

# Impact of damping on superconducting gap oscillations induced by Terahertz pulses

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*University of Minnesota*

Xu Yang, Chirag Vaswani, Jigang Wang, Peter P. Orth

*Iowa State University and Ames Laboratory*



# Equilibrium superconductivity

- [1] R. Matsunaga and R. Shimano, *Phys. Scr.*, **92**(2), 24003 (2017).
- [2] A. F. Kemper, M. A. Sentef, B. Moritz, J. K. Freericks, and T. P. Devereaux, *Phys. Rev. B*, **92**, 224517 (2015).

# Equilibrium superconductivity

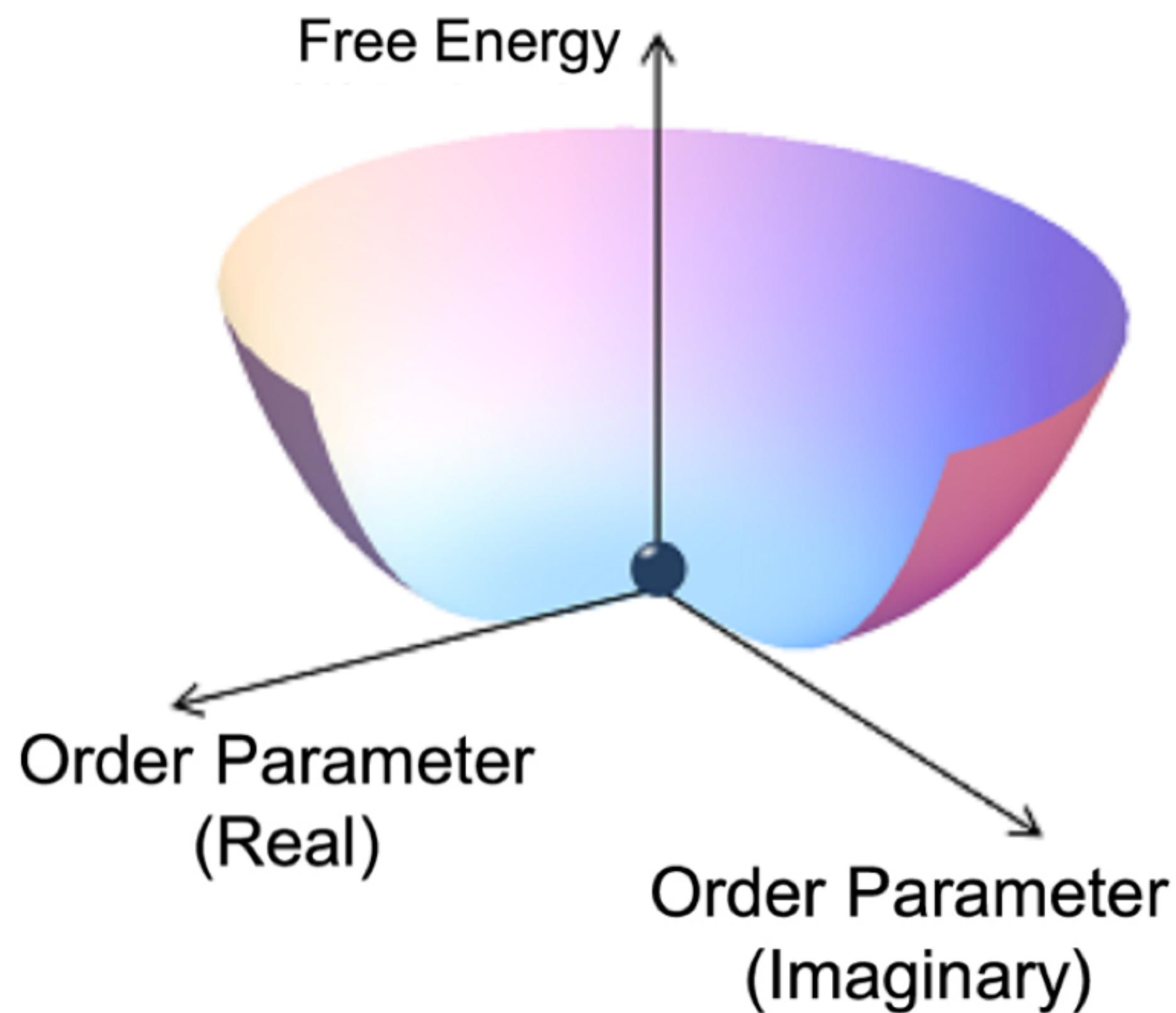
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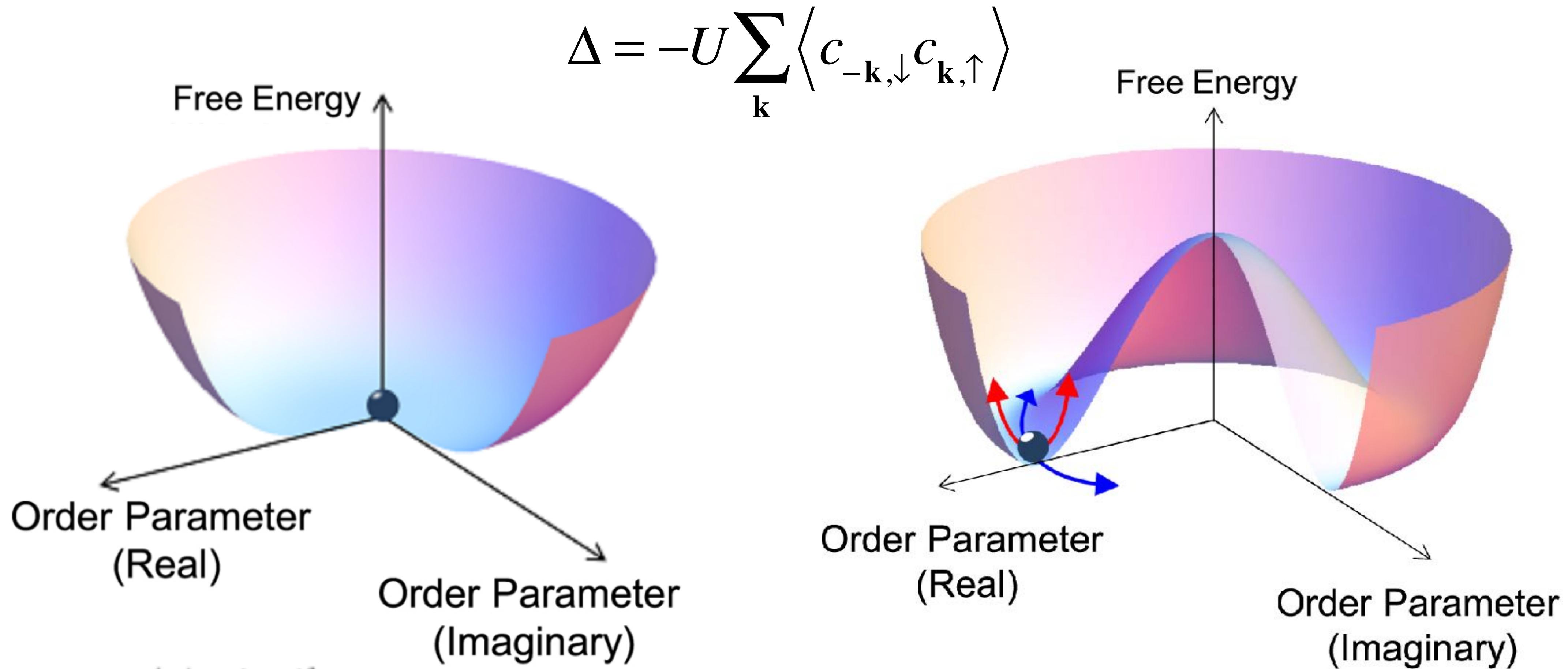
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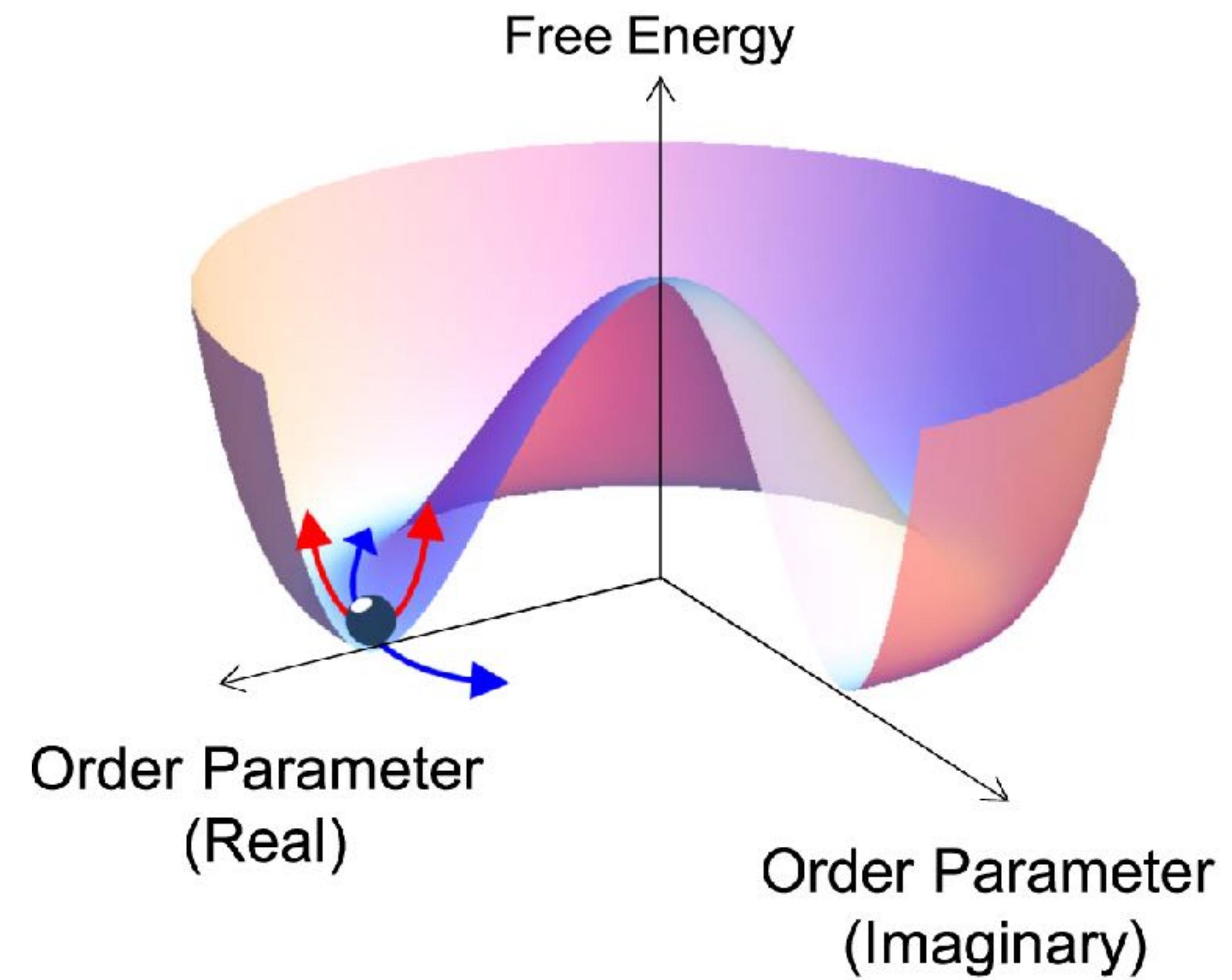
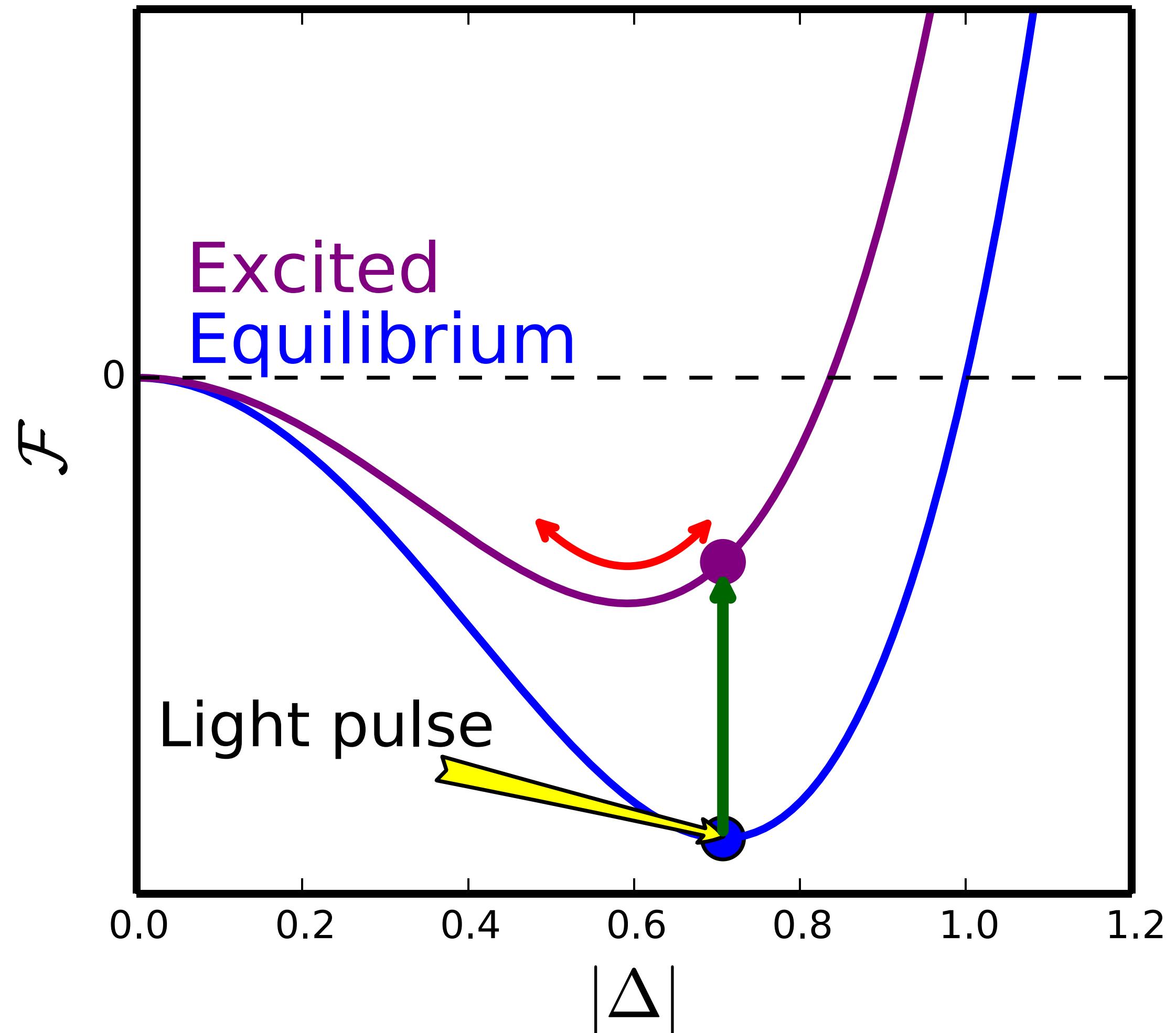
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# Non-equilibrium superconductivity



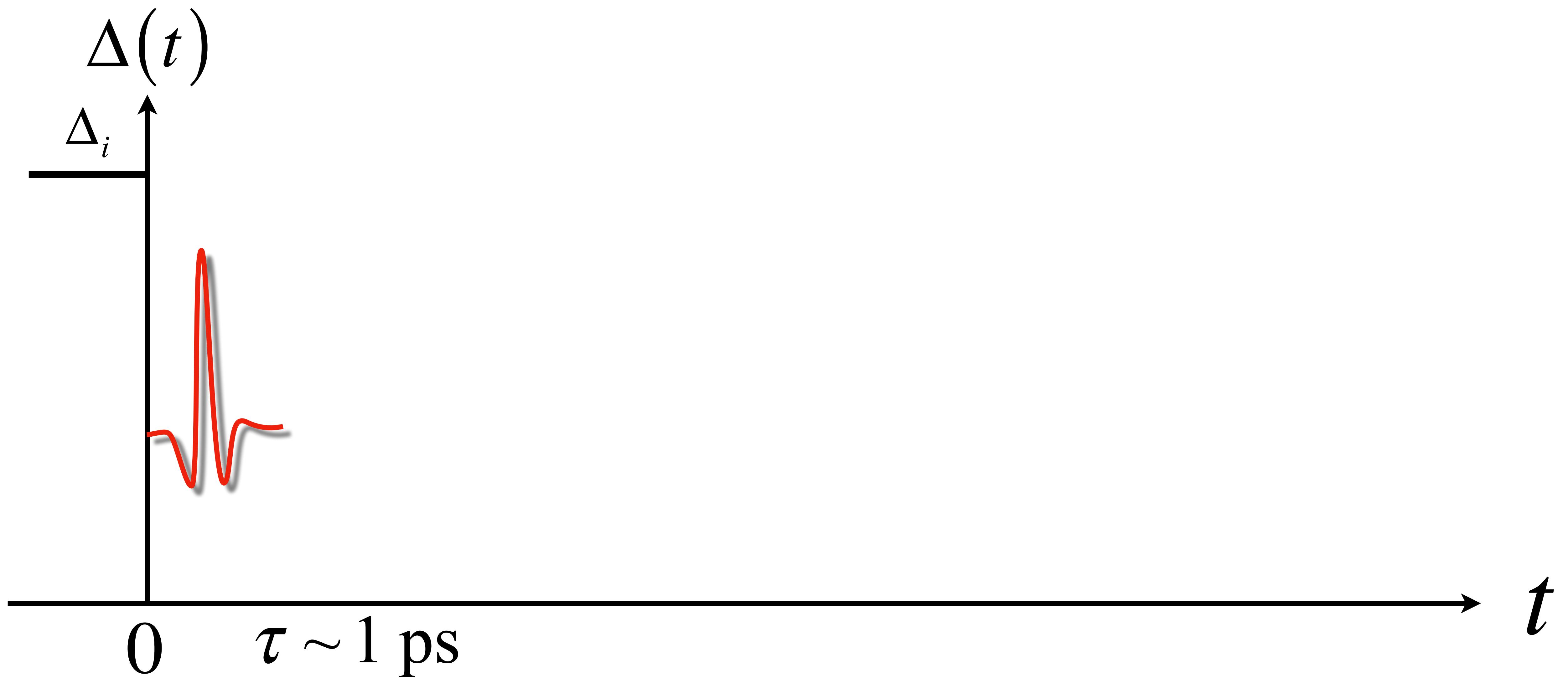
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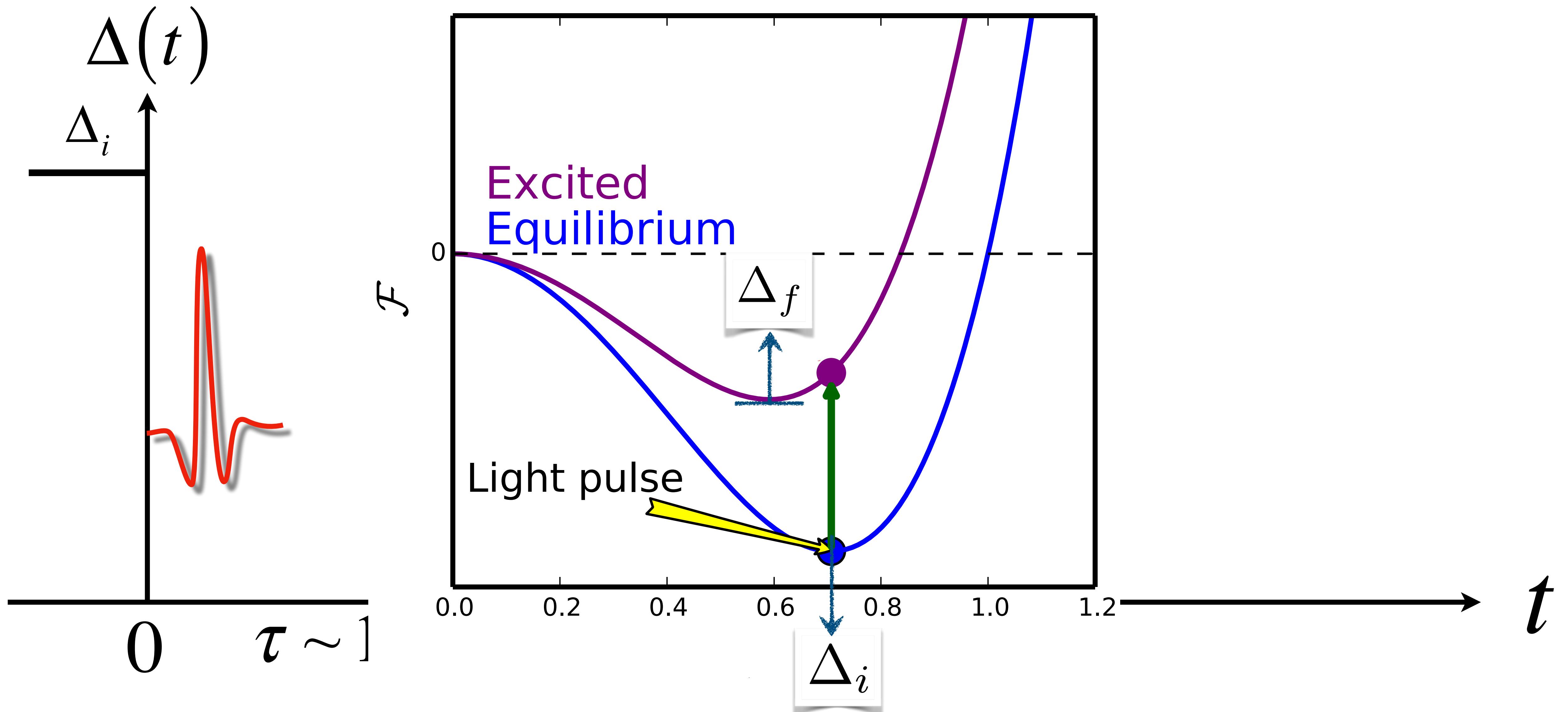
## Typical time scales



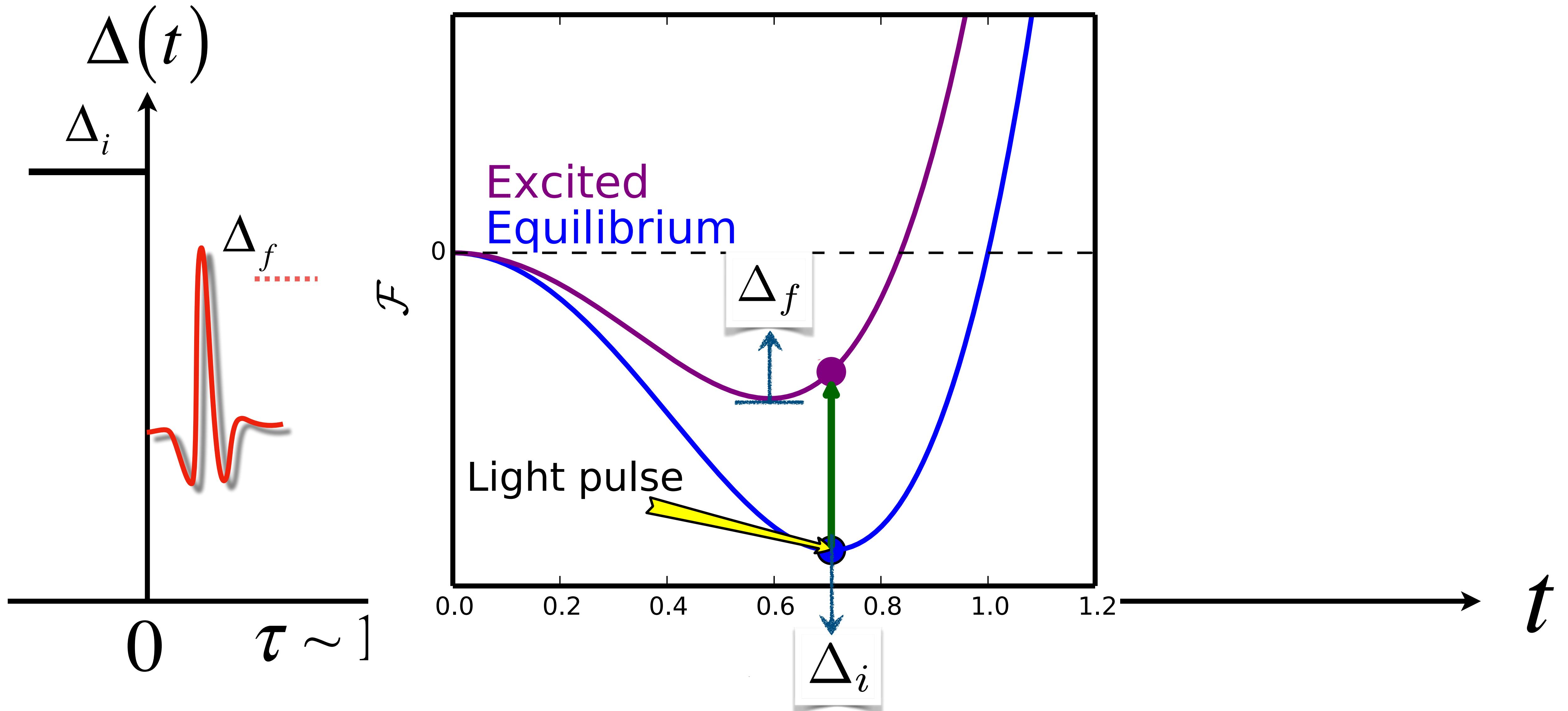
## Typical time scales



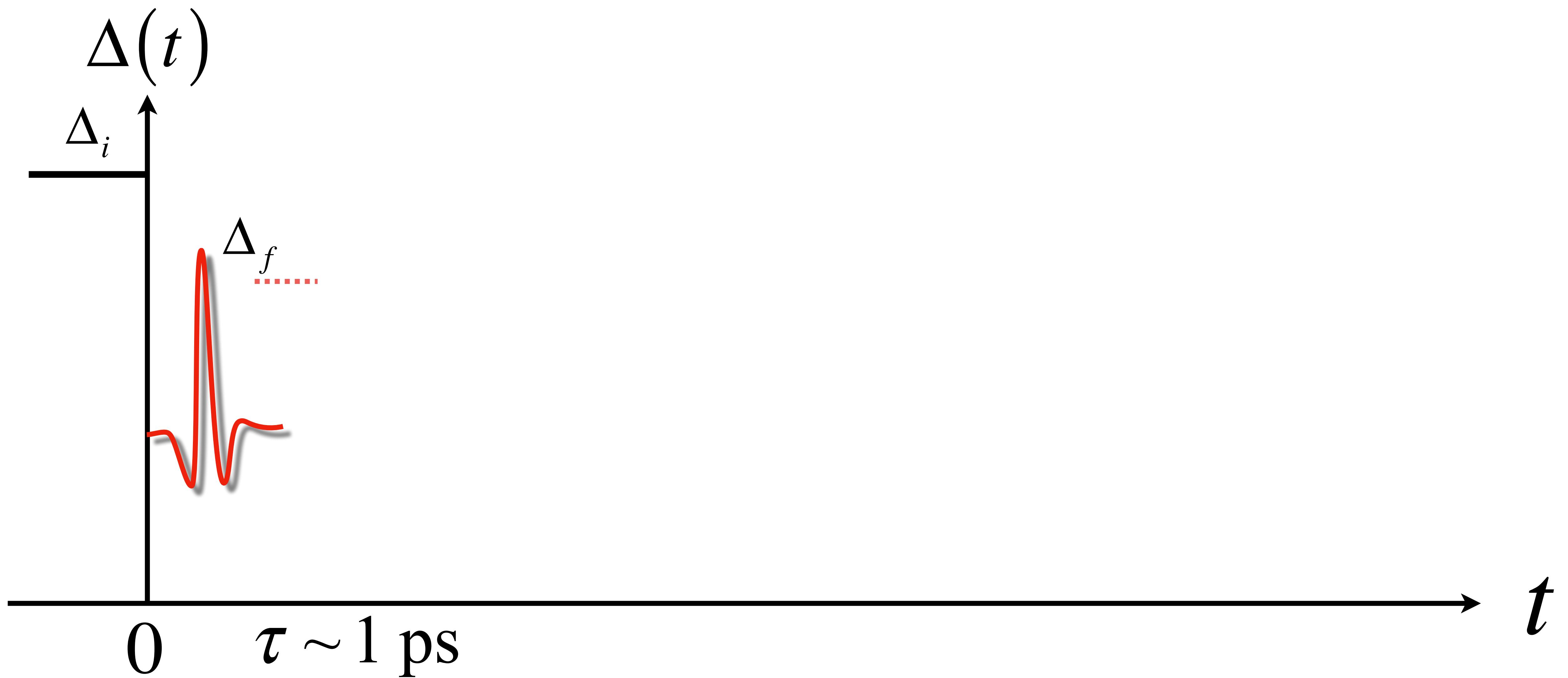
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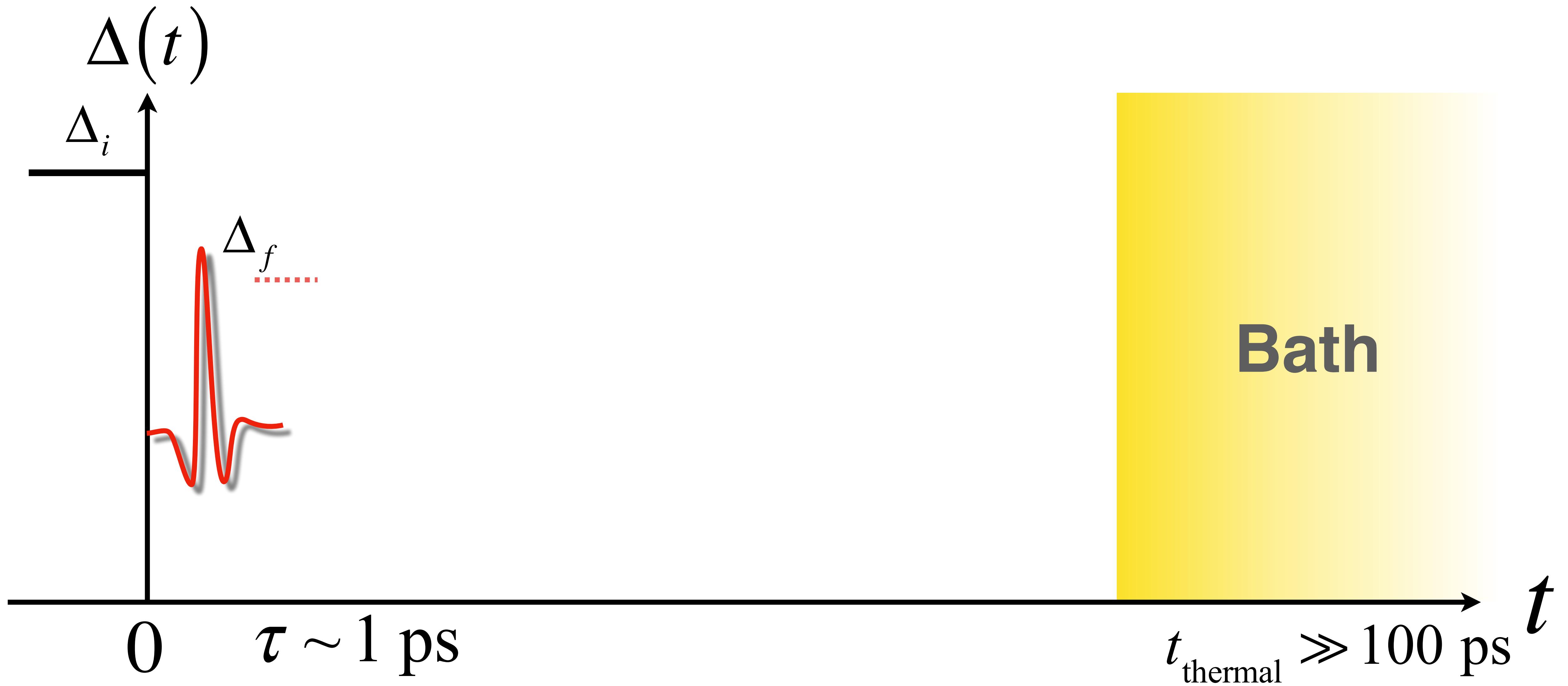
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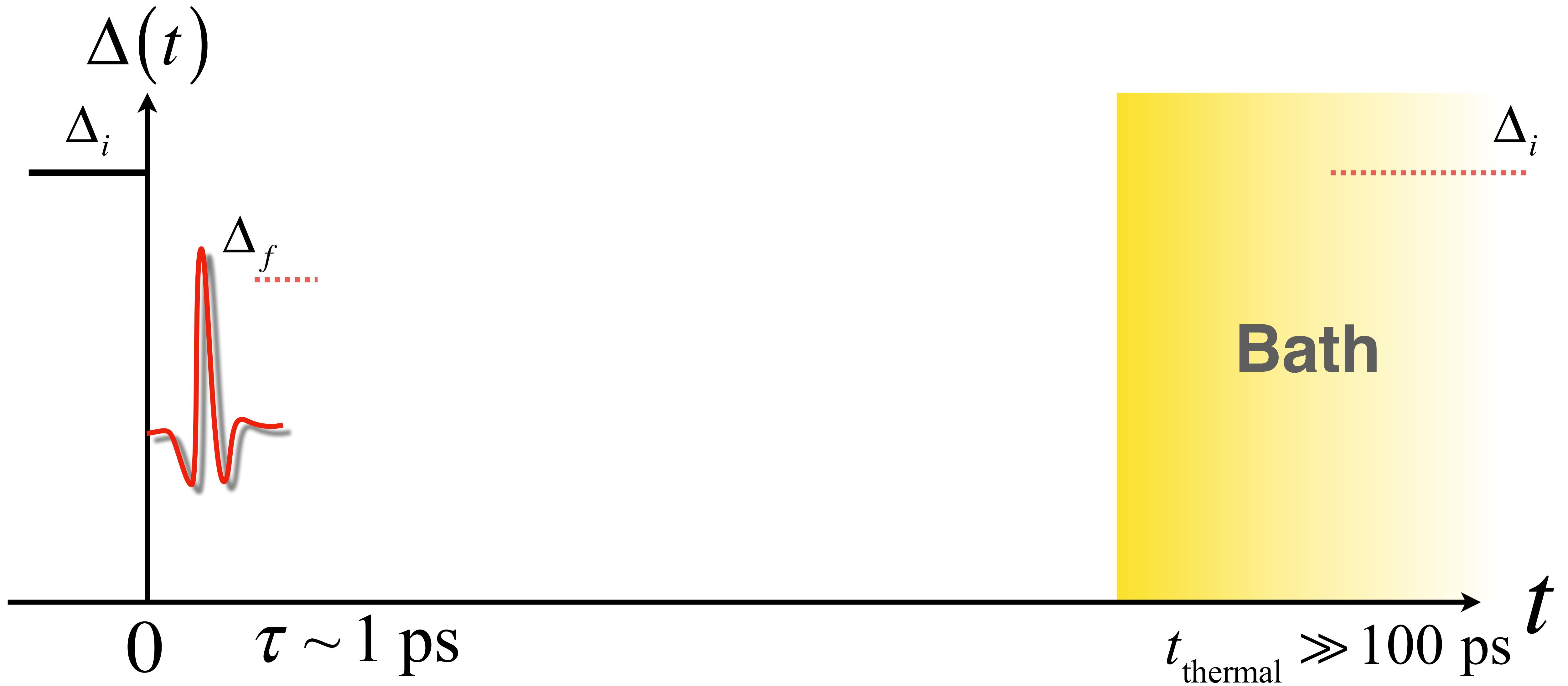
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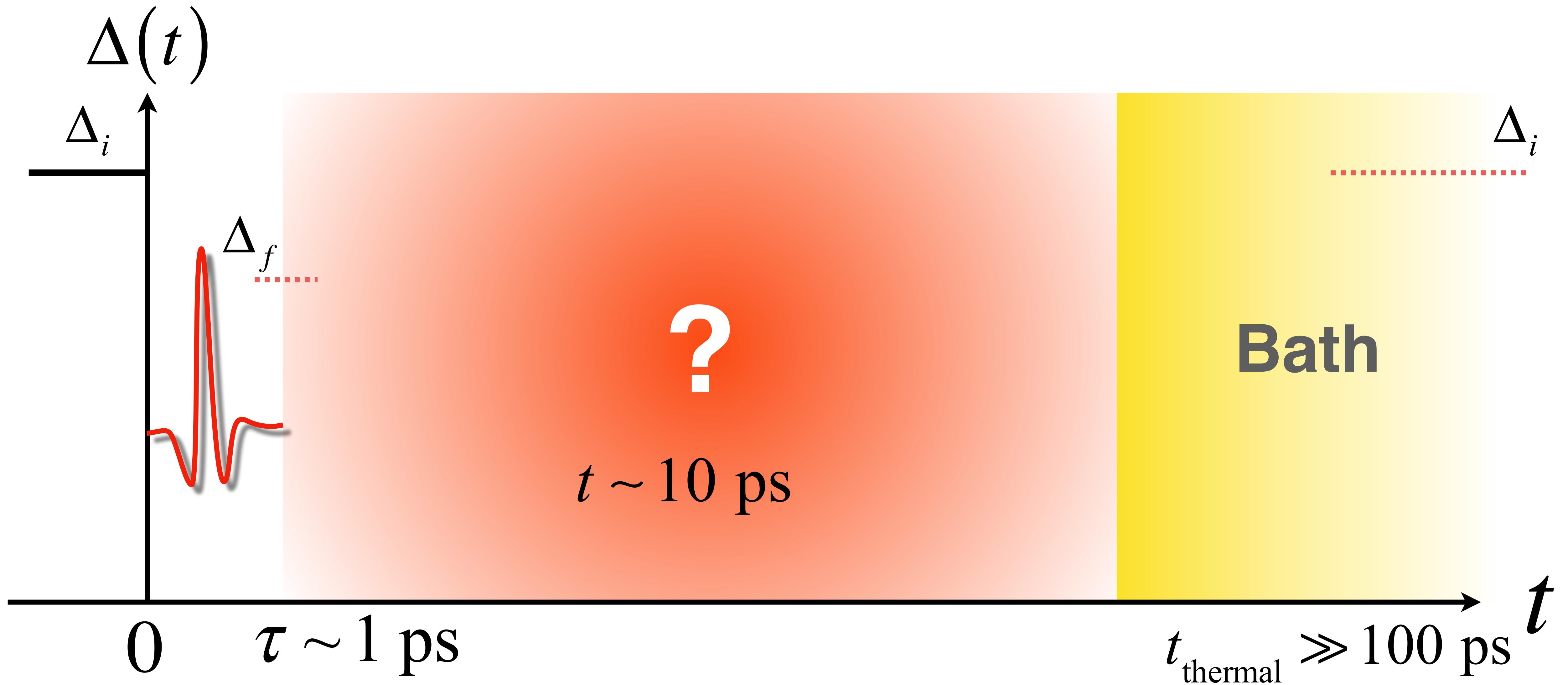
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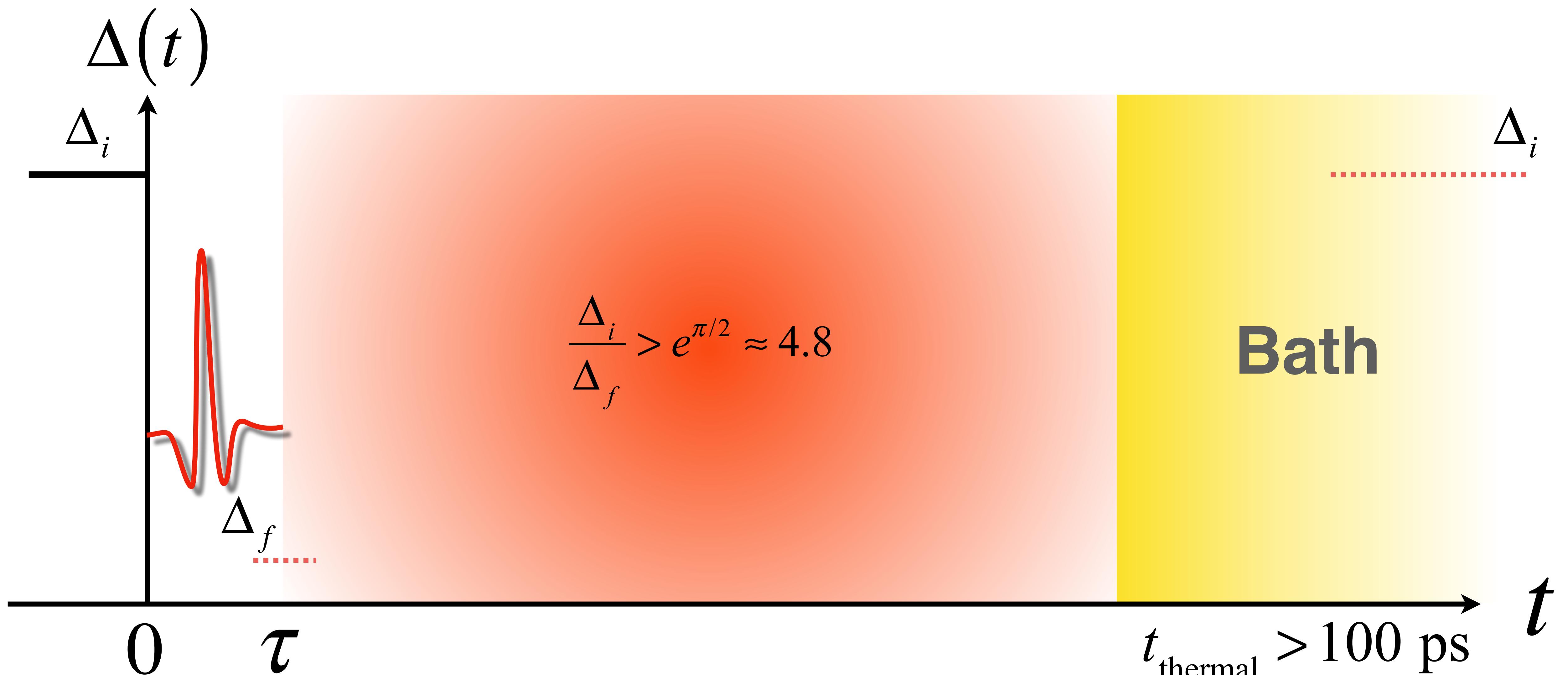
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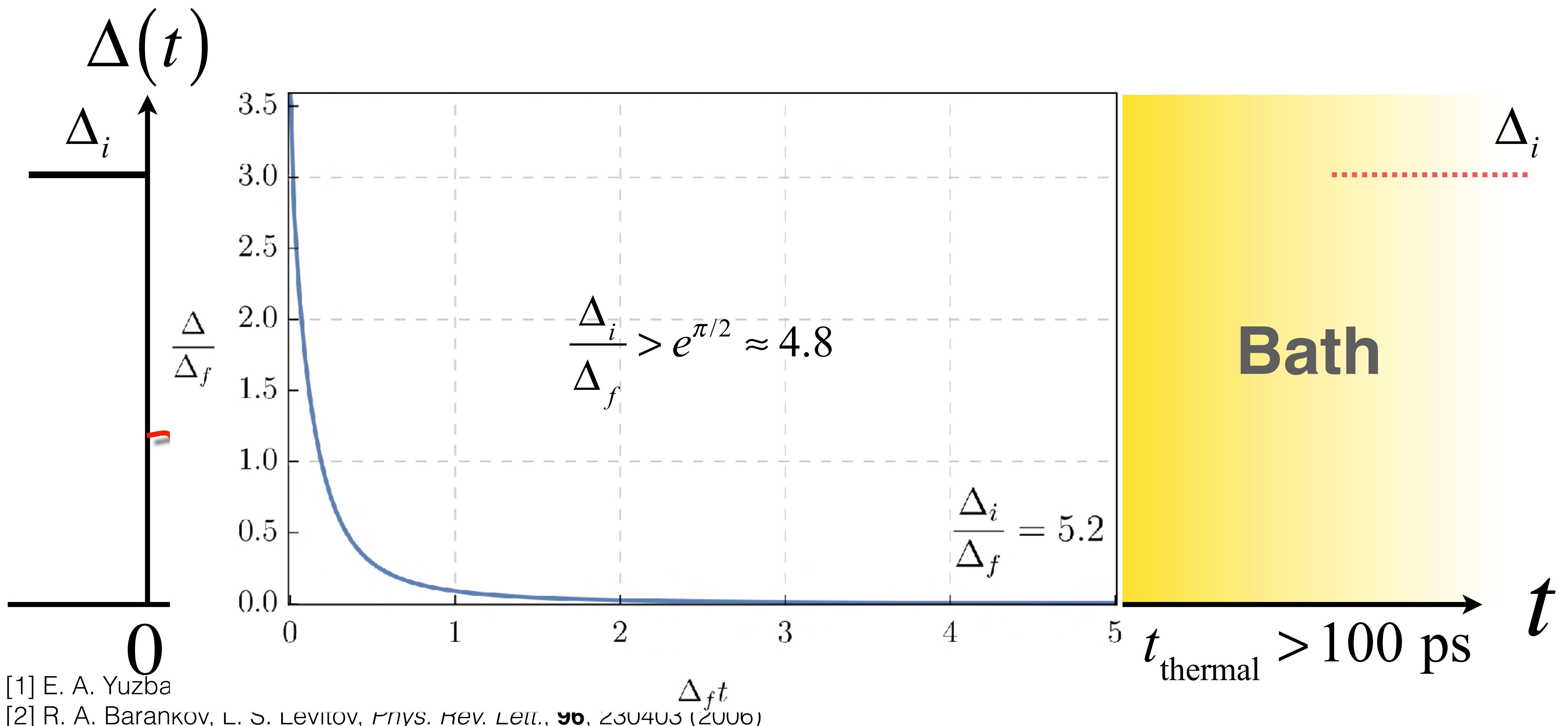


# Transient dynamics

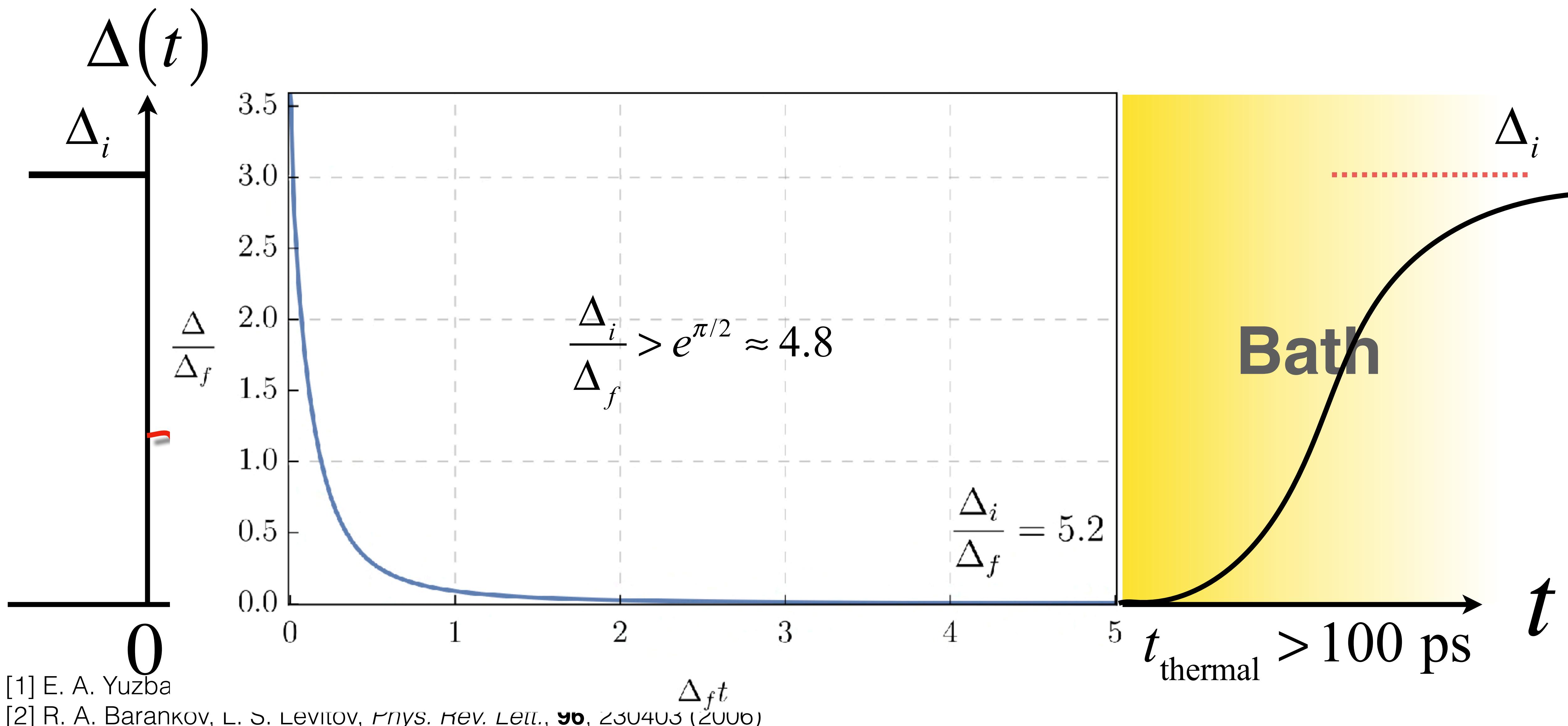


- [1] E. A. Yuzbashyan, B. L. Altshuler, V. B. Kuznetsov, V. Z. Enolskii, *Phys. Rev. B*, **72**, 220503 (2005)  
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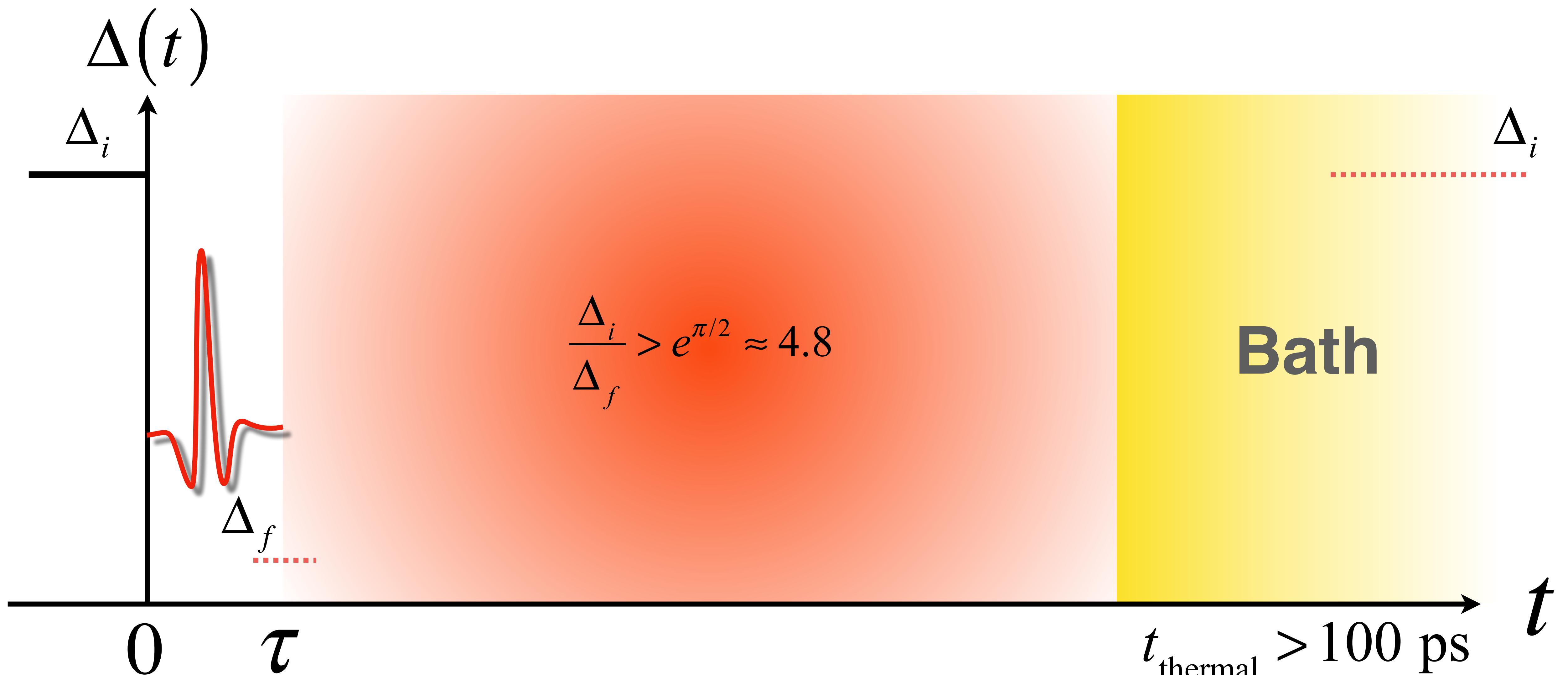
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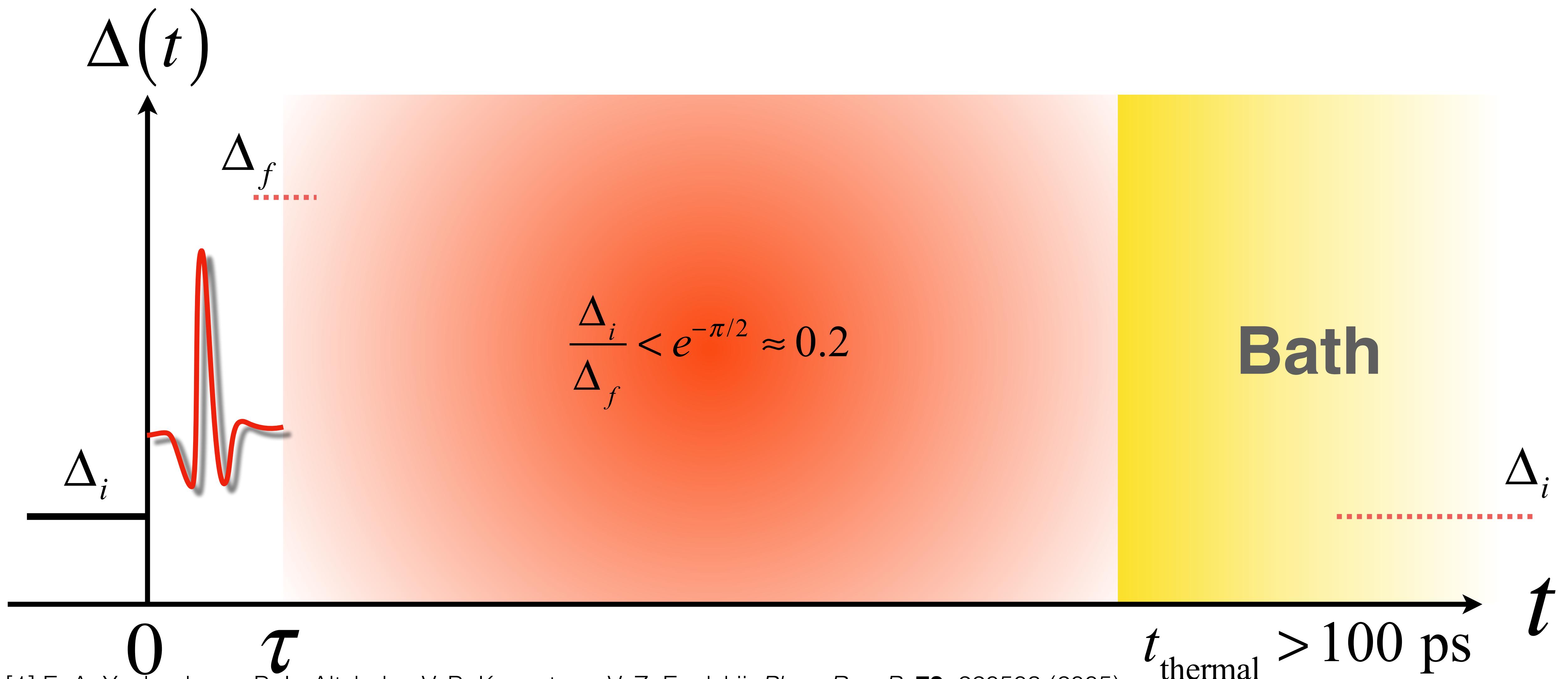


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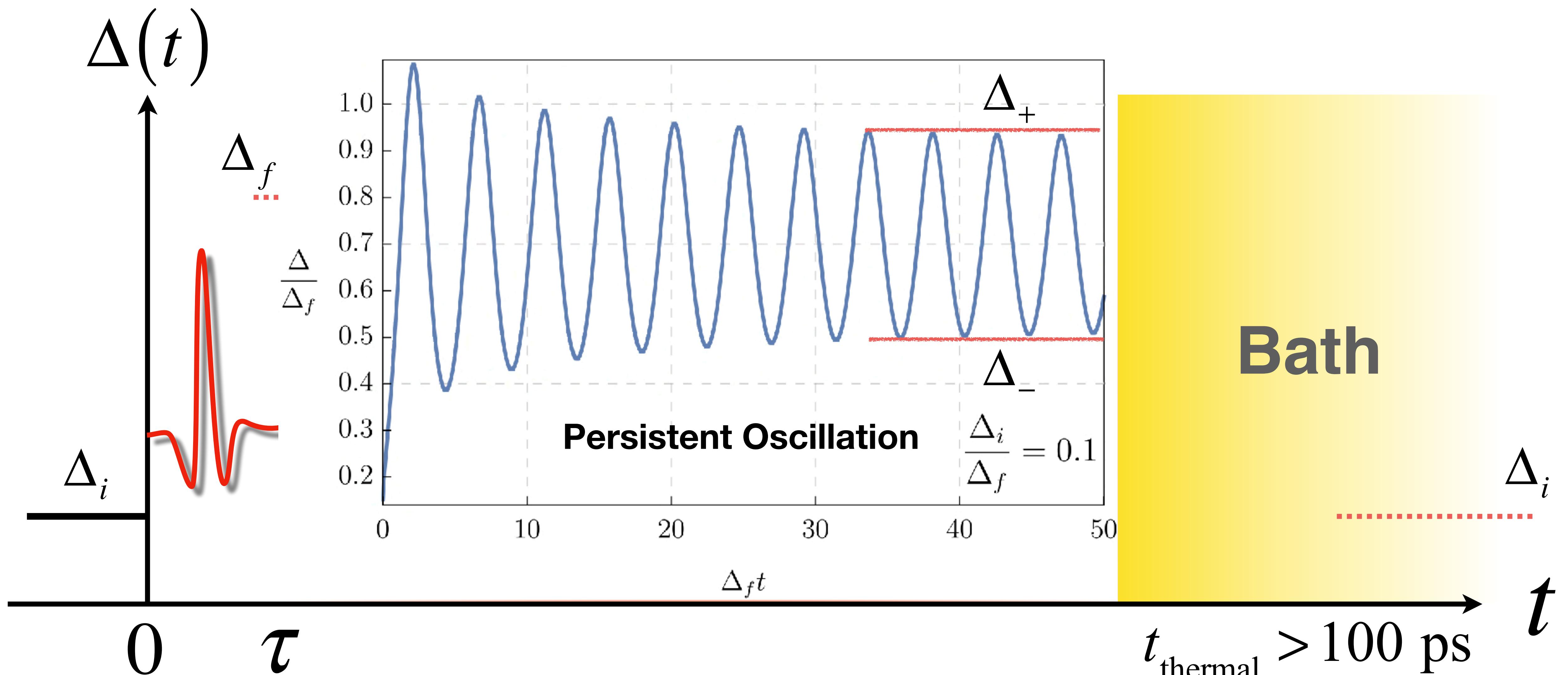
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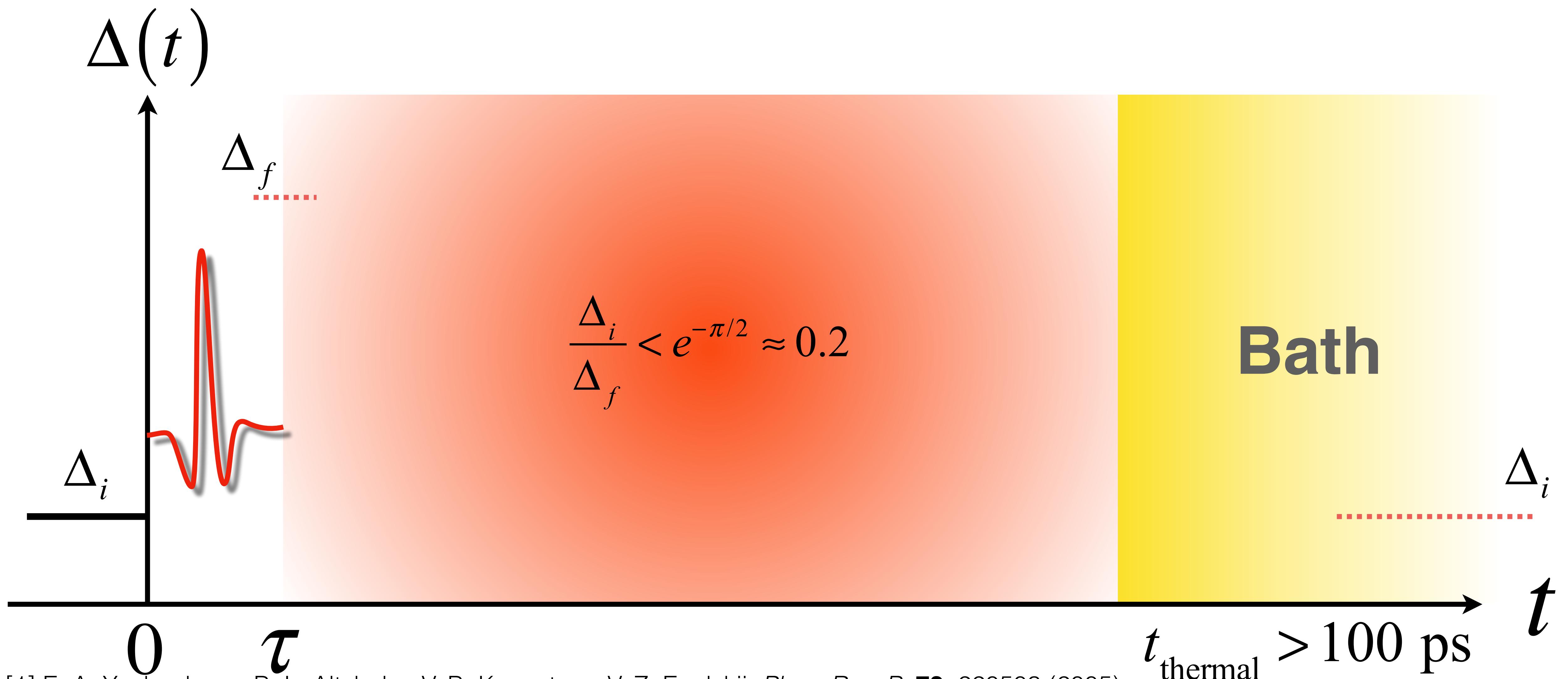
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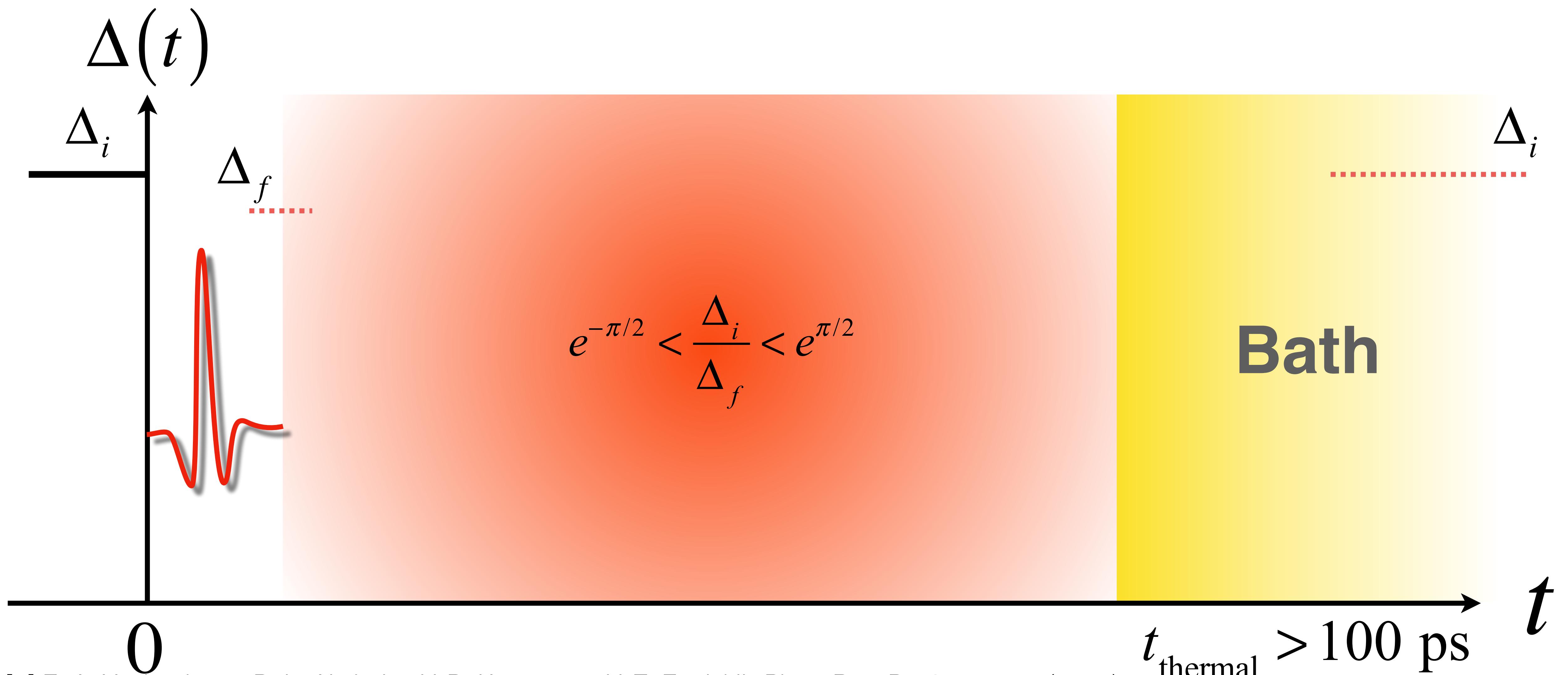
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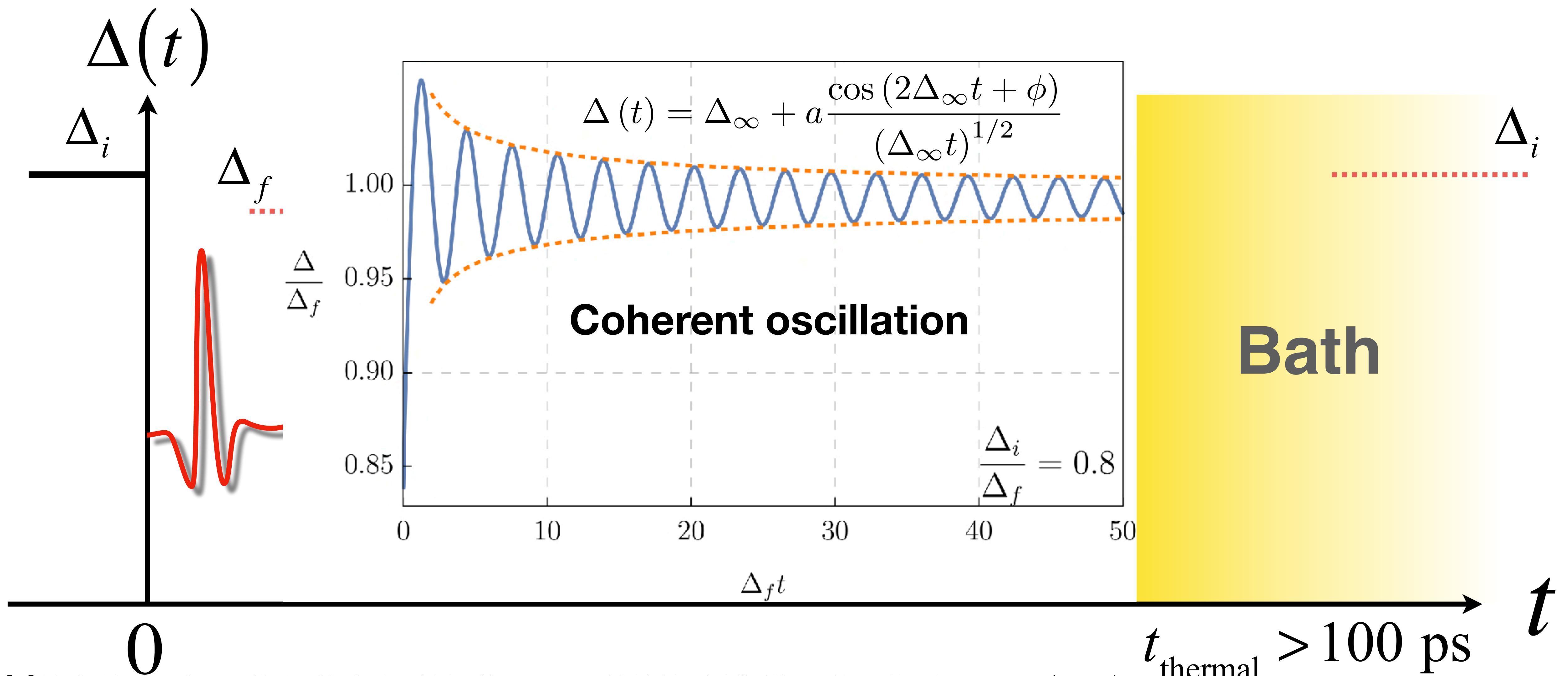
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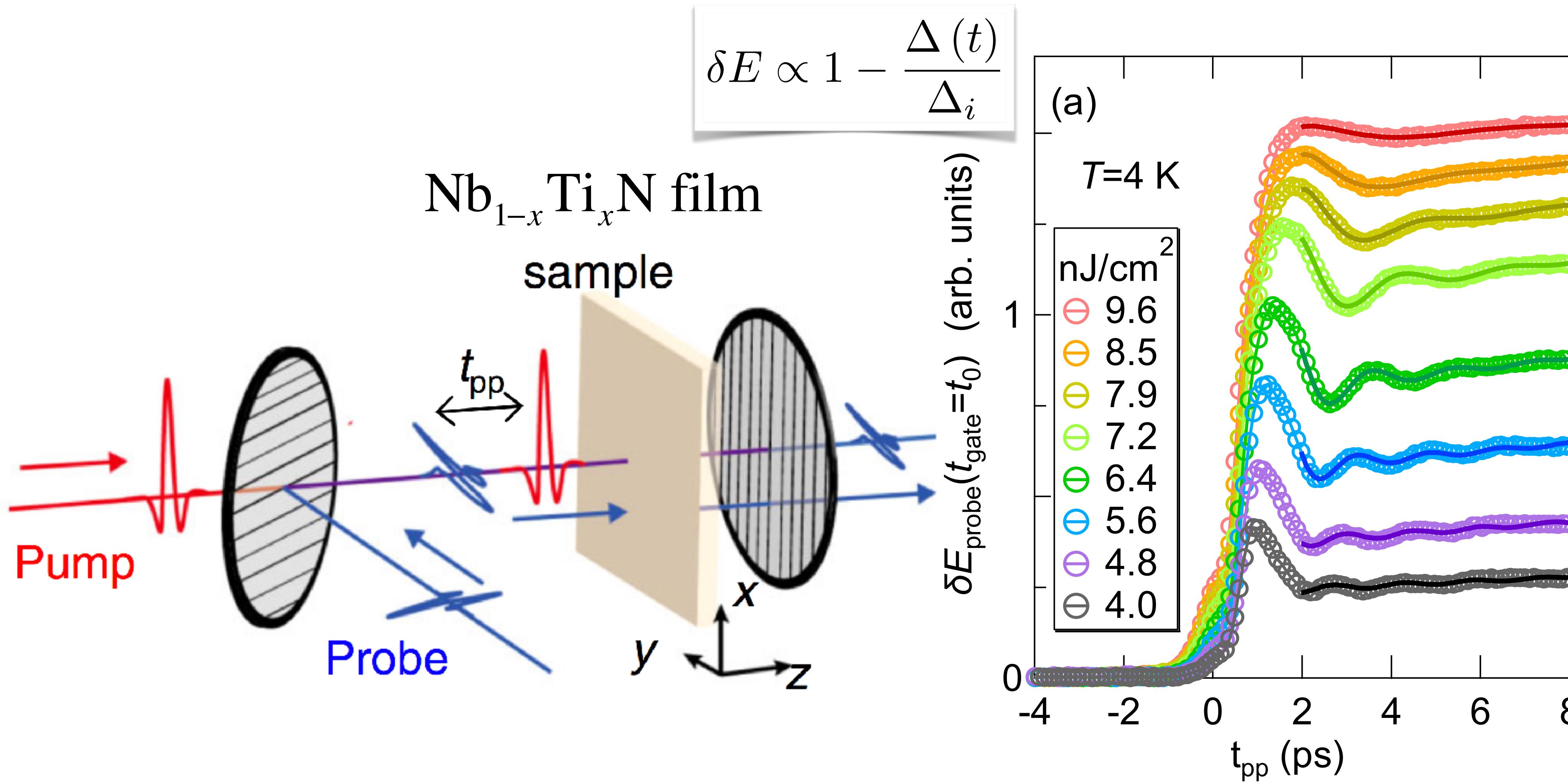
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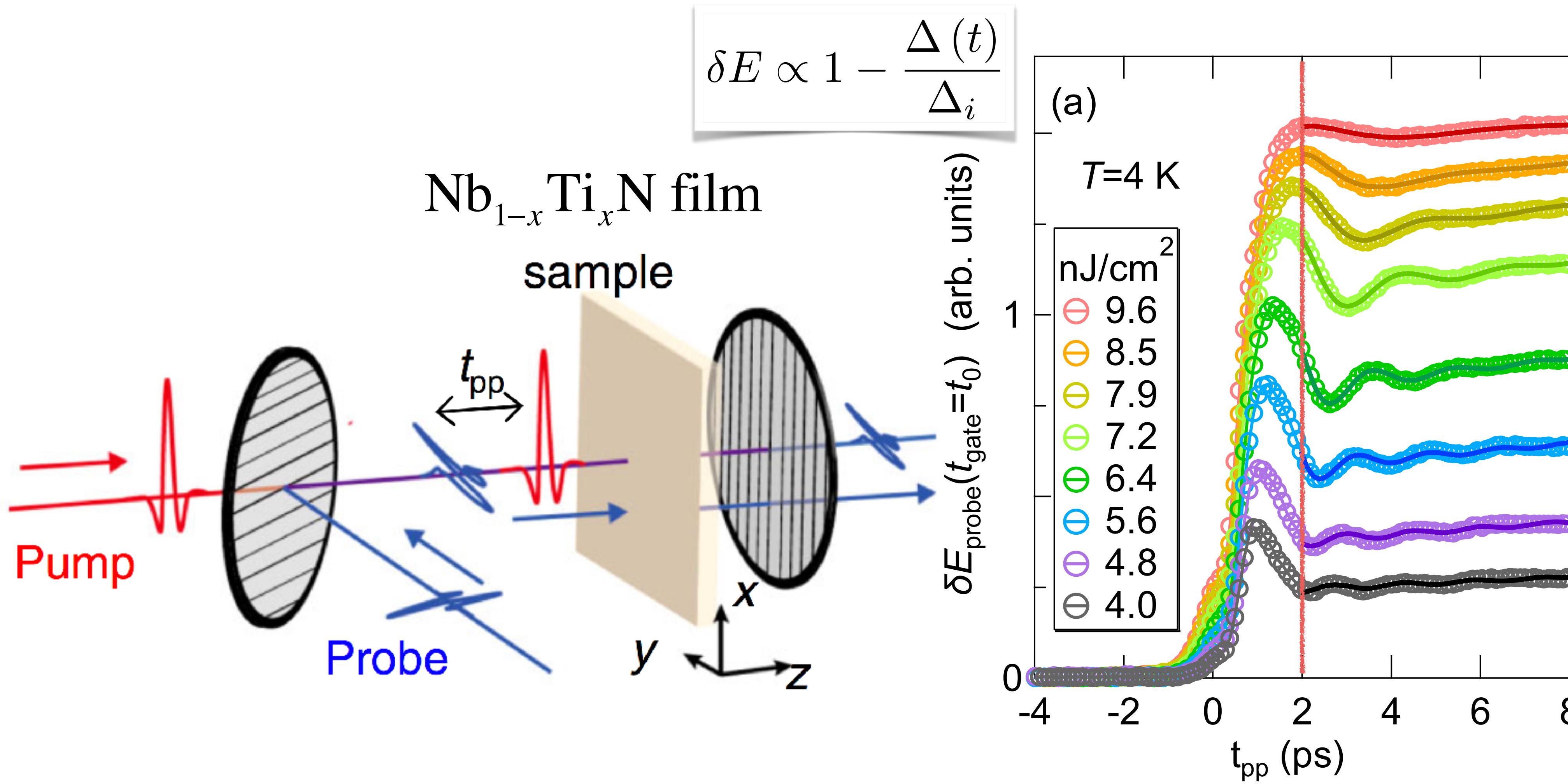
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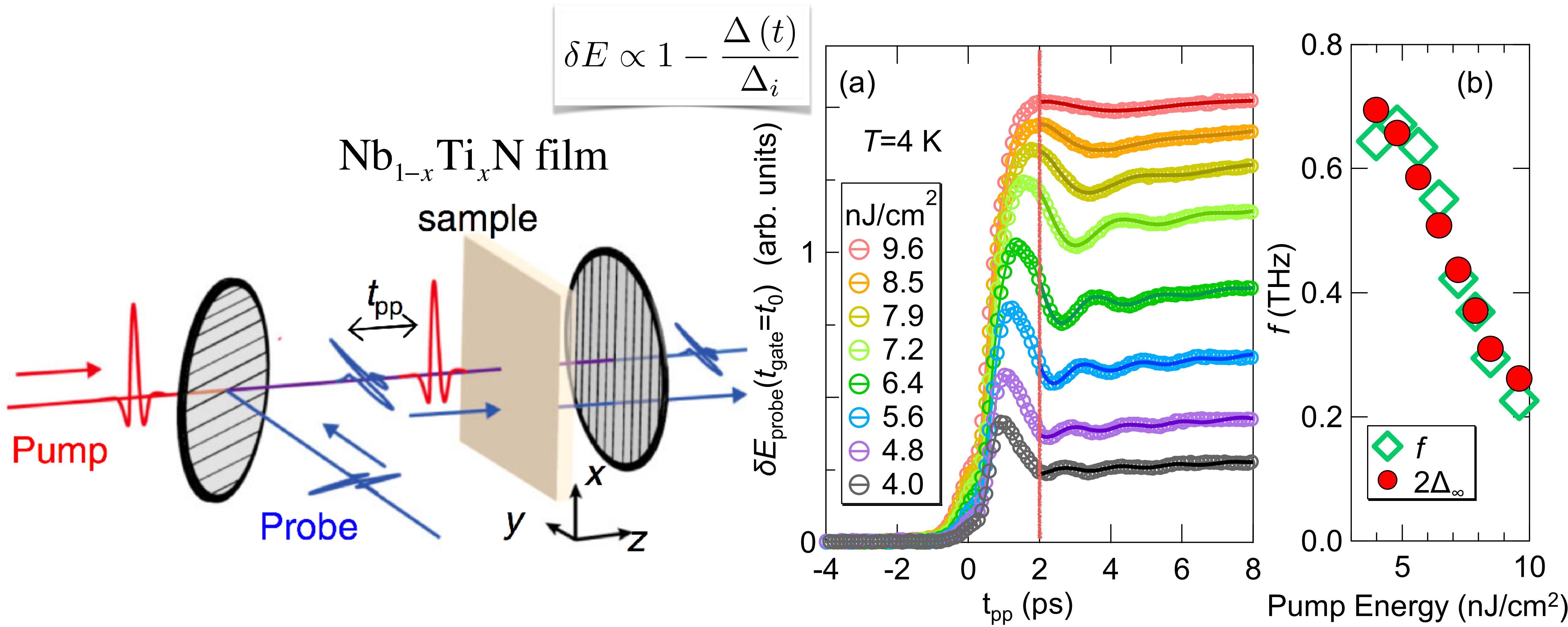
# THz-pump-probe-experiment



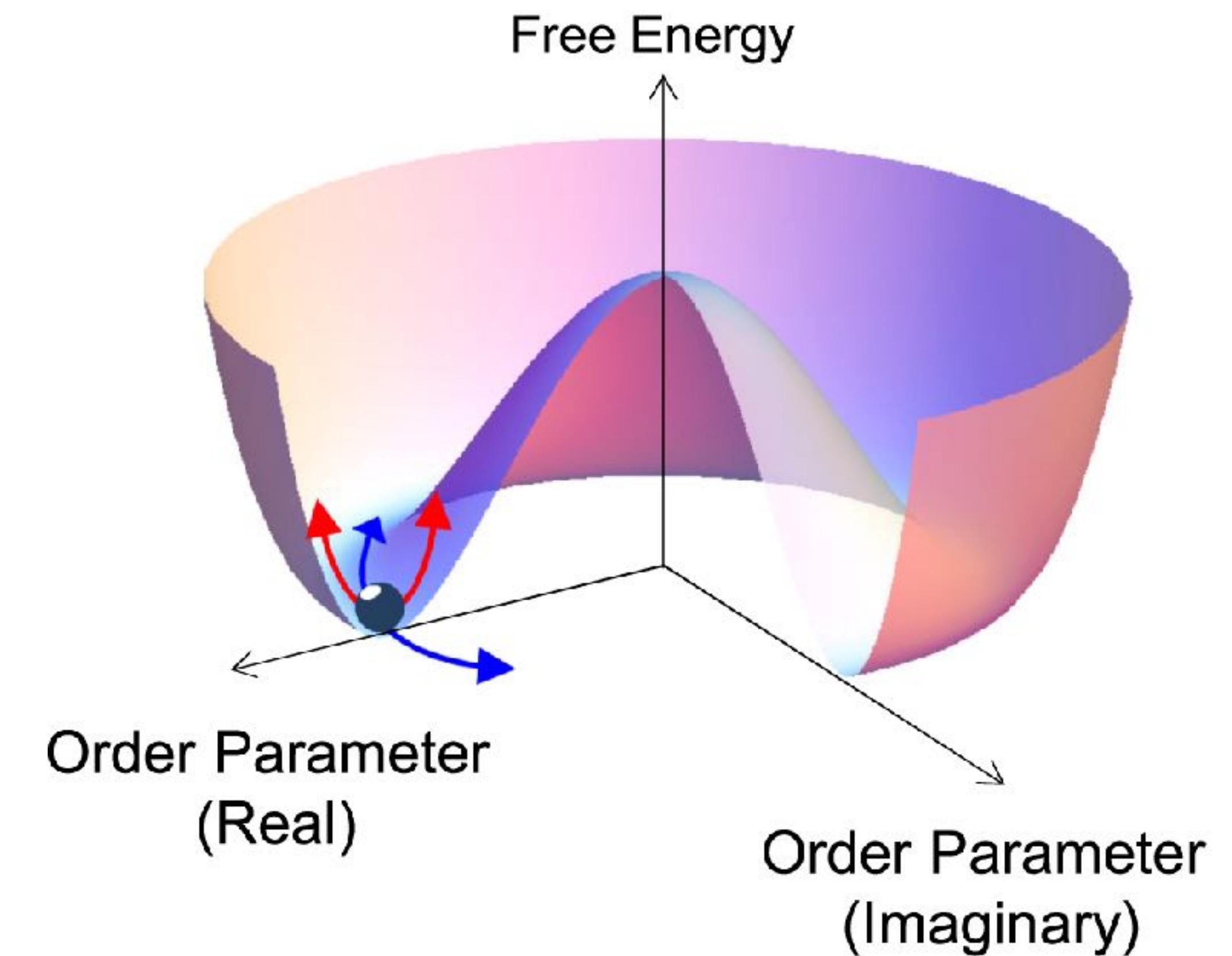
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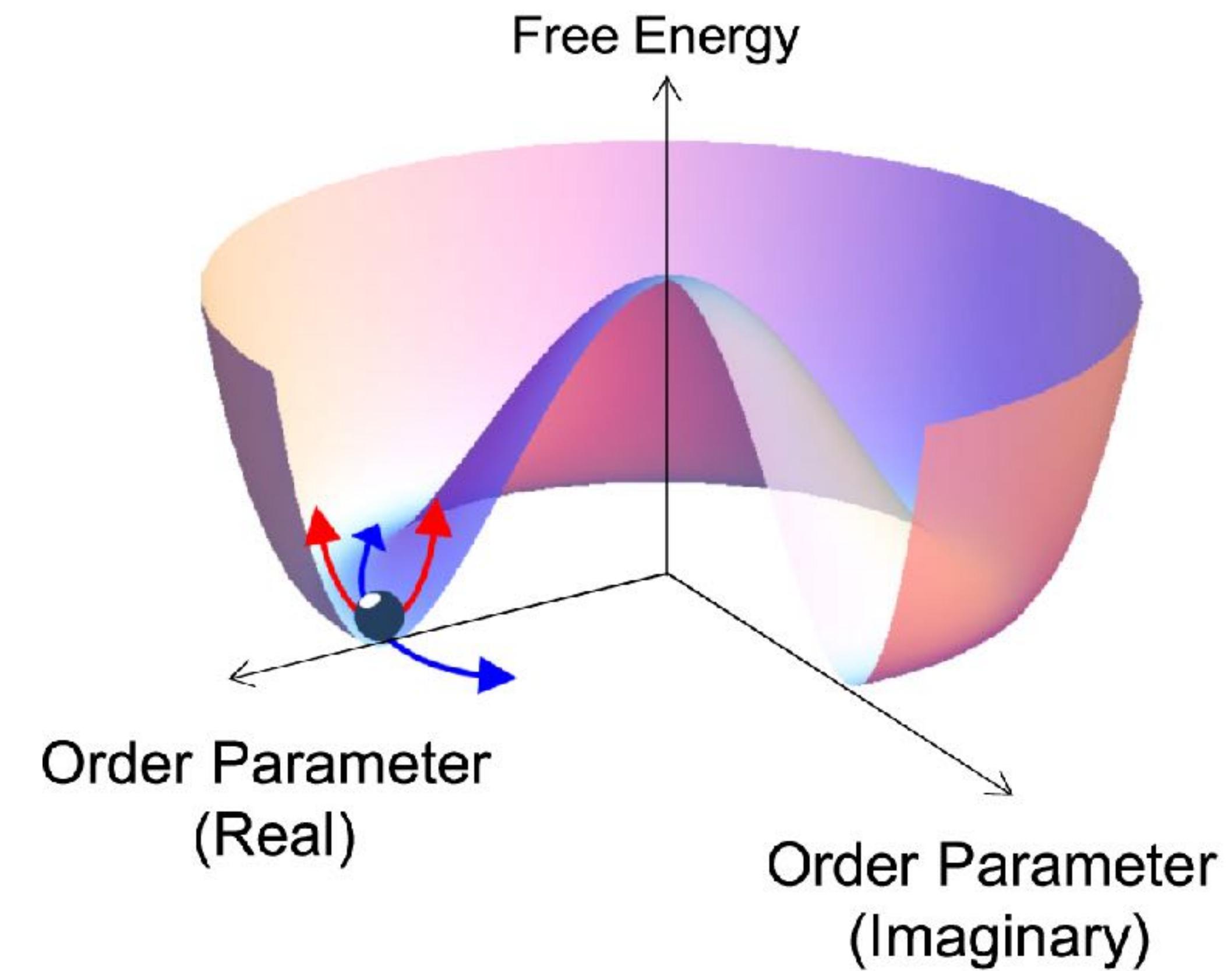
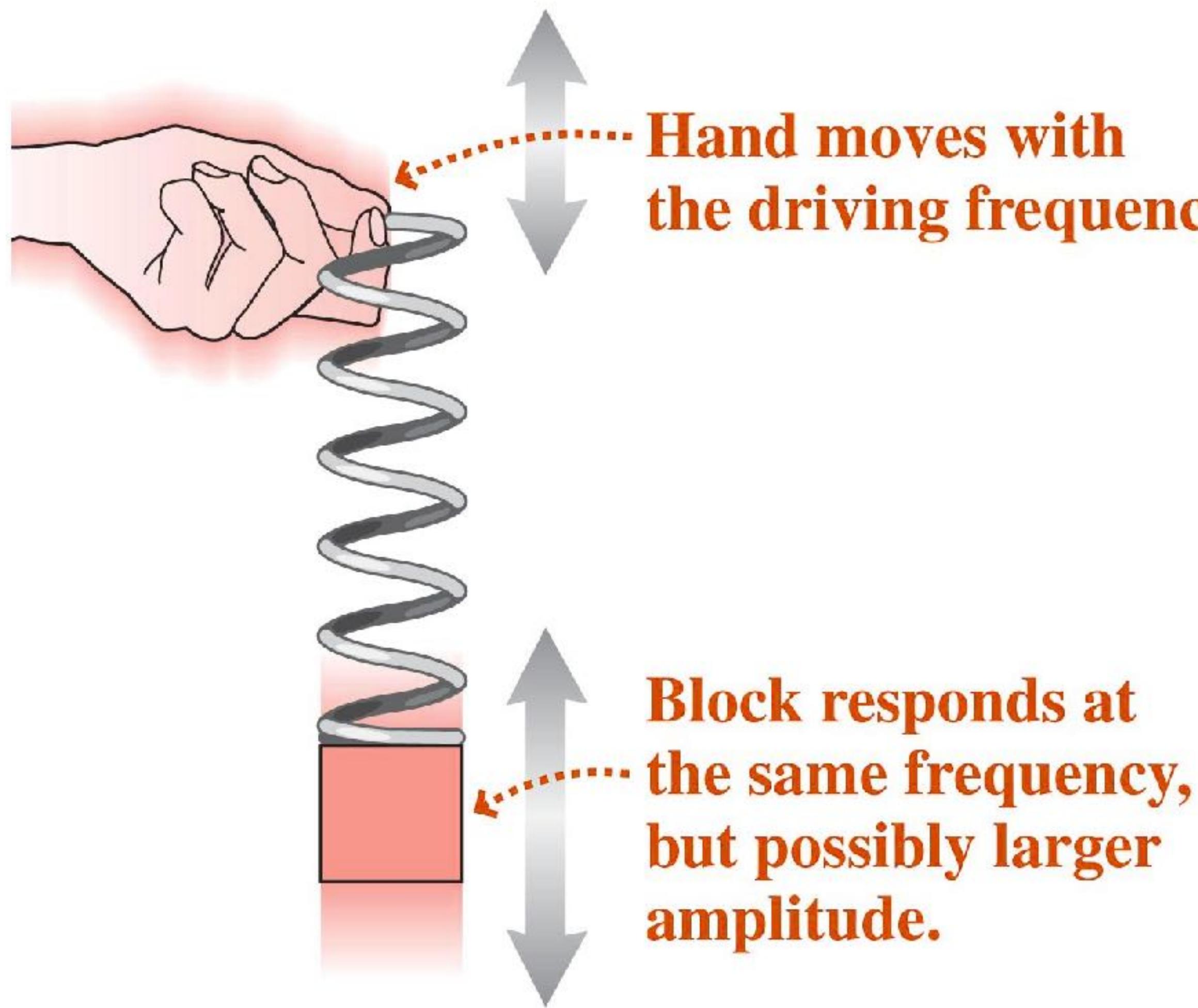
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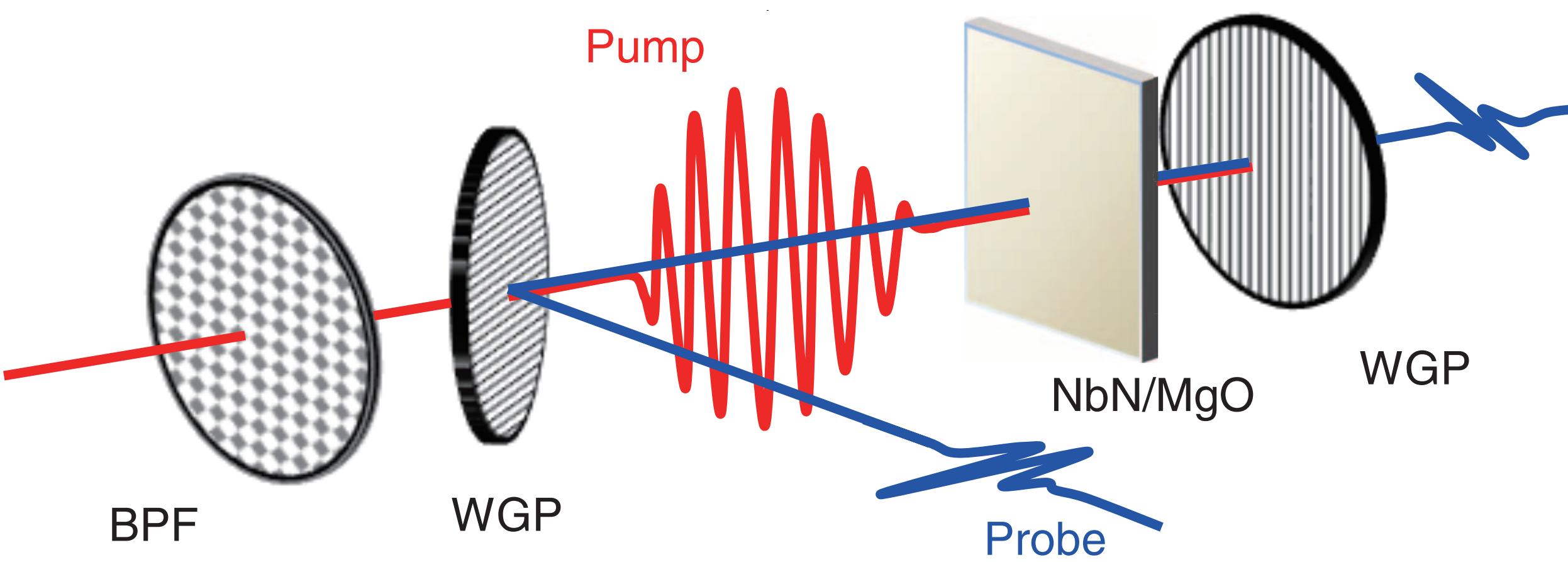
# Resonantly driven amplitude mode



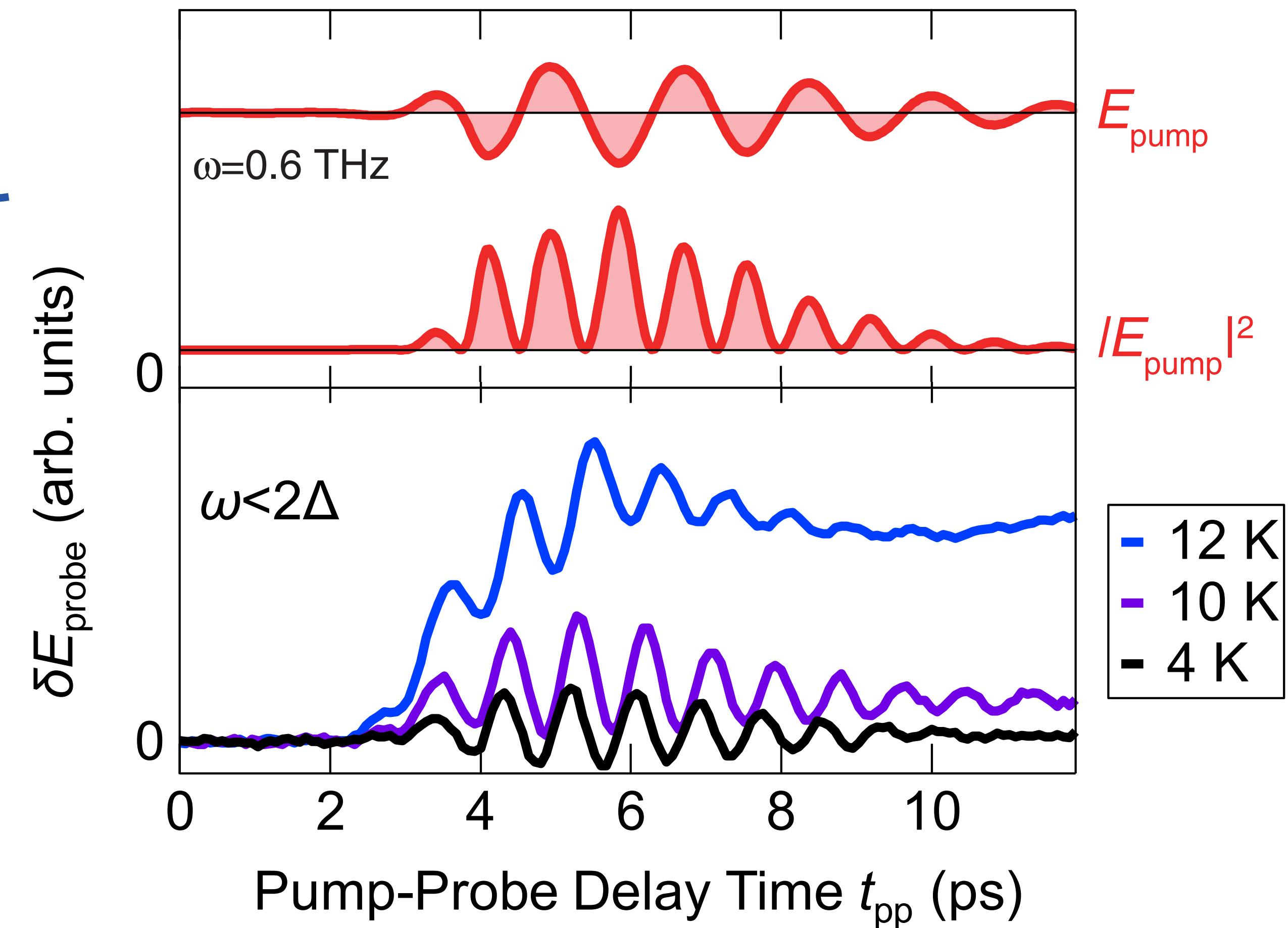
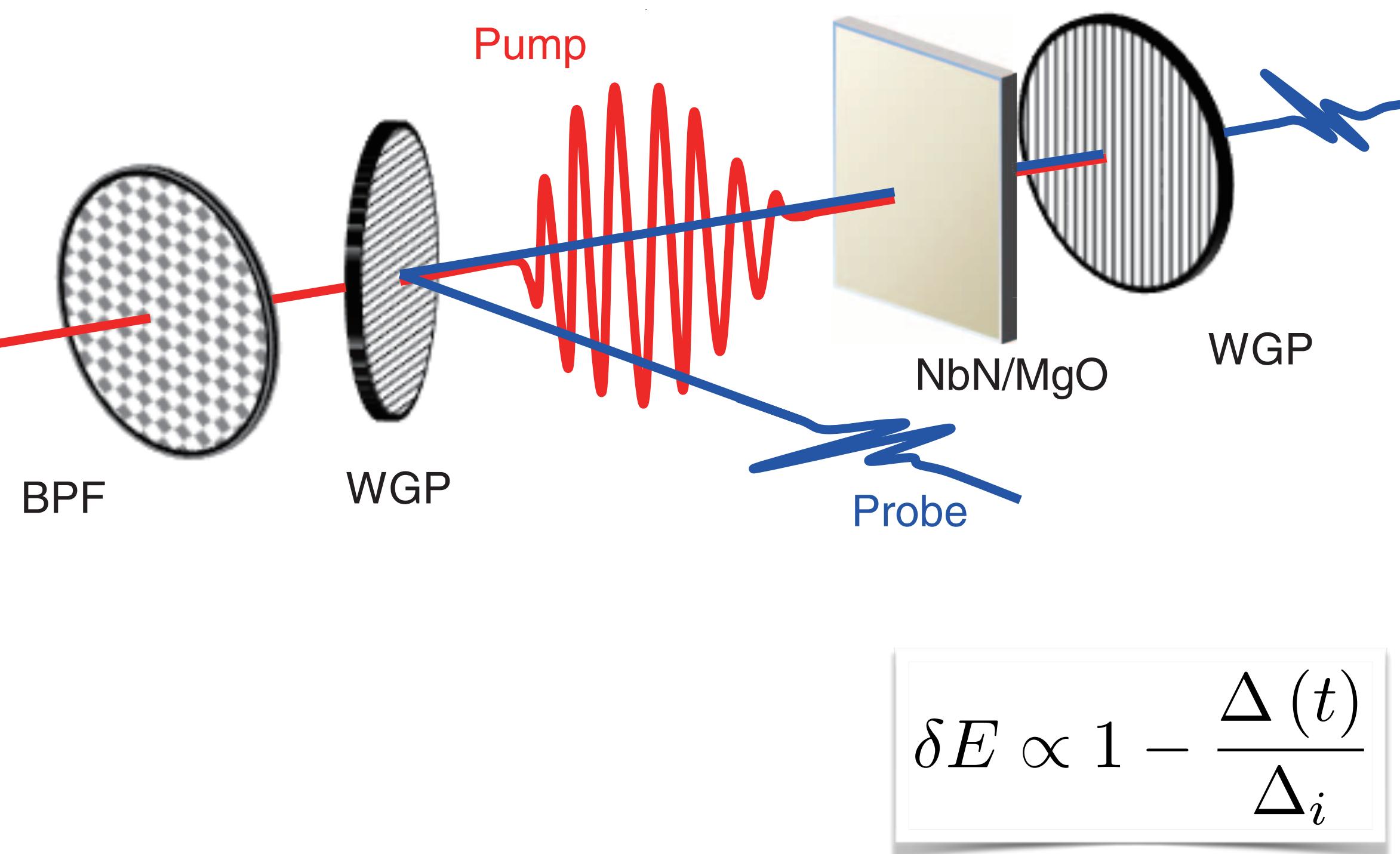
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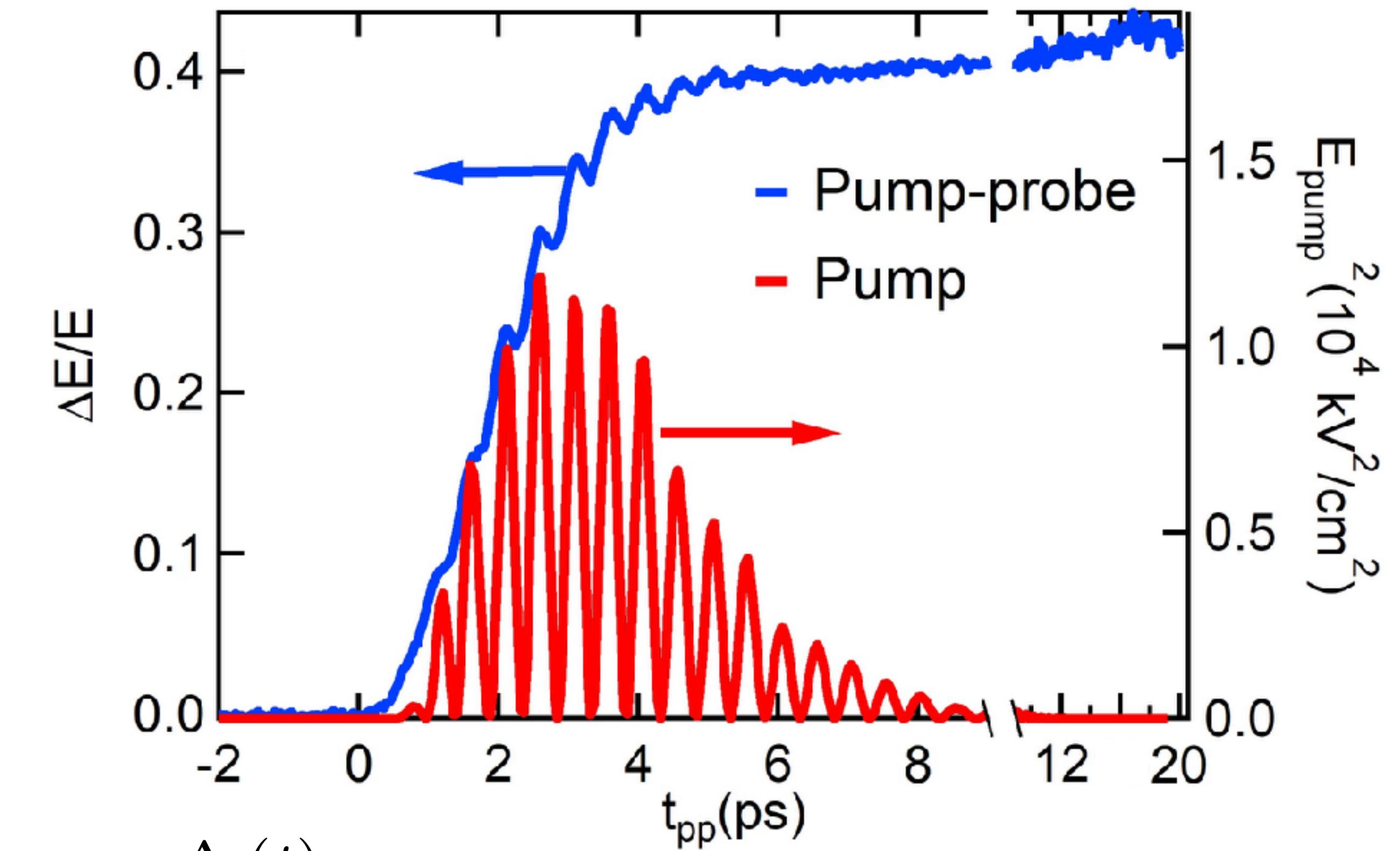
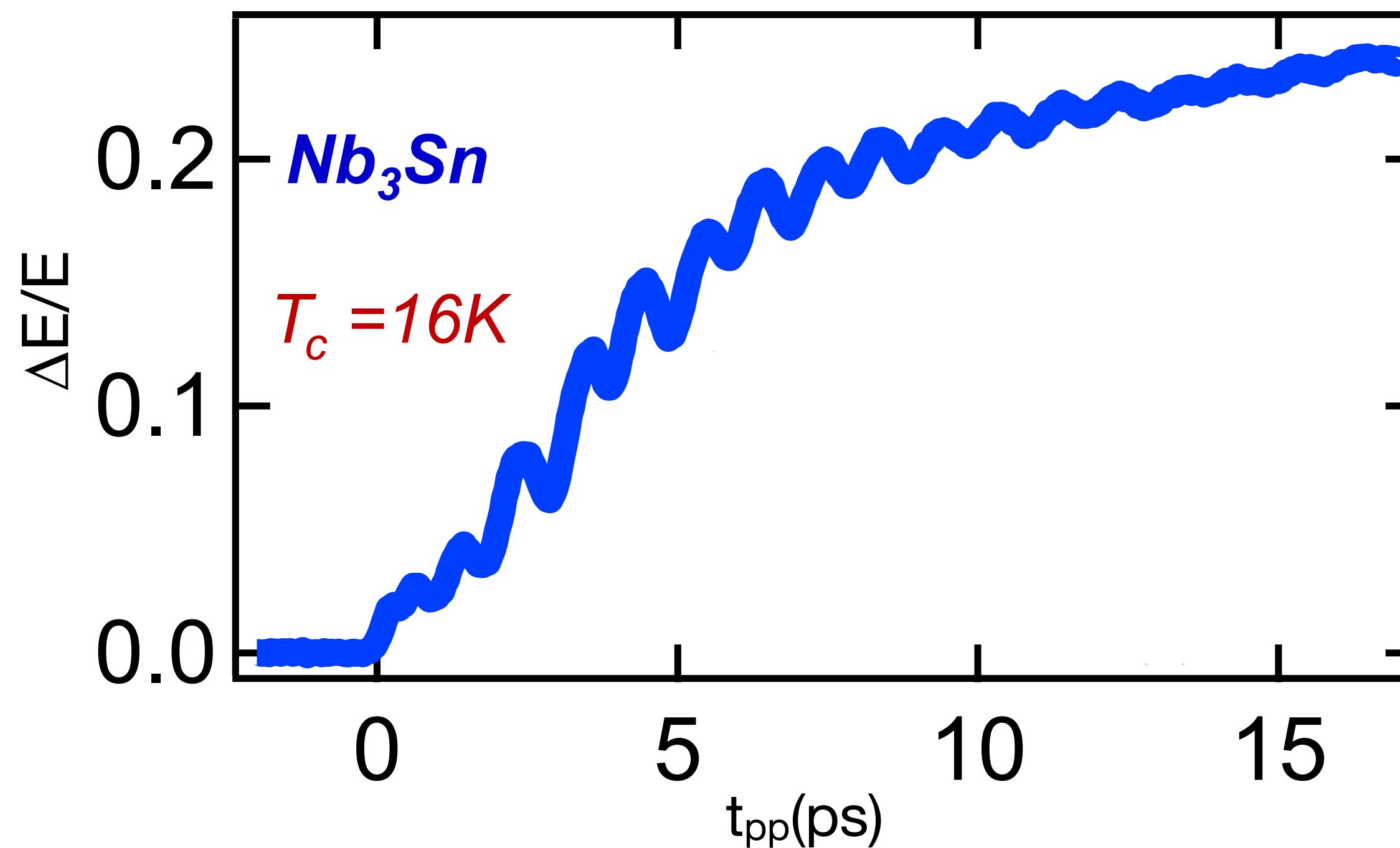
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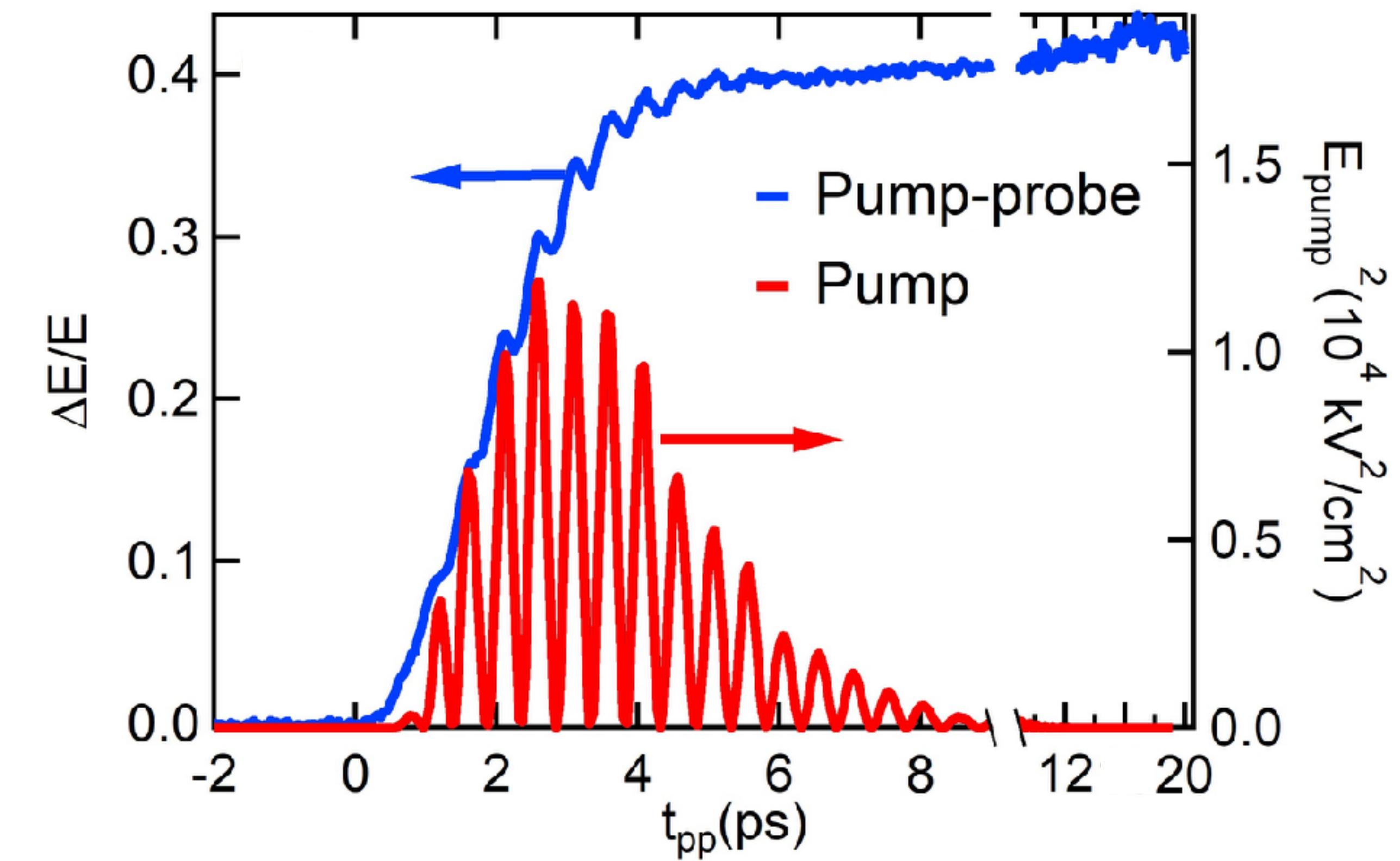
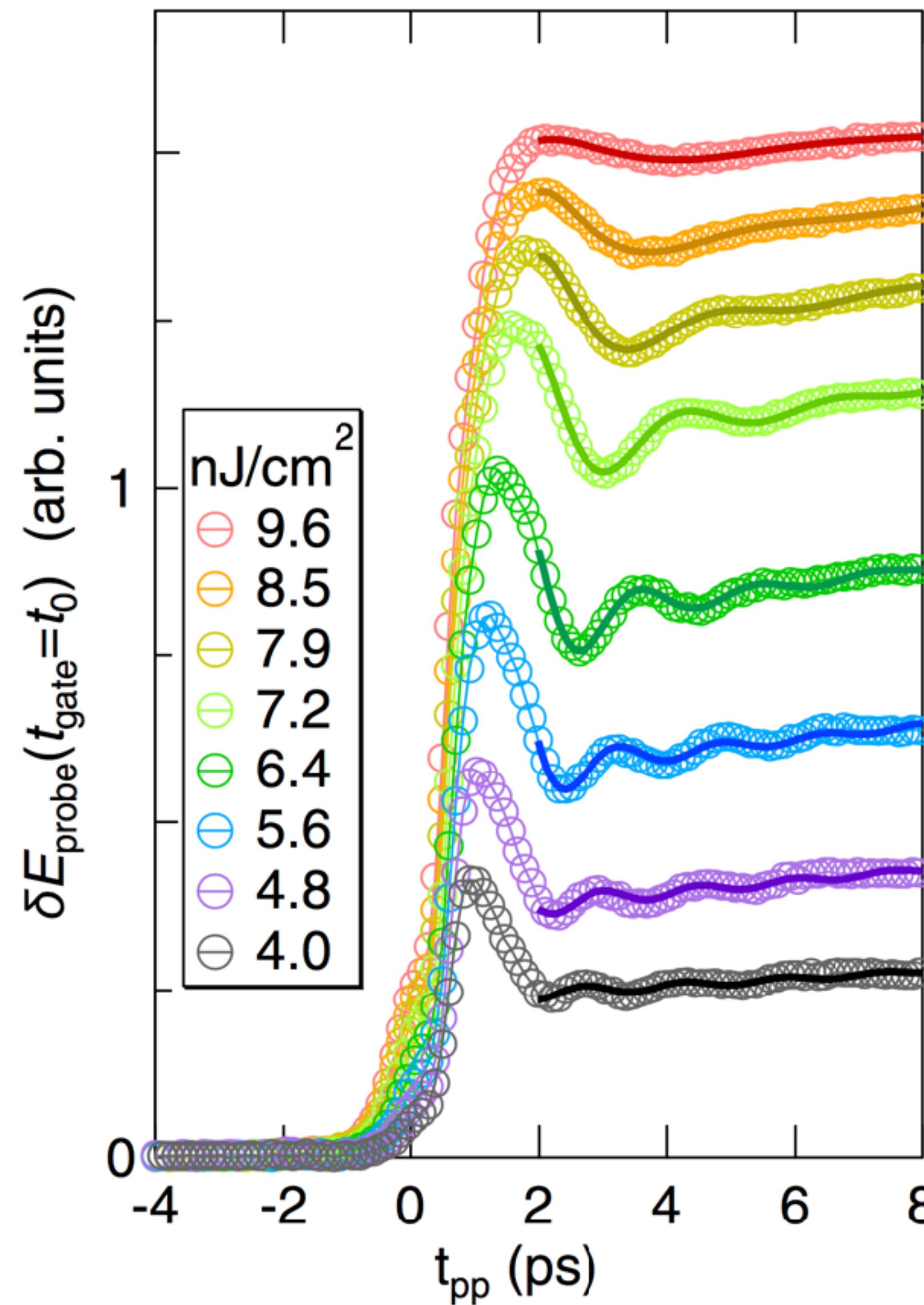


# Resonantly driven amplitude mode



$$\Delta E/E \propto 1 - \frac{\Delta(t)}{\Delta_i}$$

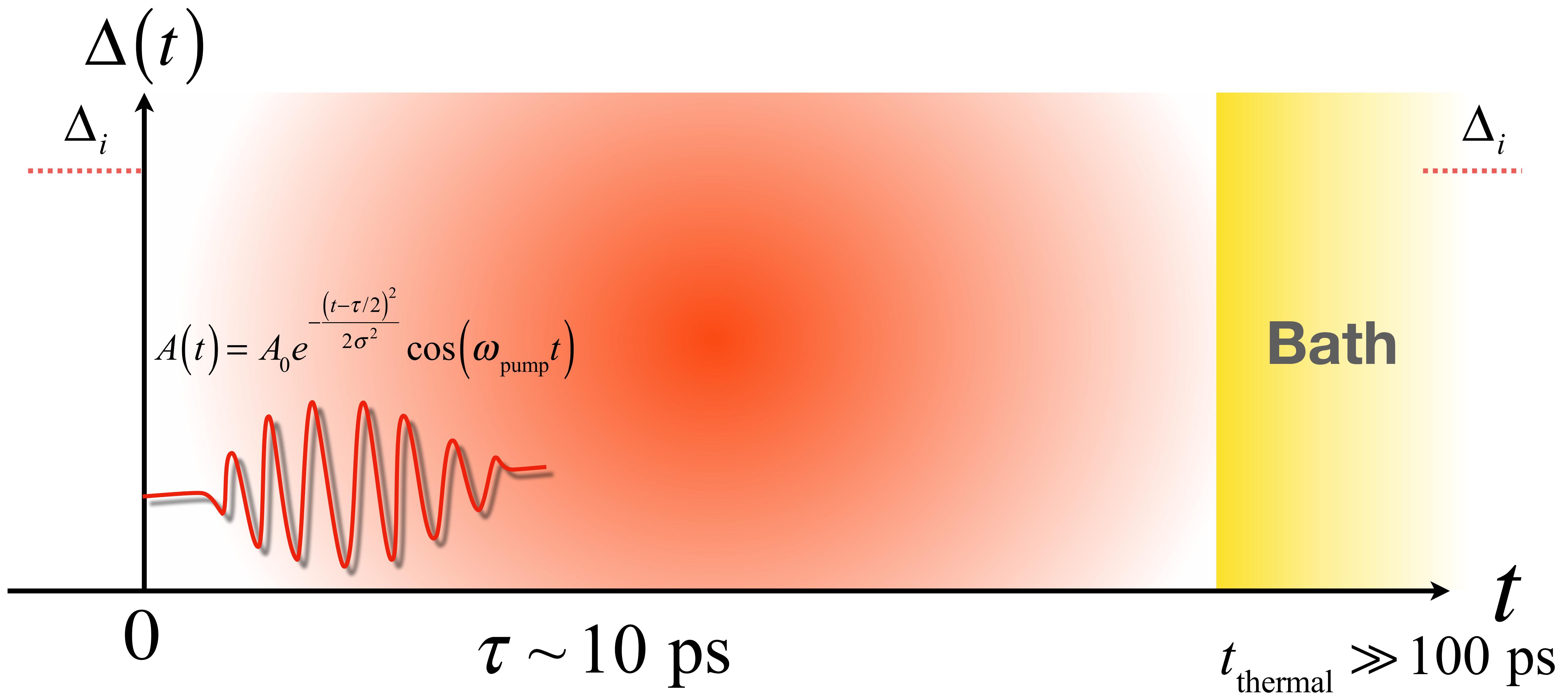
# Quench vs Resonant driving



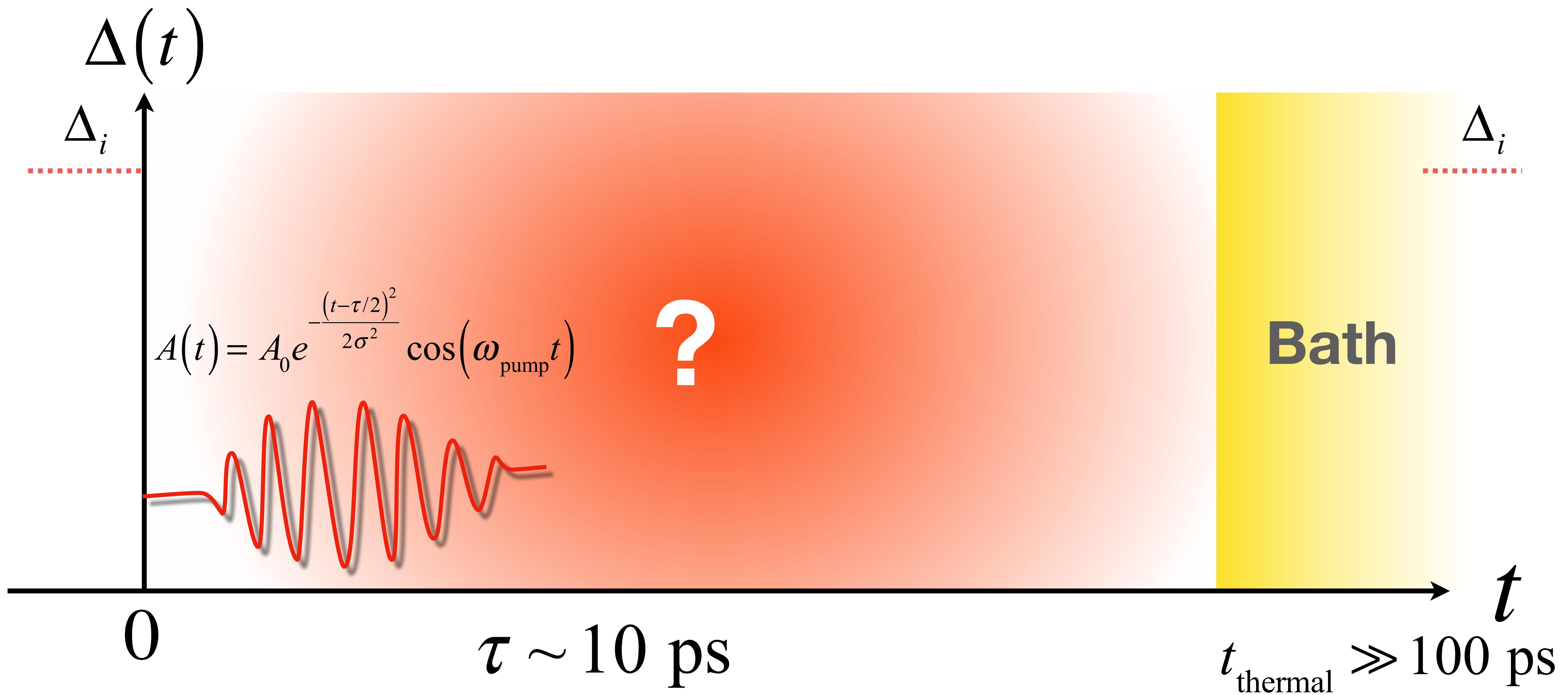
# Transient dynamics



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# Pseudo-spin formalism

$$H_{\text{BCS}} = \sum_{\mathbf{k}, \sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}, \sigma}^\dagger c_{\mathbf{k}, \sigma} + U \sum_{\mathbf{k}, \mathbf{k}'} c_{\mathbf{k}, \uparrow}^\dagger c_{-\mathbf{k}, \downarrow}^\dagger c_{-\mathbf{k}, \downarrow} c_{\mathbf{k}, \uparrow}$$
$$\Delta = -U \sum_{\mathbf{k}} \langle c_{-\mathbf{k}, \downarrow} c_{\mathbf{k}, \uparrow} \rangle$$

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$$\Delta = -U \sum_{\mathbf{k}} \langle c_{-\mathbf{k}, \downarrow} c_{\mathbf{k}, \uparrow} \rangle$$

$$\hat{S}_{\mathbf{k}}^z = \frac{1}{2} \left( c_{\mathbf{k}, \uparrow}^\dagger c_{\mathbf{k}, \uparrow} + c_{-\mathbf{k}, \downarrow}^\dagger c_{-\mathbf{k}, \downarrow} - 1 \right)$$

$$\hat{S}_{\mathbf{k}}^+ = c_{\mathbf{k}, \uparrow}^\dagger c_{-\mathbf{k}, \downarrow}^\dagger$$

$$\hat{S}_{\mathbf{k}}^- = c_{-\mathbf{k}, \downarrow} c_{\mathbf{k}, \uparrow}$$

# Pseudo-spin formalism

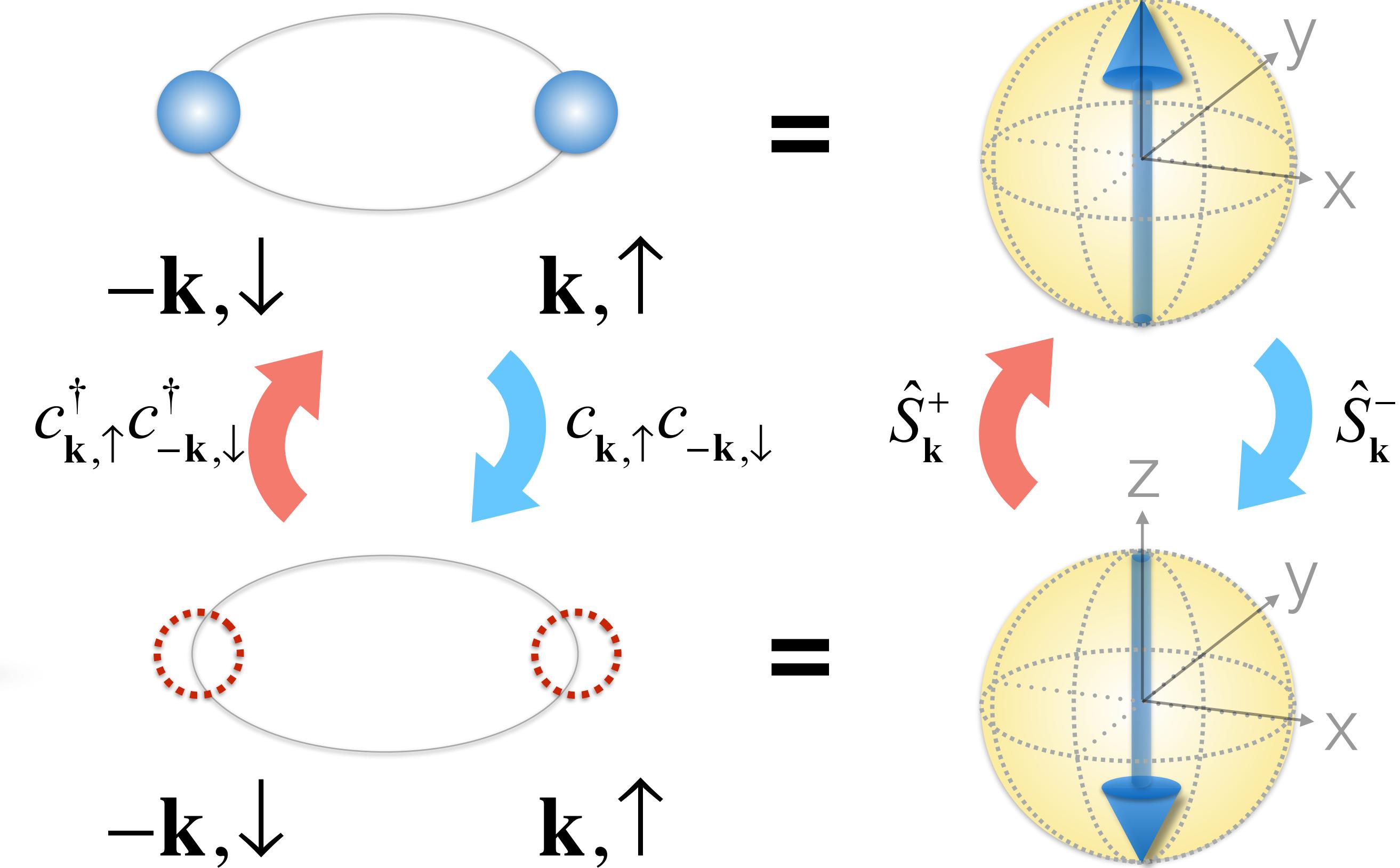
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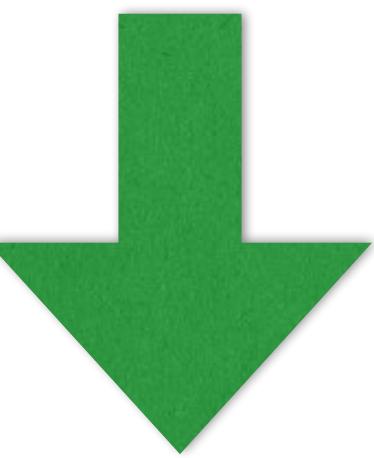
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$$H_{\text{BCS}} = - \sum_{\mathbf{k}} \mathbf{B}_{\mathbf{k}} \cdot \hat{\mathbf{S}}_{\mathbf{k}}$$

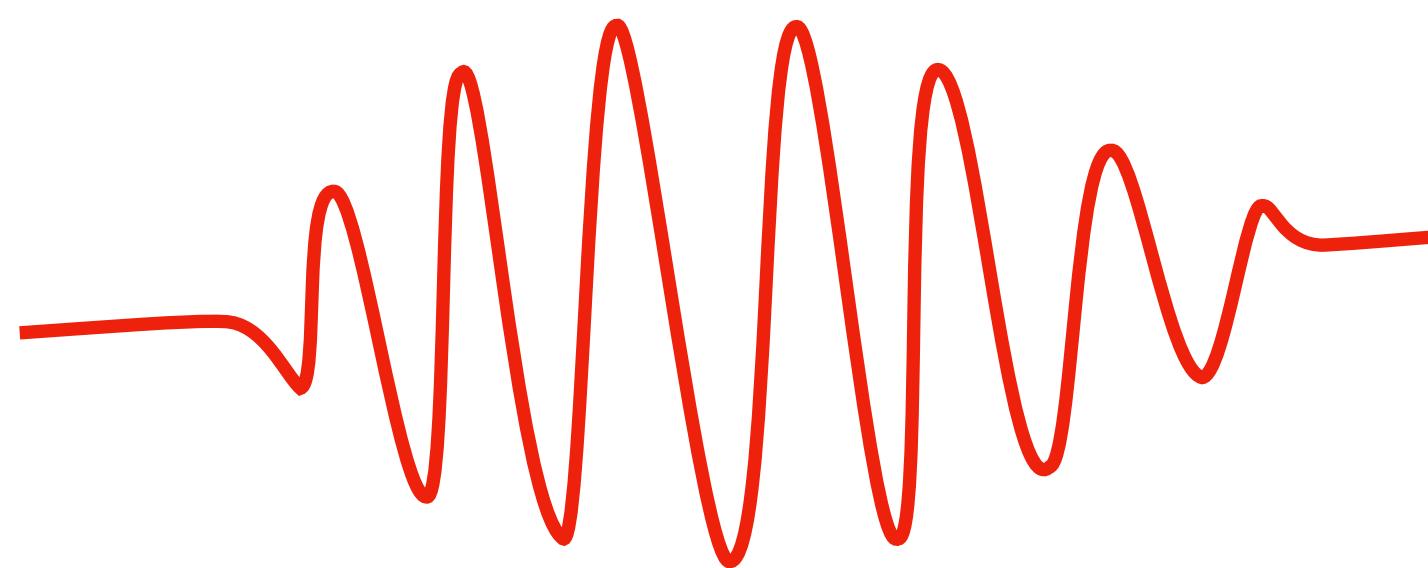
$$\mathbf{B}_{\mathbf{k}} = (2\text{Re}(\Delta), -2\text{Im}(\Delta), -2\epsilon_{\mathbf{k}})$$
$$\Delta = -U \sum_{\mathbf{k}} S_{\mathbf{k}}^-$$

# Couple BCS to Light

$$H_{\text{BCS}} = - \sum_{\mathbf{k}} \mathbf{B}_{\mathbf{k}} \cdot \hat{\mathbf{S}}_{\mathbf{k}}$$

$$\Delta = -U \sum_{\mathbf{k}} S_{\mathbf{k}}^-$$

# Couple BCS to Light



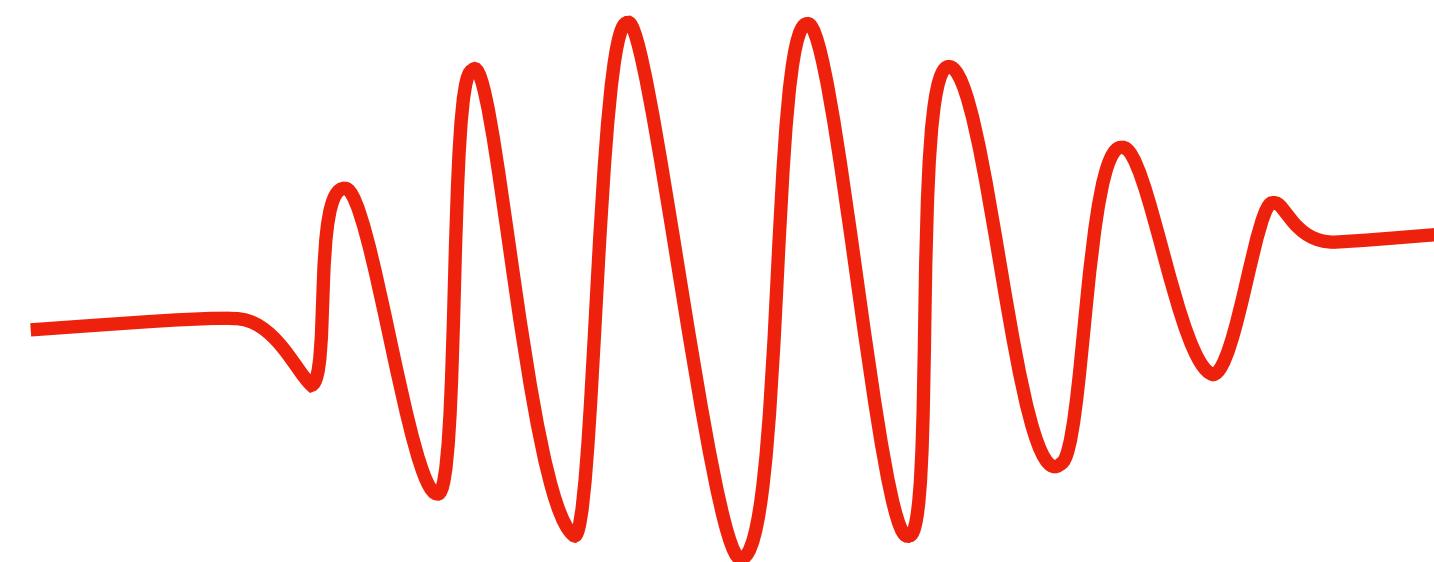
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# Couple BCS to Light

Peierls Substitution:

$$\epsilon_{\mathbf{k}} \rightarrow \epsilon_{\mathbf{k} - e\mathbf{A}(t)}$$



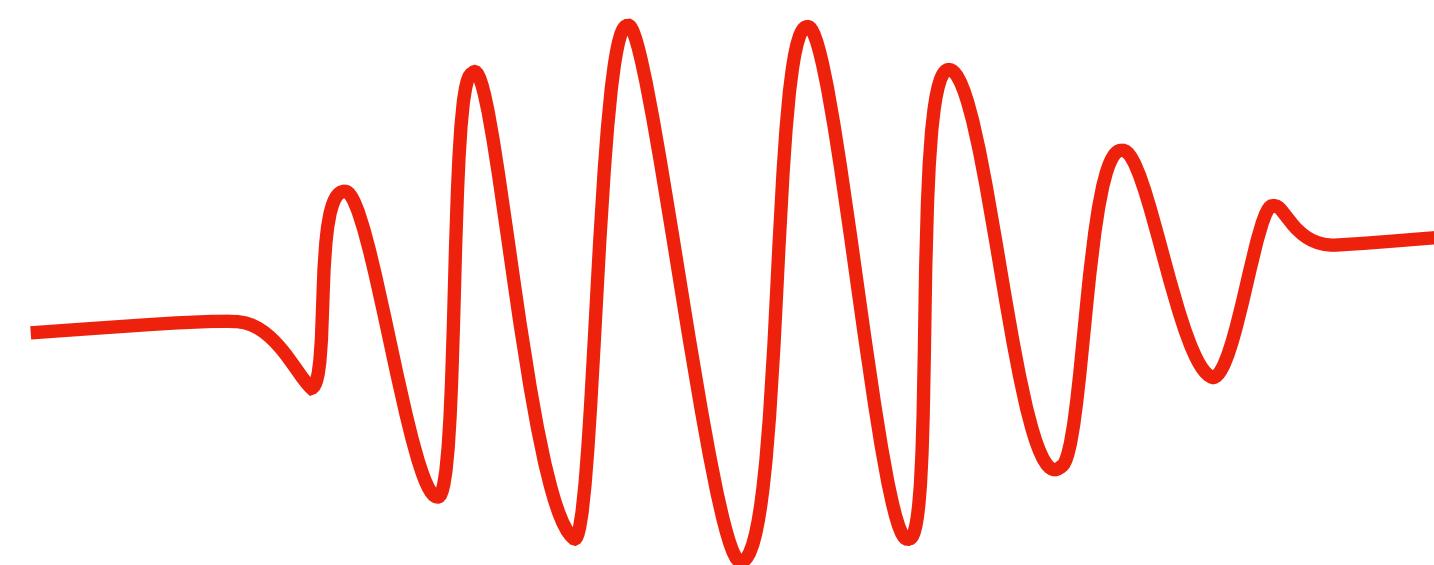
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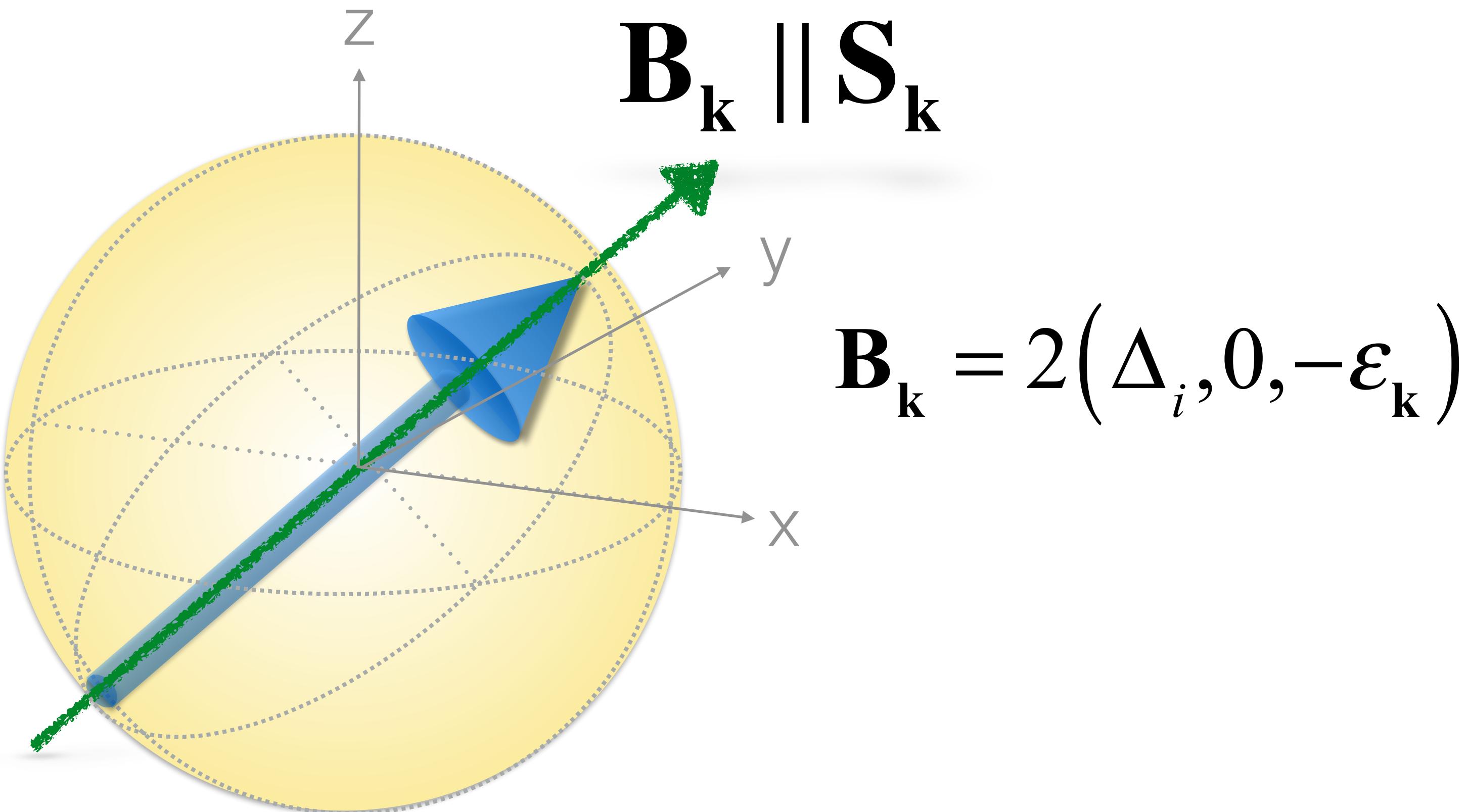
$$H_{\text{BCS}} = - \sum_{\mathbf{k}} \mathbf{B}_{\mathbf{k}} \cdot \hat{\mathbf{S}}_{\mathbf{k}}$$

$$\Delta = -U \sum_{\mathbf{k}} S_{\mathbf{k}}^-$$

$$\mathbf{B}_{\mathbf{k}} = 2 \left( \text{Re}(\Delta), -\text{Im}(\Delta), -\frac{\epsilon_{\mathbf{k}-e\mathbf{A}(t)} + \epsilon_{\mathbf{k}+e\mathbf{A}(t)}}{2} \right)$$

# Pseudo-spin dynamics

$$H_{\text{BCS}} = - \sum_{\mathbf{k}} \mathbf{B}_{\mathbf{k}} \cdot \hat{\mathbf{S}}_{\mathbf{k}}$$



Initial conditions:

$$S_{\mathbf{k}}^x(0^-) = \frac{\Delta_i}{2\sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_i^2}}$$

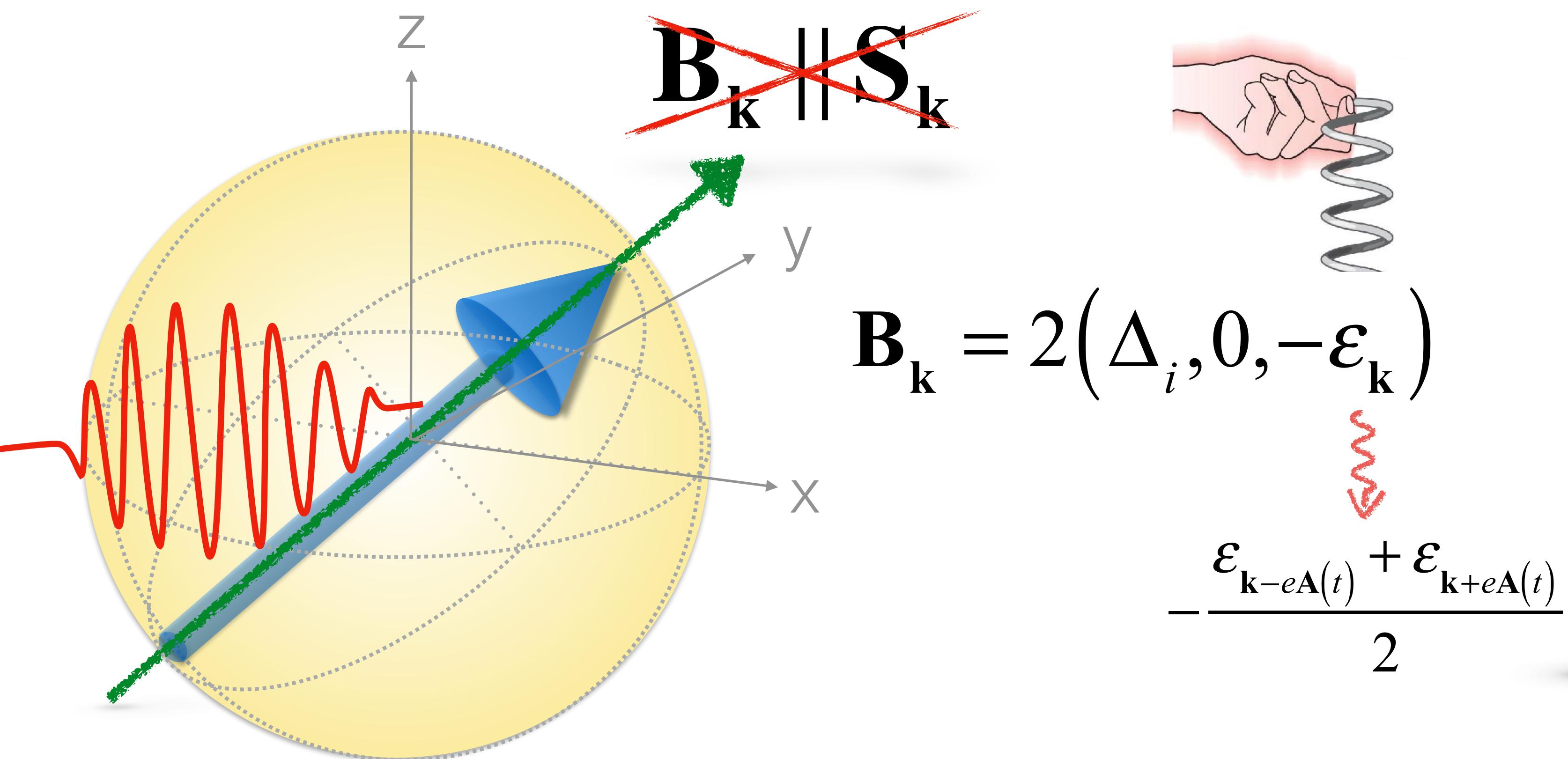
$$S_{\mathbf{k}}^y(0^-) = 0$$

$$S_{\mathbf{k}}^z(0^-) = \frac{-\varepsilon_{\mathbf{k}}}{2\sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_i^2}}$$

# Pseudo-spin dynamics

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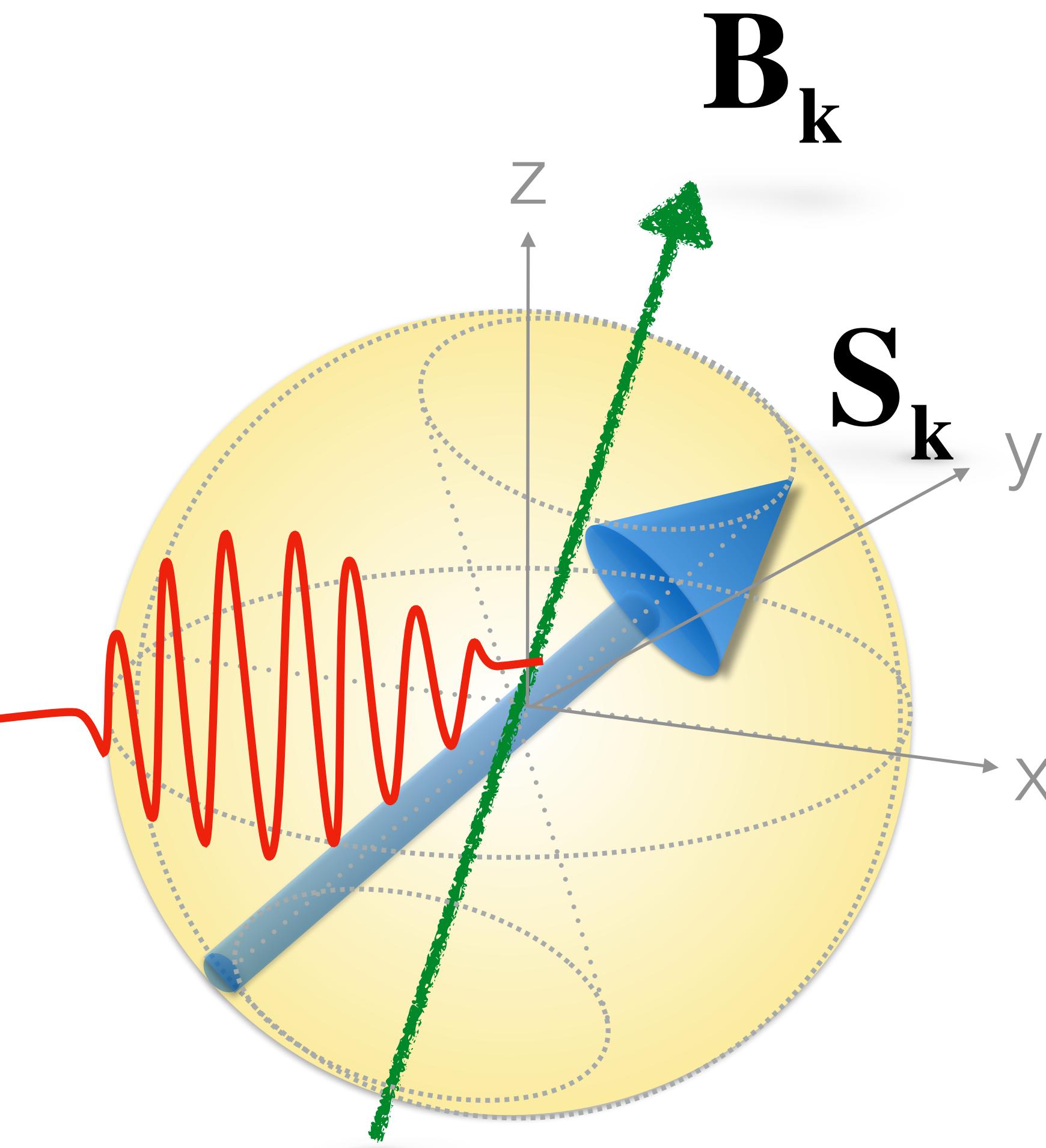


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$$S_{\mathbf{k}}^y(0^-) = 0$$

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# Pseudo-spin dynamics

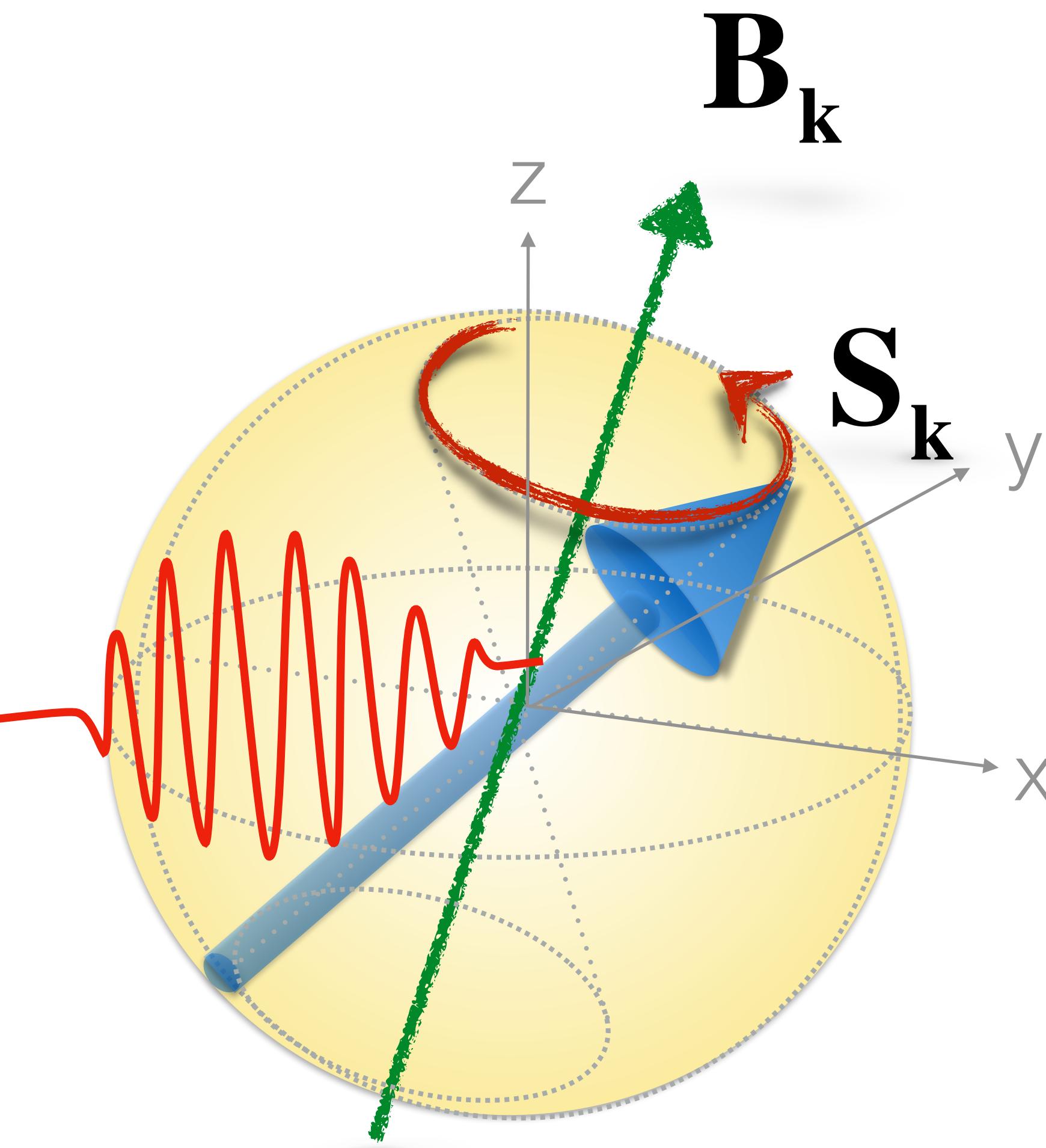


Equations of motion:

$$\frac{d}{dt} \mathbf{S}_k = i \left\langle [H_{\text{BCS}}, \hat{\mathbf{S}}_k] \right\rangle = -\mathbf{B}_k \times \mathbf{S}_k$$

$$\Delta(t) = -U \sum_{\mathbf{k}} S_{\mathbf{k}}^-$$

# Pseudo-spin dynamics

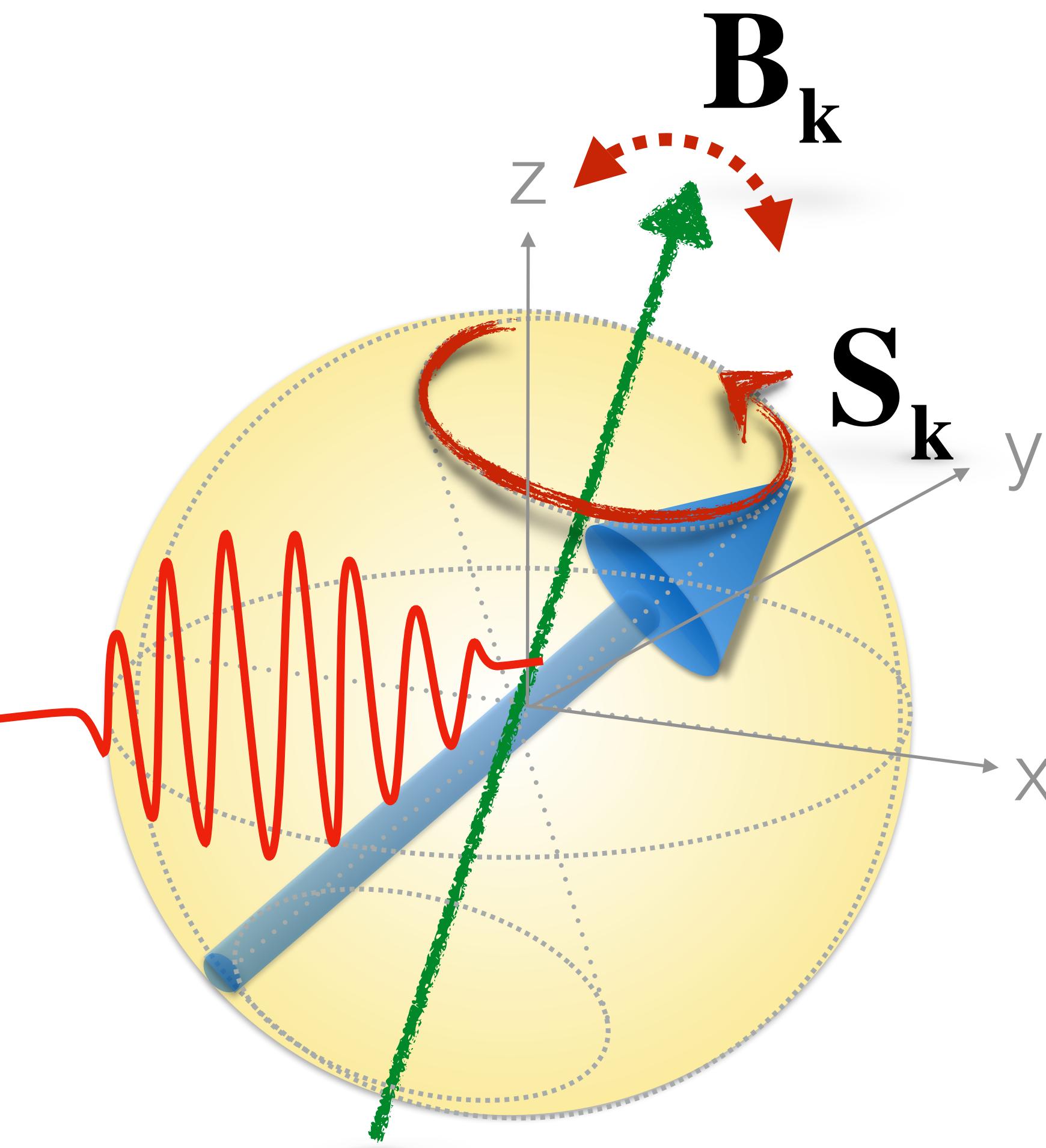


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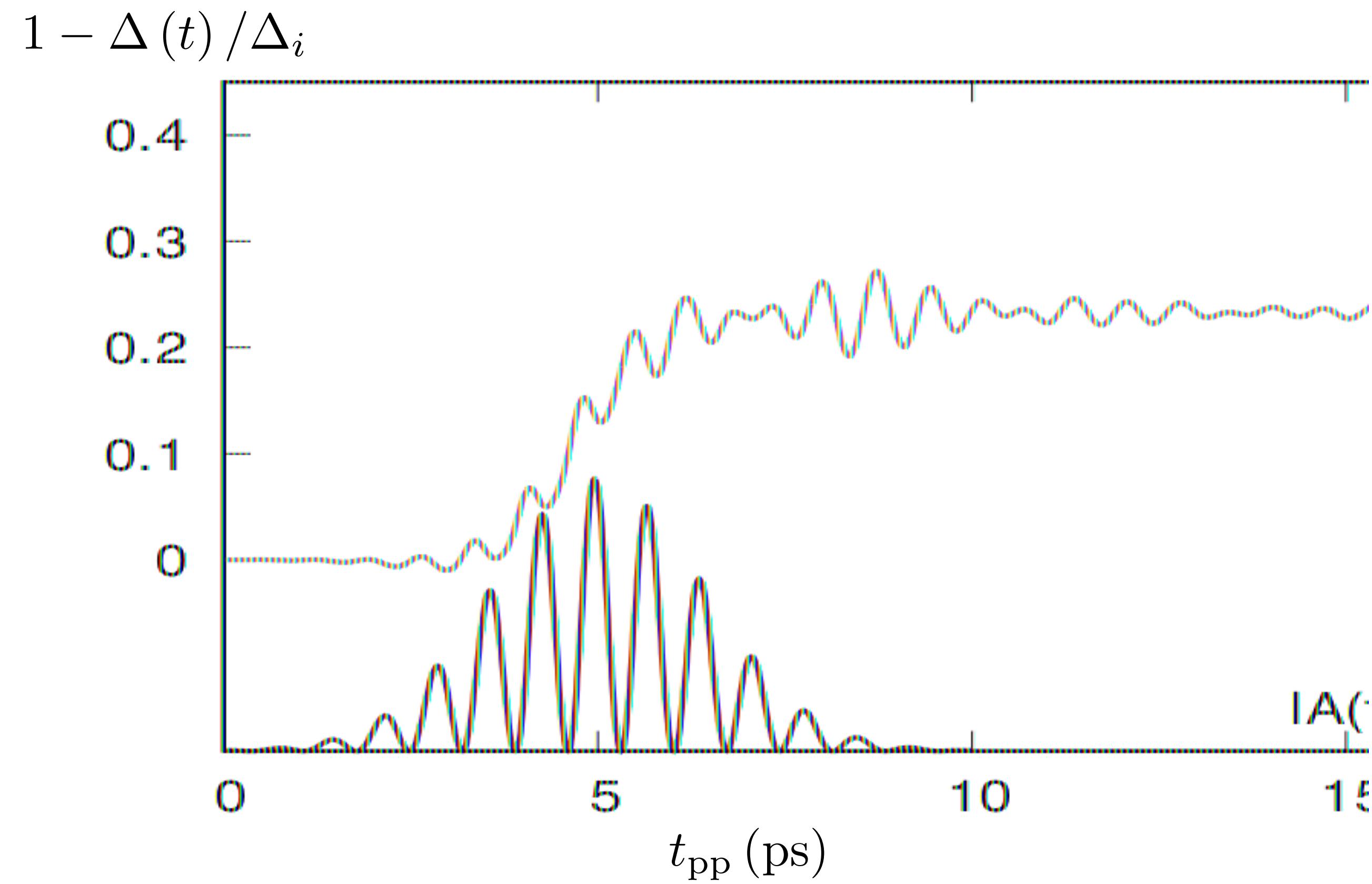


Equations of motion:

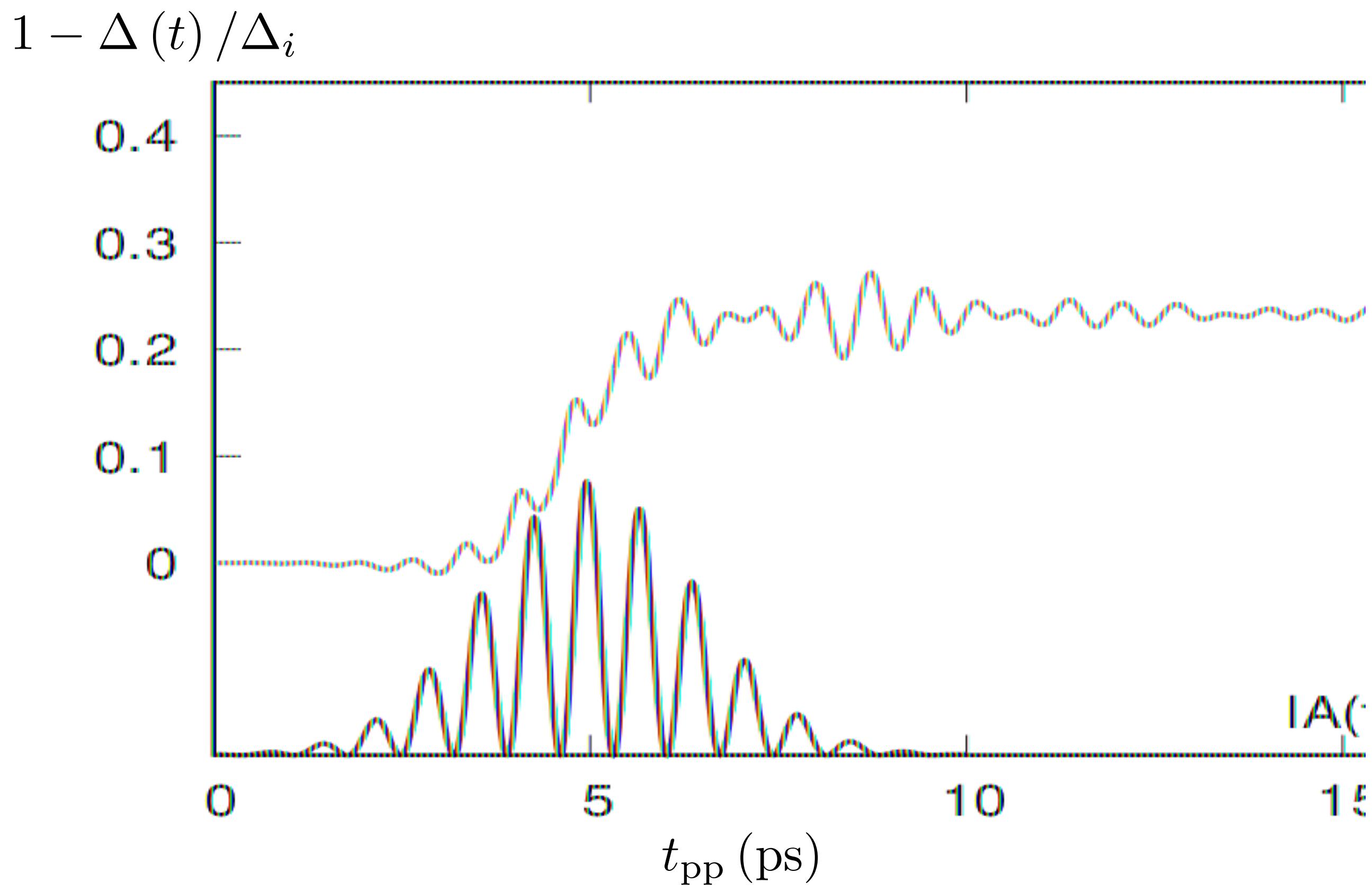
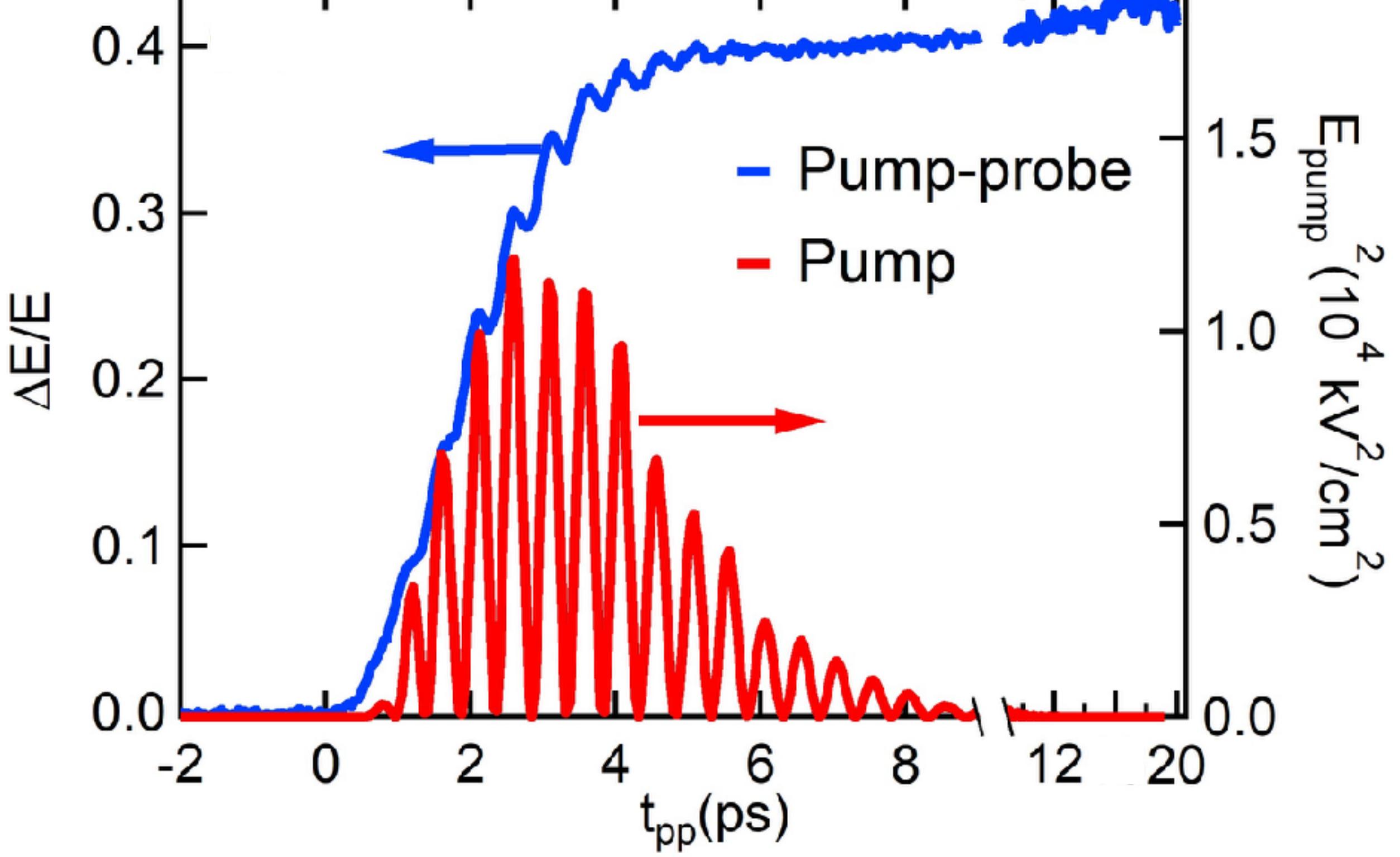
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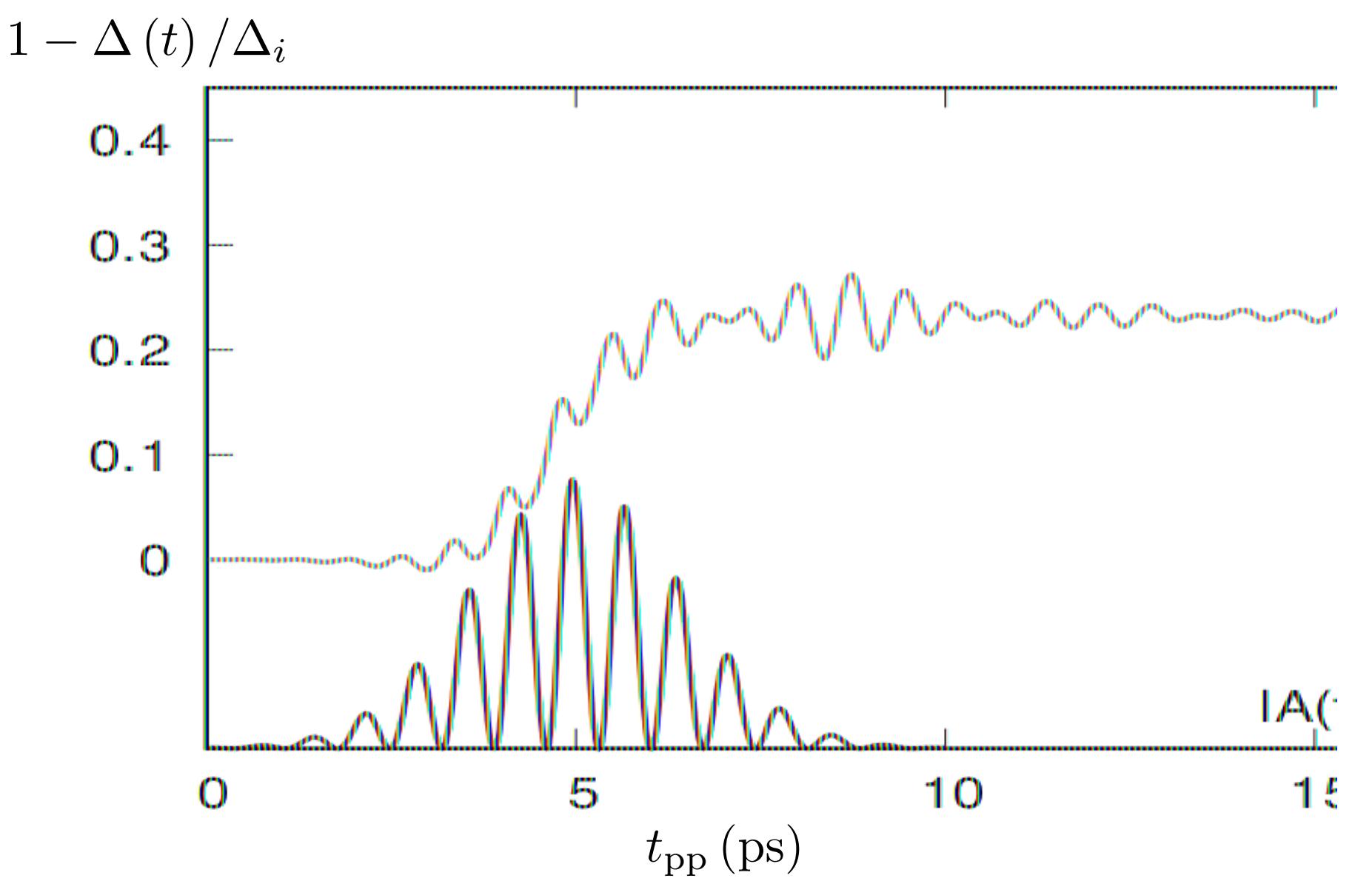
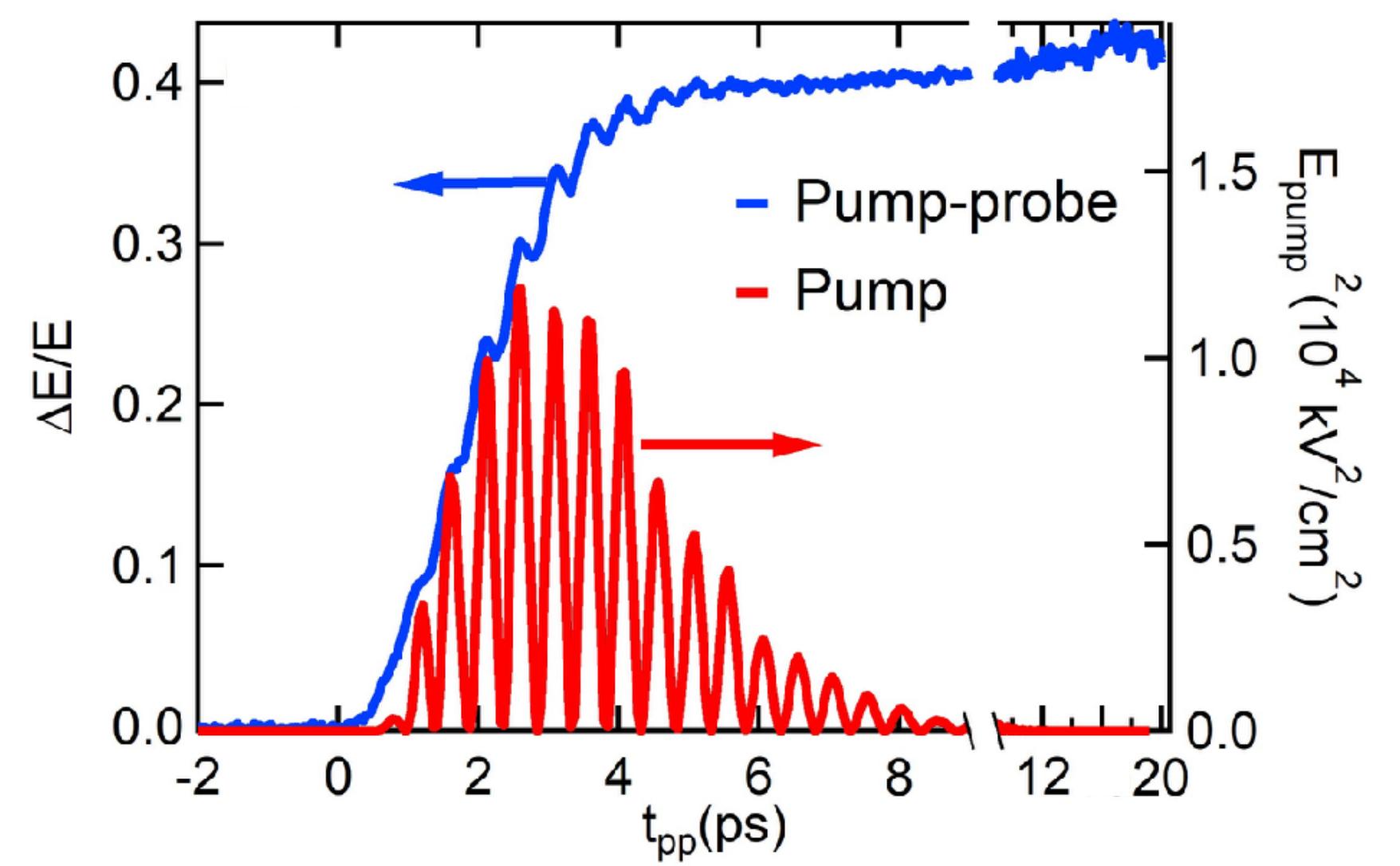
# Results



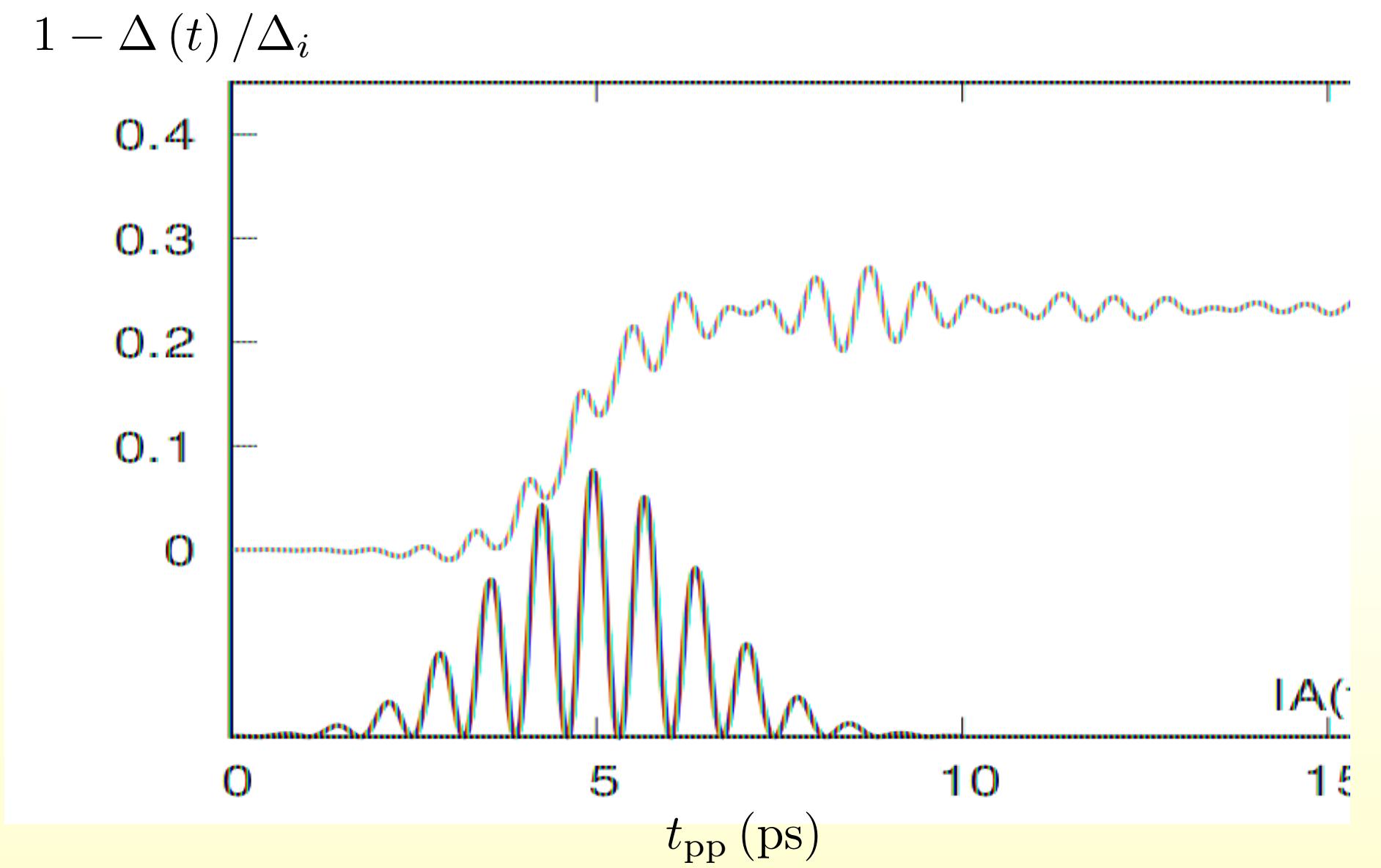
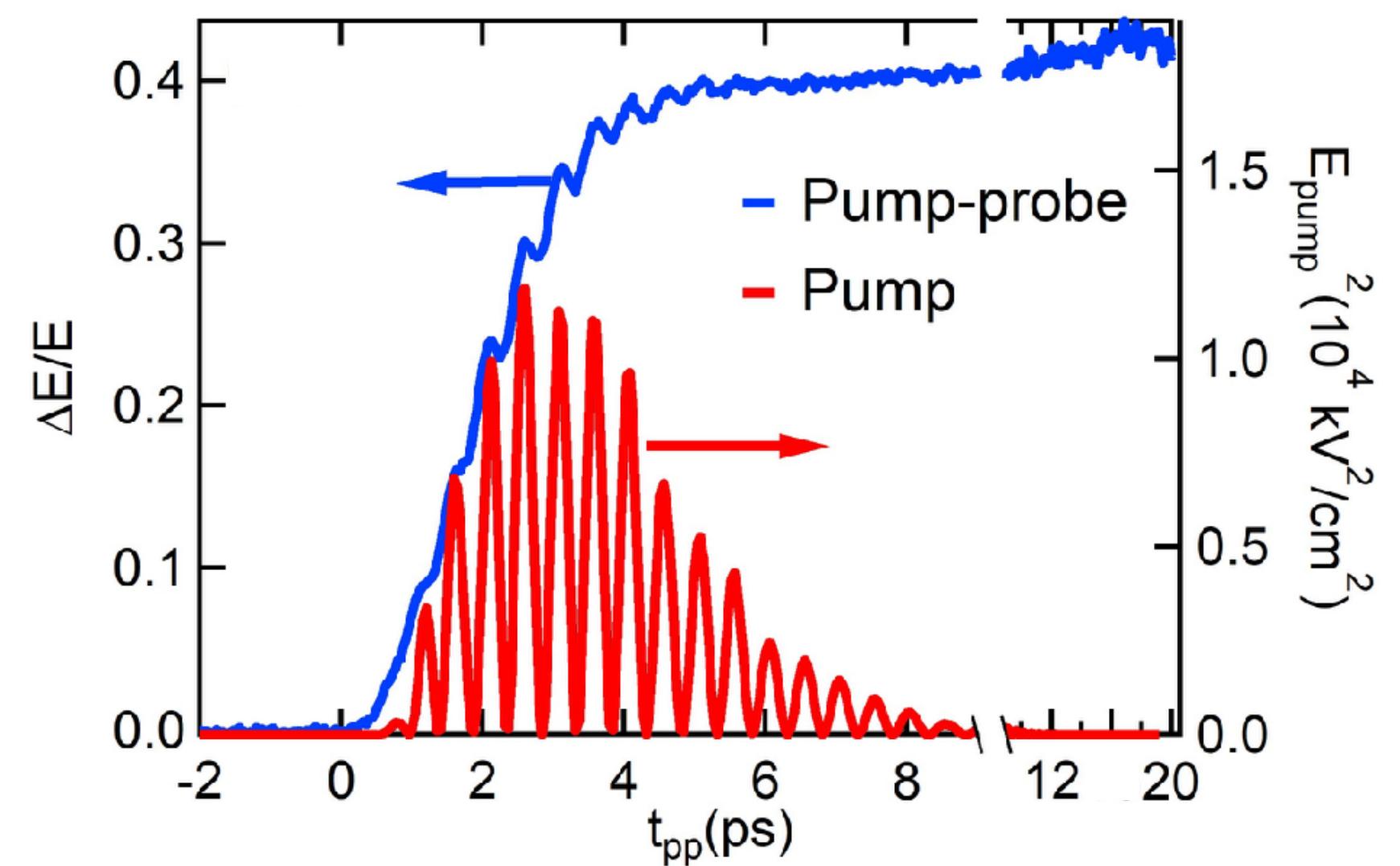
# Results



# Reality vs BCS

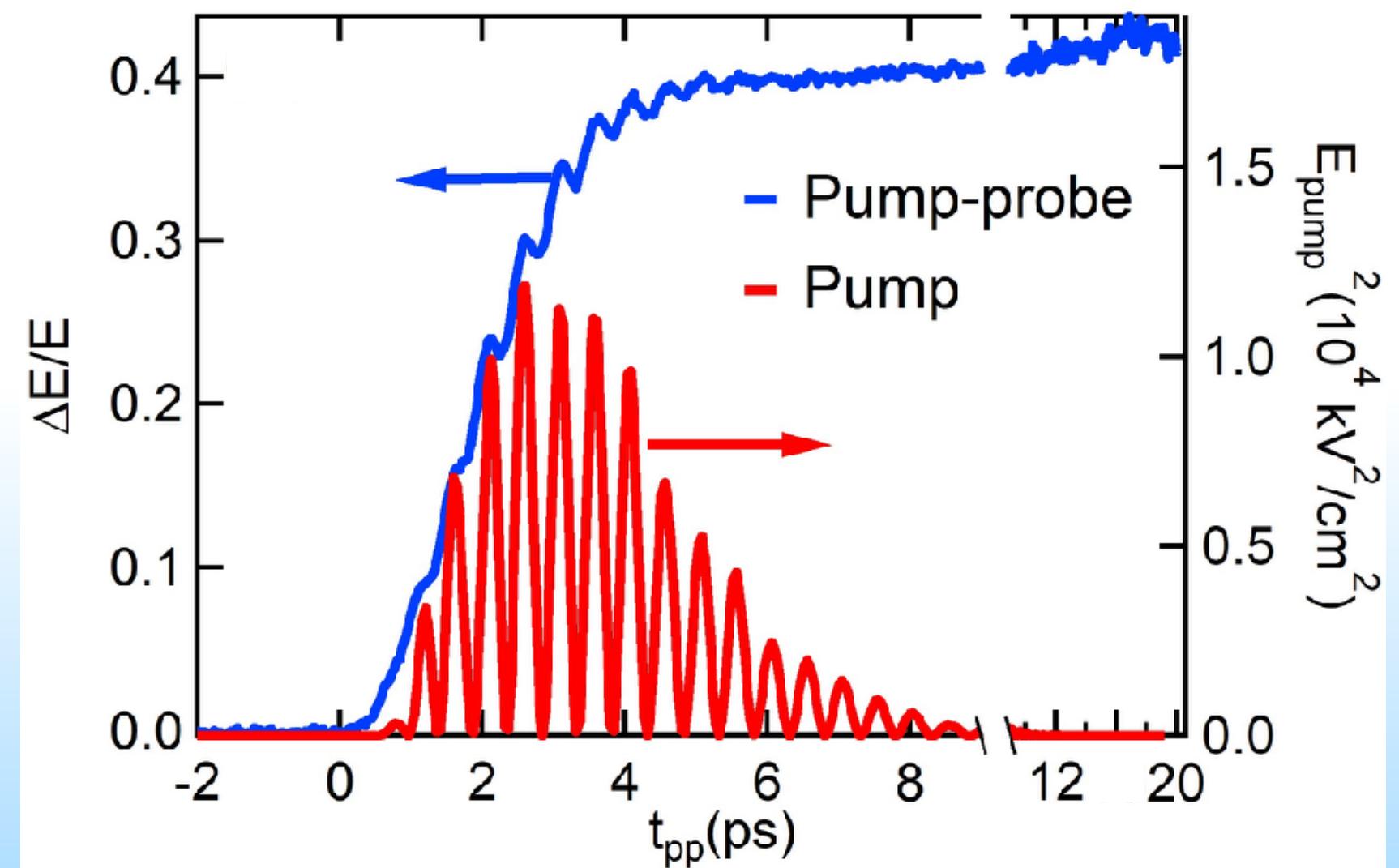


# Reality vs BCS

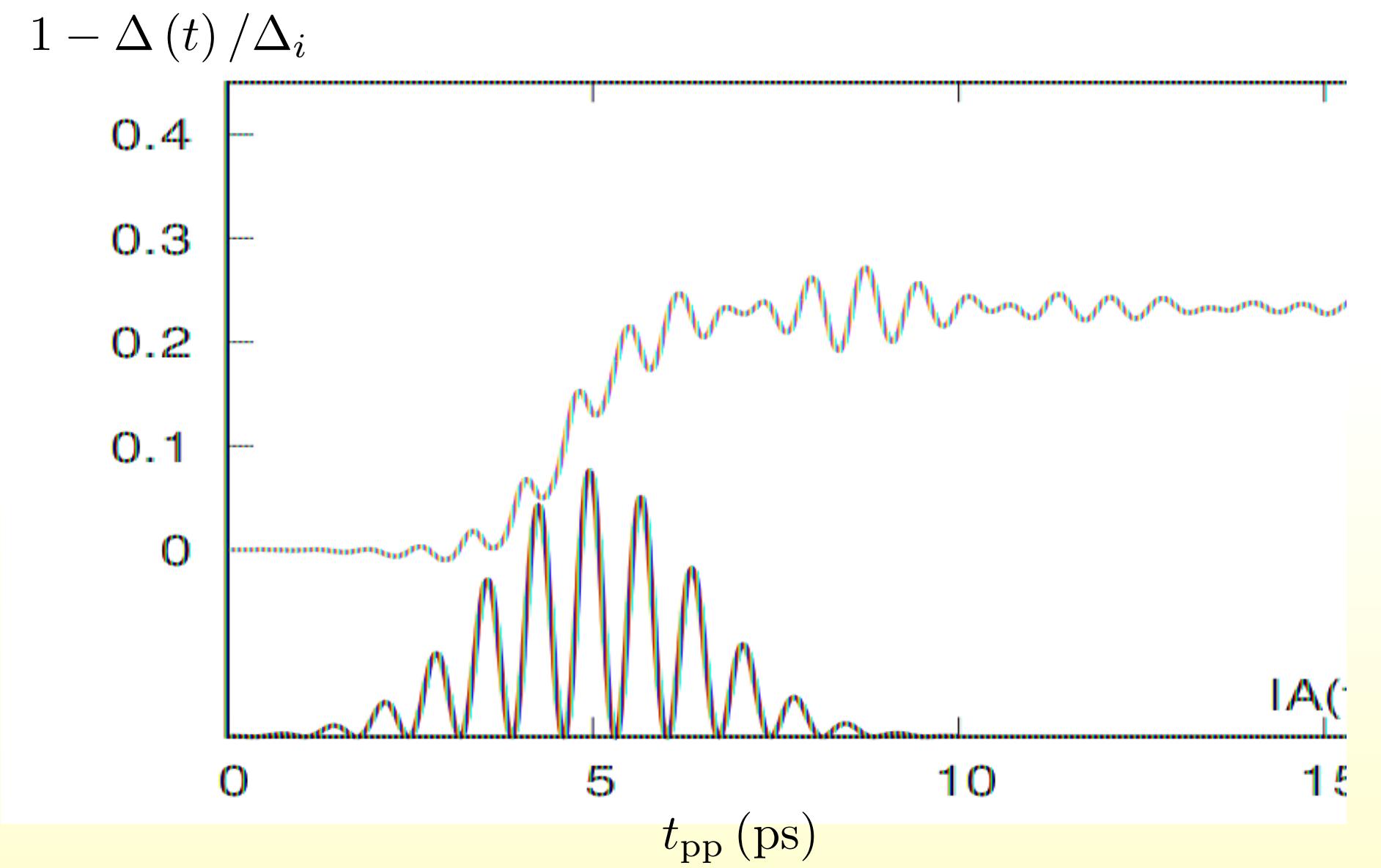


- Integrable
- Collisionless

# Reality vs BCS

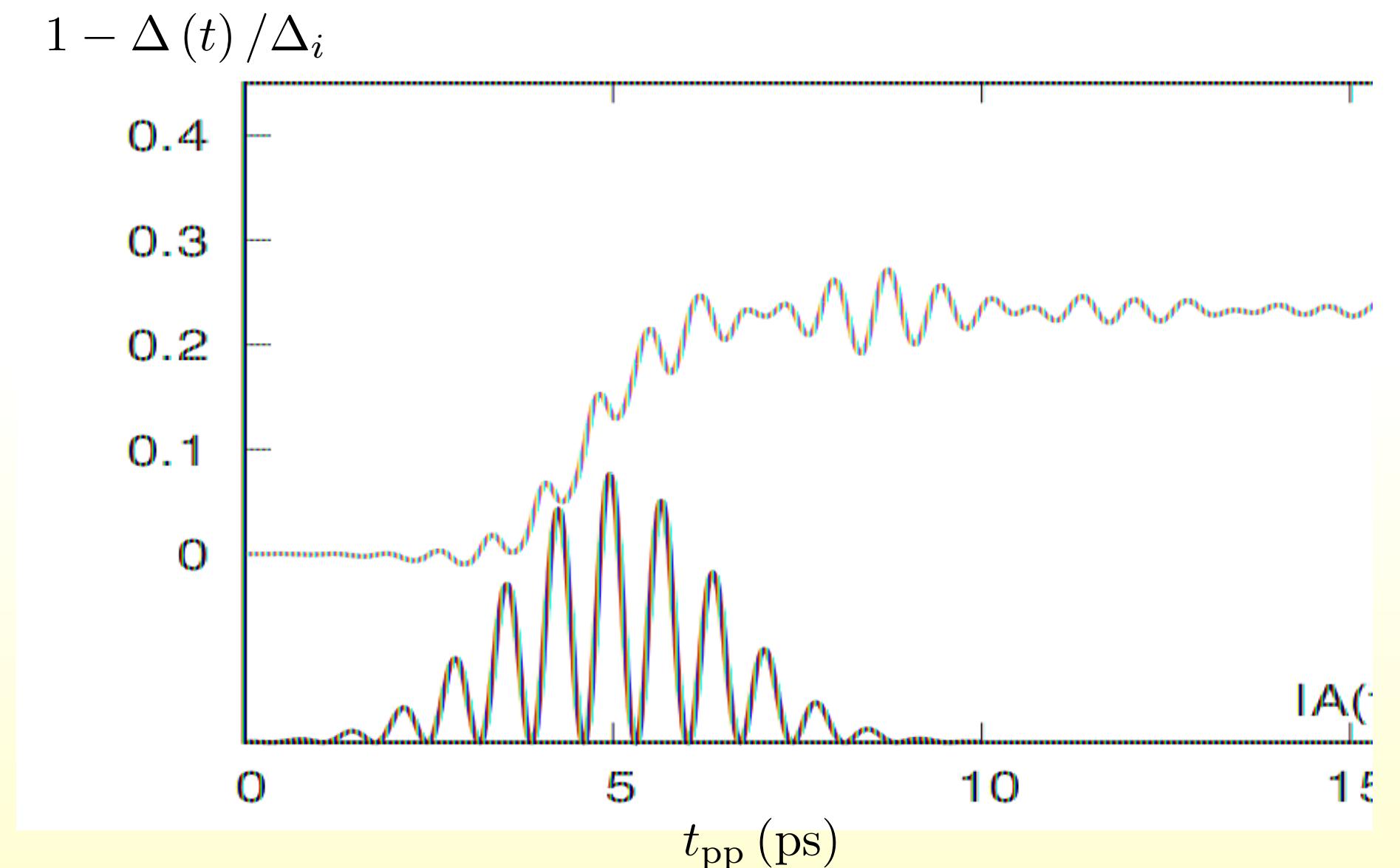
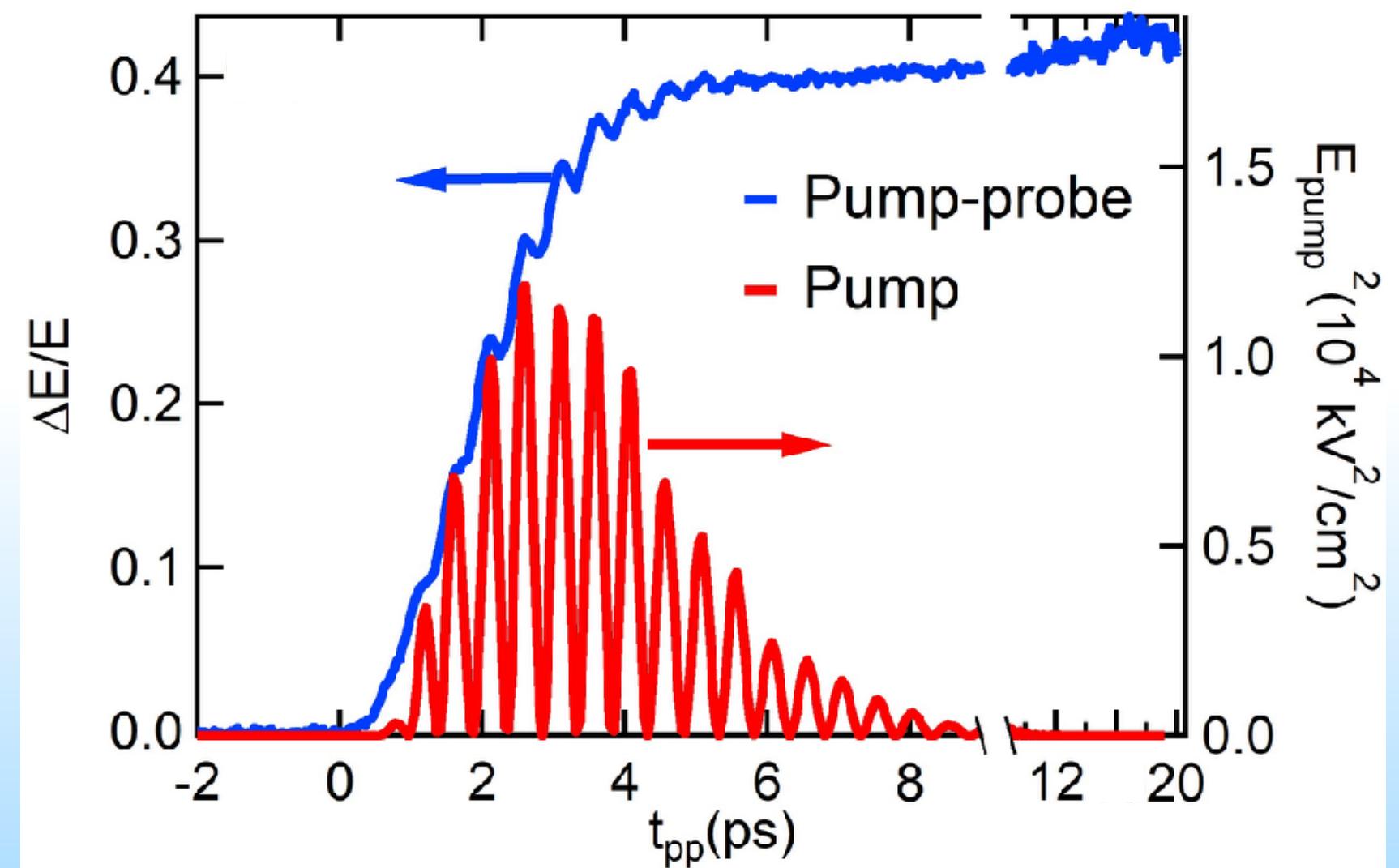


- Non-integrable
- Collision



- Integrable
- Collisionless

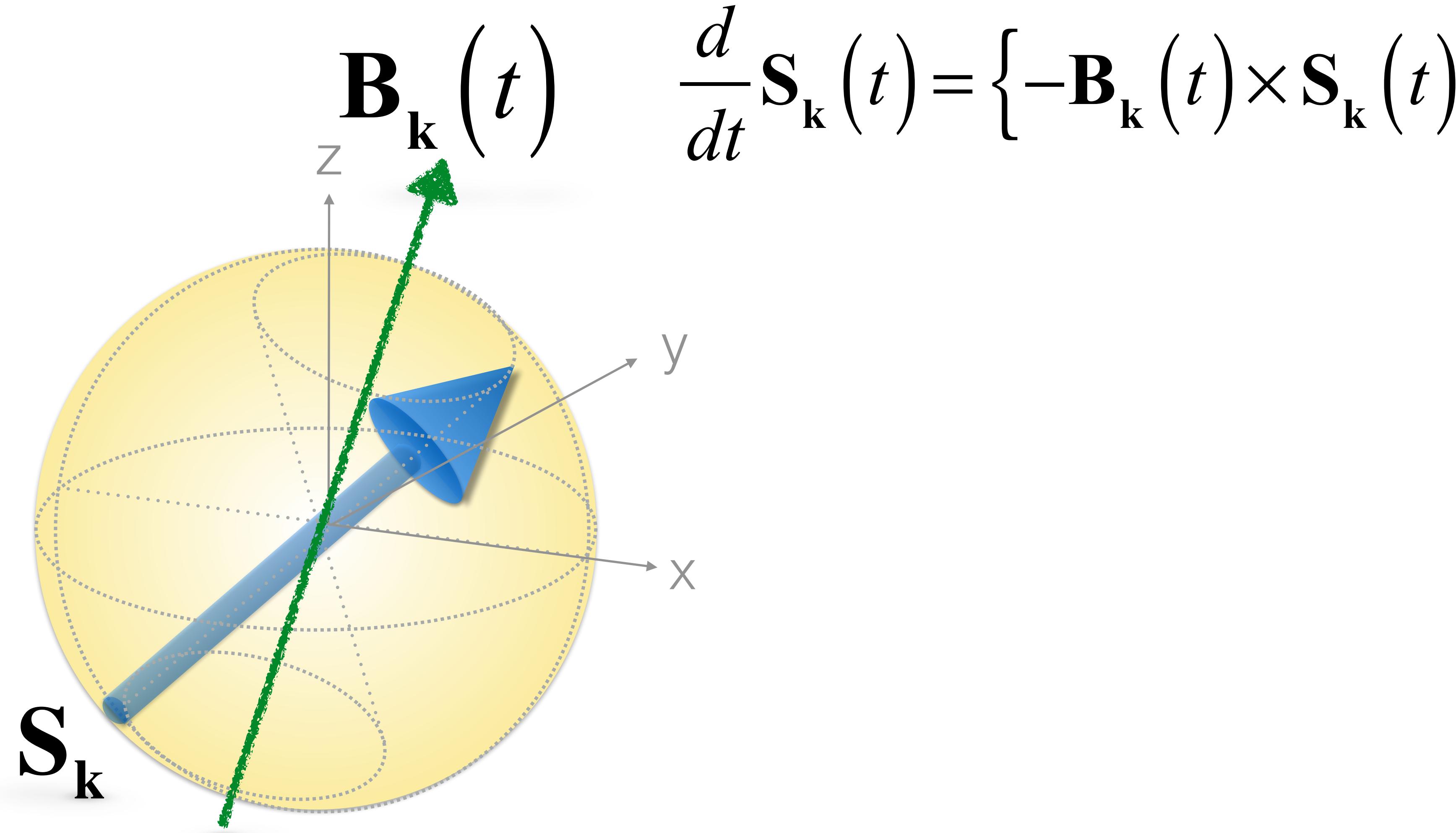
# Reality vs BCS



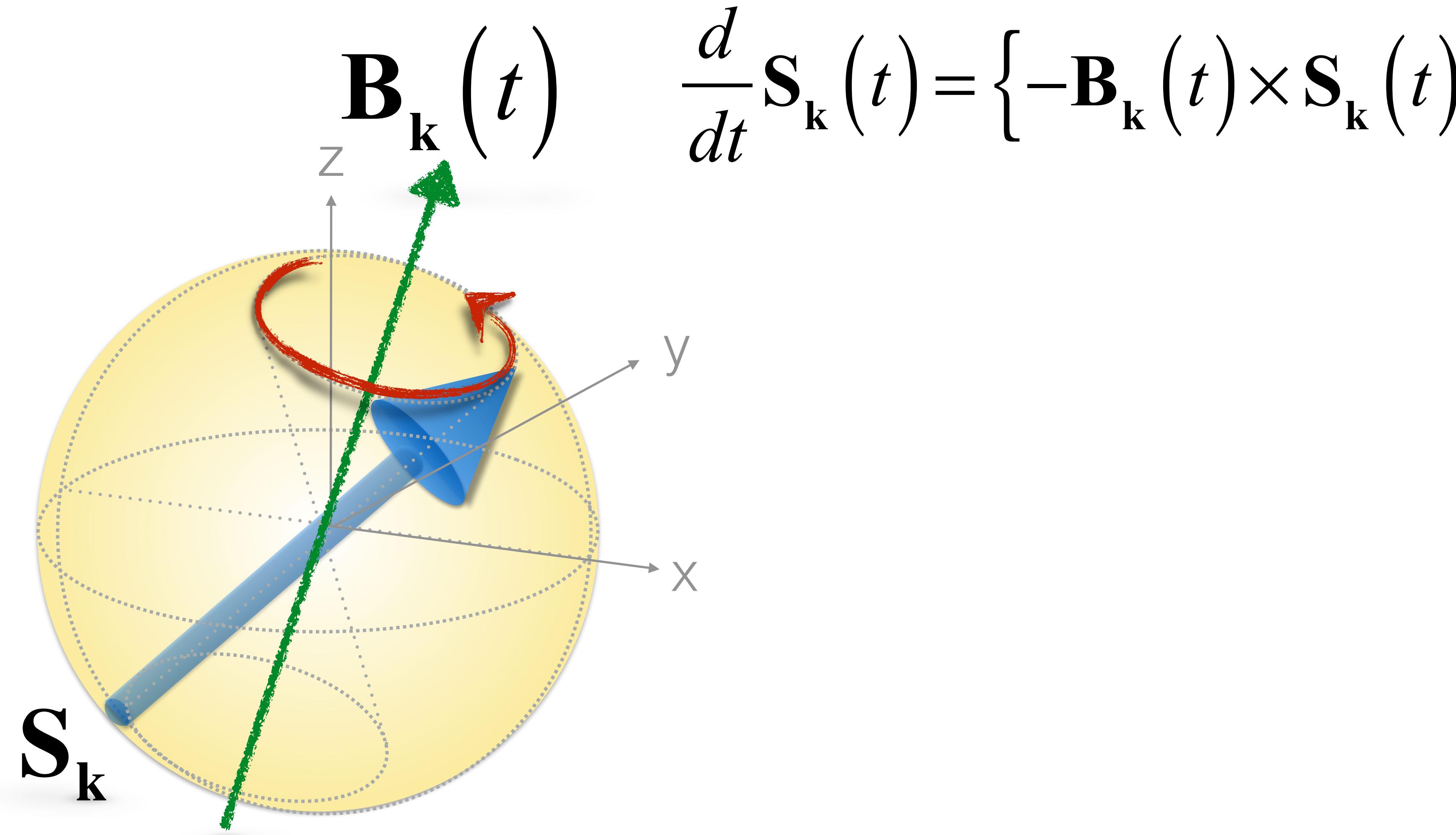
- Non-integrable
- Collision

- Integrable
- Collisionless

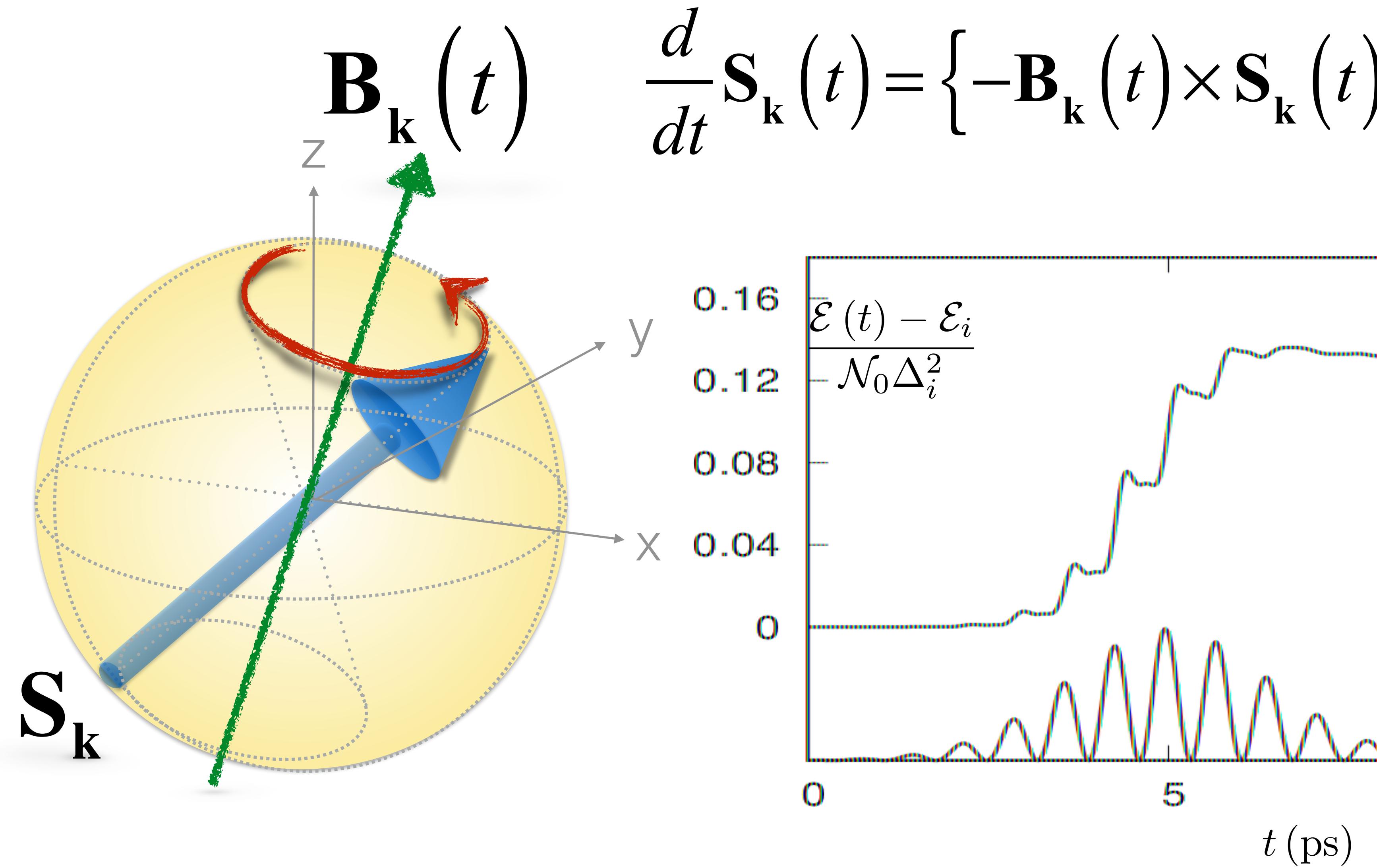
# Relaxation of the pseudo-spins



# Relaxation of the pseudo-spins

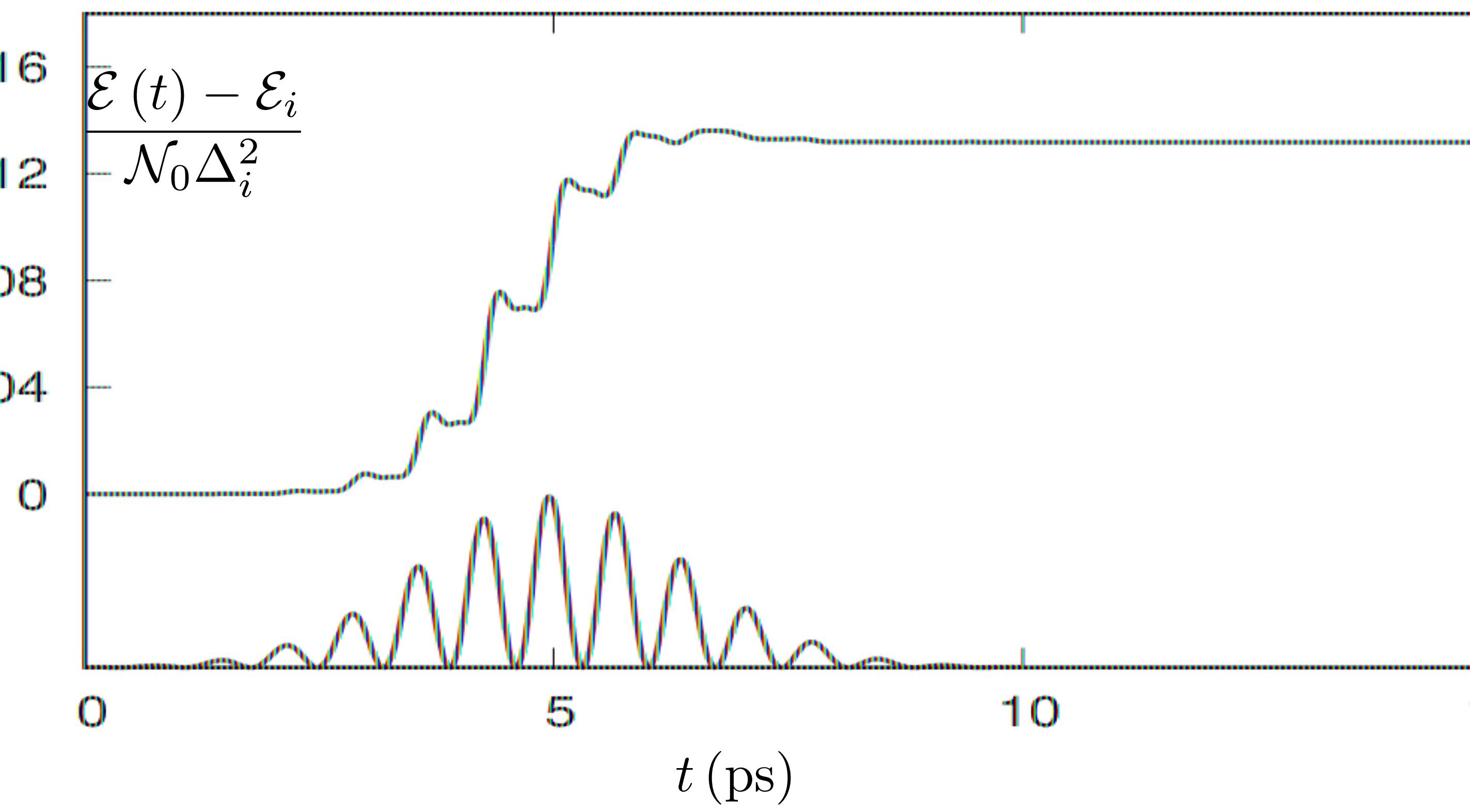
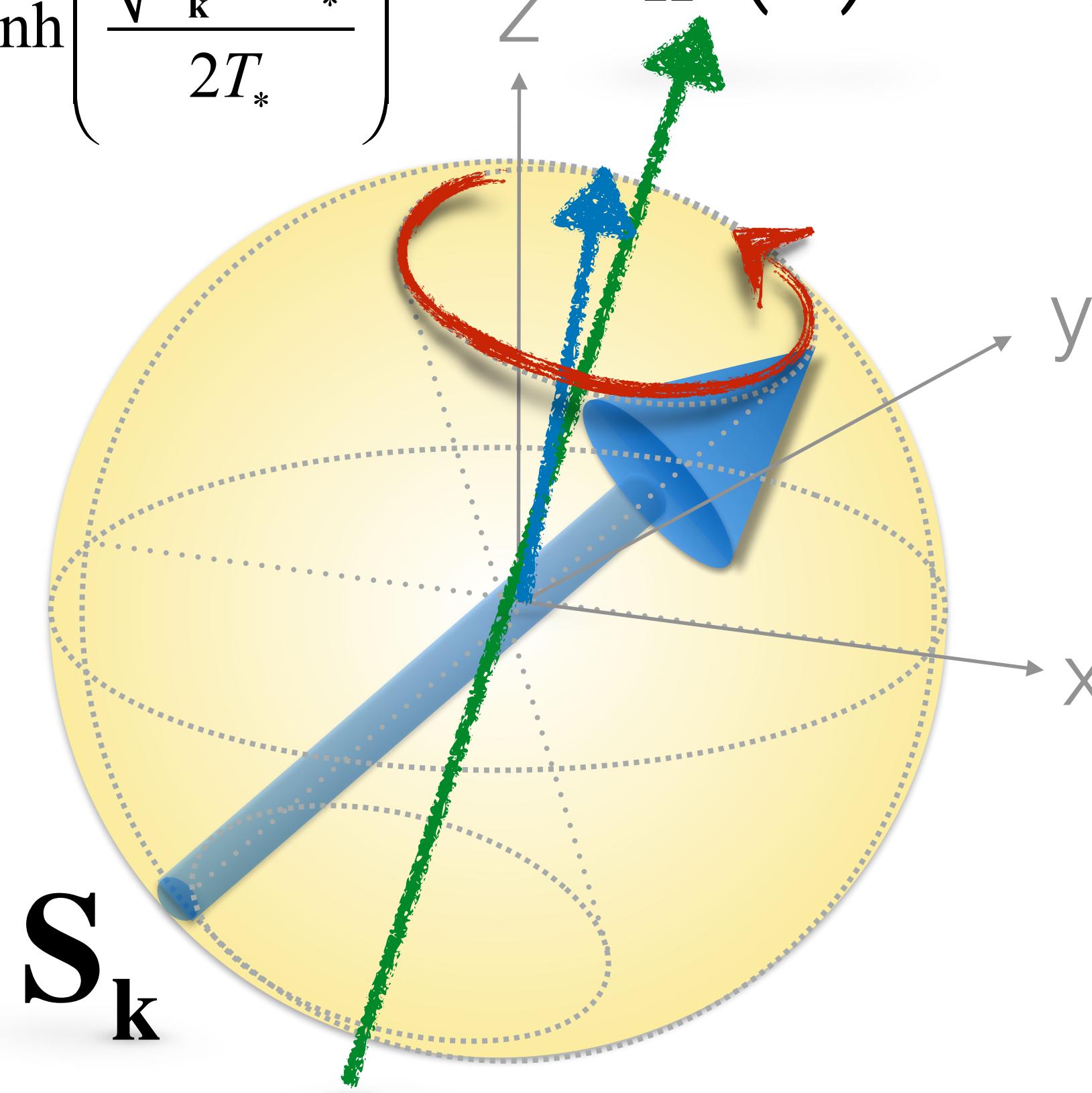


# Relaxation of the pseudo-spins



# Relaxation of the pseudo-spins

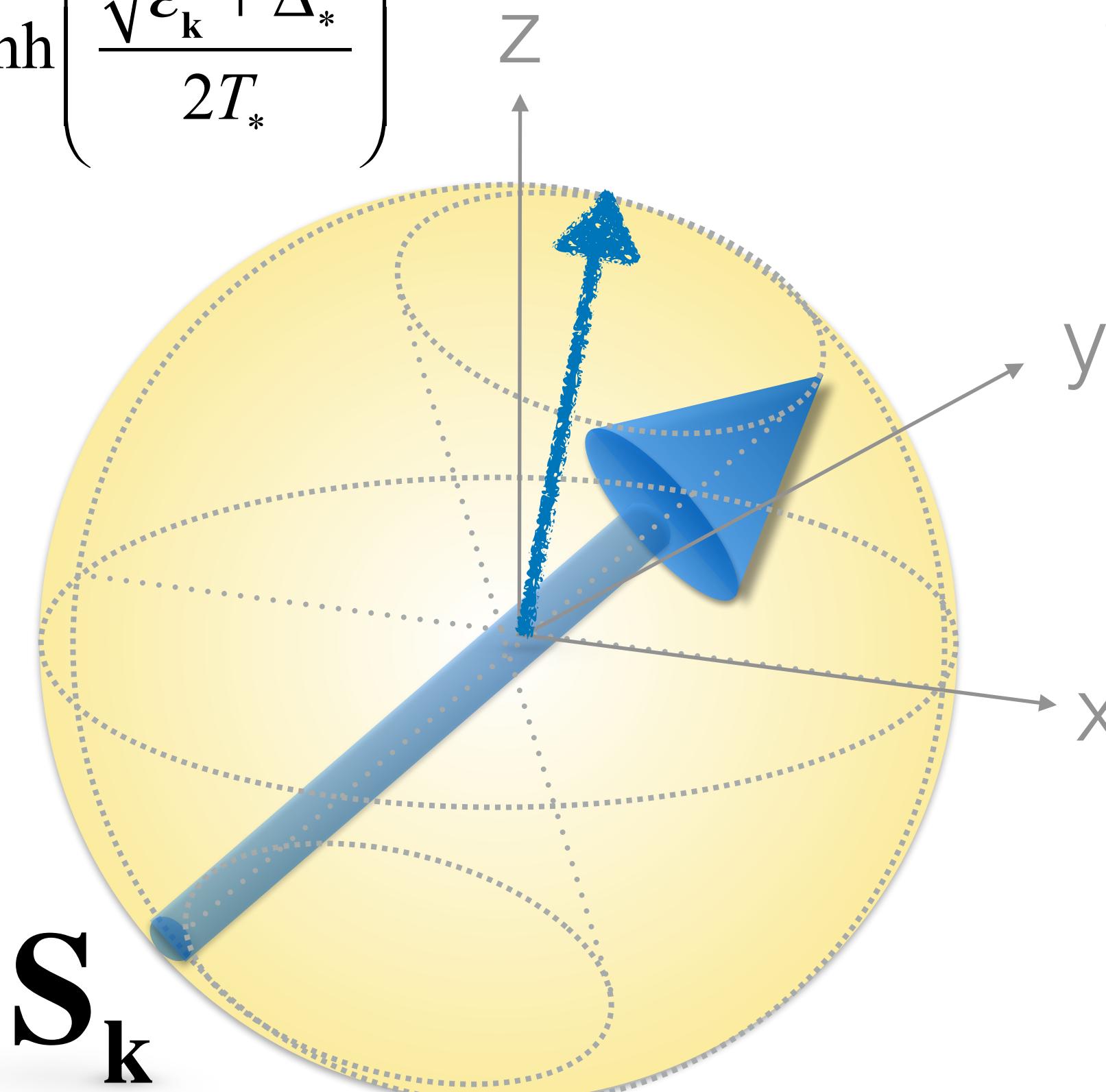
$$\mathbf{S}_k^* = \frac{\hat{s}_k^*}{2} \tanh\left(\frac{\sqrt{\varepsilon_k^2 + \Delta_*^2}}{2T_*}\right)$$
$$\mathbf{B}_k(t) \quad \frac{d}{dt} \mathbf{S}_k(t) = \{-\mathbf{B}_k(t) \times \mathbf{S}_k(t)\}$$



# Relaxation of the pseudo-spins

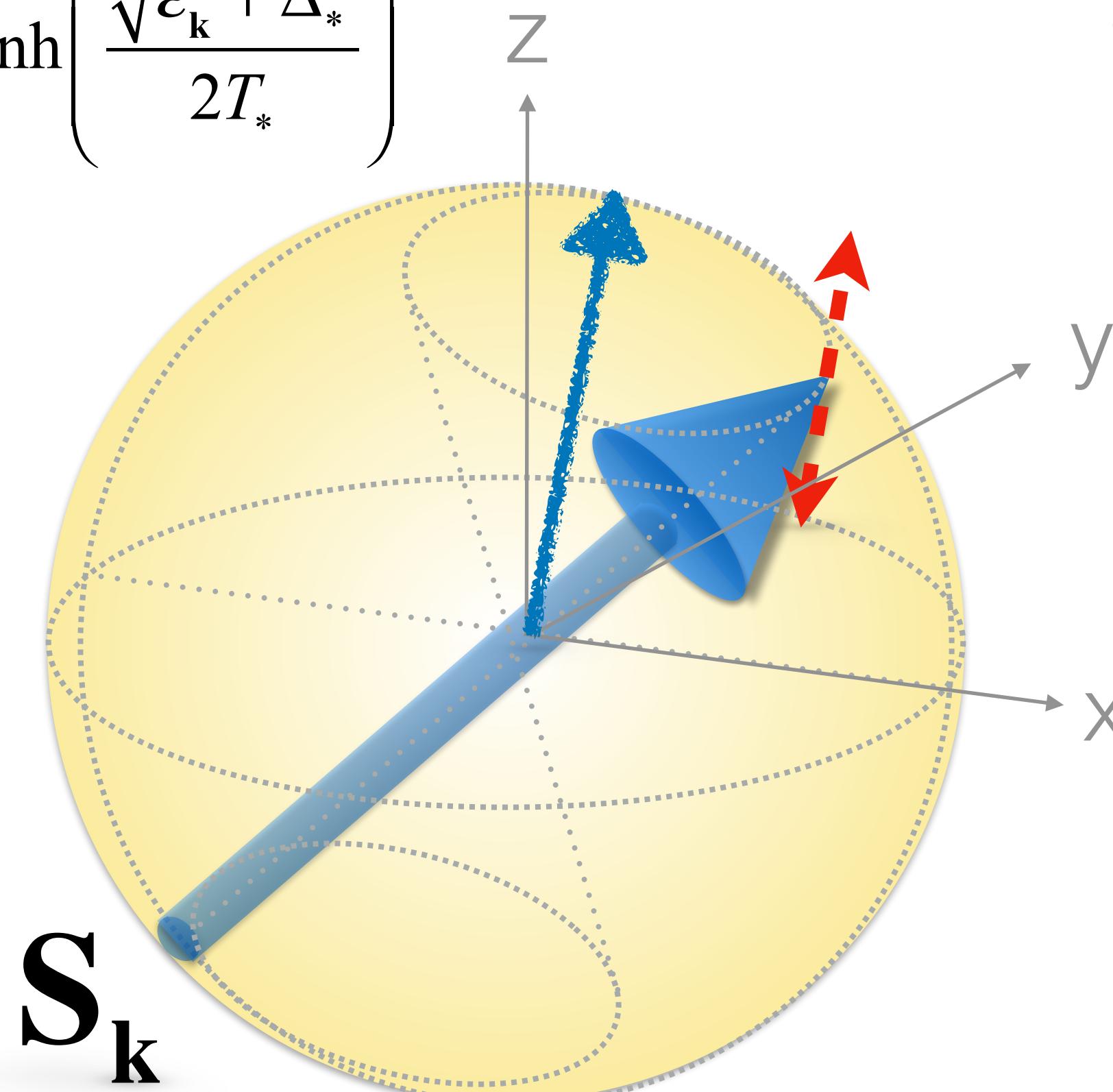
$$\mathbf{S}_k^* = \frac{\hat{s}_k^*}{2} \tanh\left(\frac{\sqrt{\varepsilon_k^2 + \Delta_*^2}}{2T_*}\right)$$

$$\frac{d}{dt} \mathbf{S}_k(t) = \{-\mathbf{B}_k(t) \times \mathbf{S}_k(t)\}$$



# Relaxation of the pseudo-spins

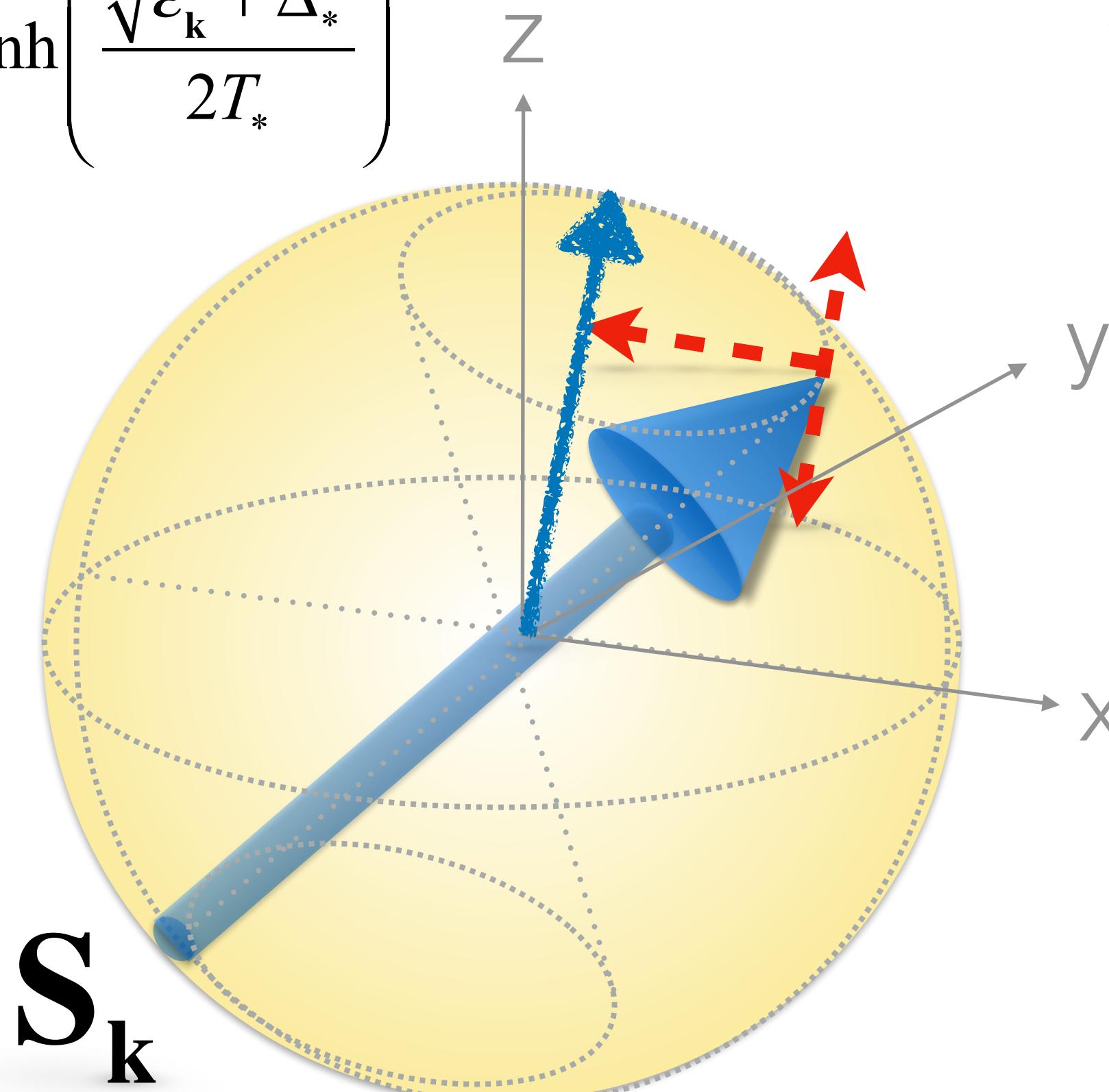
$$\mathbf{S}_k^* = \frac{\hat{s}_k^*}{2} \tanh\left(\frac{\sqrt{\varepsilon_k^2 + \Delta_*^2}}{2T_*}\right)$$



$$\frac{d}{dt} \mathbf{S}_k(t) = \left\{ -\mathbf{B}_k(t) \times \mathbf{S}_k(t) \right. \\ \left. - \frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\parallel} - |\mathbf{S}_k^*|}{T_1} \hat{s}_k^{*\parallel} \right\}$$

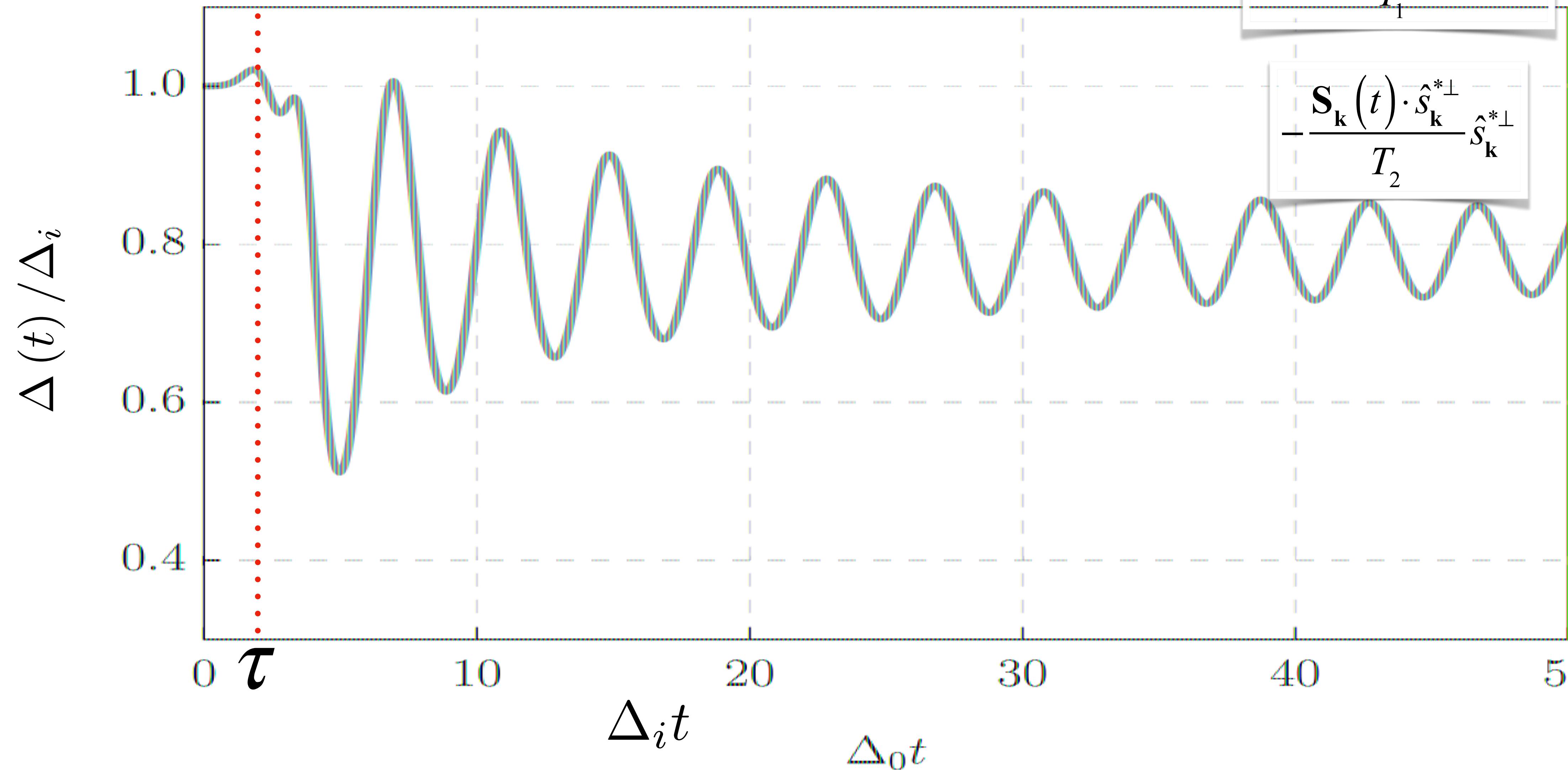
# Relaxation of the pseudo-spins

$$S_k^* = \frac{\hat{s}_k^*}{2} \tanh\left(\frac{\sqrt{\varepsilon_k^2 + \Delta_*^2}}{2T_*}\right)$$

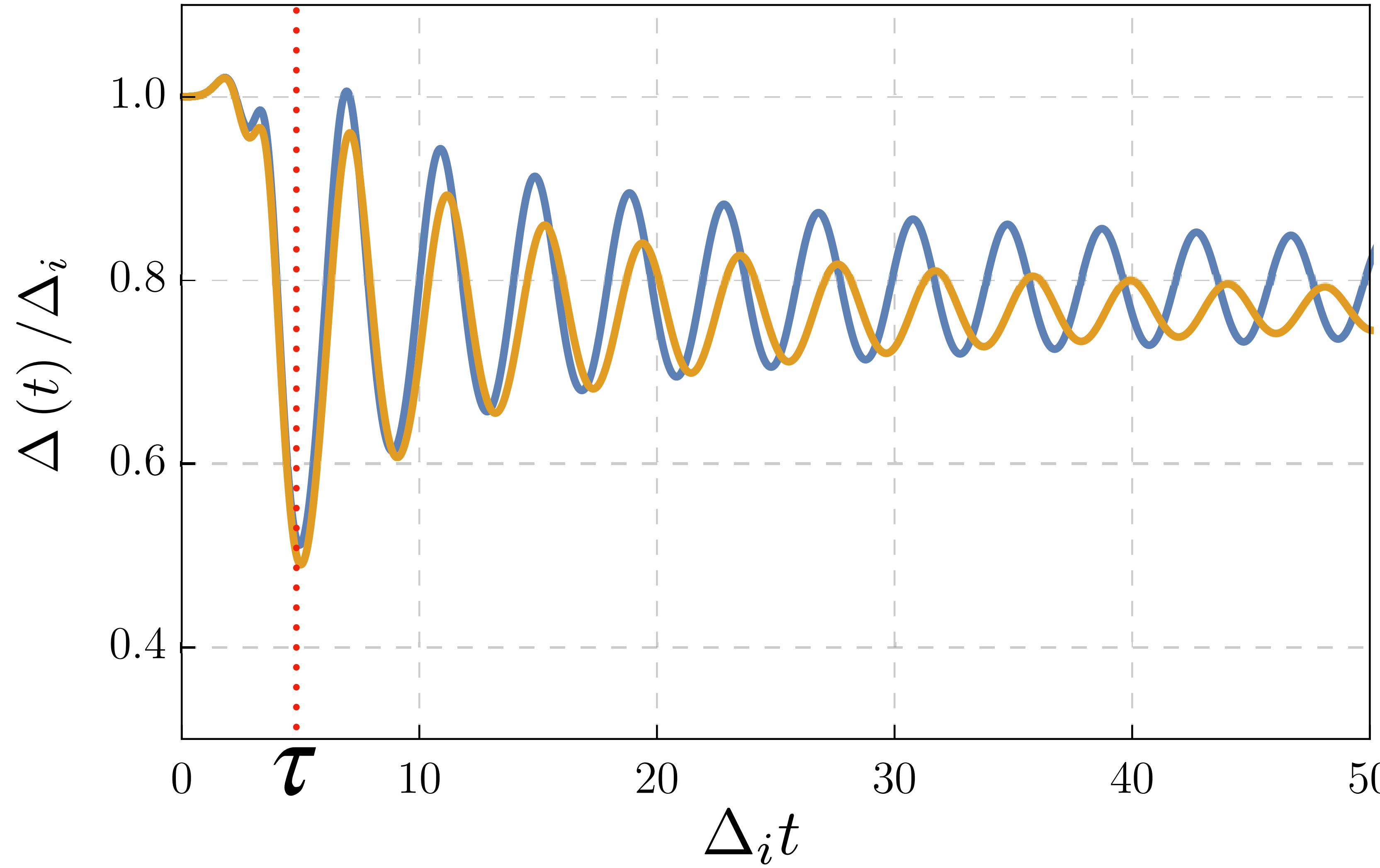


$$\begin{aligned} \frac{d}{dt} S_k(t) = & \left\{ -B_k(t) \times S_k(t) \right. \\ & - \frac{S_k(t) \cdot \hat{s}_k^{*\parallel} - |S_k^*|}{T_1} \hat{s}_k^{*\parallel} \\ & \left. - \frac{S_k(t) \cdot \hat{s}_k^{*\perp}}{T_2} \hat{s}_k^{*\perp} \right\} \end{aligned}$$

# Damped quench dynamics



# Damped quench dynamics

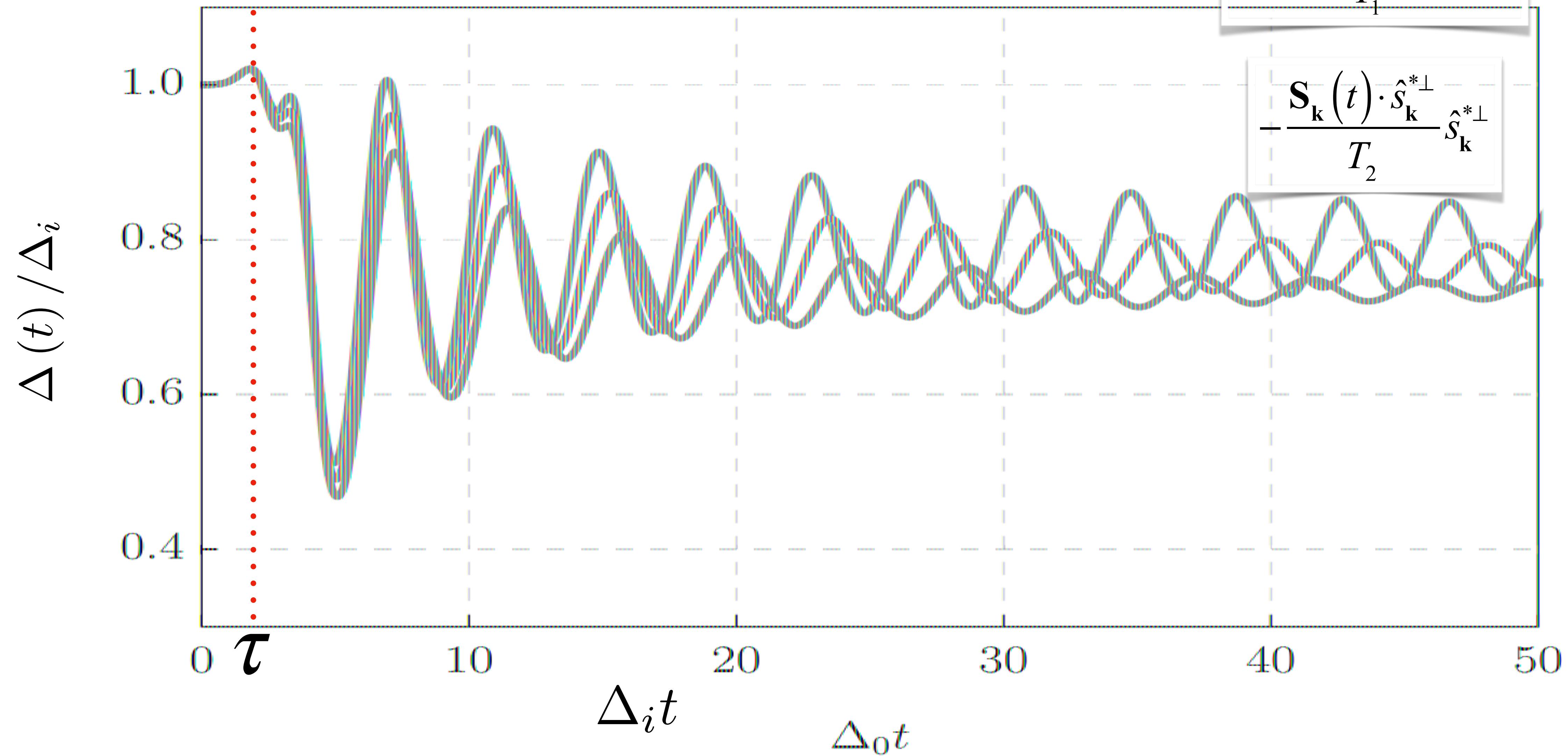


$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\parallel} - |\mathbf{S}_k^*|}{T_1} \hat{s}_k^{*\parallel}$$

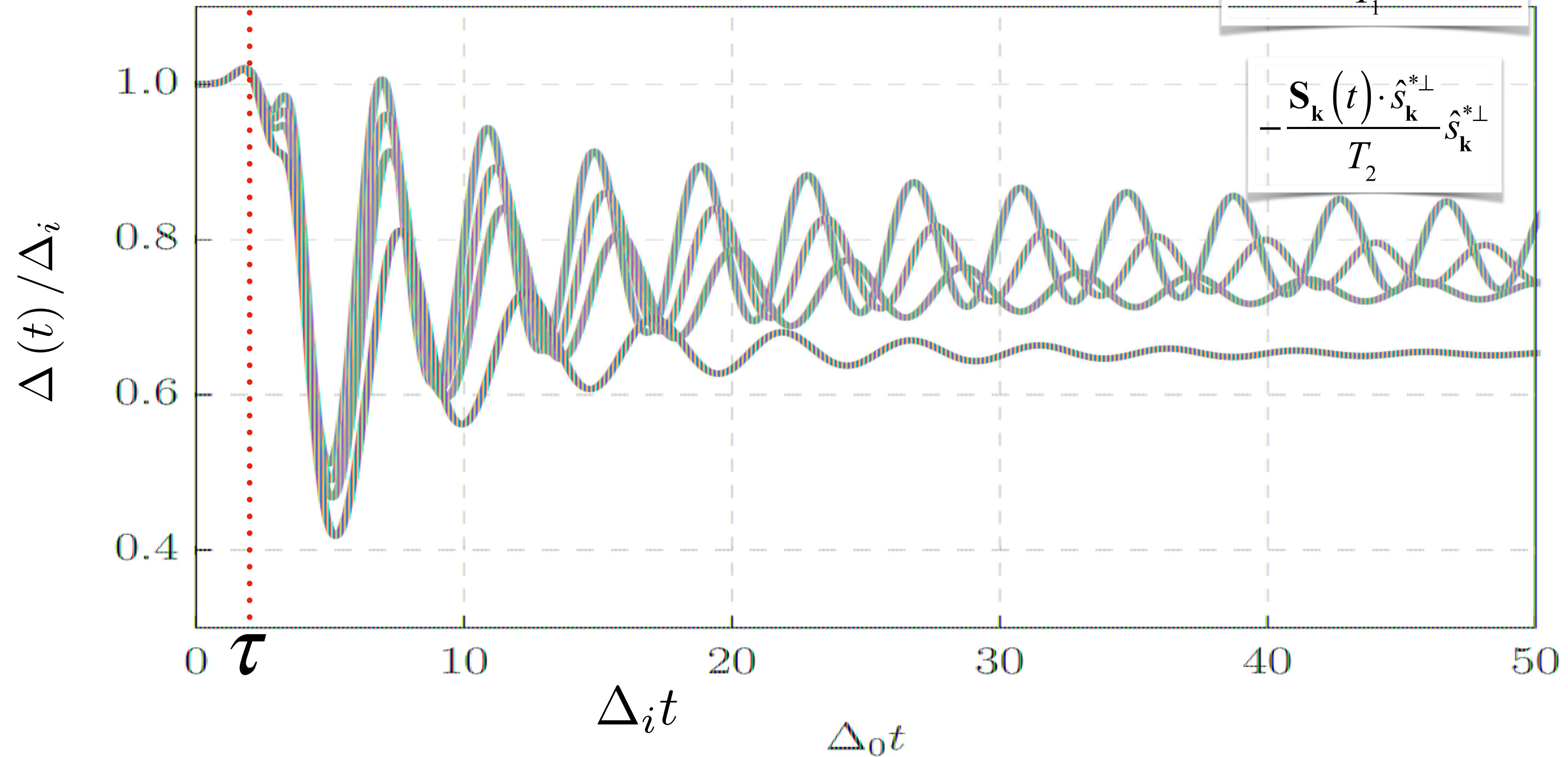
$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\perp}}{T_2} \hat{s}_k^{*\perp}$$

- $T_1 = T_2 = \infty$
- $T_1 = T_2 = 10\tau$

# Damped quench dynamics



# Damped quench dynamics

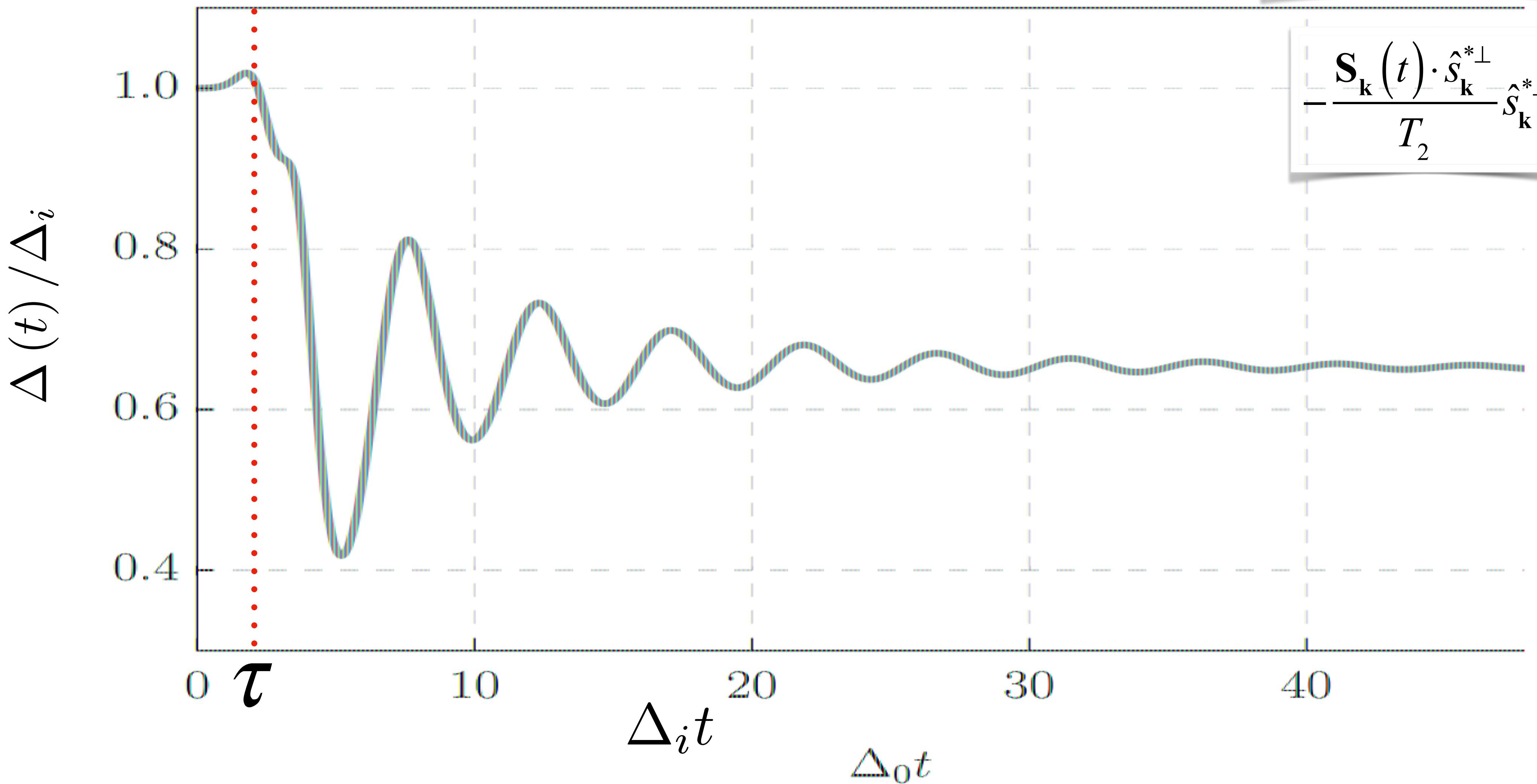


# Impact of T1 relaxation process

$$T_2 = 2.5\tau$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\parallel} - |\mathbf{S}_k^*|}{T_1} \hat{s}_k^{*\parallel}$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\perp}}{T_2} \hat{s}_k^{*\perp}$$

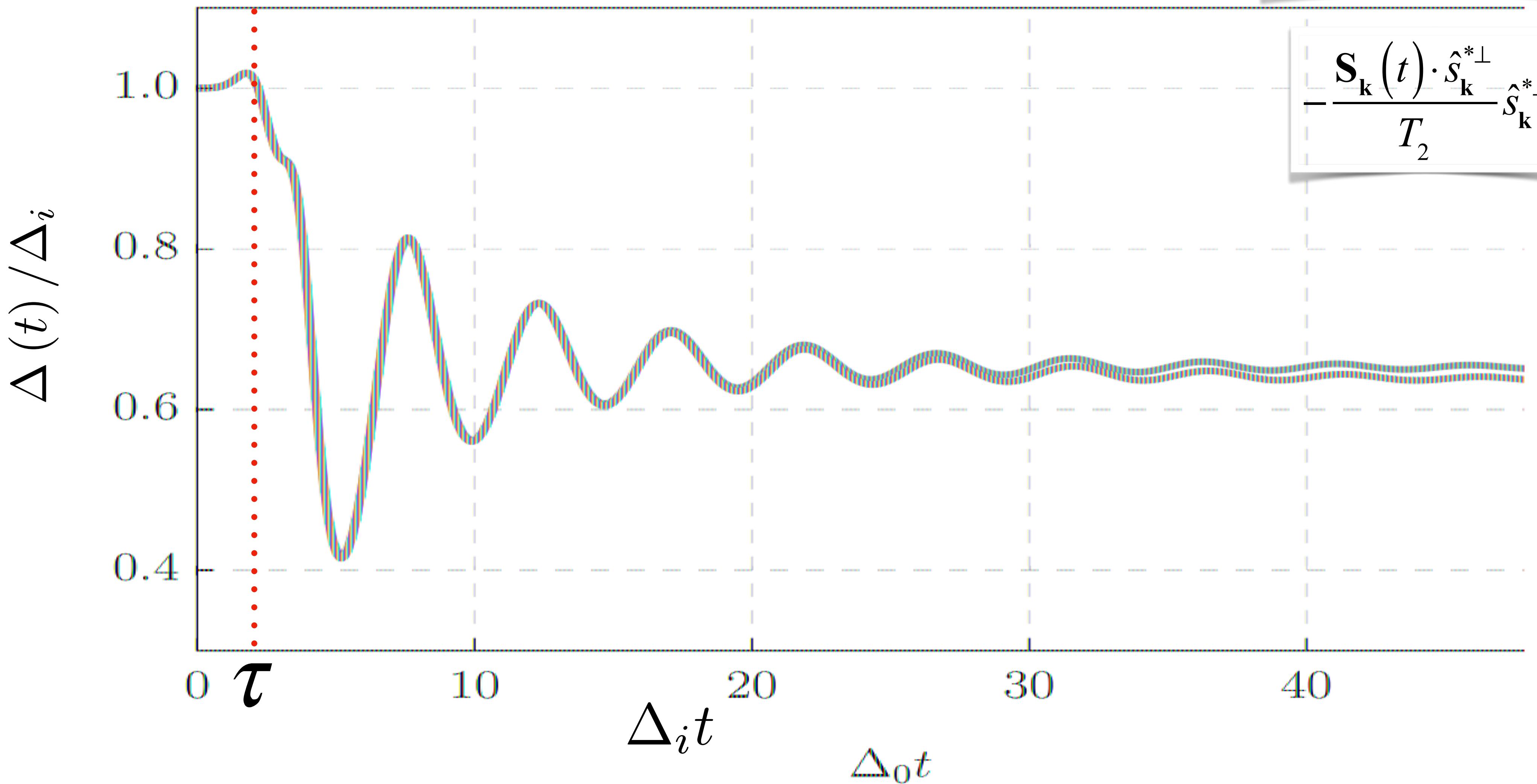


# Impact of T1 relaxation process

$$T_2 = 2.5\tau$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\parallel} - |\mathbf{S}_k^*|}{T_1} \hat{s}_k^{*\parallel}$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\perp}}{T_2} \hat{s}_k^{*\perp}$$

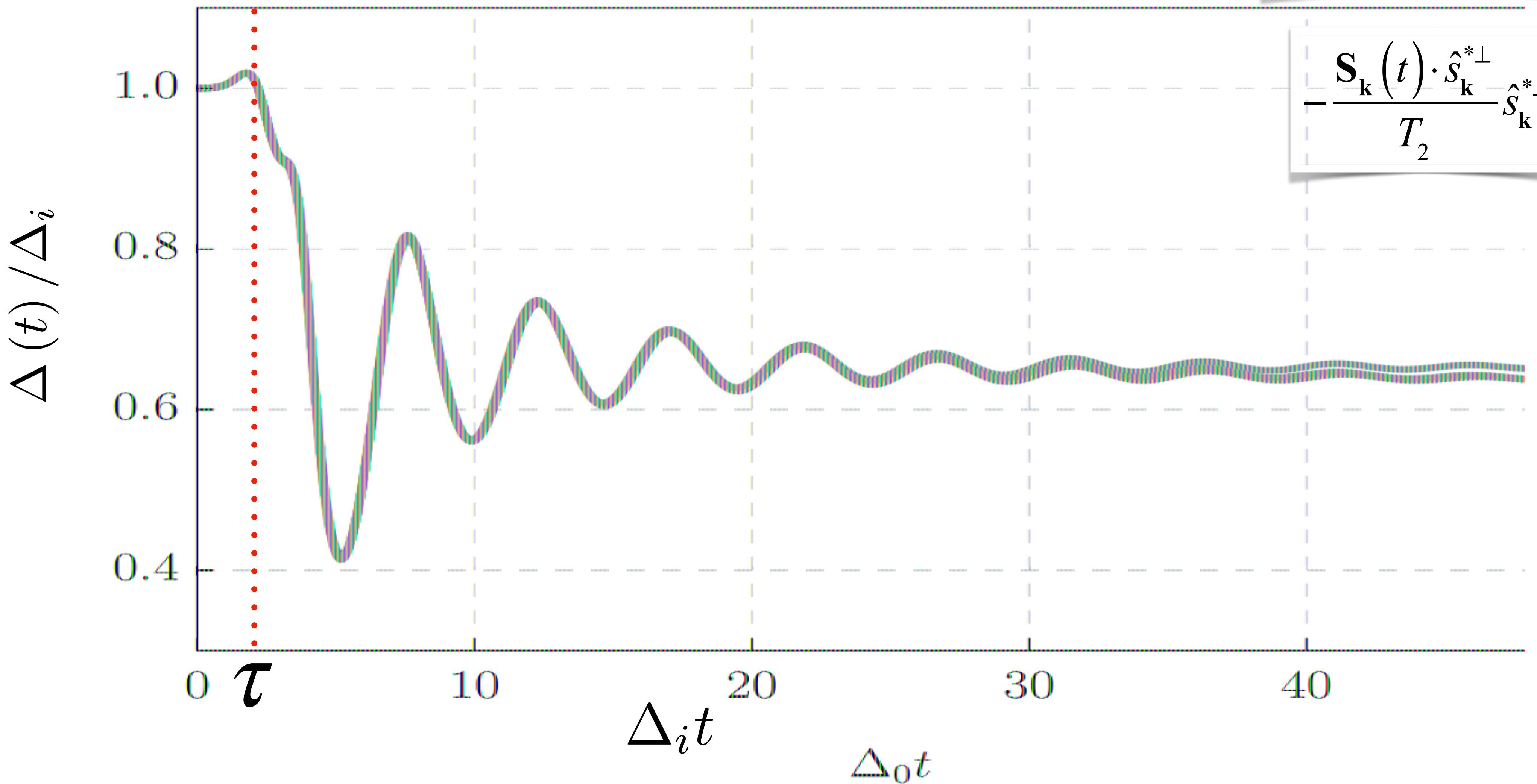


# Impact of T1 relaxation process

$$T_2 = 2.5\tau$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\parallel} - |\mathbf{S}_k^*|}{T_1} \hat{s}_k^{*\parallel}$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\perp}}{T_2} \hat{s}_k^{*\perp}$$

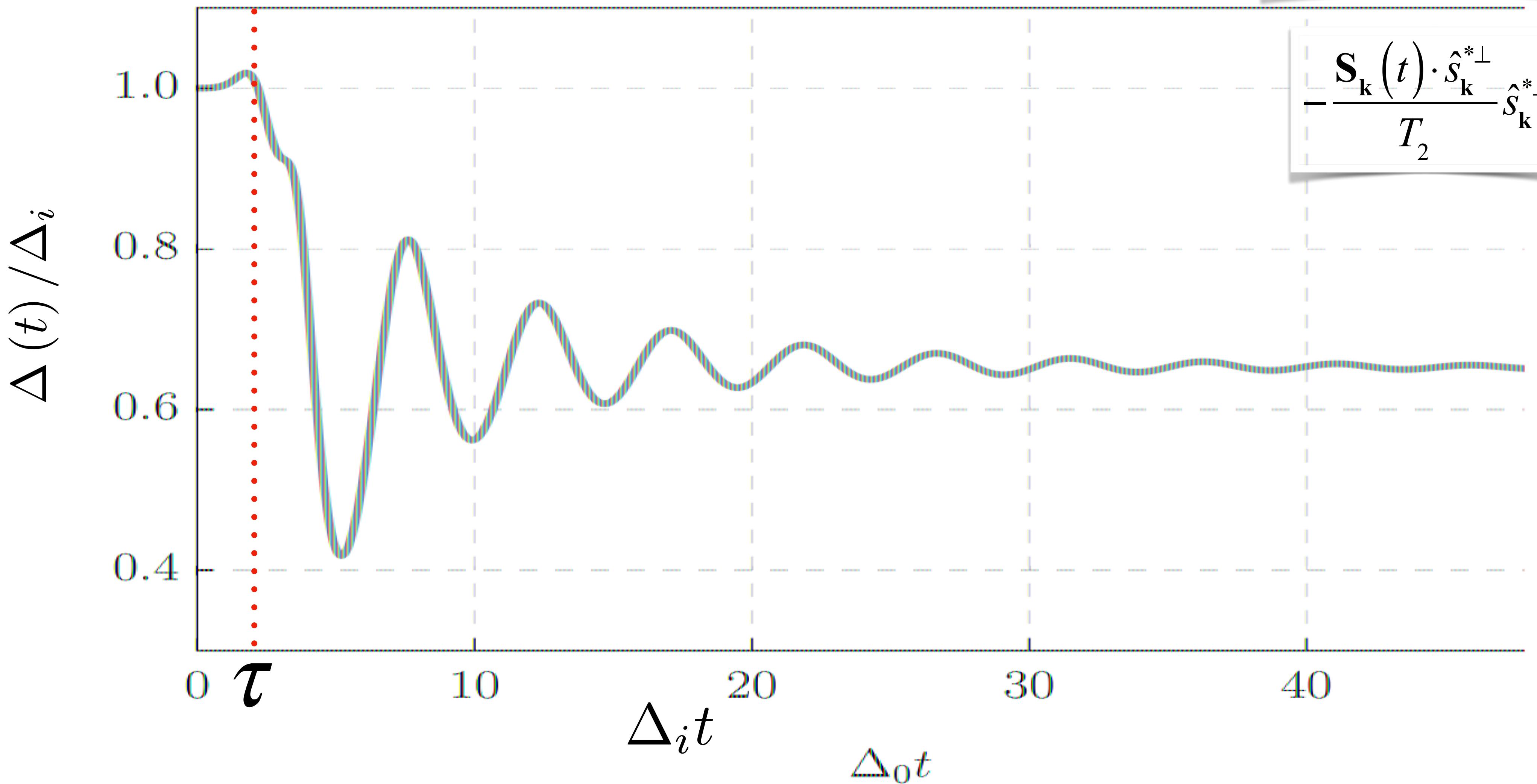


# Impact of T2 relaxation process

$$T_1 = 2.5\tau$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\parallel} - |\mathbf{S}_k^*|}{T_1} \hat{s}_k^{*\parallel}$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\perp}}{T_2} \hat{s}_k^{*\perp}$$

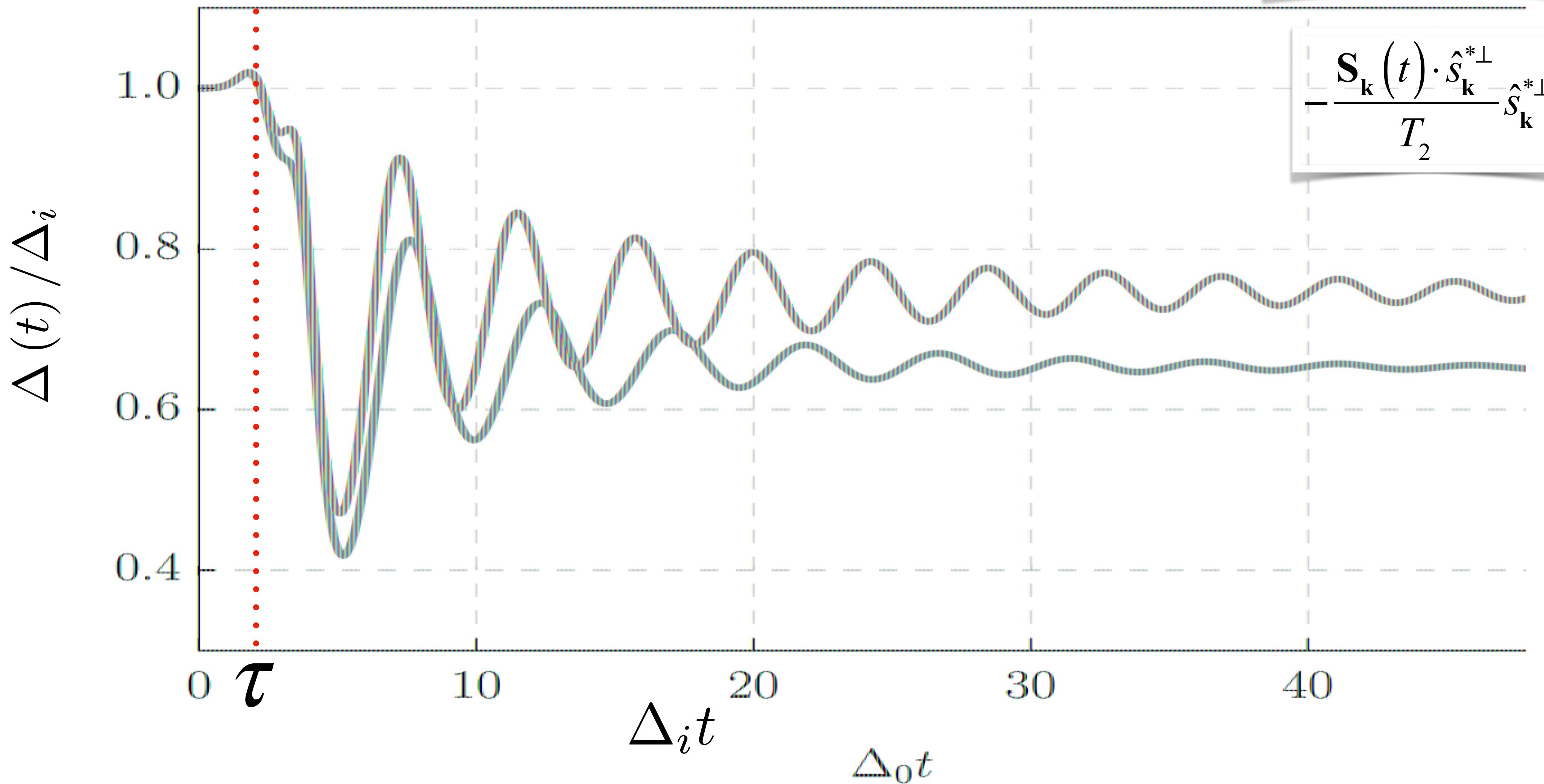


# Impact of T2 relaxation process

$$T_1 = 2.5\tau$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\parallel} - |\mathbf{S}_k^*|}{T_1} \hat{s}_k^{*\parallel}$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\perp}}{T_2} \hat{s}_k^{*\perp}$$

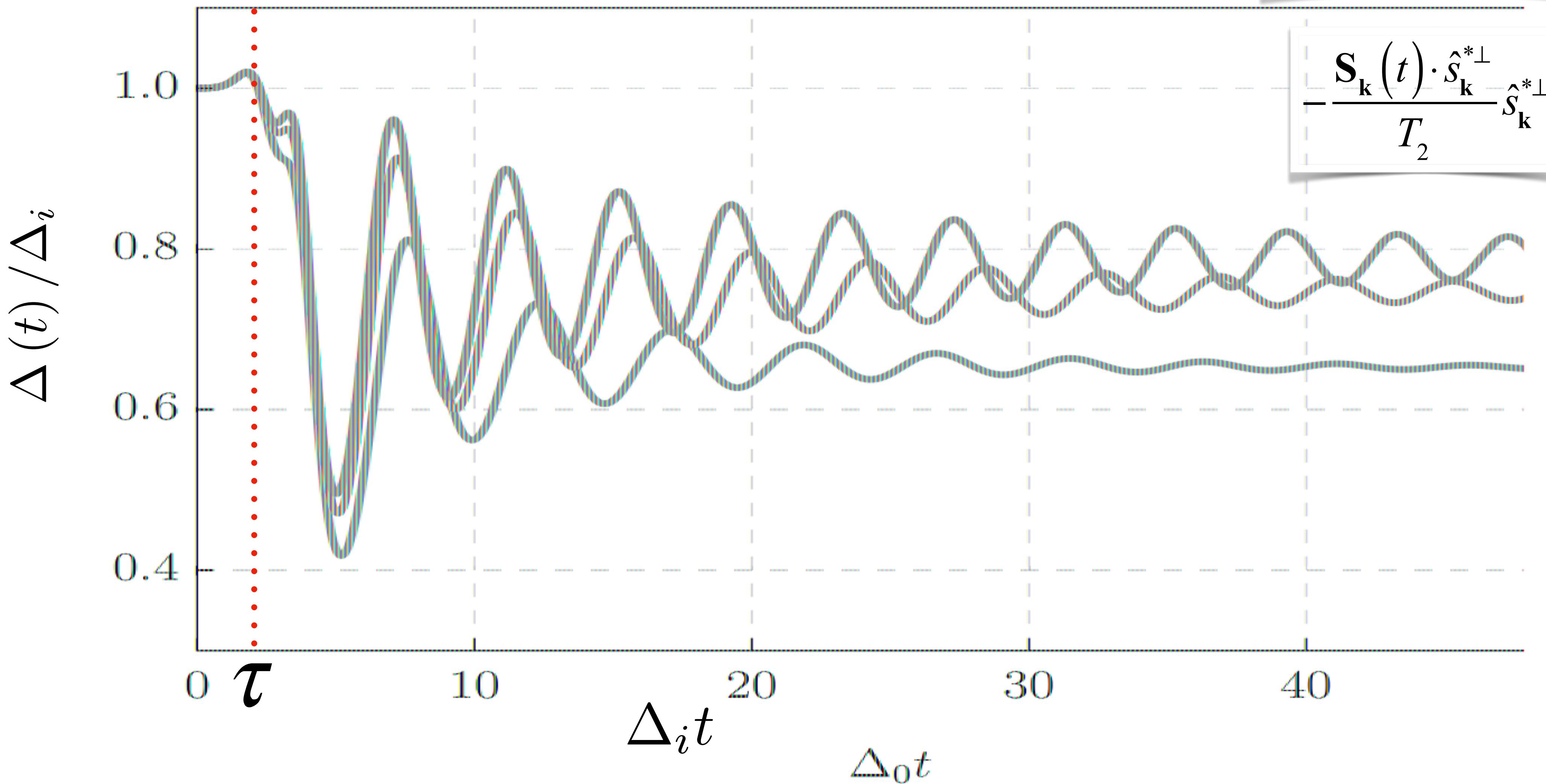


# Impact of T2 relaxation process

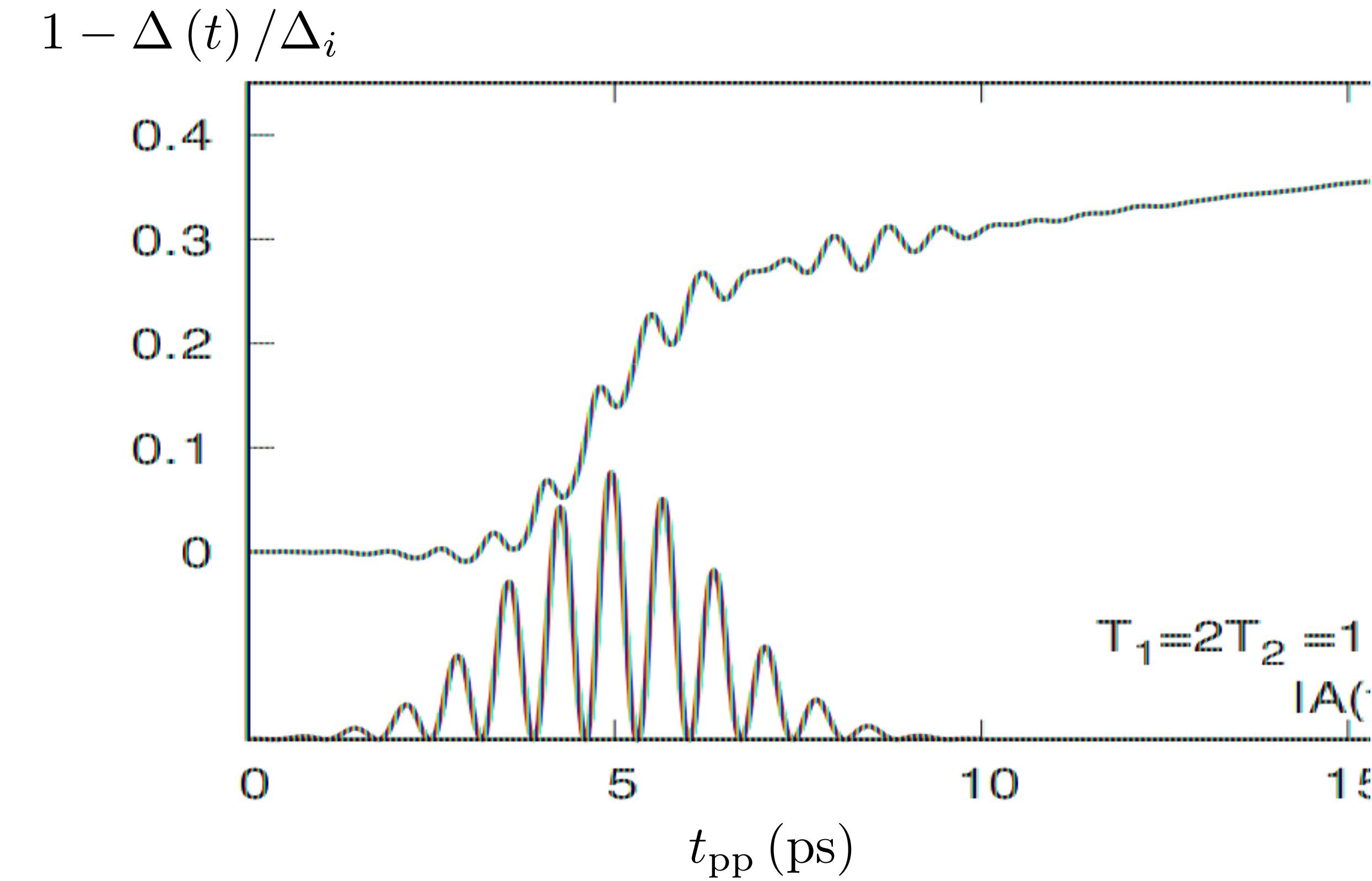
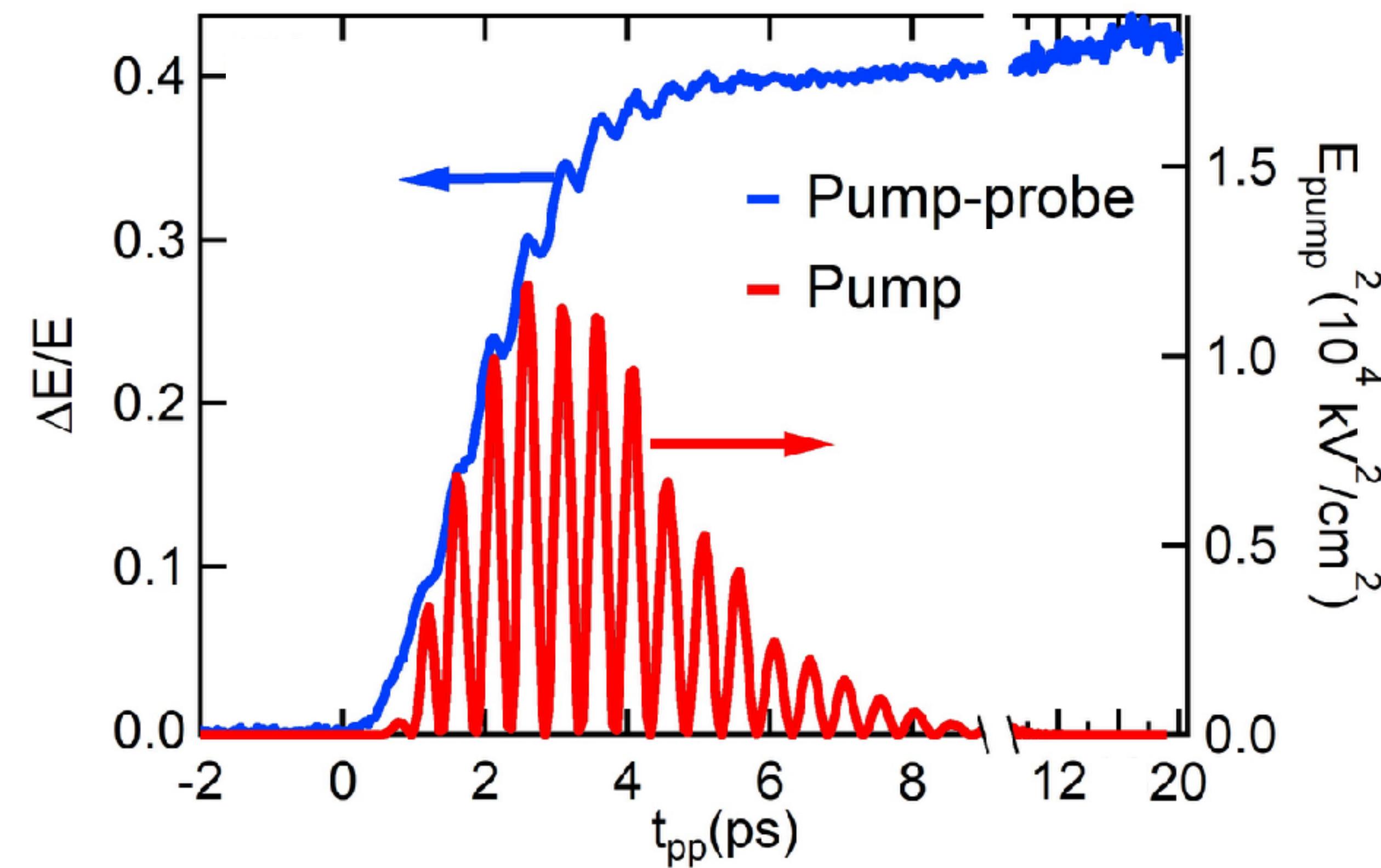
$$T_1 = 2.5\tau$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\parallel} - |\mathbf{S}_k^*|}{T_1} \hat{s}_k^{*\parallel}$$

$$-\frac{\mathbf{S}_k(t) \cdot \hat{s}_k^{*\perp}}{T_2} \hat{s}_k^{*\perp}$$



# Results with damping effects



## Conclusions

- A semi-phenomenological framework is established to incorporate **damping** in the **transient** dynamics of the superconducting gap within the **electronic subsystem**.
- Application of this approach to **different** superconductors will allow one to **distinguish** the type of relaxation processes **dominant** in each system.
- Future work: **microscopic models** for different relaxation processes.