Rare Region Effects on the Ising-nematic Quantum Phase Transitions

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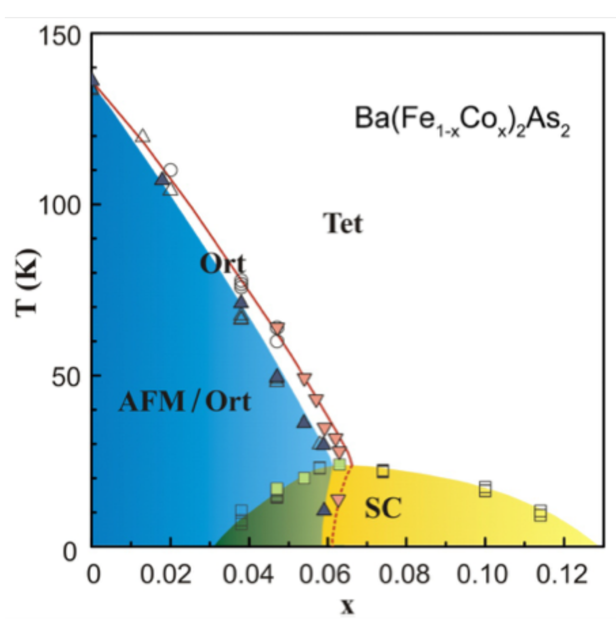
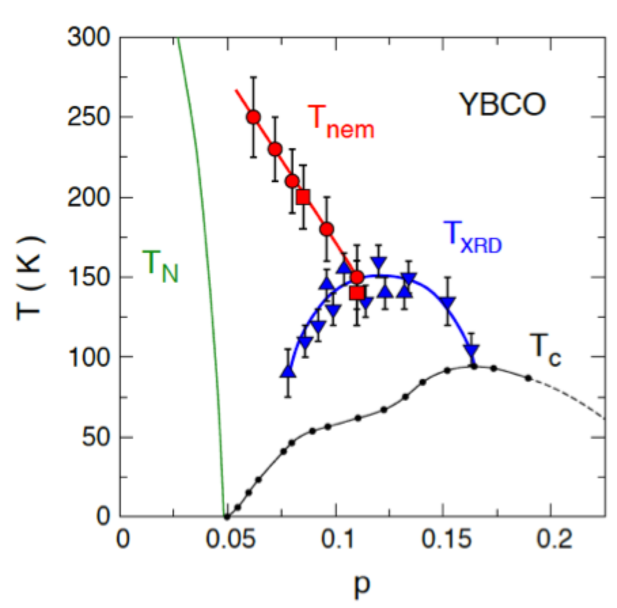
Introduction

Phase Diagram of Unconventional Superconductors



Nandi *et al.*, PRL **104**, 057006 (2010)

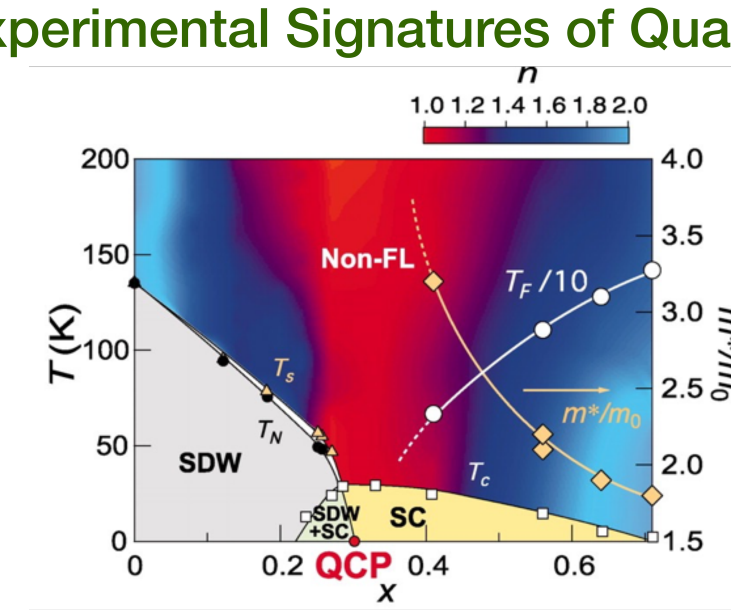


Cyr-Choinière *et al.*, PRB **92**, 224502 (2015)

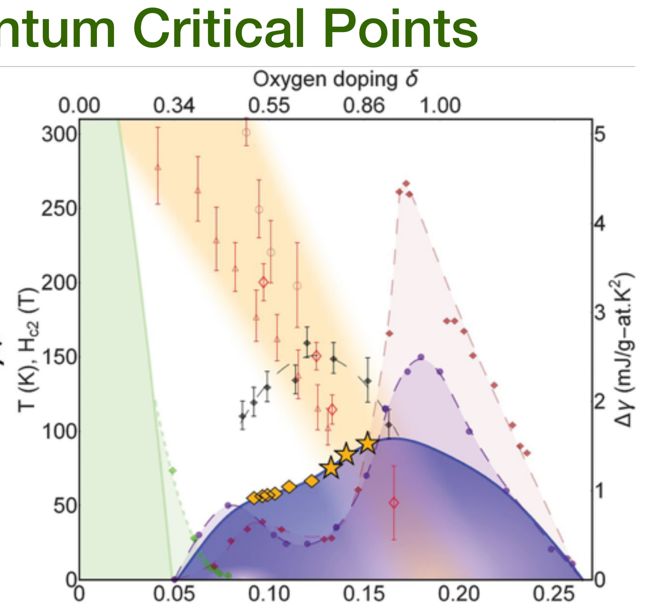
Ising-nematic Phase

- Electronic order breaks lattice symmetry.
- Vestigial order: partially melting a spin-density wave (iron-pnictide) or a incommensurate charge-density wave (cuprate).
- What is the fate of the Ising-nematic order at T=0? Can the Ising-nematic order give rise to quantum criticality?

Experimental Signatures of Quantum Critical Points



$BaFe_2(As_{1-x}P_x)_2$
Hashimoto *et al*, Science, **336**, 6088 (2012)

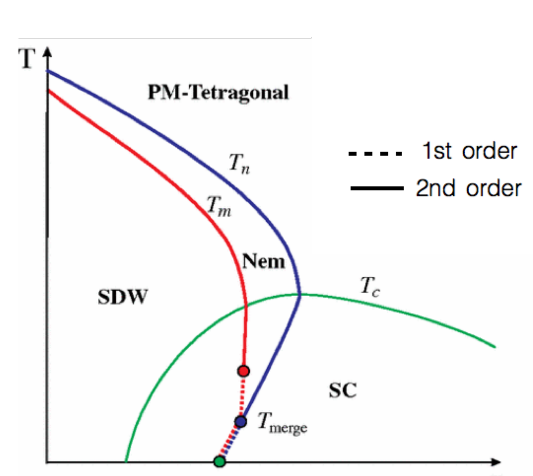


$YBa_2Cu_3O_{6+\delta}$
Ramshaw *et al*, Science, **348**, 6232 (2015)

- Non-Fermi liquid behavior due to strong quantum fluctuations indicates the existence of a quantum critical point (QCP) near optimal doping.
- The Ising-namtic transition line seems to end also in the vicinity of the optimal doping at T=0.
- Is this a Ising-nematic QCP?

Results From Theory[1]

- Ising-nematic and magnetic transitions merge when temperature decreases.
- At T=0, Ising-nematic and magnetic transitions happen simultaneously and they are first-order.
- No Ising-nematic QCP.



This Study: Quenched Disorder Effects

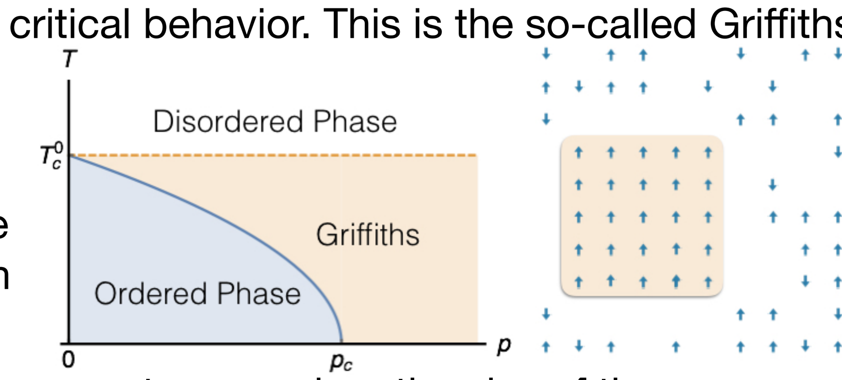
✧ How does quenched disorder change the critical behavior of the Ising-nematic quantum phase transition?

Rare Region Effects

Griffiths Phase

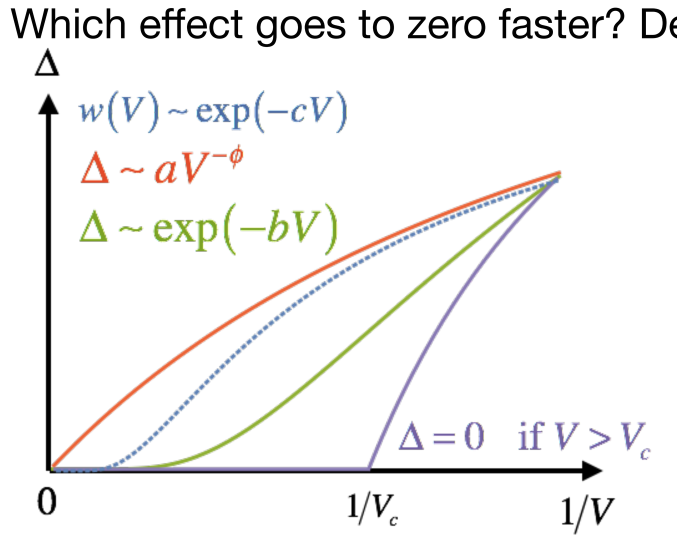
- Rare regions: Large spatial regions that are devoid of impurities.
- Statistically, their contribution is characterized by the probability weight, which is exponentially small.
- Thermodynamically, their contribution is inversely proportional to the energy gap of the rare region, Δ (i.e. the characteristic energy scale of fluctuations above the ground state), which can be sizable.
- If the thermodynamic contribution overcomes the statistical suppression, the rare regions will significantly change the critical behavior. This is the so-called Griffiths phase.

Classification



Compare the dimensionality of the rare region with the lower critical dimension of the transition.

- In the Griffiths phase, the energy gap goes to zero when the size of the rare region goes to infinity.
- The probability weight also goes to zero when the size of the rare region goes to infinity.
- Which effect goes to zero faster? Depends on the dimensionality.



Class	Rare Region Dimension	Rare Region Effects
A	$d_{RR}<d_c^-$	Weak exponential
B	$d_{RR}=d_c^-$	Strong power law
C	$d_{RR}>d_c^-$	Rare regions is ordered

Quantum Griffiths Phase

At T=0, the dimensionality of the rare region becomes $d_{RR}=d+z$.
The spatial dimension of the rare region, depends on the topology of the defects. For point defects, the rare region is confined in all the spatial dimensions, so $d_{RR}=z$
The dynamical critical exponent depends on the model. For Heisenberg model in itinerate systems, $z=2$.

- Ising-nematic order: lower critical dimension is 1. (Class C)
- Magnetic order: lower critical dimension is 2. (Class B)

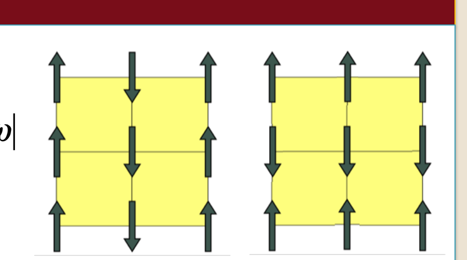
Ising-nematic quantum phase transition splits from the magnetic transition.

Model and Strategy

Landau-Ginzburg-Wilson Theory

$$S[\mathbf{M}_X,\mathbf{M}_Y]=\int_{q,\omega}\left\{\chi^{-1}_{q,\omega}(\mathbf{M}_X^2+\mathbf{M}_Y^2)+\frac{u}{2}(\mathbf{M}_X^2+\mathbf{M}_Y^2)^2-\frac{g}{2}(\mathbf{M}_X^2-\mathbf{M}_Y^2)^2\right\}$$

$$\chi^{-1}_{q,\omega}=r_0+q^2+\gamma|\omega|,\quad u>0,\quad g>0$$



Integrating out the magnetic order parameter fields after applying Hubbard-Stratanovich transformation, we arrive at the effective action for the Ising-nematic order inside a finite size rare region.

Effective Action for a Finite Size Rare Region

$$S_{eff}[\psi,\phi]=\frac{1}{V}\sum_q\int\frac{d\omega}{2\pi}\left\{\frac{\phi^2}{2g}-\frac{\psi^2}{2u}+\frac{1}{2}\ln\left[\left(\chi^{-1}_{q,\omega}+\psi\right)^2-\phi^2\right]\right\}\qquad \mathbf{q}=\frac{2\pi}{L}\mathbf{n}\qquad V\equiv L^2$$

- Ising-nematic order parameter: $\phi\propto g\langle\mathbf{M}_X^2-\mathbf{M}_Y^2\rangle$
- Magnetic Fluctuation: $\psi\propto u\langle\mathbf{M}_X^2+\mathbf{M}_Y^2\rangle$
- Momentum is not continuous due to spatial confinement.

Strategy

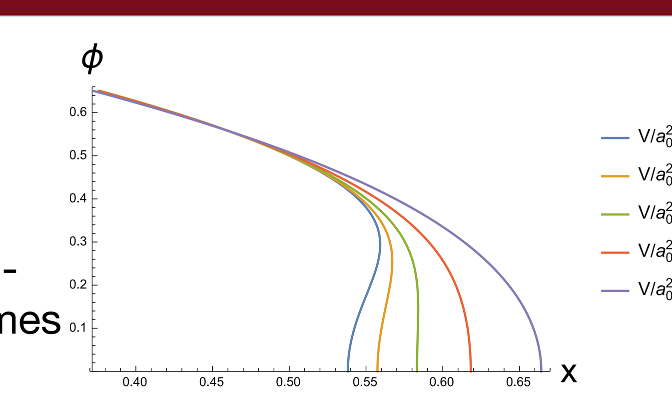
i. We use the saddle-point condition to solve for the Ising-nematic order parameter as a function of control parameter for different sizes of rare regions, $\phi(x,V)$.

ii. Average over all the possible rare regions by integrating over the probability weight: $\langle\phi(x)\rangle\sim\int P(V)\phi(x,V)dV$

iii. To make sure the rare region is clean inside and isolated from the dirty system, we estimate the probability weight by $P(V)\sim p^{2\pi\sqrt{V}/a_0}(1-p)^{V/a_0^2}$

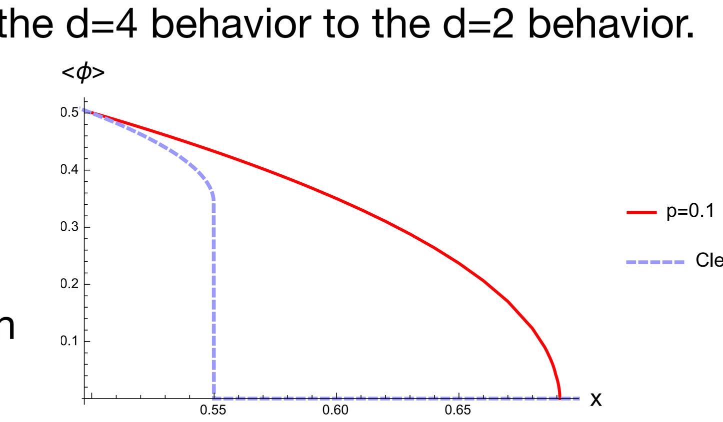
Results

Single Finite Size Rare Region



- For large rare regions, the Ising-nematic quantum phase transition is first-order.
- For relatively small rare regions, the Ising-nematic quantum phase transition becomes continuous.
- Upon decreasing the size of the rare region, the Ising-nematic quantum phase transition undergoes a crossover from the d=4 behavior to the d=2 behavior.

Statistical Average



- Relatively small rare regions dominate the average, since they are statistically more favorable.
- Ising-nematic quantum phase transition becomes continuous.

Summary and References

- Rare region effects due to quenched disorder split the Ising-nematic and magnetic phase transitions at T=0.
- Rare regions effects dramatically change the Ising-nematic quantum critical behavior and give rise to a Ising-nematic QCP.

[1] R. M. Fernandes, S. Maiti, P. Woelfle, and A. V Chubukov, *Phys. Rev. Lett.* **111**, 057001 (2013).
[2] R.M. Fernandes, A.V. Chubukov, J. Knolle, I. Eremin, and J. Schmalian, *Phys. Rev. B* **85**, 024534 (2012).
[3] R. M. Fernandes, A. V. Chubukov, and J. Schmalian, *Nat. Phys.* **10**, 97 (2014).
[4] T. Vojta, *J. Phys. A. Math. Gen.* **39**, R143 (2006).
[5] T. Vojta and J. Schmalian, *Phys. Rev. B* **72**, 045438 (2005).