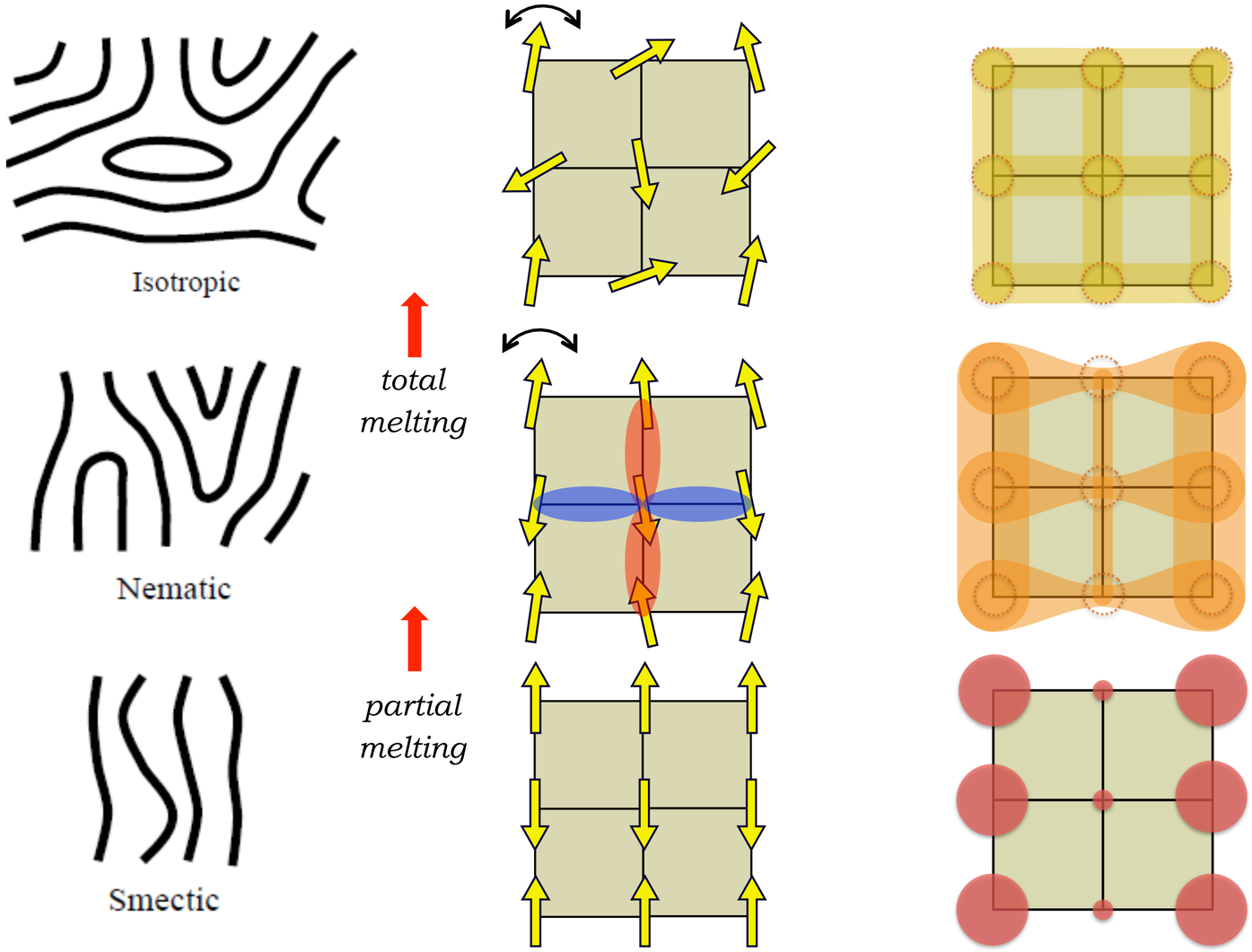

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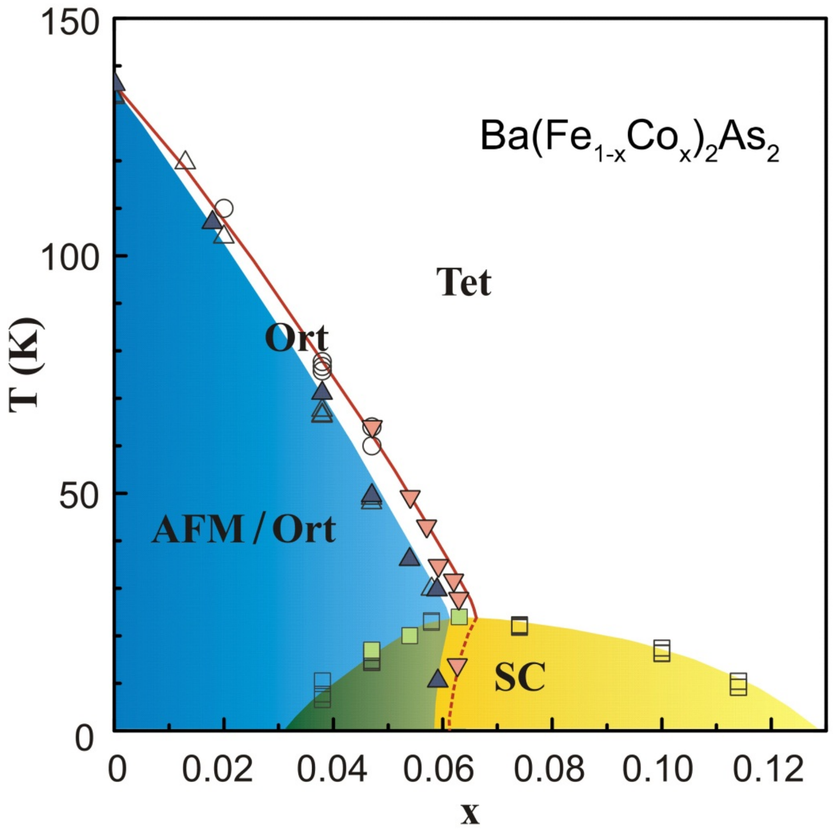
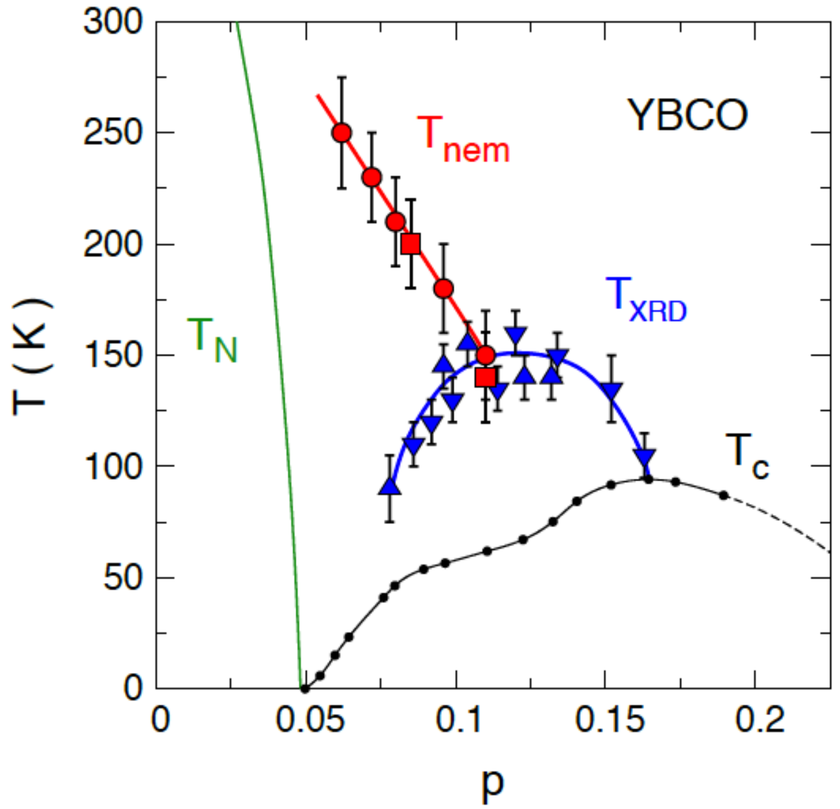
Phase Diagrams of High-Tc Superconductors

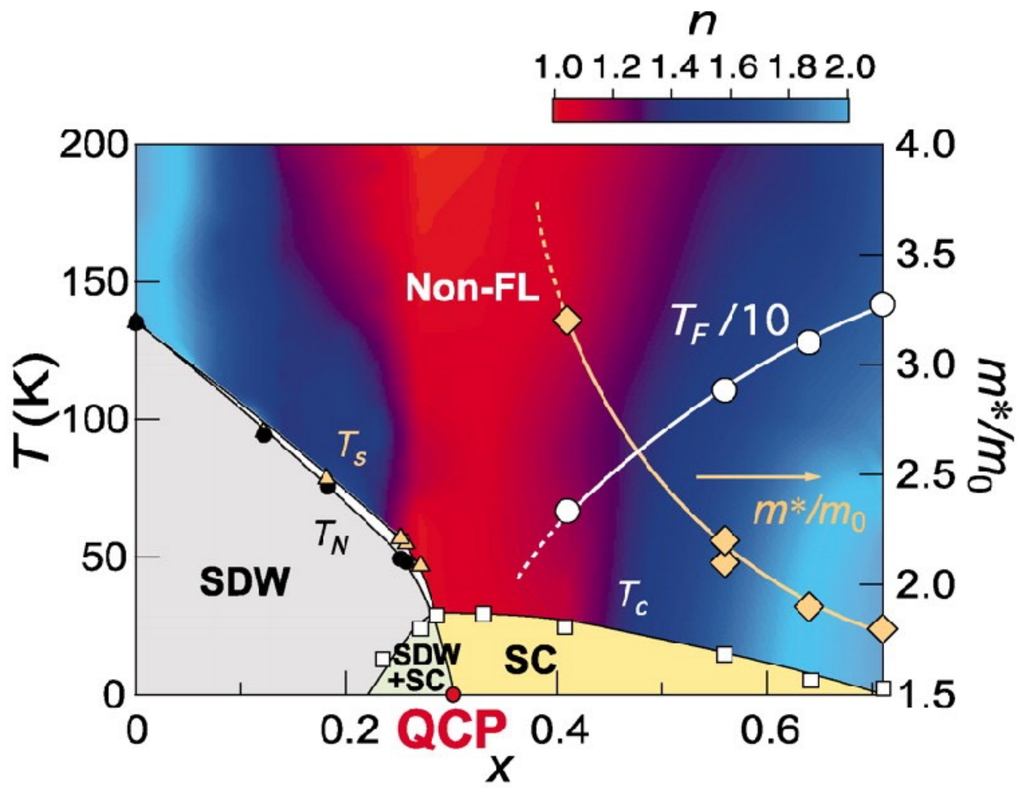




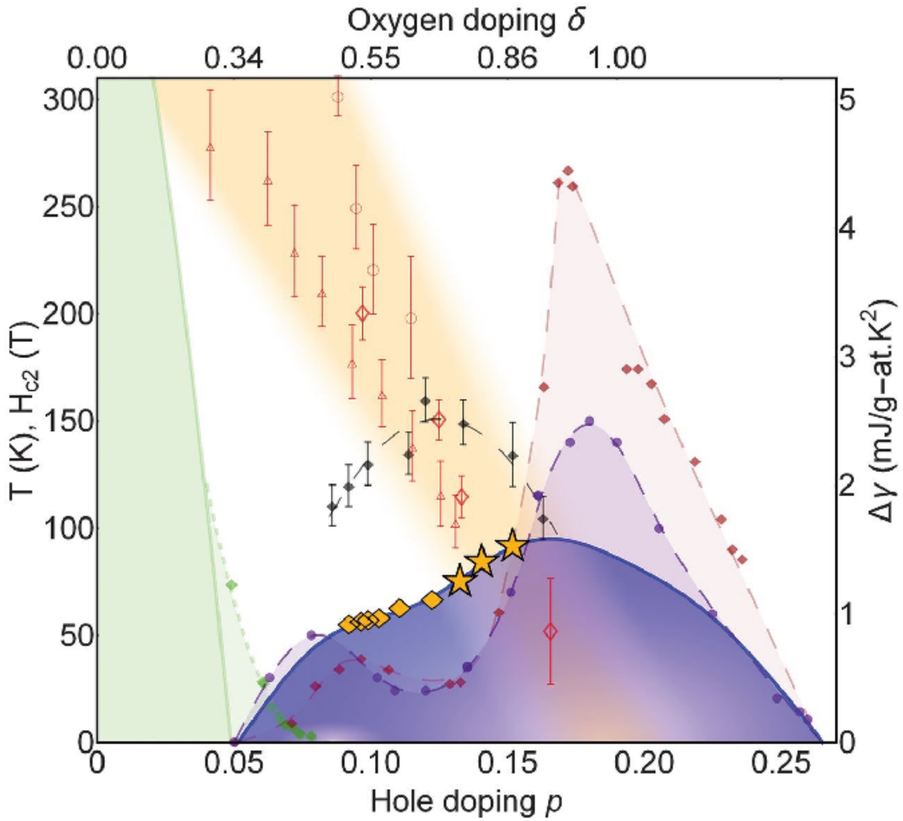
Nematic phase: Electronic order breaks lattice symmetry

[1] S. Nandi *et al*, Phys. Rev. Lett. **104**, 57006 (2010).
[2] O. Cyr-Choinière *et al*, Phys. Rev. B **92**, 224502 (2015).

Motivation



$BaFe_2\left(As_{1-x}P_x\right)_2$



$YBa_2Cu_3O_{6+\delta}$

[1] K. Hashimoto *et al*, Science, **336**(6088):1554–1557, (2012).
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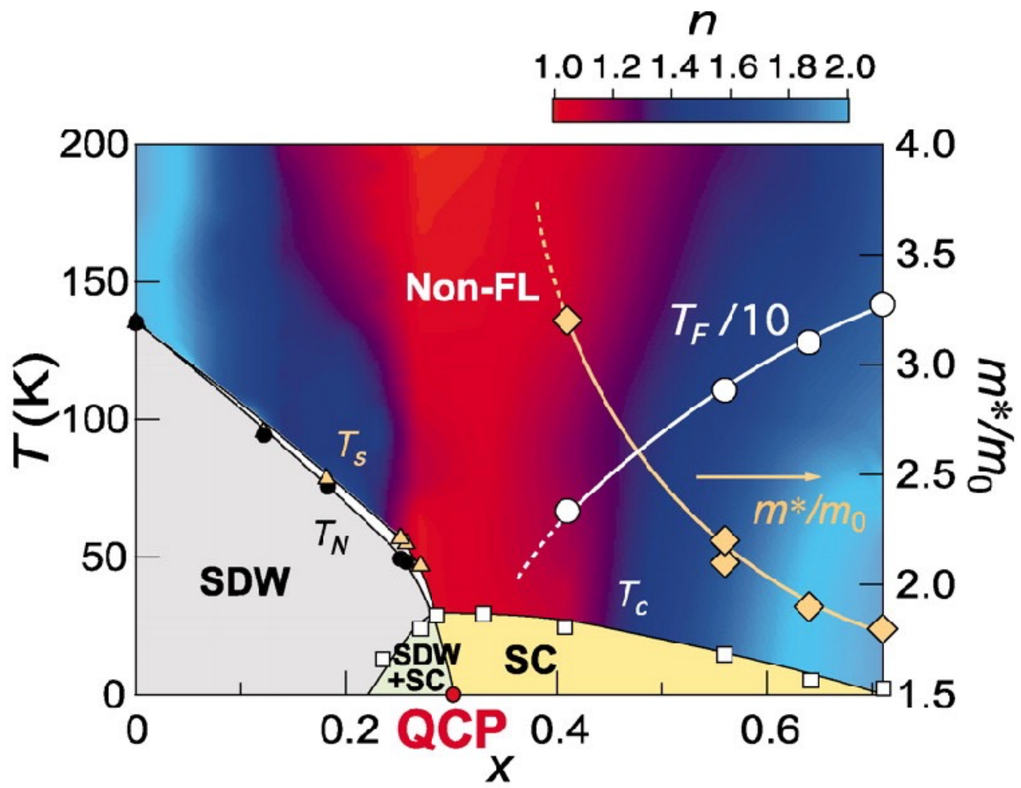

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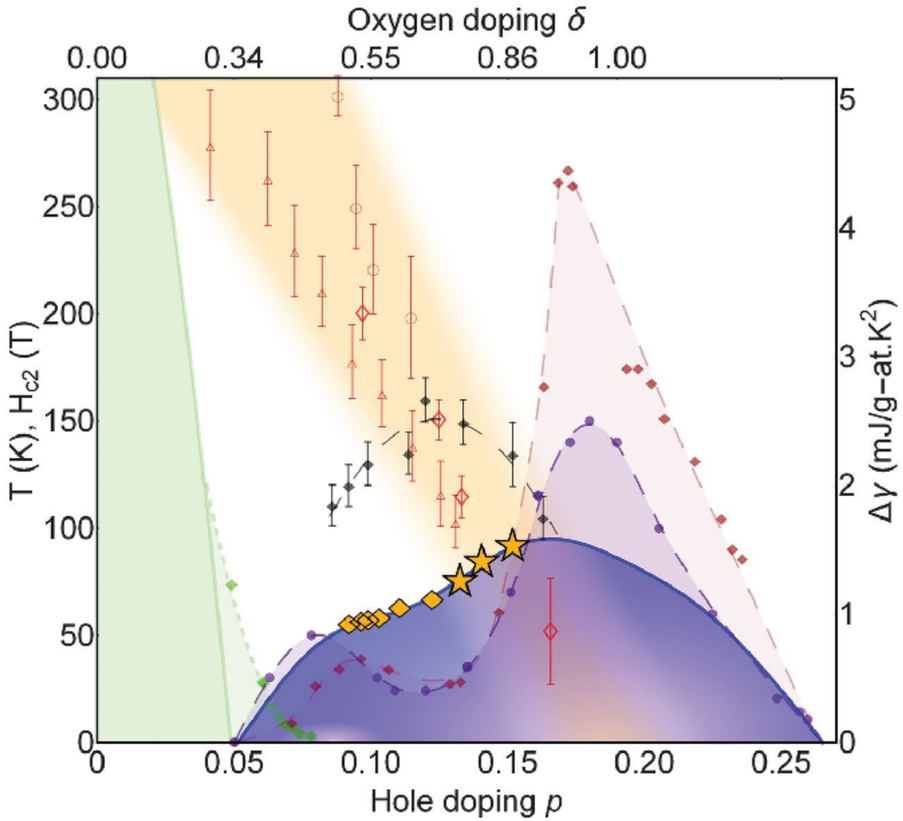

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Can this QCP originate from Ising-nematic order?

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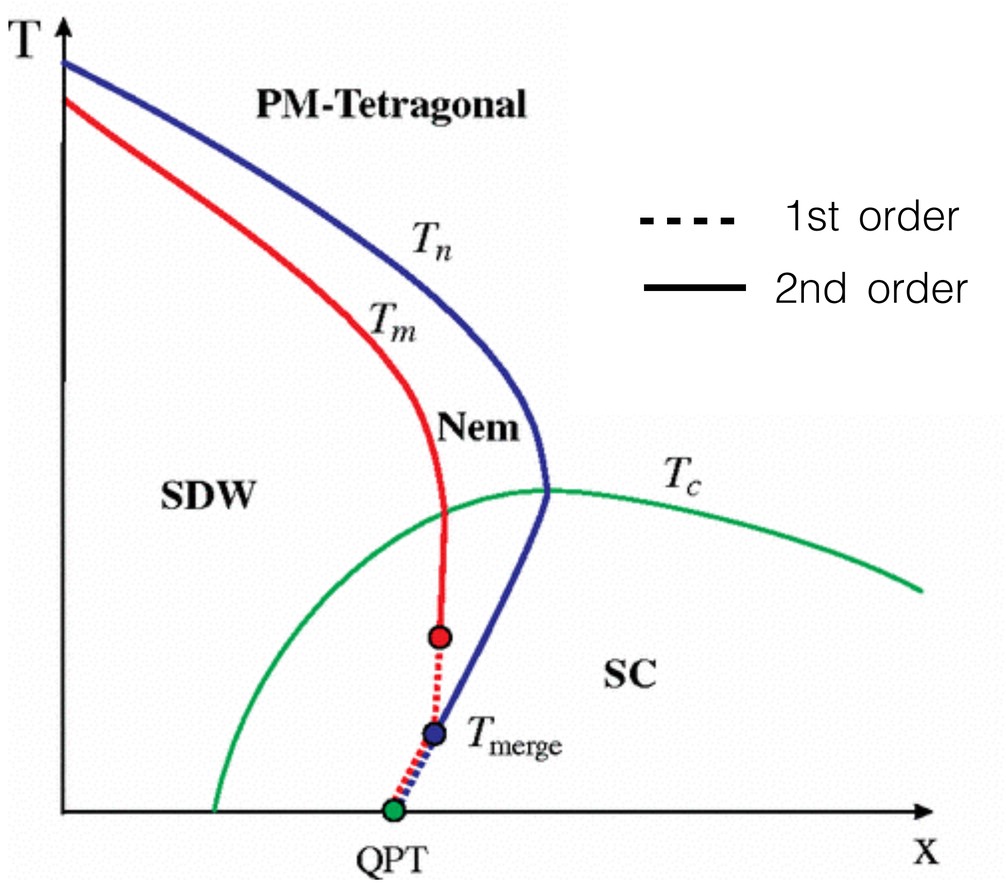

[1] R. M. Fernandes, S. Maiti, P. Woelfle, and A. V Chubukov, Phys. Rev. Lett. **111**, 057001 (2013).

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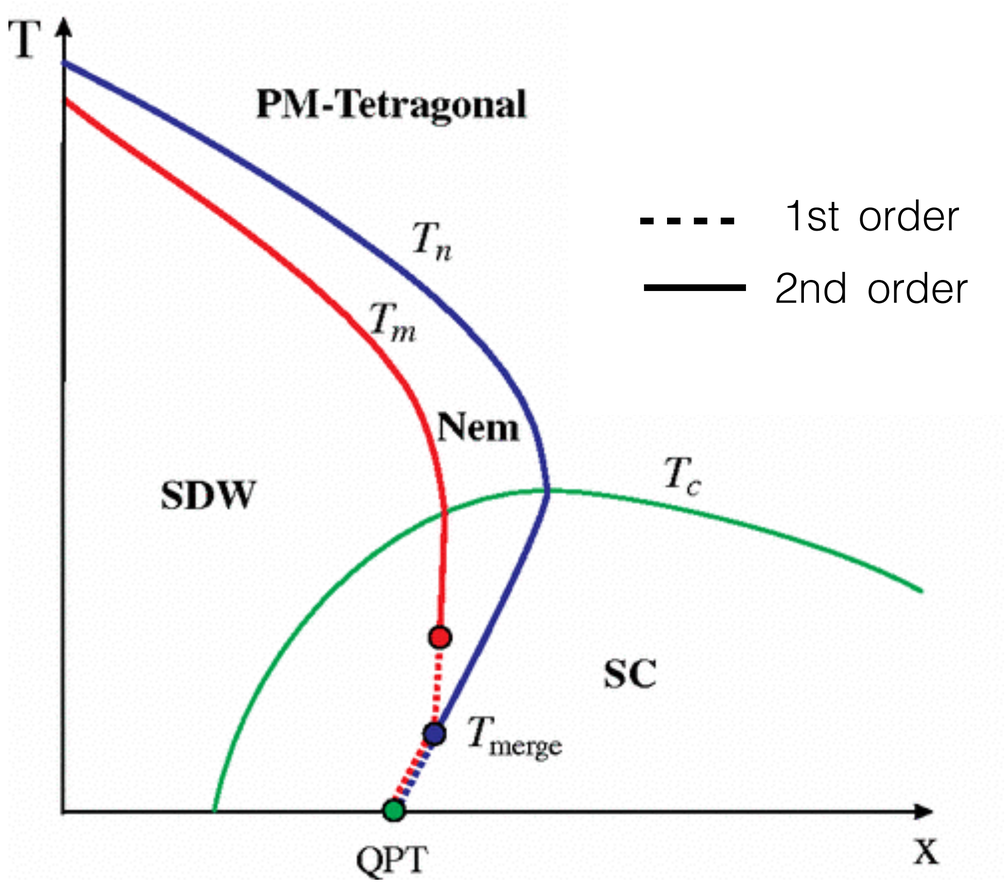

Split or simultaneous?

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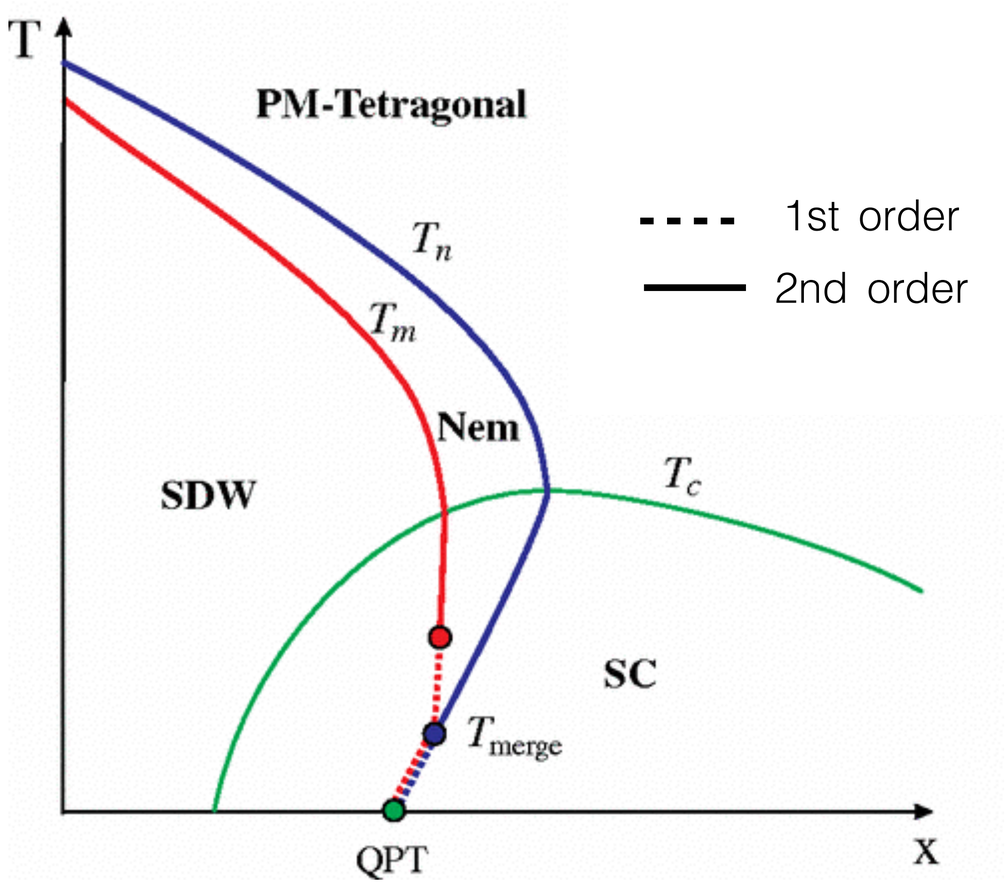

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First or second order?

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Theoretical Difficulties

Randomness:
Introduce random variables into the theory

Inhomogeneity:
Spatial translation invariance is broken

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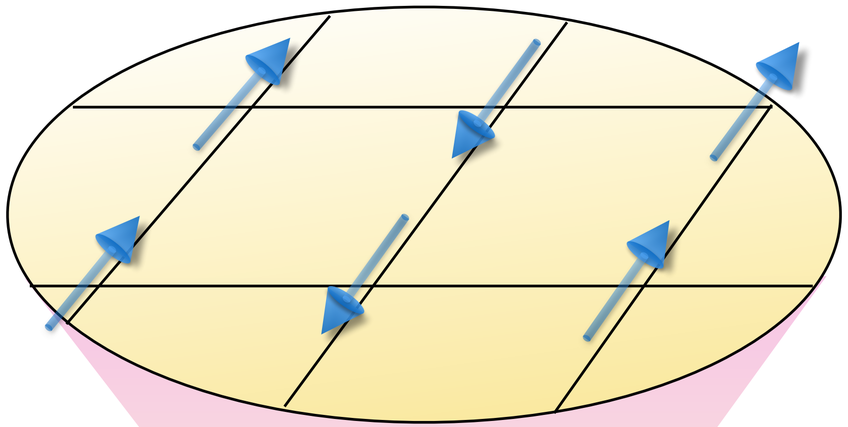
Thinking in real space! → Rare region effects

[1] R. B. Griffiths, Phys. Rev. Lett. **23**(1), 17–19 (1969)

Landau-Ginzburg-Wilson (Clean)

$$S = \int \frac{d^2q}{(2\pi)^2} \int \frac{d\omega}{2\pi} \left\{ \chi^{-1}_{q,\omega} \left(\mathbf{M}_X^2 + \mathbf{M}_Y^2 \right) + \frac{u}{2} \left(\mathbf{M}_X^2 + \mathbf{M}_Y^2 \right)^2 - \frac{g}{2} \left(\mathbf{M}_X^2 - \mathbf{M}_Y^2 \right)^2 \right\}$$



$$\chi^{-1}_{q,\omega} = r_0 + q^2 + \gamma |\omega|^{2/z}$$

$$z = 2$$

$$u > 0 \qquad g > 0$$

$$\mathbf{M}_X \neq 0$$

$$\mathbf{M}_Y = 0$$

Clean System

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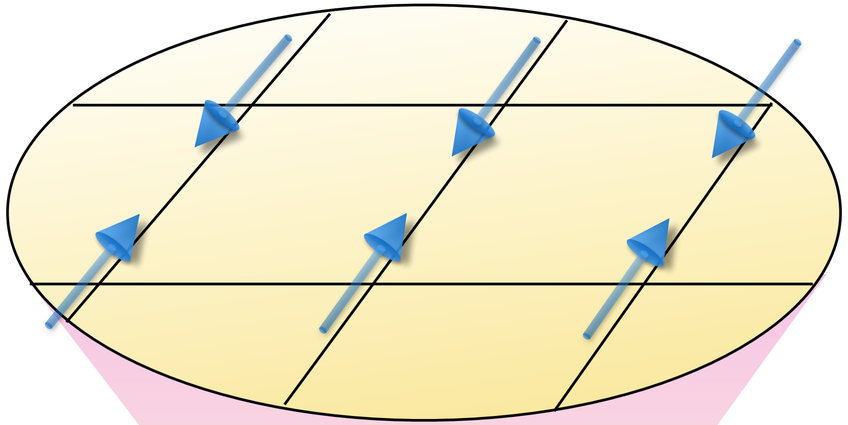

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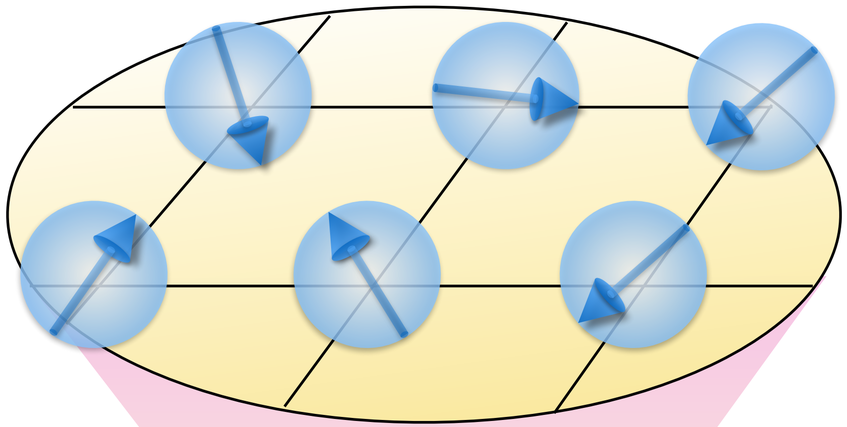

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Clean System

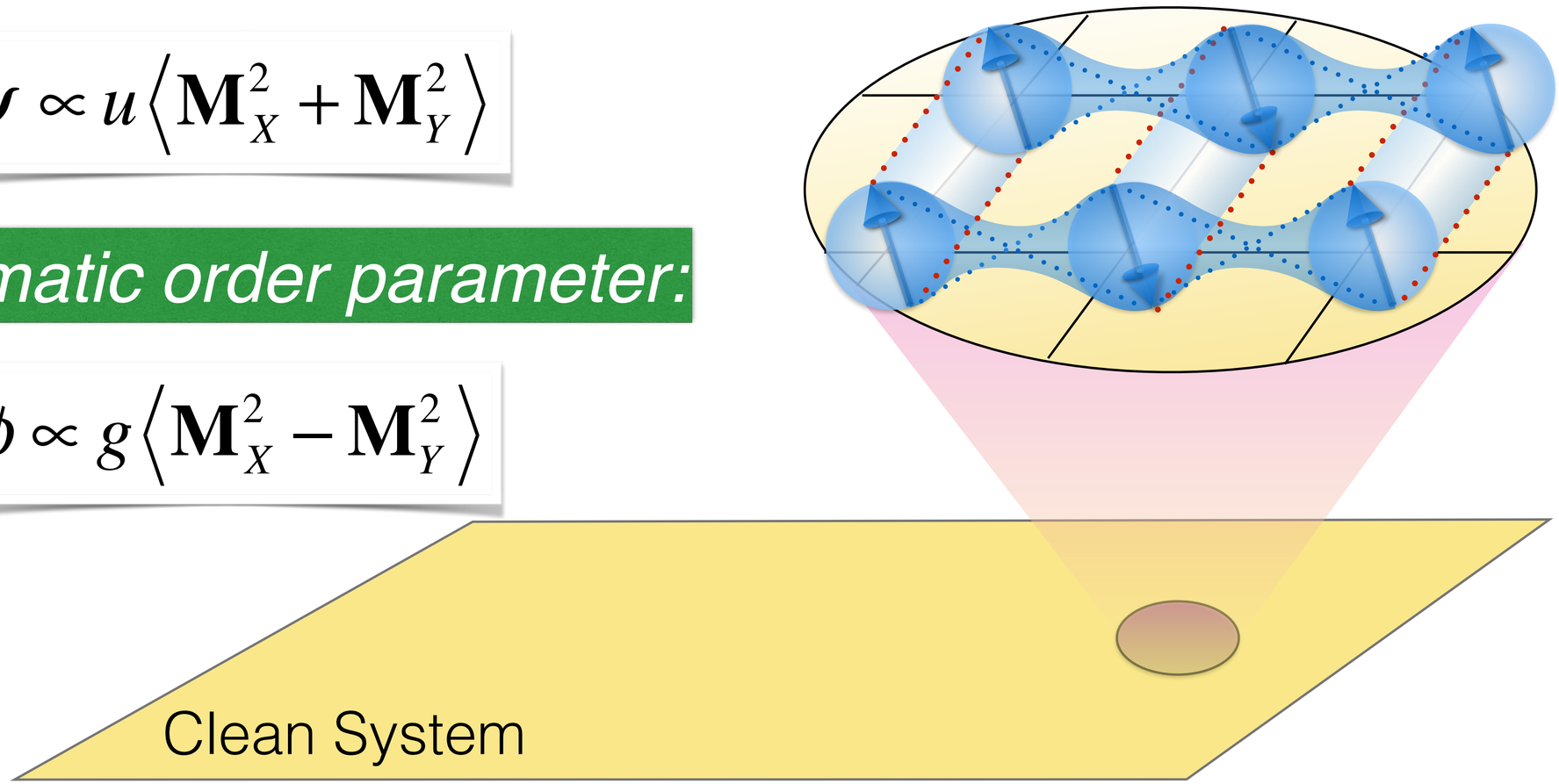
Ising-nematic Order (Clean)

Magnetic Fluctuation:

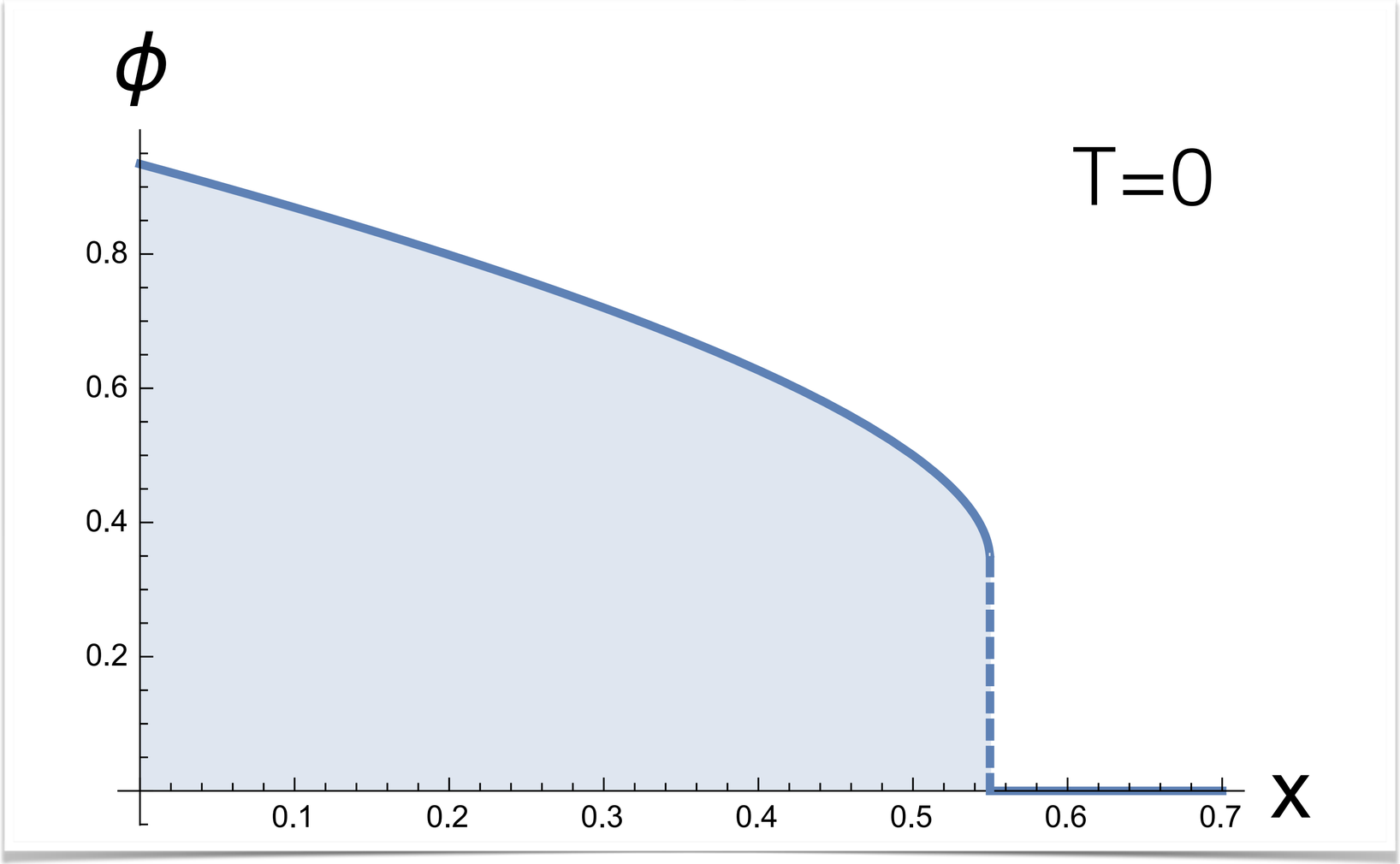
$$\psi \propto u\left\langle \mathbf{M}_X^2 + \mathbf{M}_Y^2 \right\rangle$$

Ising-nematic order parameter:

$$\phi \propto g\left\langle \mathbf{M}_X^2 - \mathbf{M}_Y^2 \right\rangle$$



The Clean Limit: First Order Transition

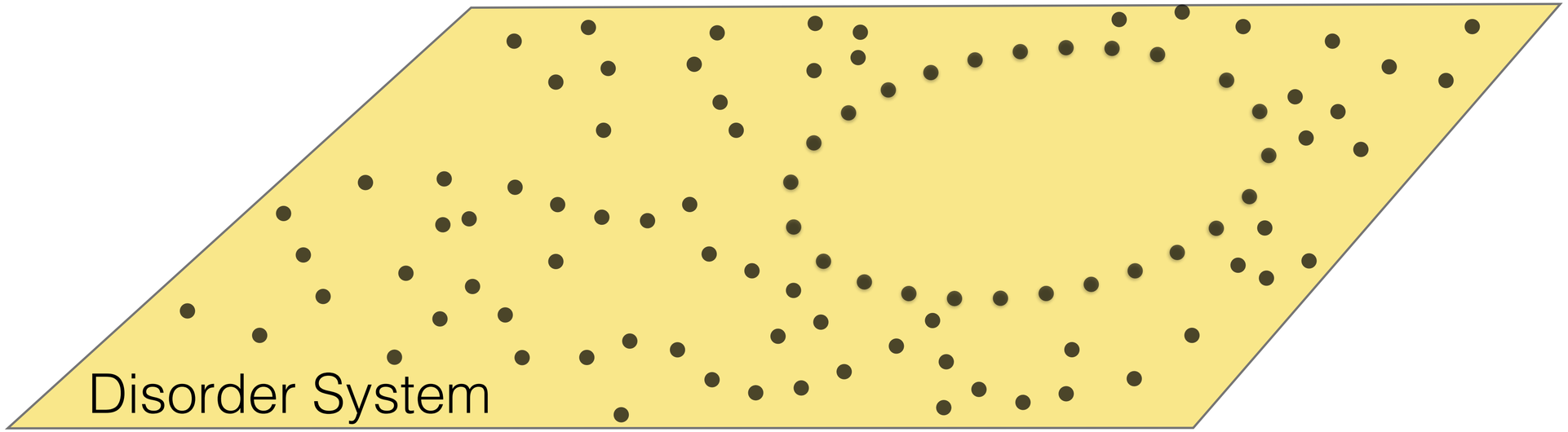


Simultaneously triggers magnetic order

Disorder: Random dilution

$$P(\varepsilon_i) = (1-p)\delta(1-\varepsilon_i) + p\delta(\varepsilon_i) \qquad \varepsilon_i = \begin{cases} 1 & \text{Occupied Site} \\ 0 & \text{Vacancy} \end{cases}$$

Impurity concentration



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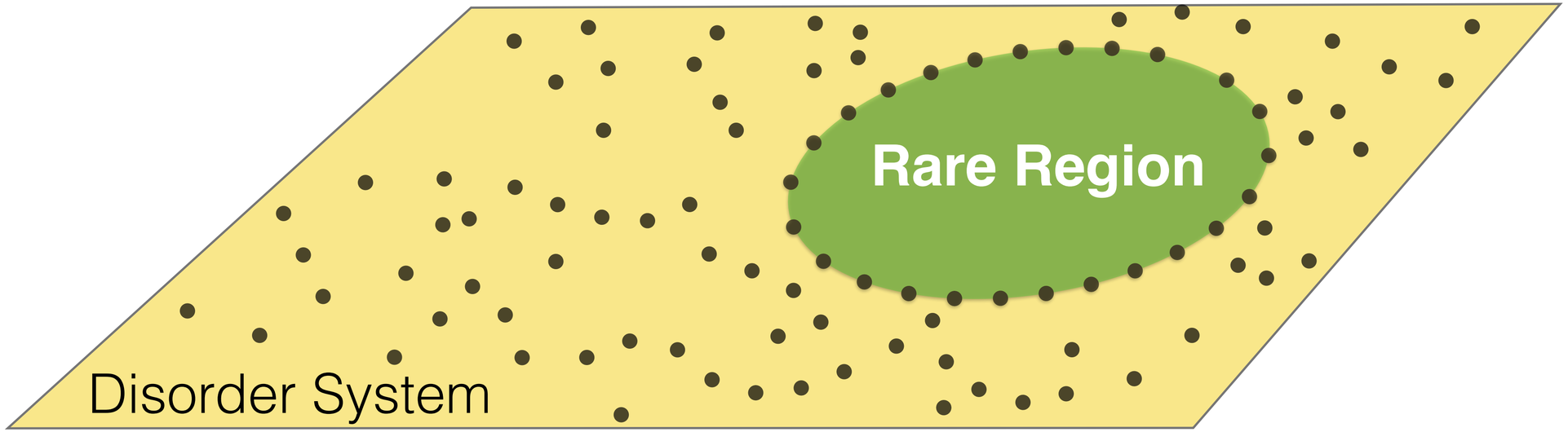

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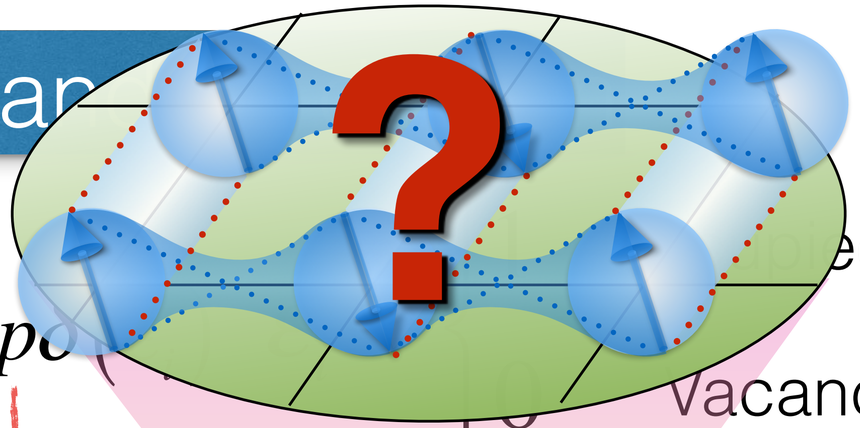

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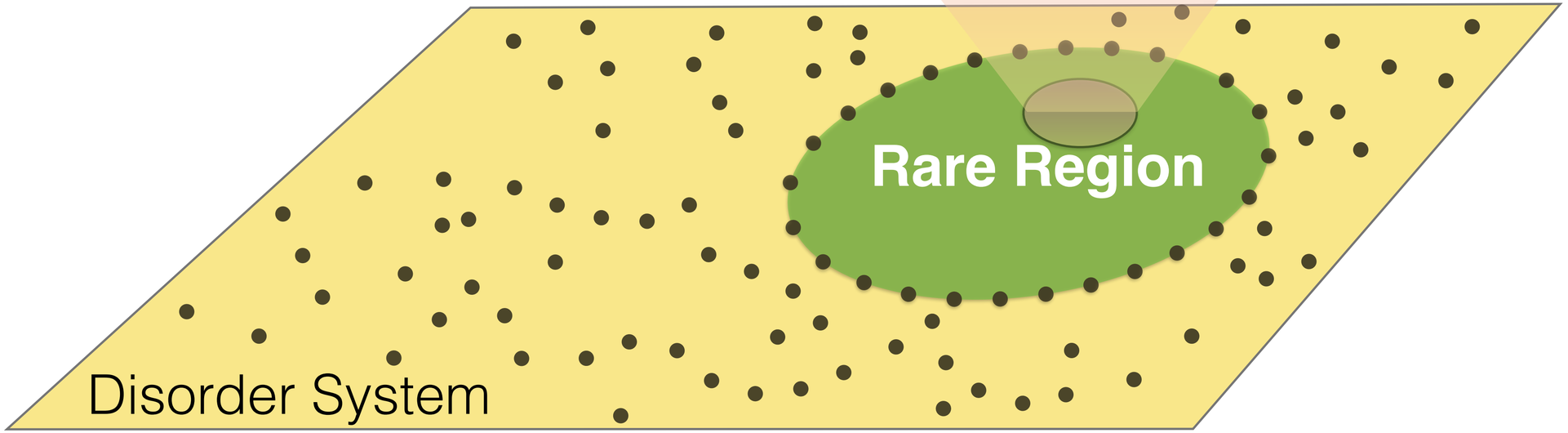

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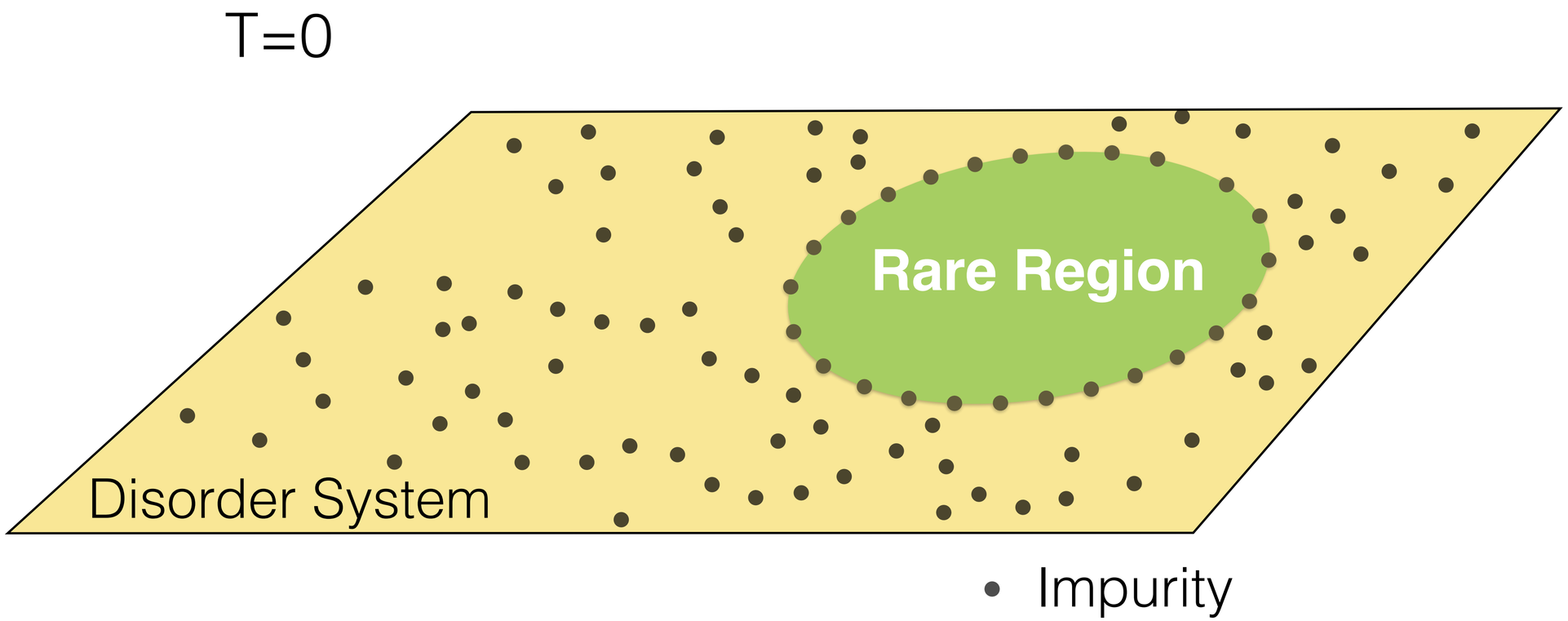
Dimensional Analysis



[1] T. Vojta, J. Phys. A: Math. Gen. **39**, R143–R205 (2006)

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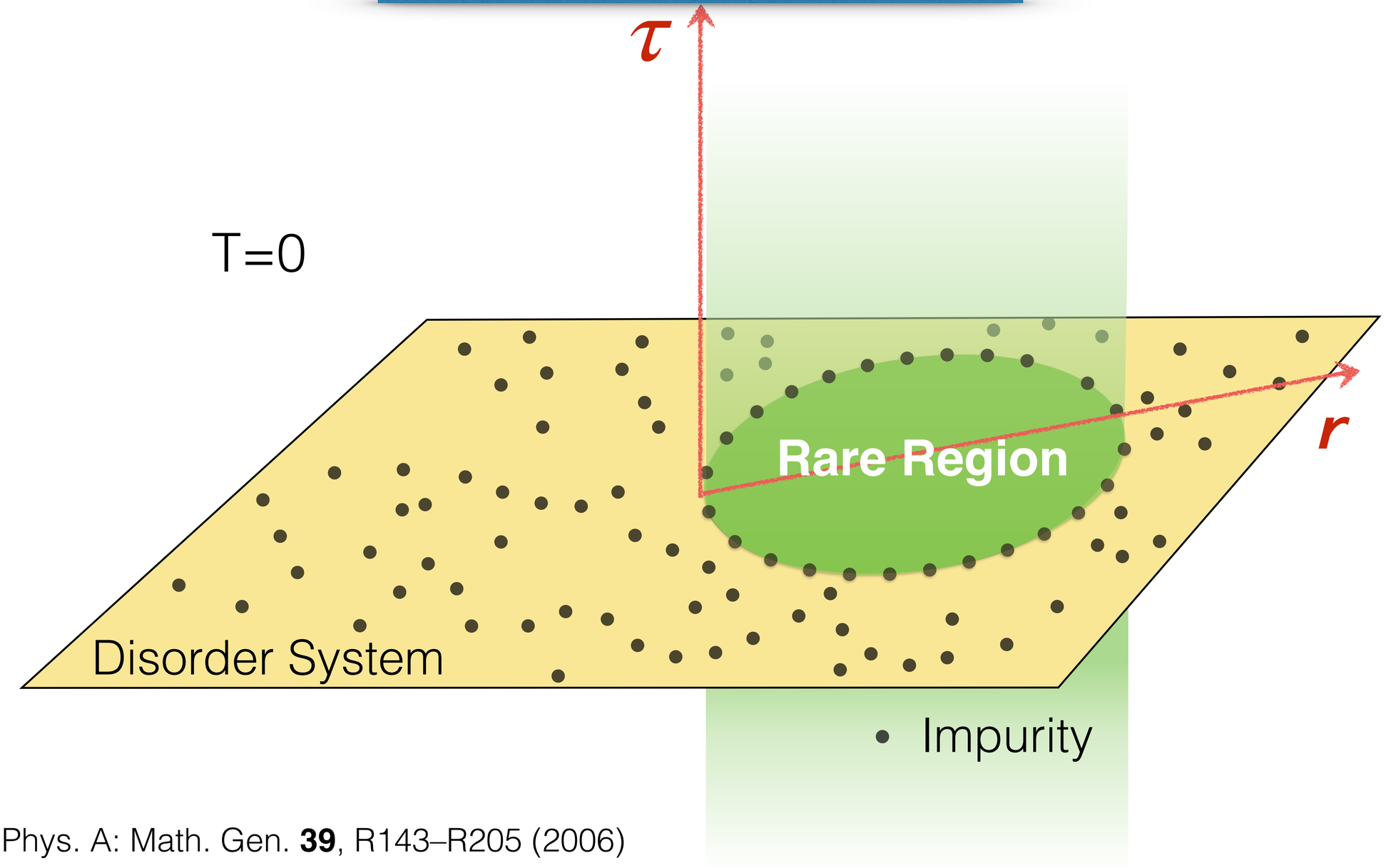

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Dimensional Analysis

	Dimension of rare-region	Lower-critical dimension	Order or not
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Split transitions

Landau-Ginzburg-Wilson (Rare Region)

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$$\int \frac{d^2q}{(2\pi)^2} \int \frac{d\omega}{2\pi} \quad \rightarrow \quad \frac{1}{V} \sum_{\mathbf{q}} \int \frac{d\omega}{2\pi}$$

$$\mathbf{q} = \frac{2\pi}{L} \mathbf{n} \qquad V = L^2$$

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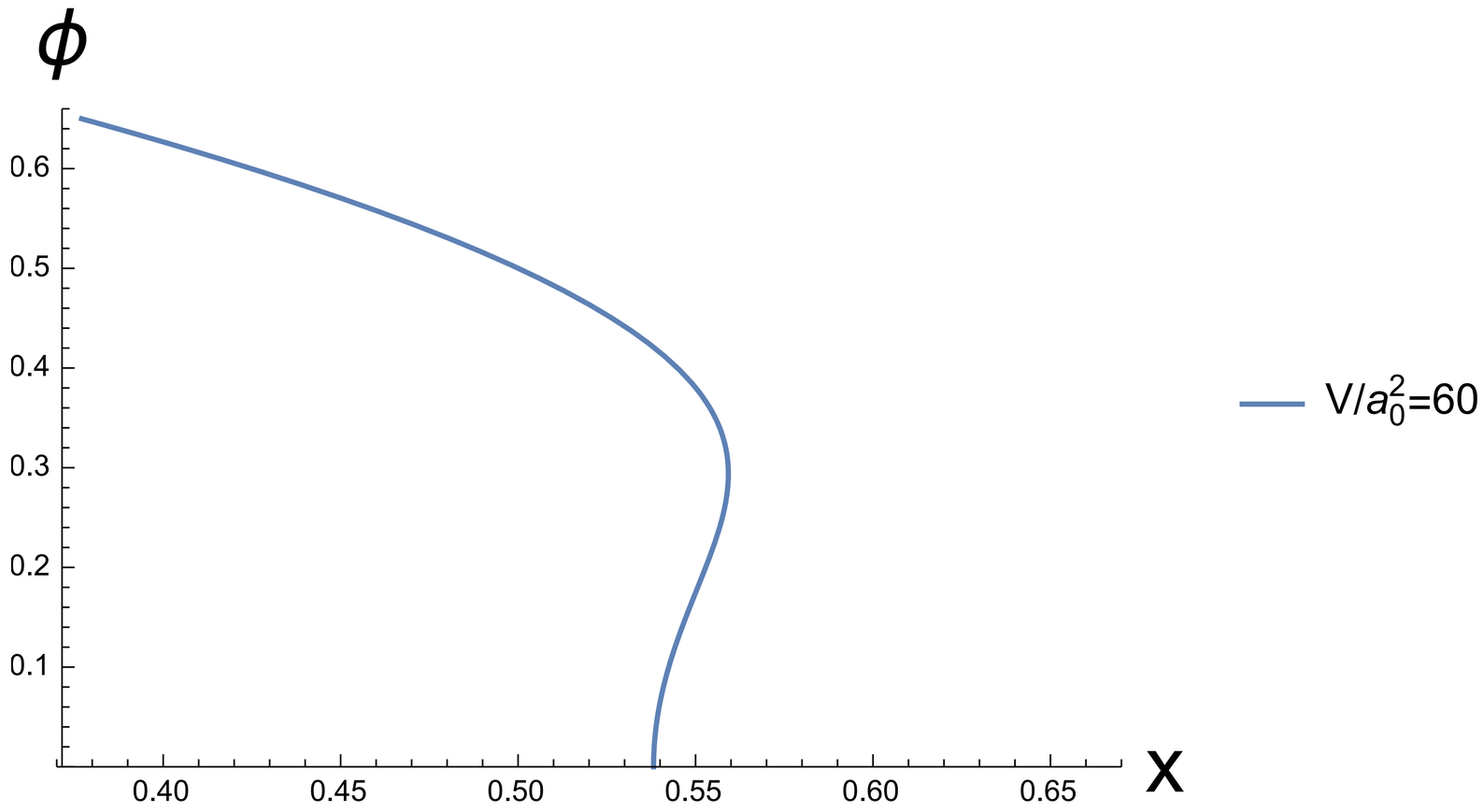
Results for finite size droplet



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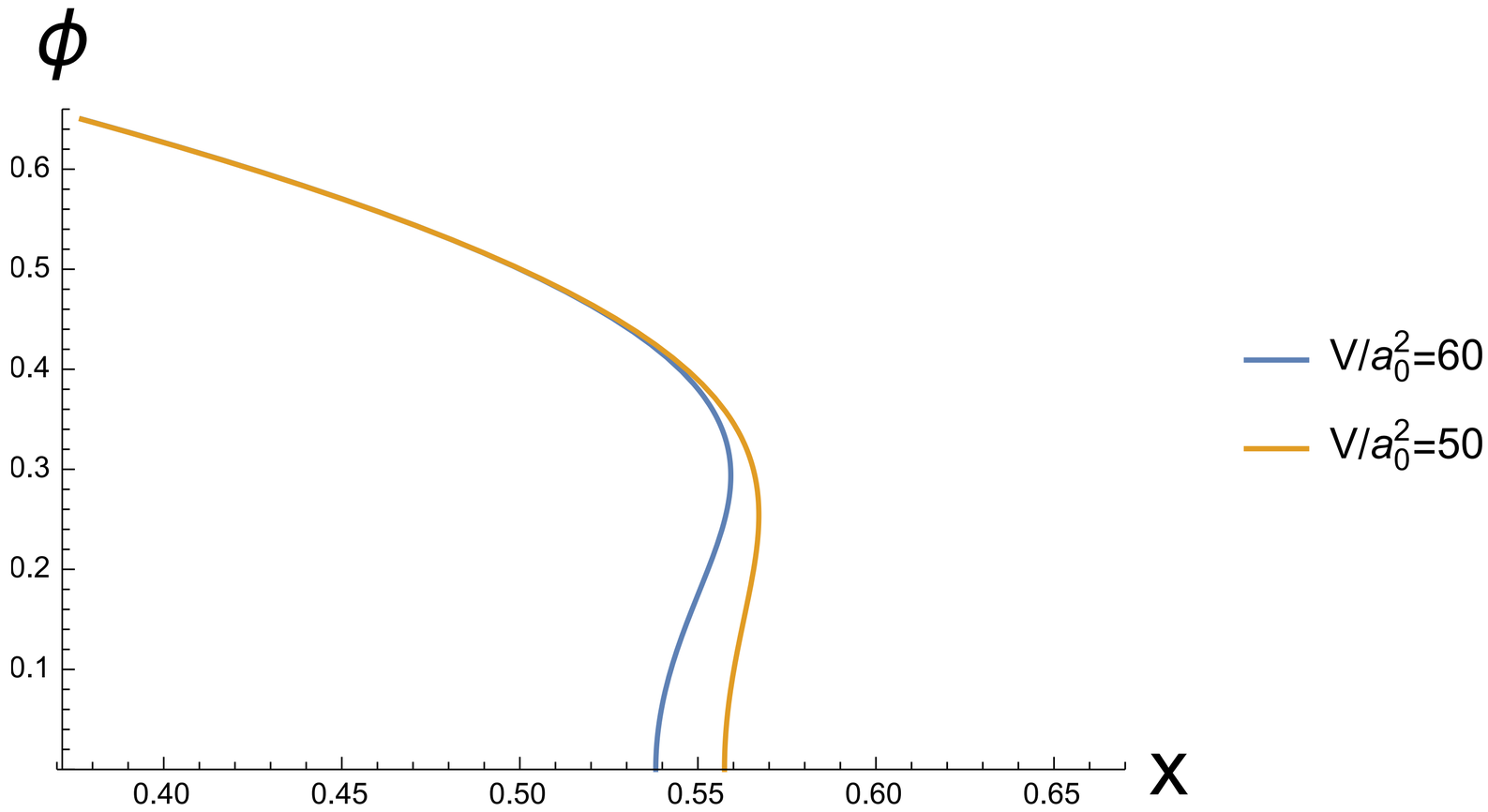

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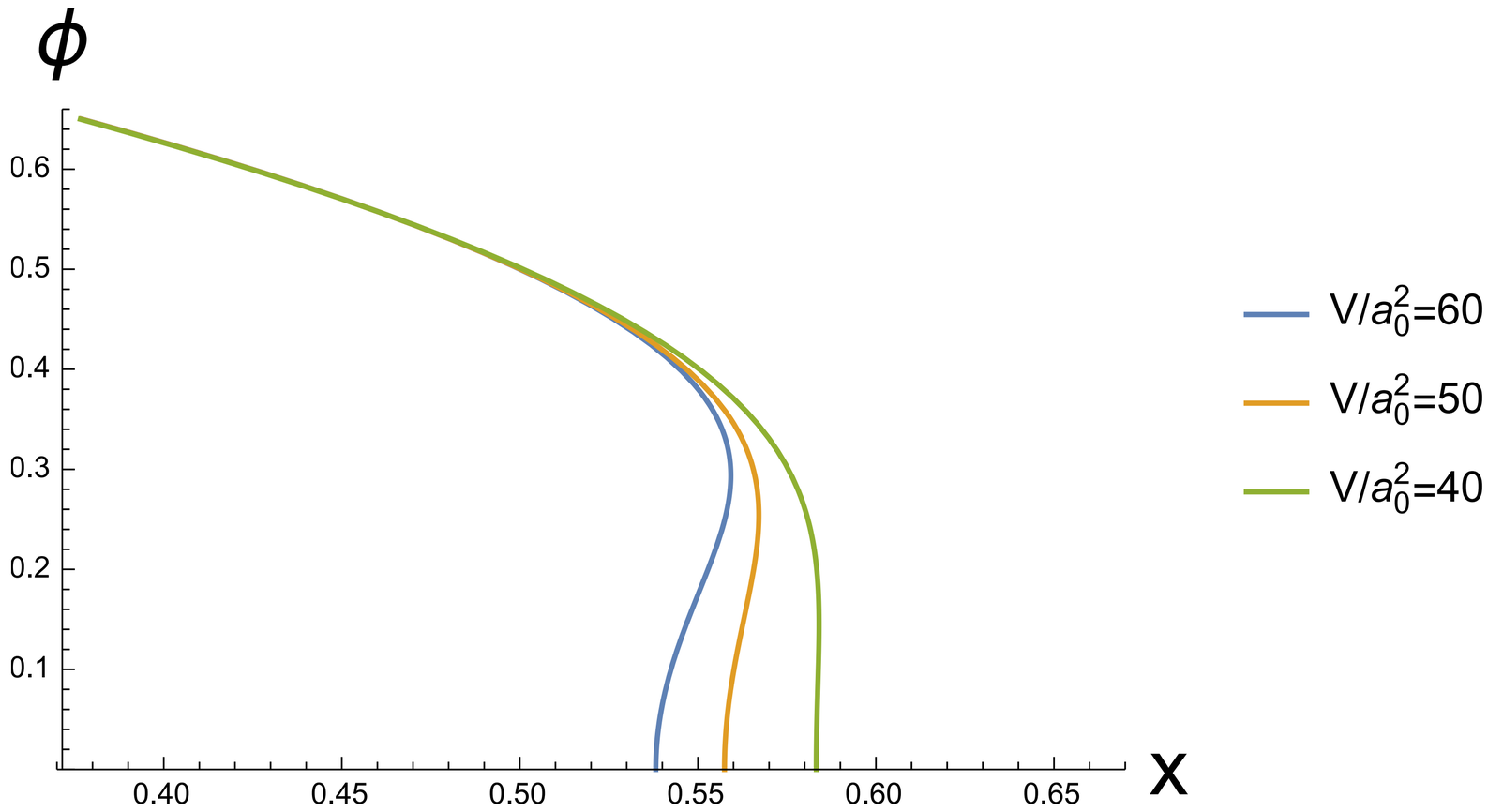

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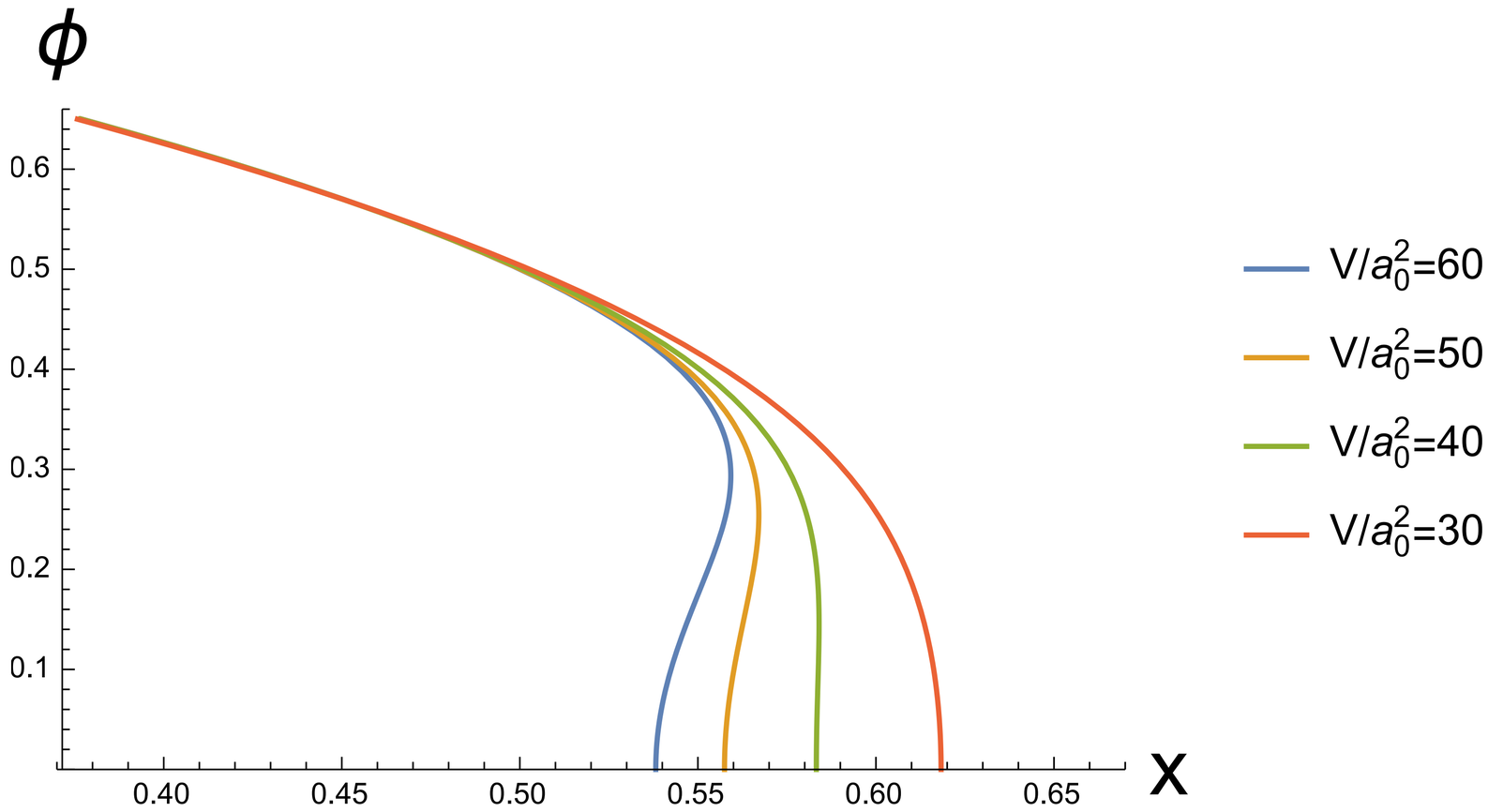

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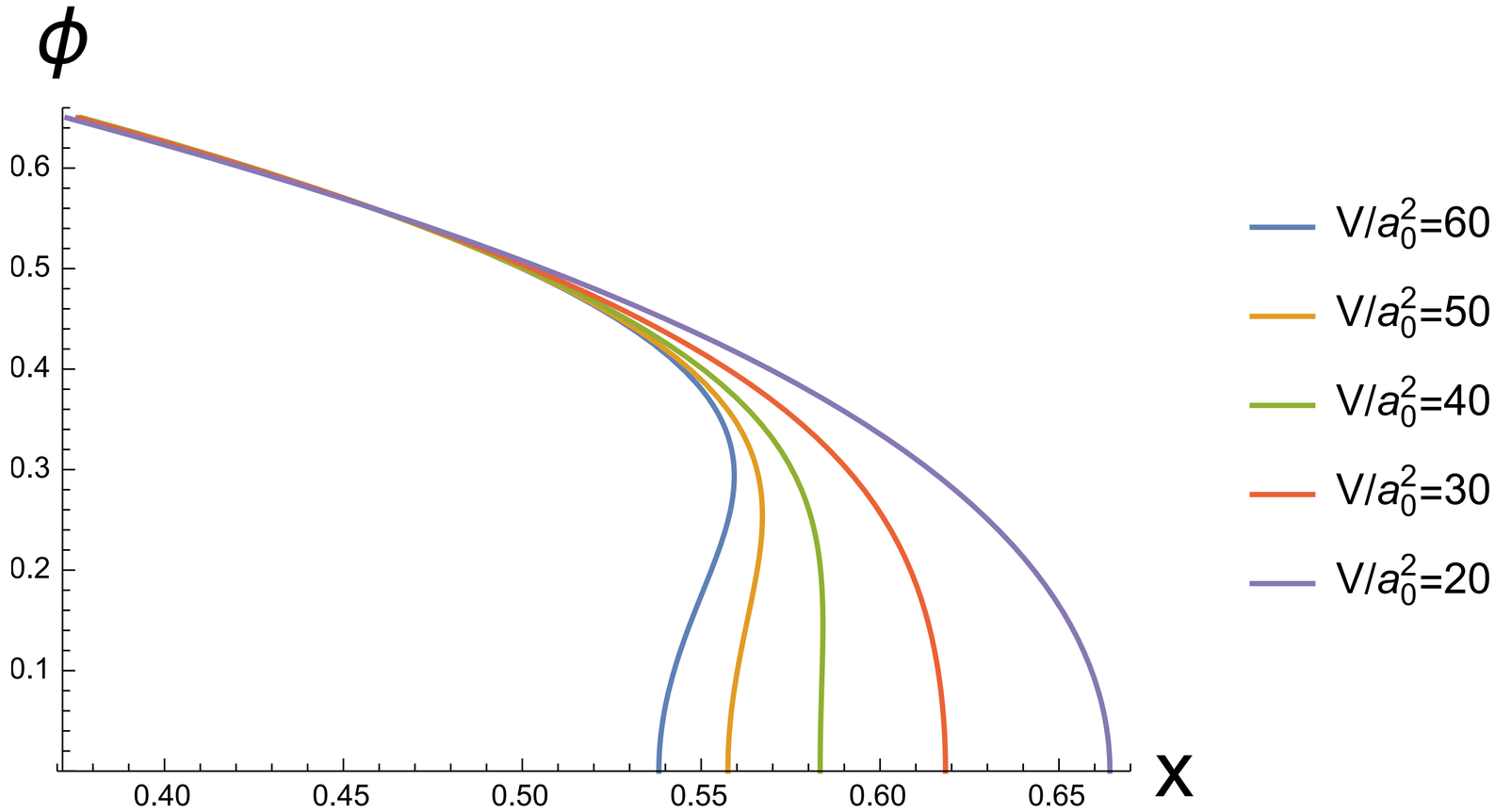


Average over all droplets

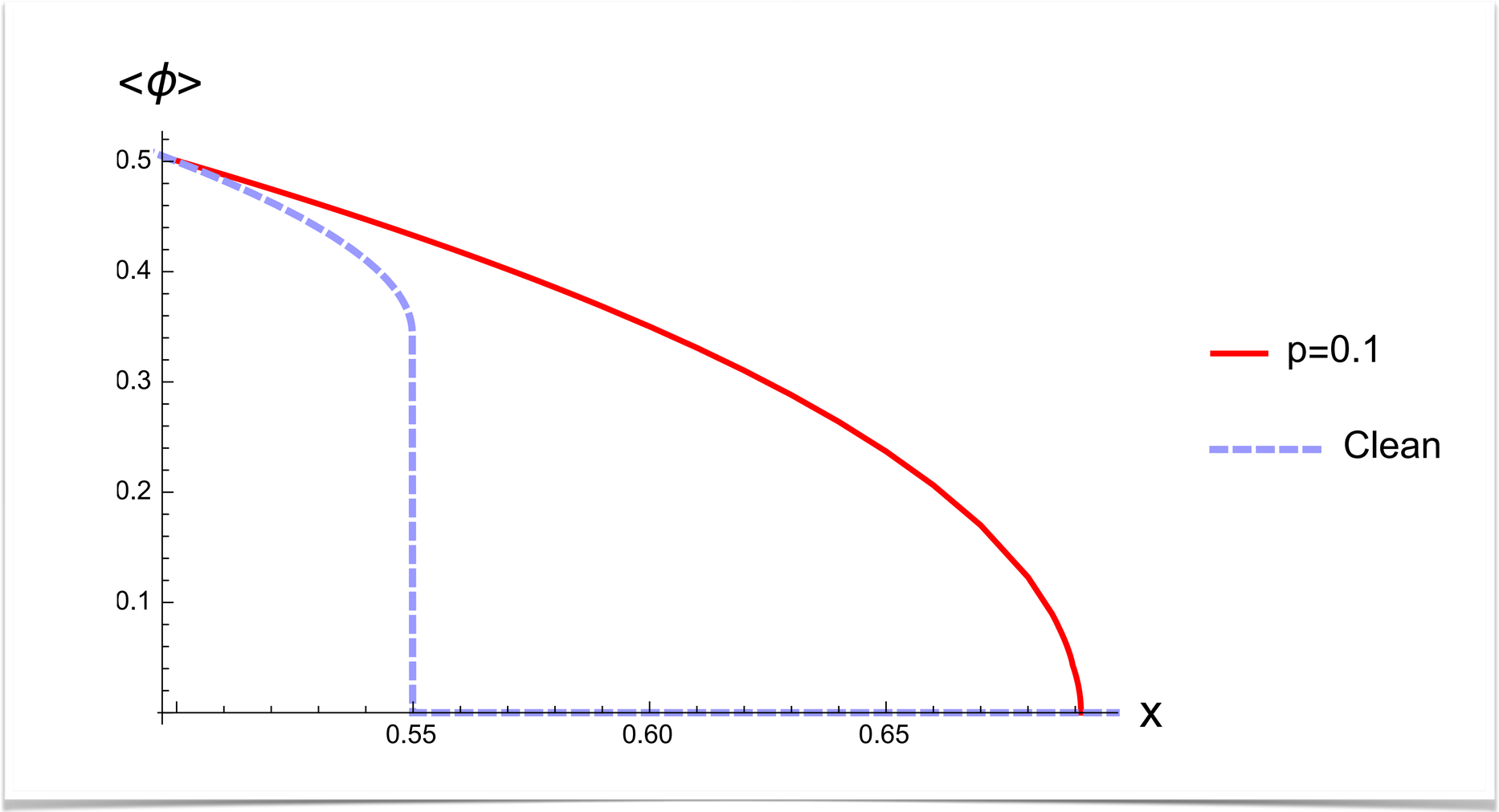
Probability of finding a droplet of size V:

$$P(V) \sim p^{2\pi\sqrt{V}/a_0}(1-p)^{V/a_0^2}$$

Averaging over all the droplets:

$$\langle \phi(x) \rangle \sim \int P(V)\phi(x,V)dV$$

Average over all droplets



Conclusions

- Droplets with larger sizes behave like the clean bulk (first order nematic transition)
- Droplets with smaller sizes undergo continuous nematic phase transitions
- Ising-nematic quantum phase transition behaves as a continuous one due to rare region effects