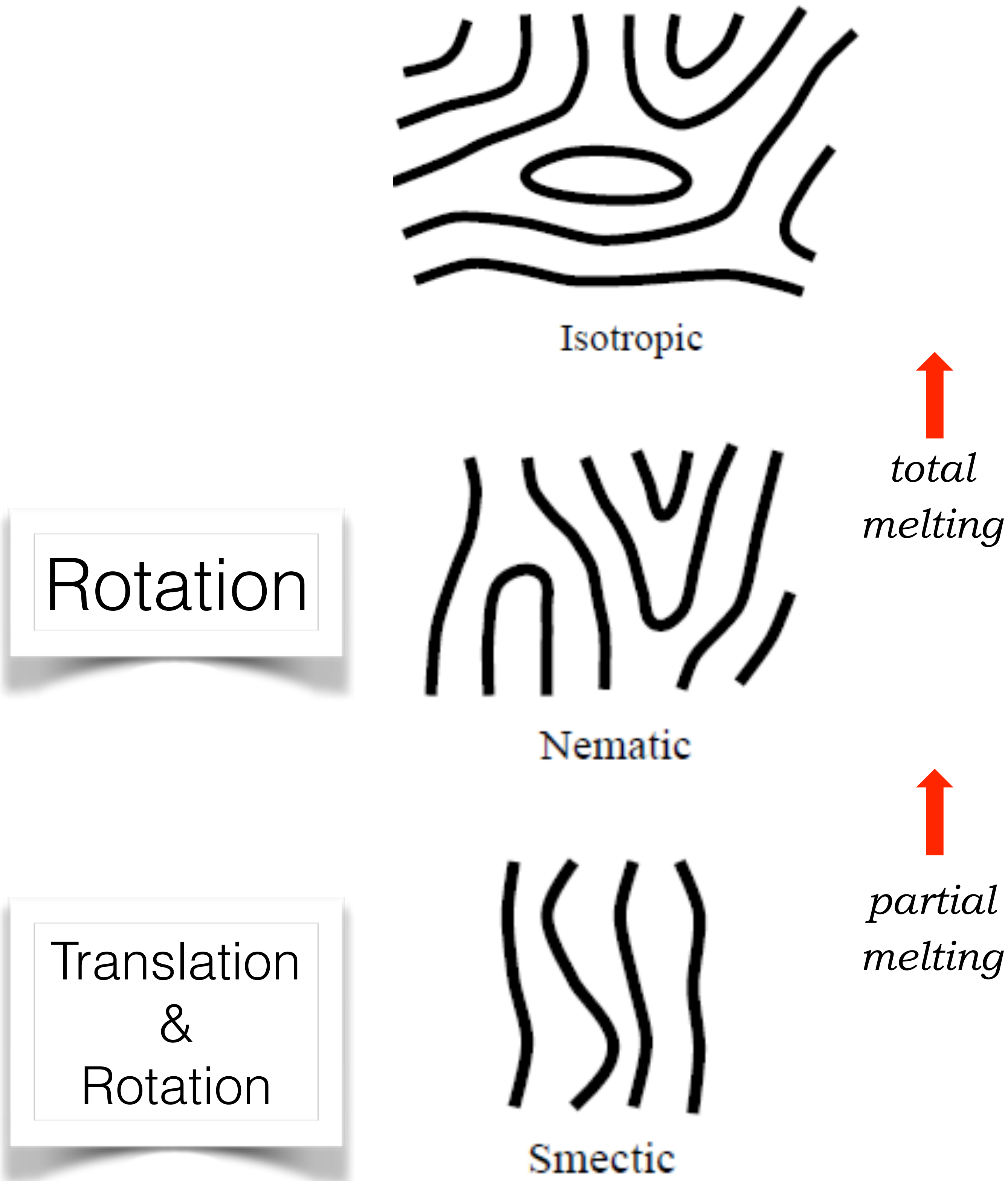
Rare Region Effects on the Ising-nematic Quantum Phase Transition

Tianbai Cui

Advisor: Prof. Rafael Fernandes

UNIVERSITY OF MINNESOTA

What is Ising-nematic phase?



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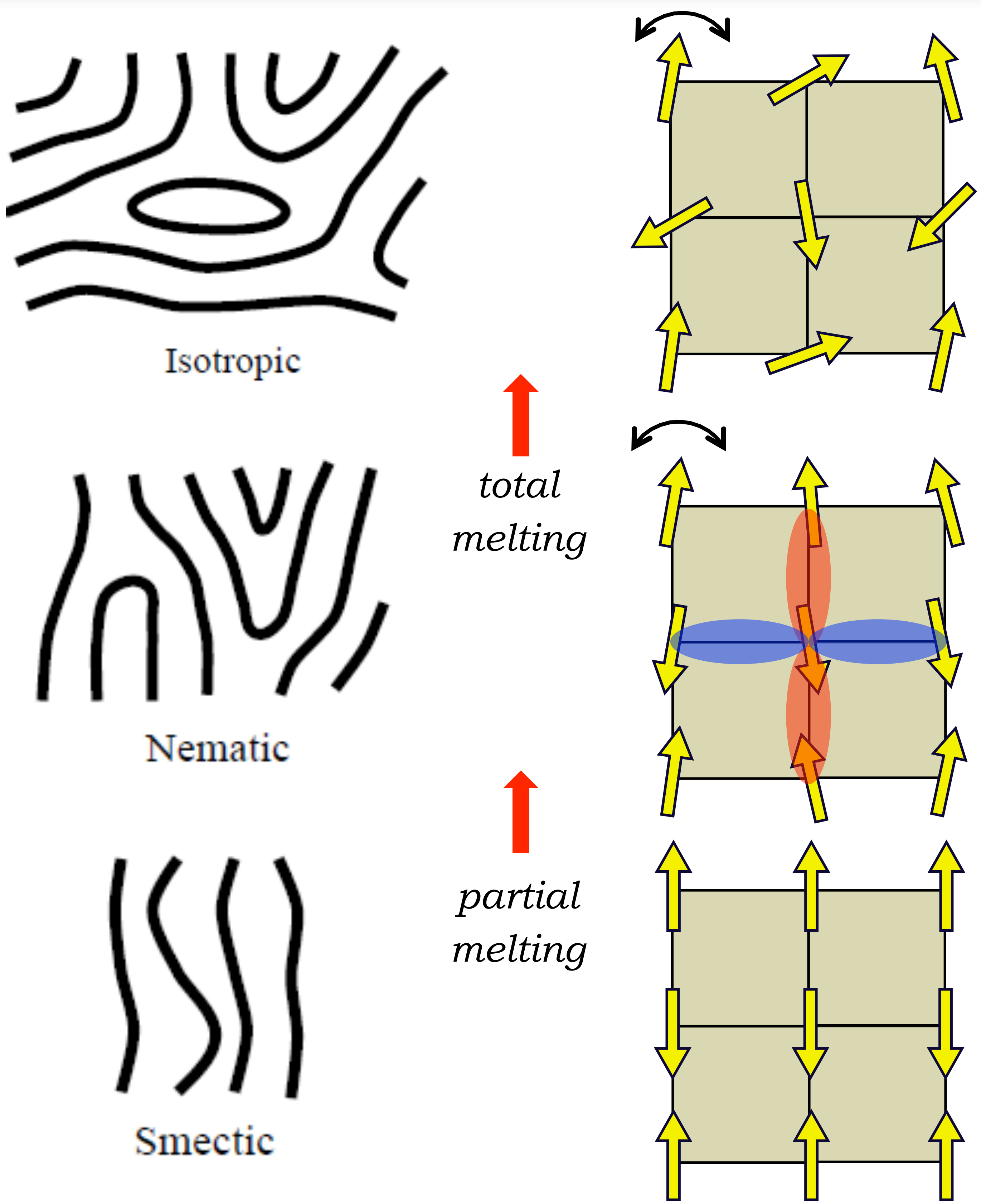


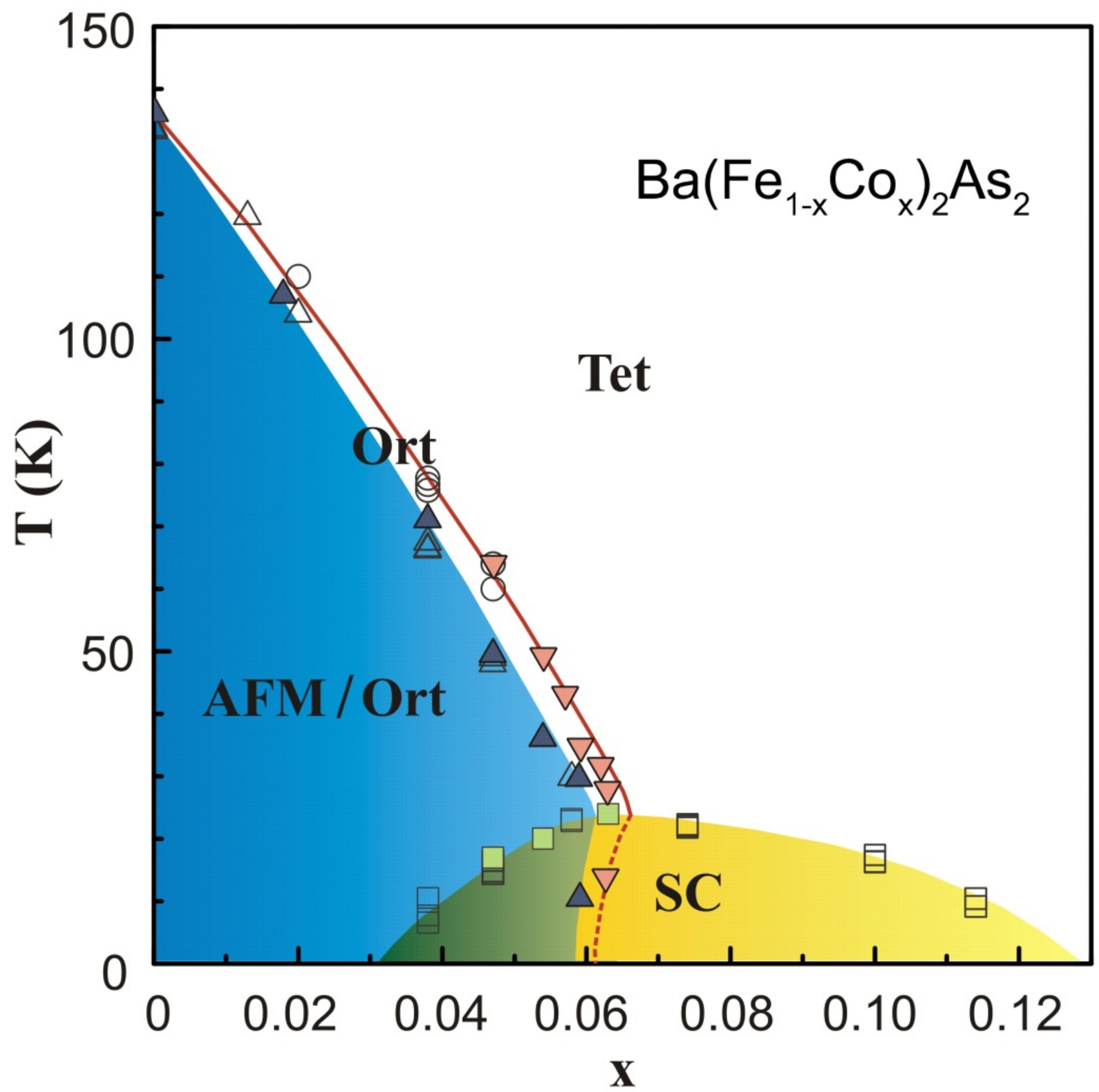
Rotation

C_4

Translation & Rotation

$O(3) \& C_4$

Phase Diagrams of High-Tc Superconductors



Nematic phase: Electronic order breaks lattice symmetry

[1] S. Nandi *et al*, Phys. Rev. Lett. **104**, 57006 (2010).
[2] O. Cyr-Choinière *et al*, Phys. Rev. B **92**, 224502 (2015).

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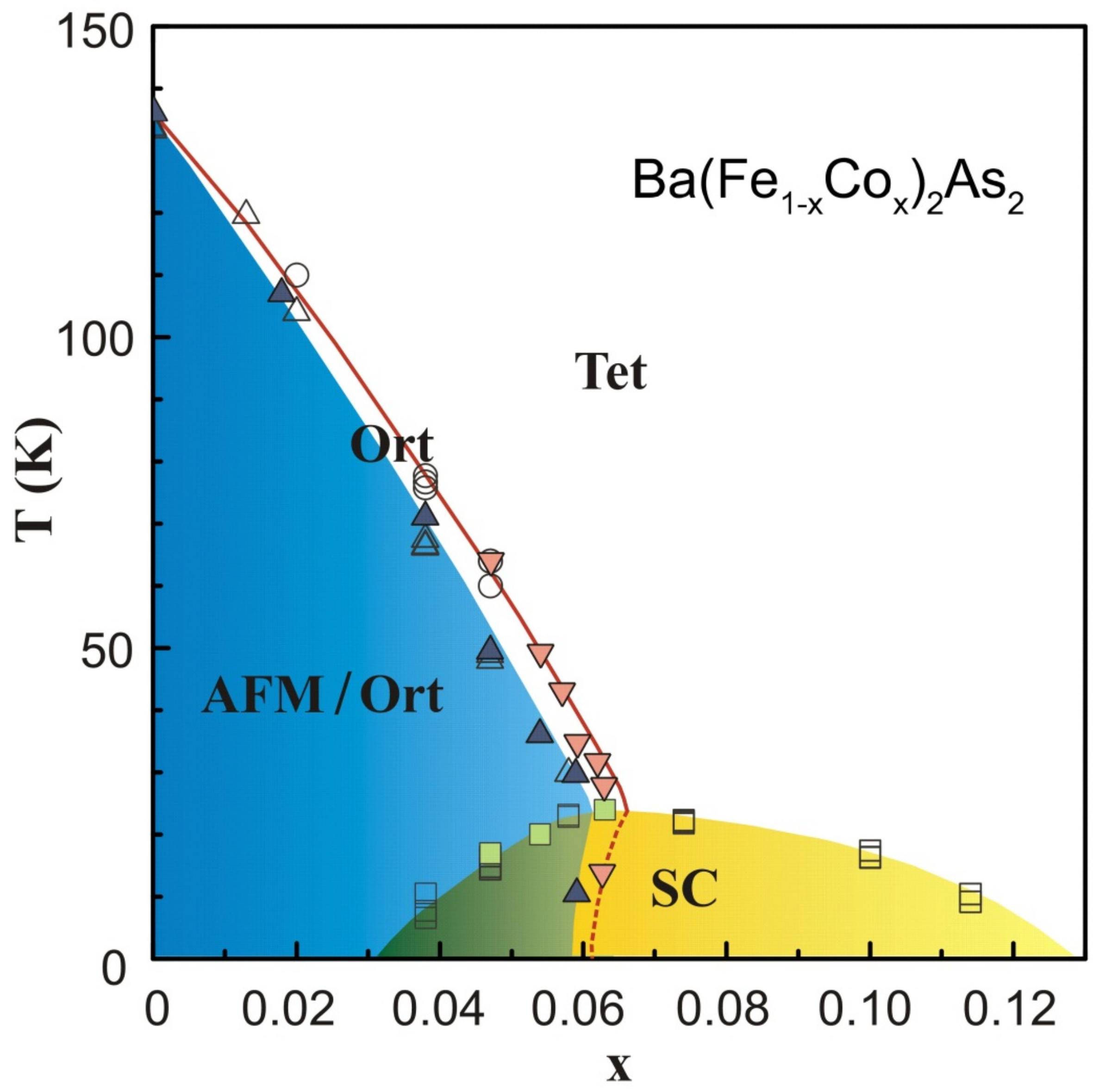

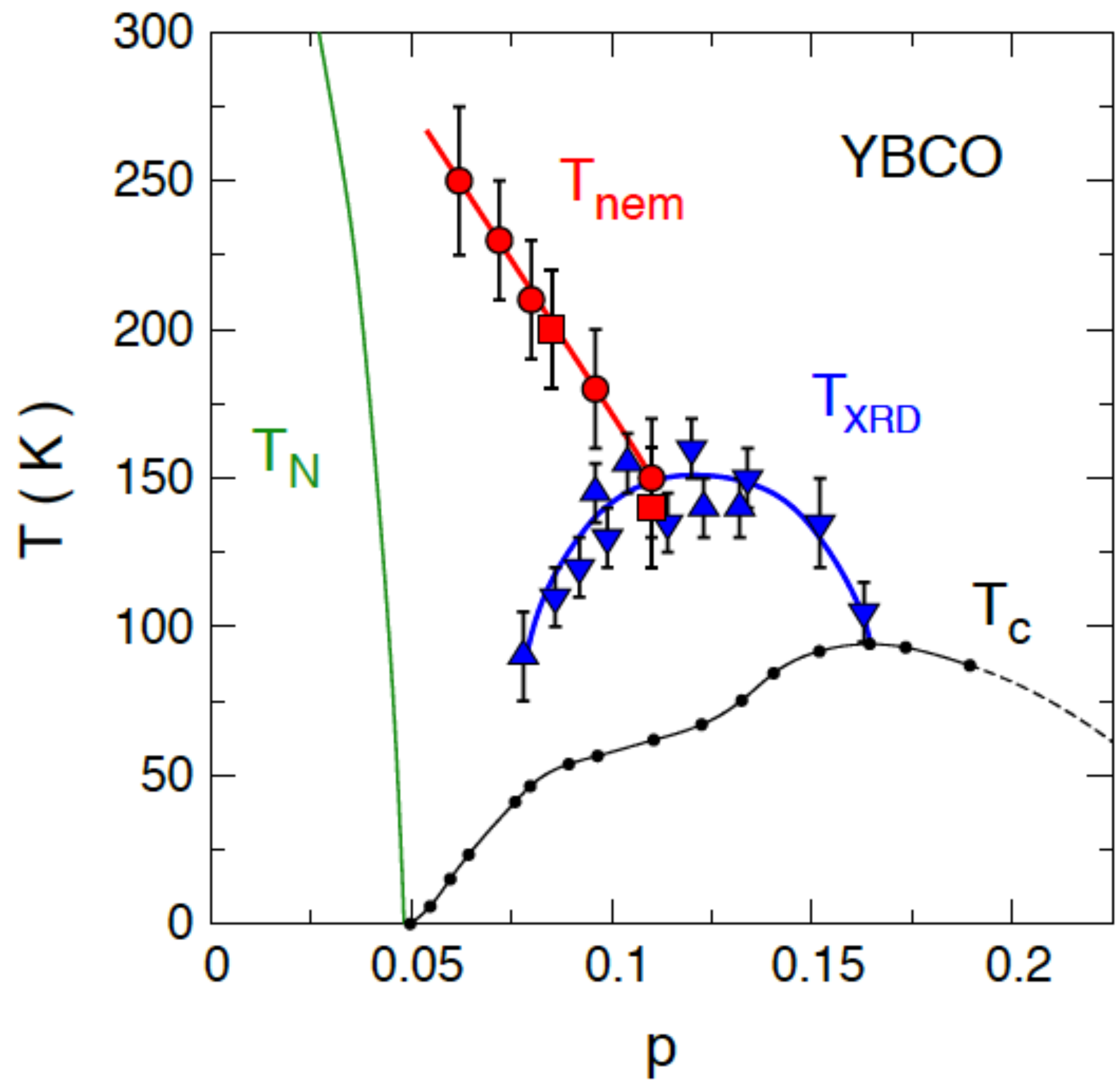

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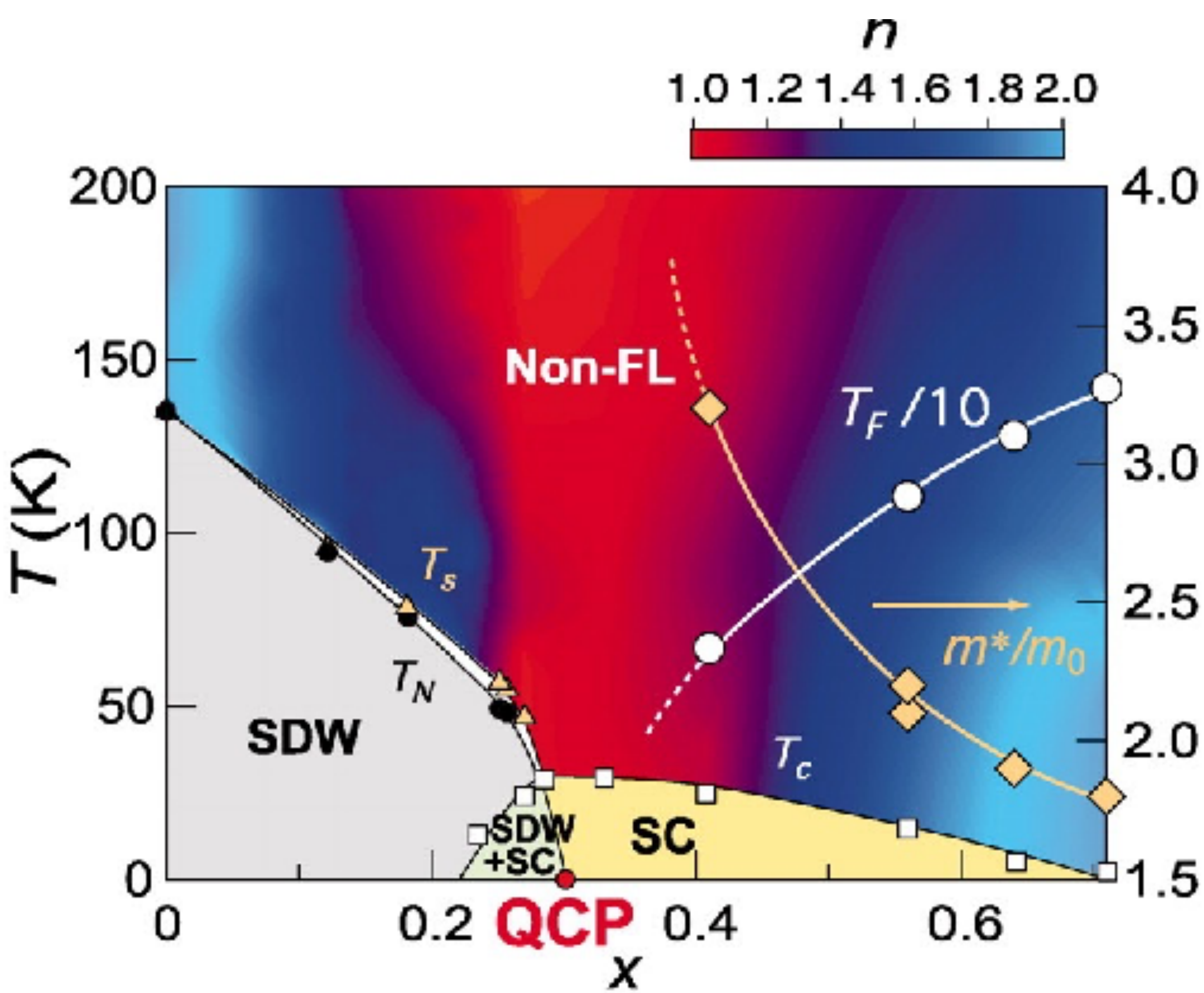
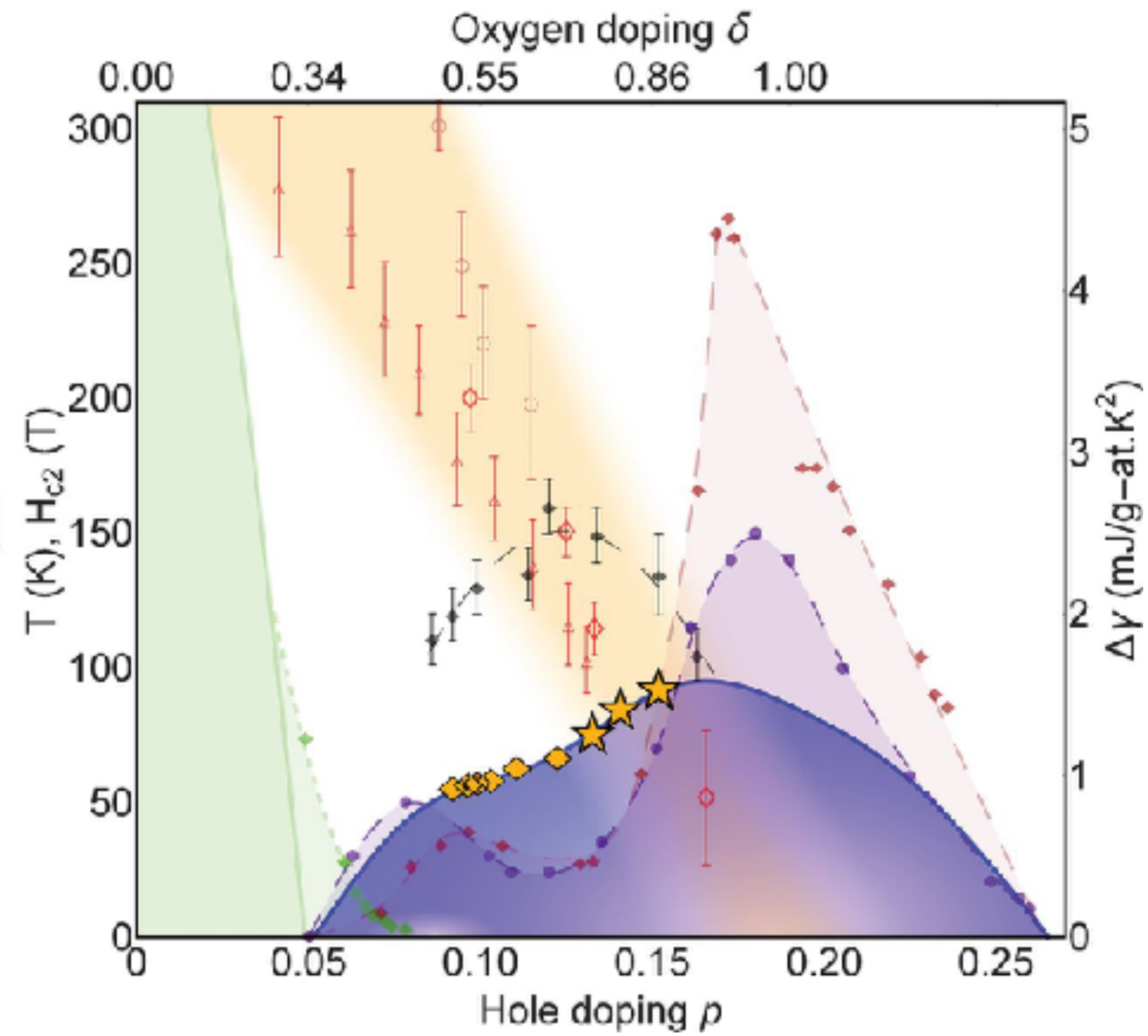
Motivation





$BaFe_2(As_{1-x}P_x)_2$ $YBa_2Cu_3O_{6+\delta}$

[1] K. Hashimoto *et al*, Science, **336**(6088):1554–1557, (2012).
[2] B. J. Ramshaw *et al*, Science, **348**(6232): 317-320 (2015).

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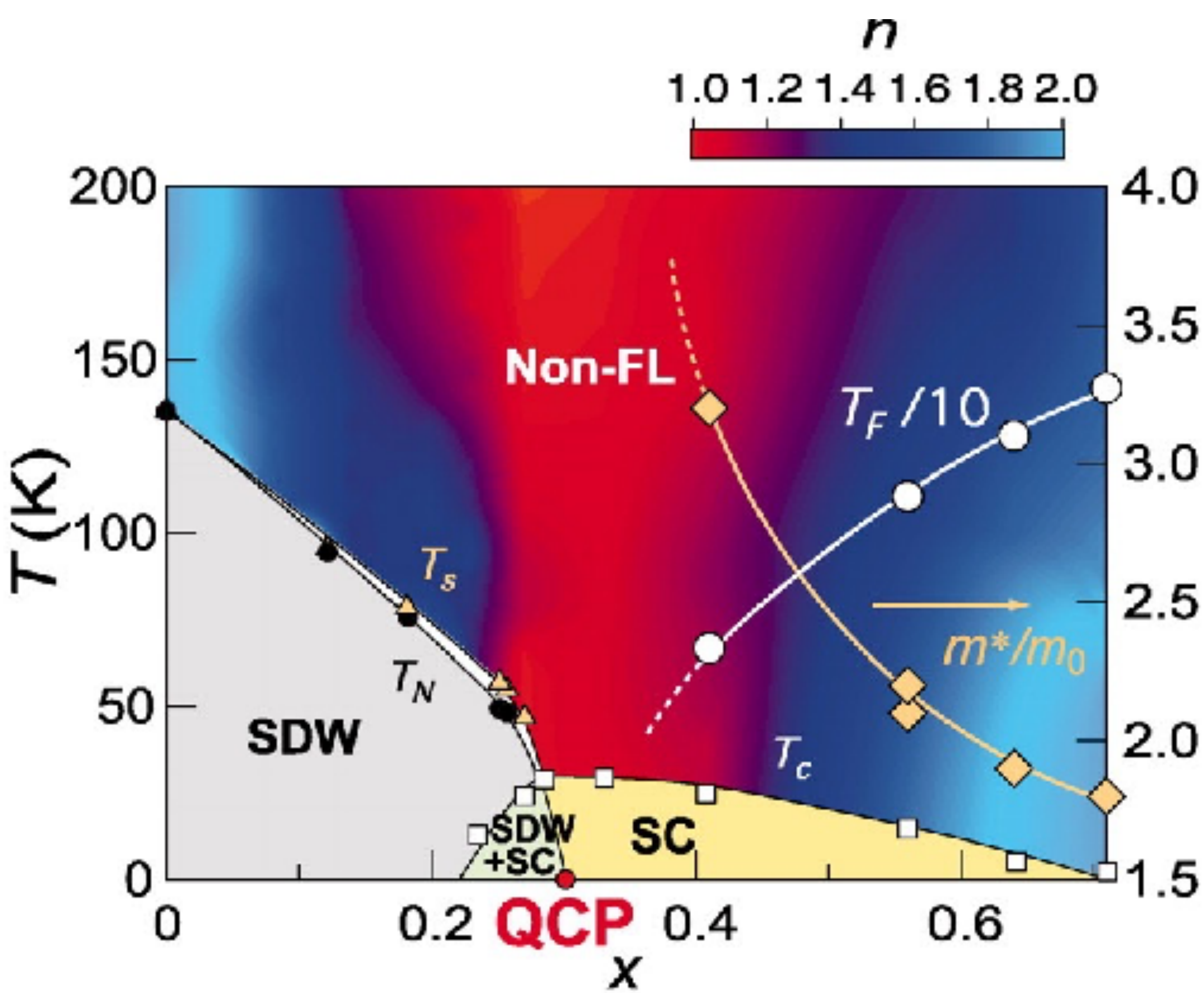

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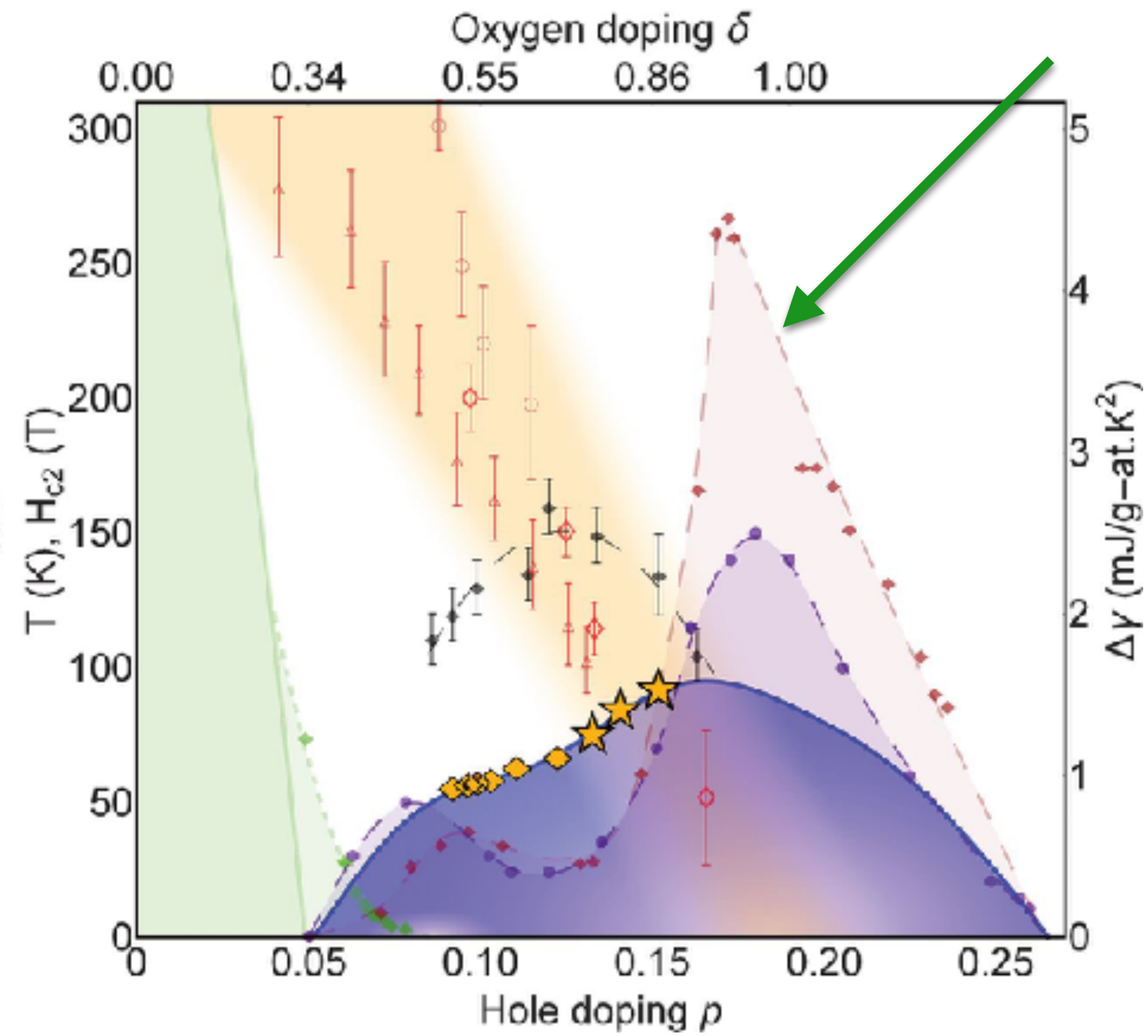

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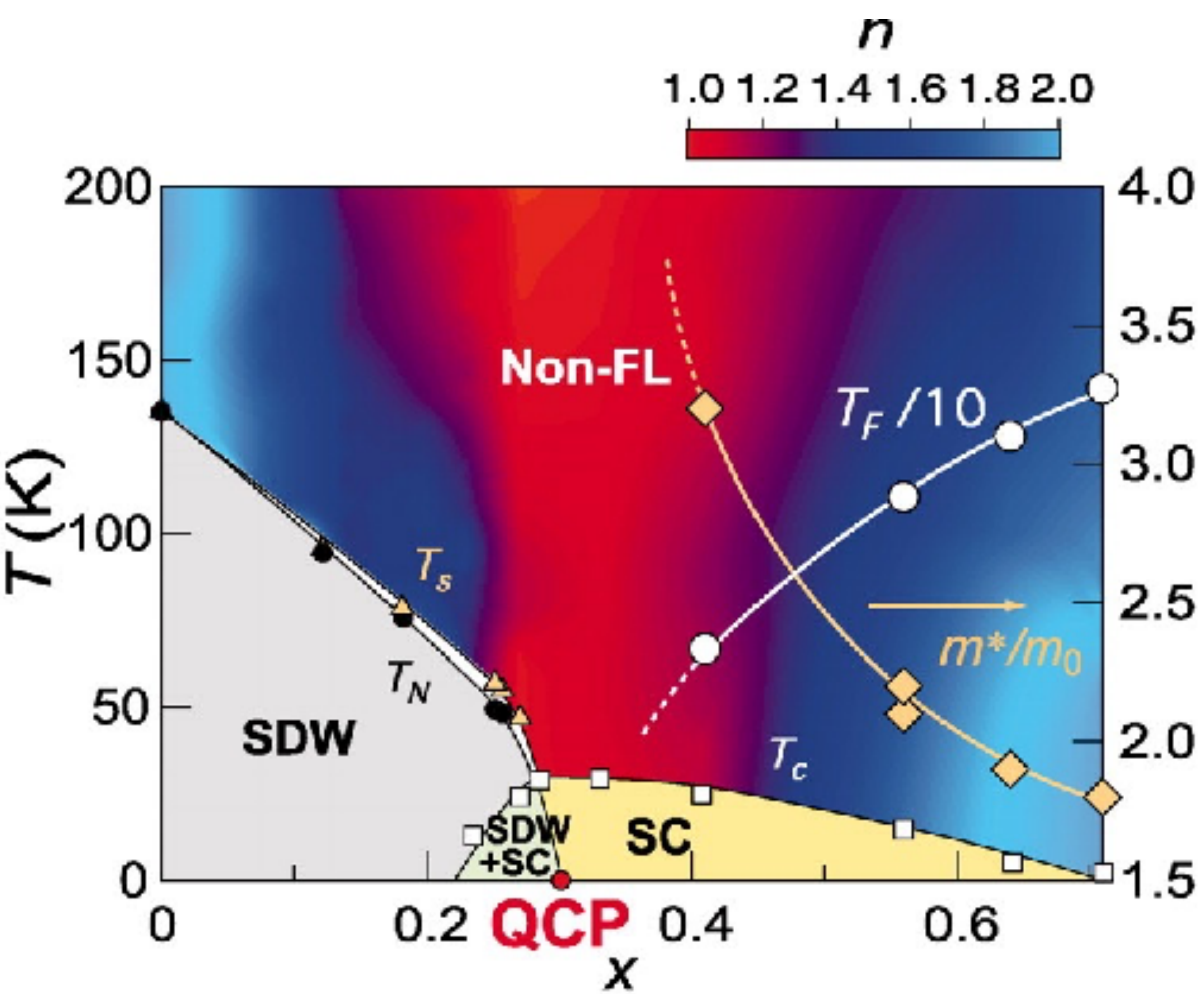

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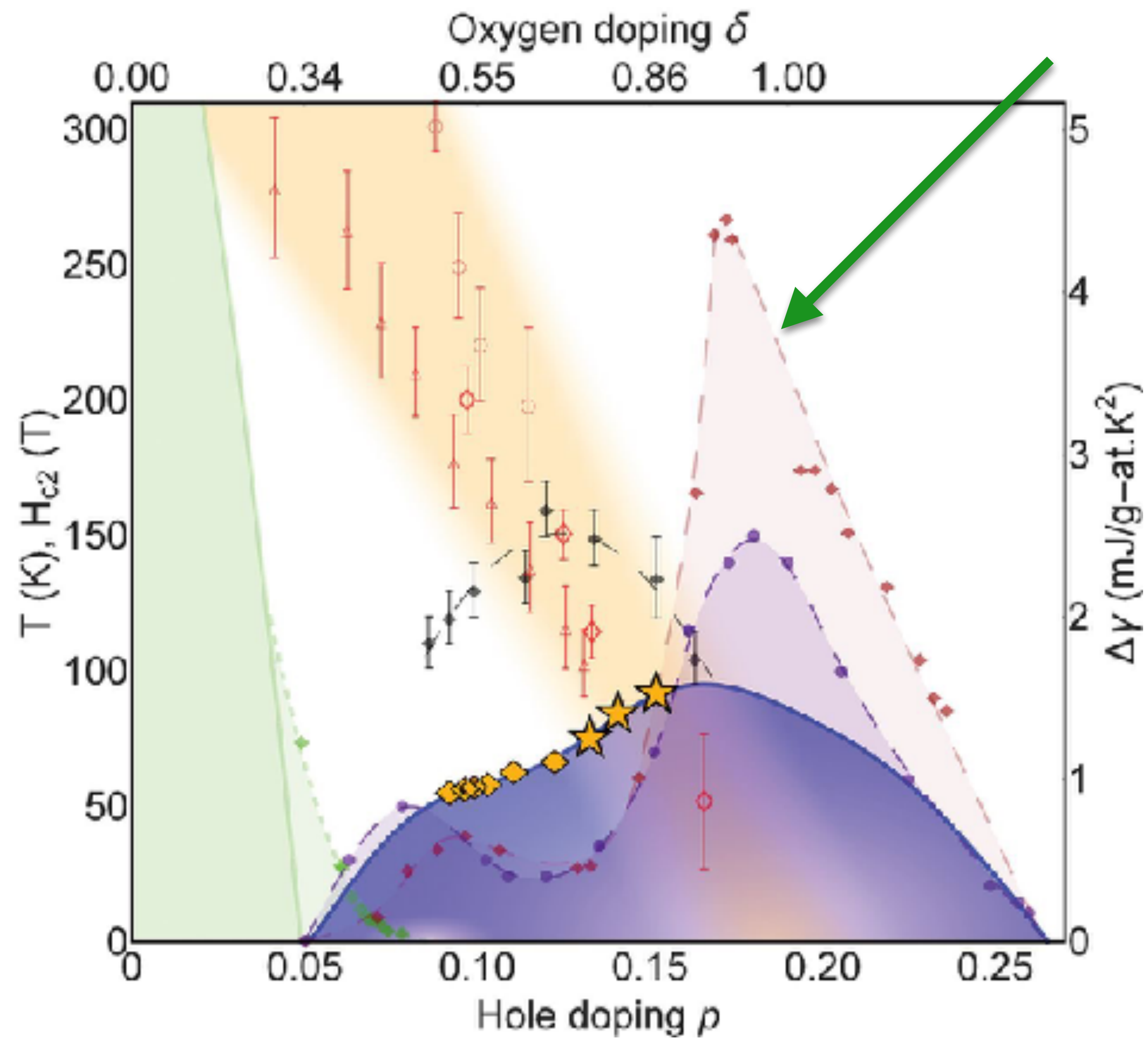

$YBa_2Cu_3O_{6+\delta}$

QCP: Second order phase transition at T=0

[1] K. Hashimoto *et al*, Science, **336**(6088):1554–1557, (2012).
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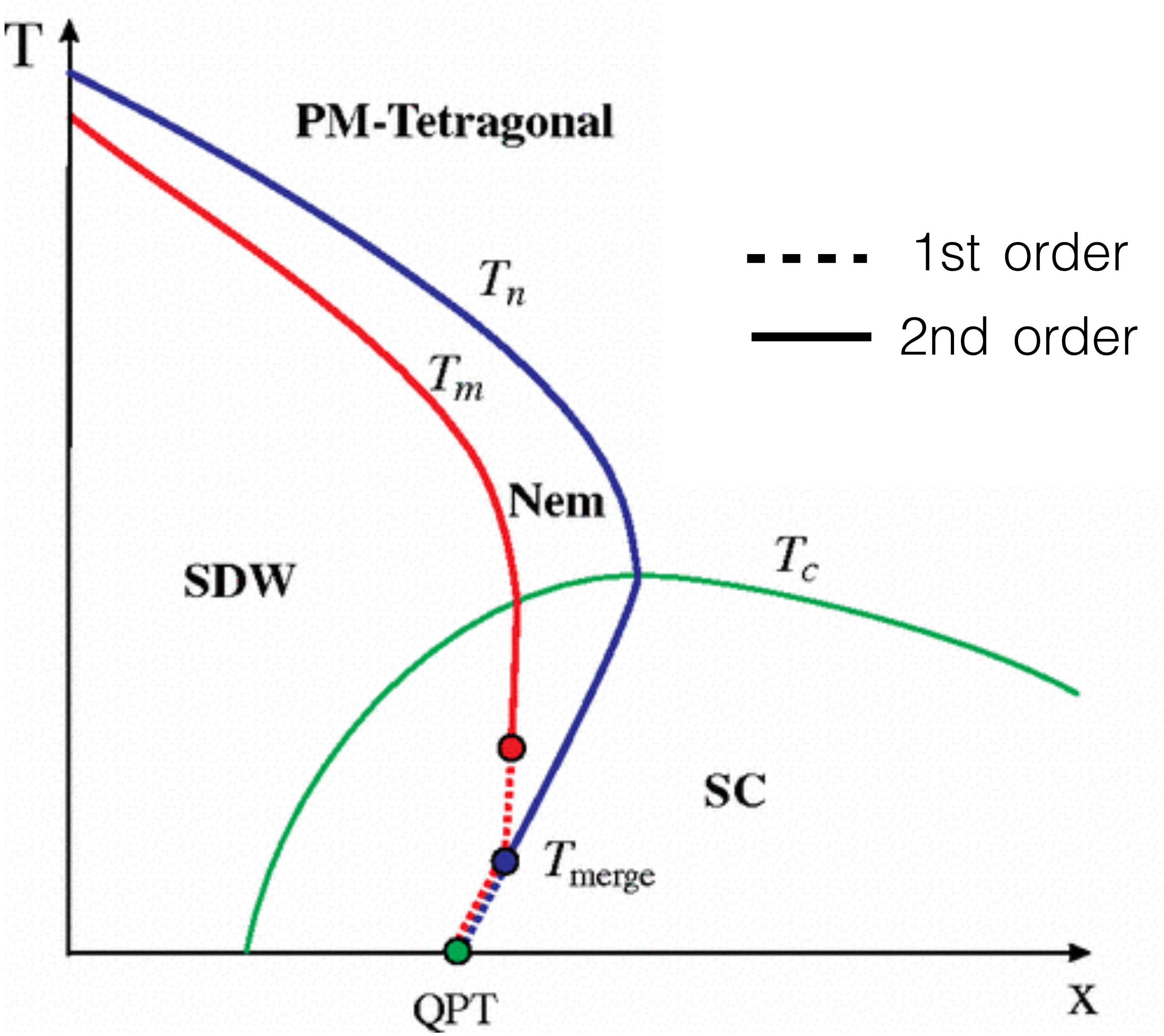

- - - - 1st order

—— 2nd order

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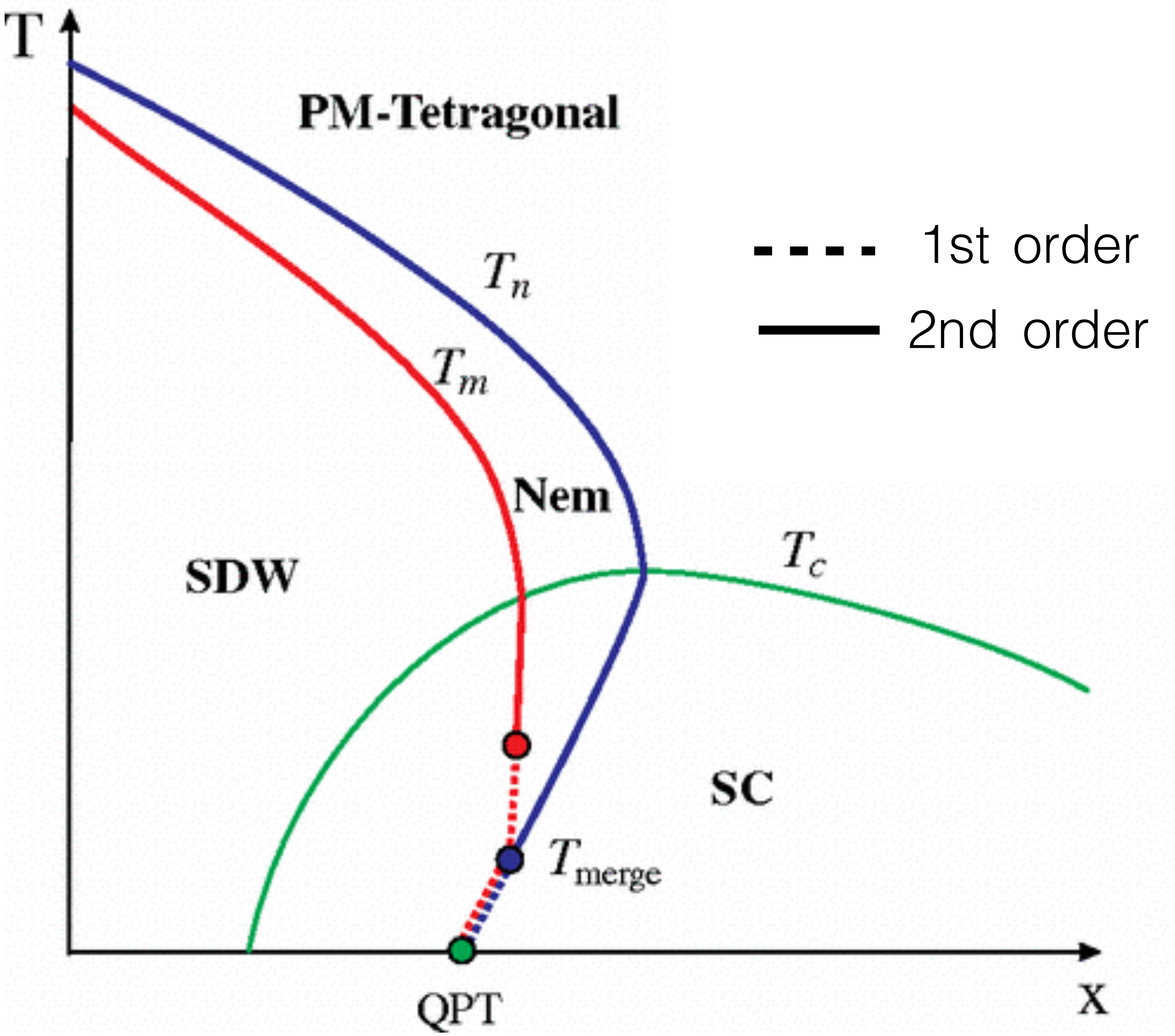

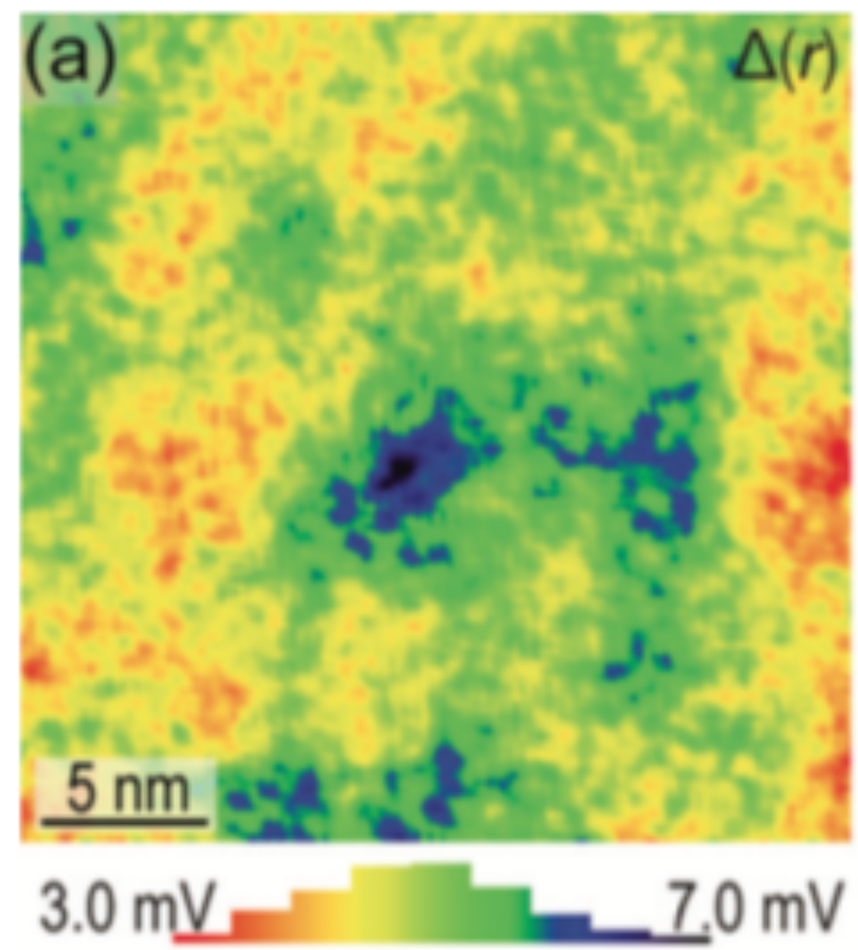

$\Delta(r)$

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$Sr_{1\text{-}x}K_xFe_2As_2$

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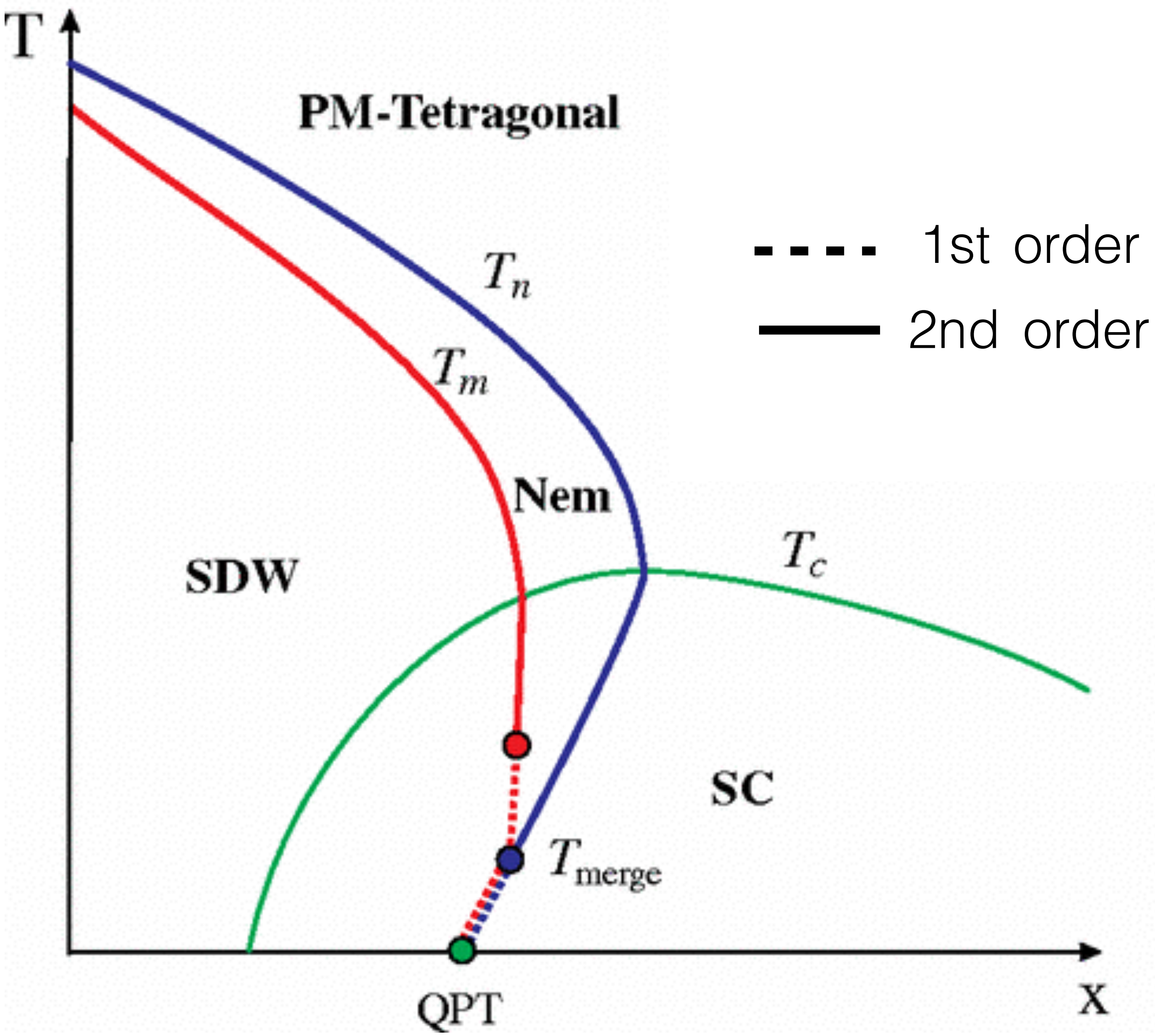

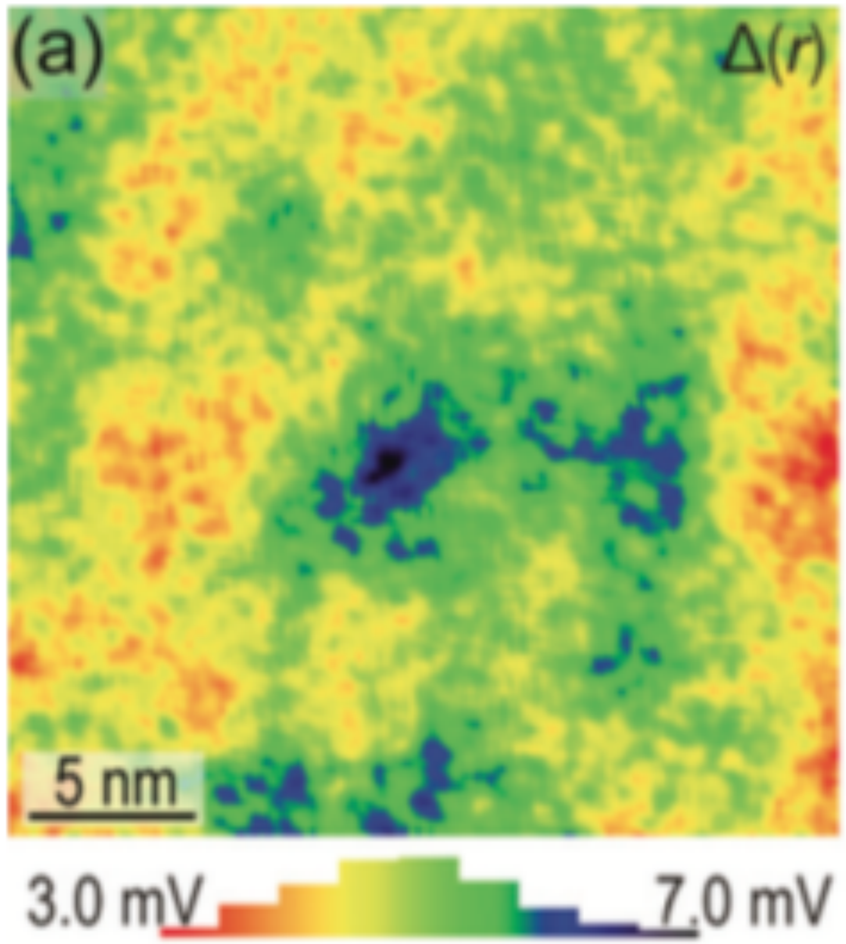

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Split or simultaneous?

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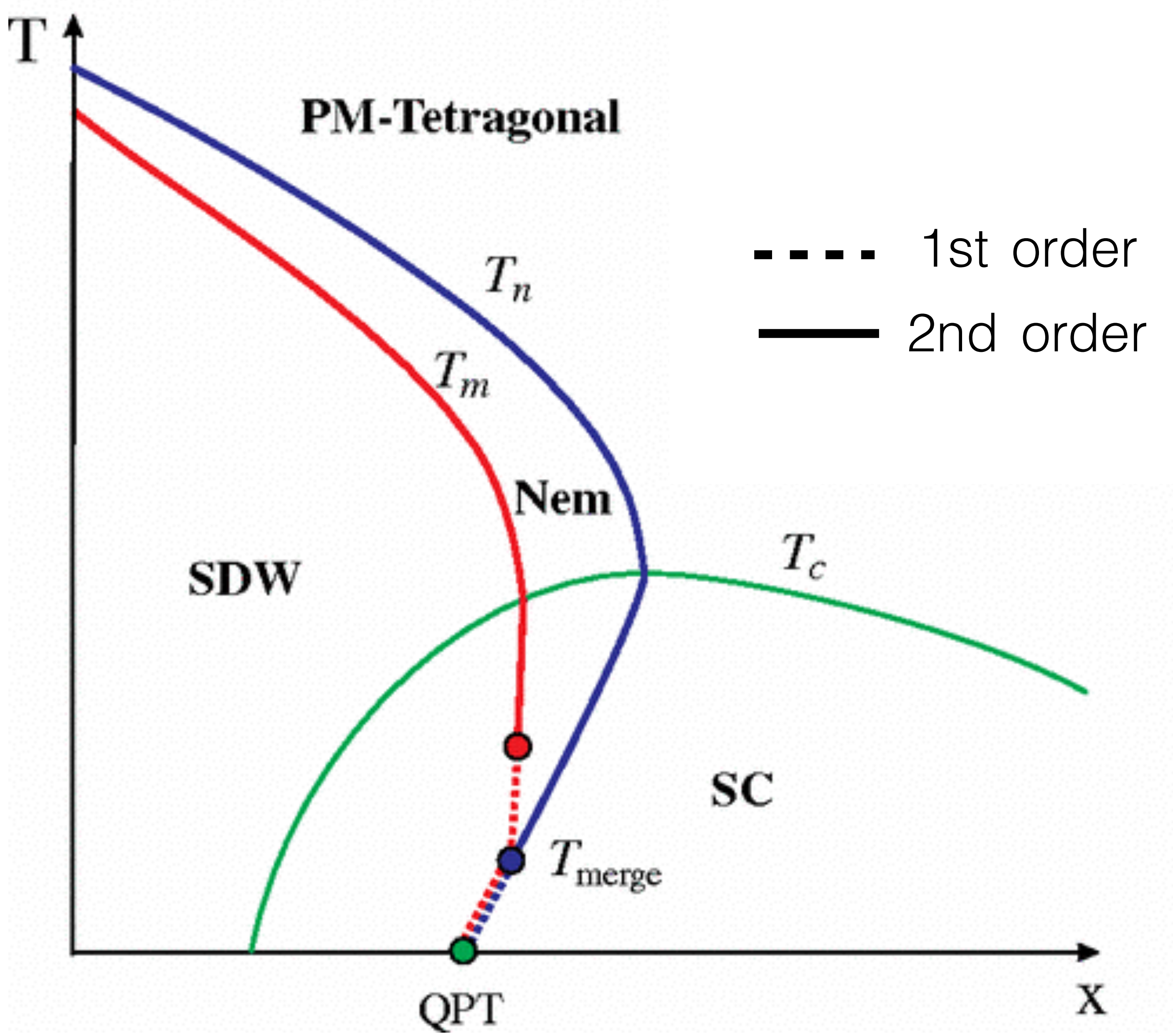

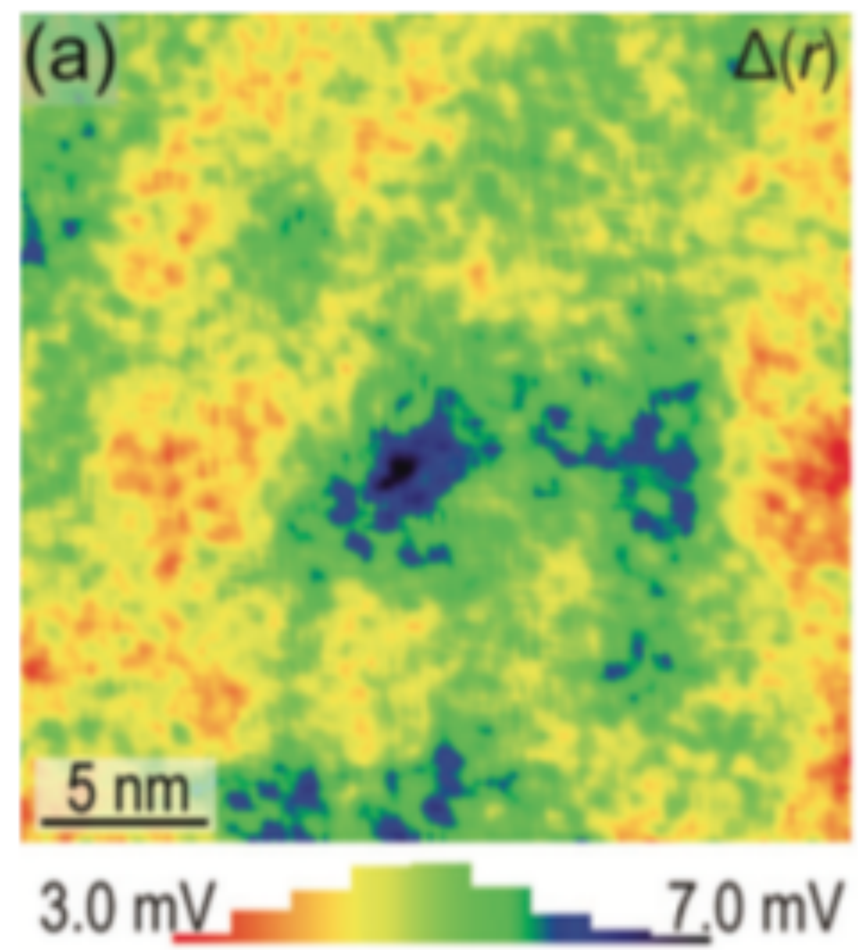

$\Delta(r)$

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$Sr_{1-x}K_xFe_2As_2$

Split or simultaneous?

First or second order?

[1] R. M. Fernandes, S. Maiti, P. Woelfle, and A. V Chubukov, Phys. Rev. Lett. **111**, (2013).

Theoretical Difficulties

Randomness:
Introduce random variables into the theory

Inhomogeneity:
Spatial translation invariance is broken

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Thinking in real space! → Rare region effects

Outline

- Rare region effects on classical phase transitions: Griffiths physics
- Rare region effects on the Ising-nematic quantum phase transition

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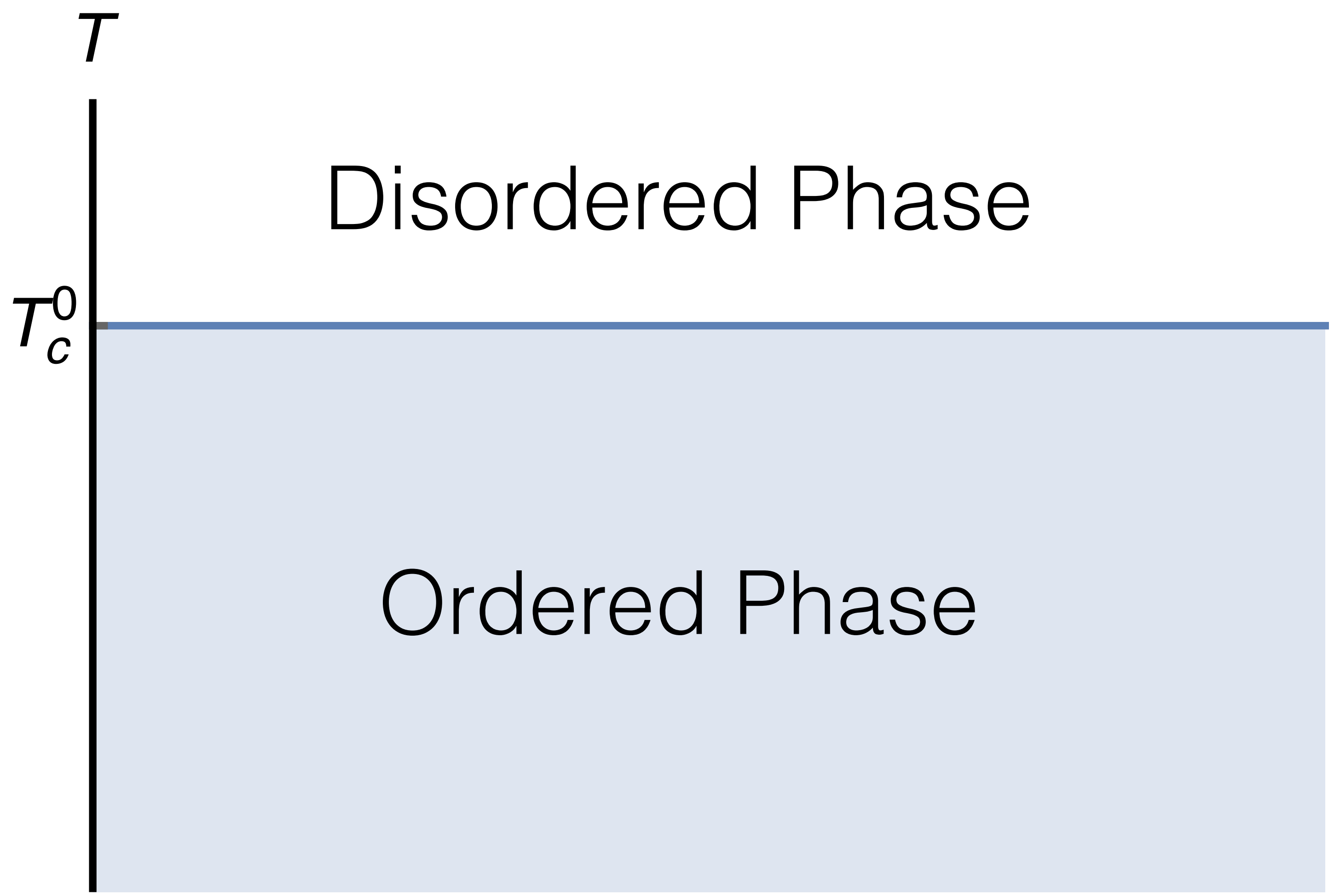
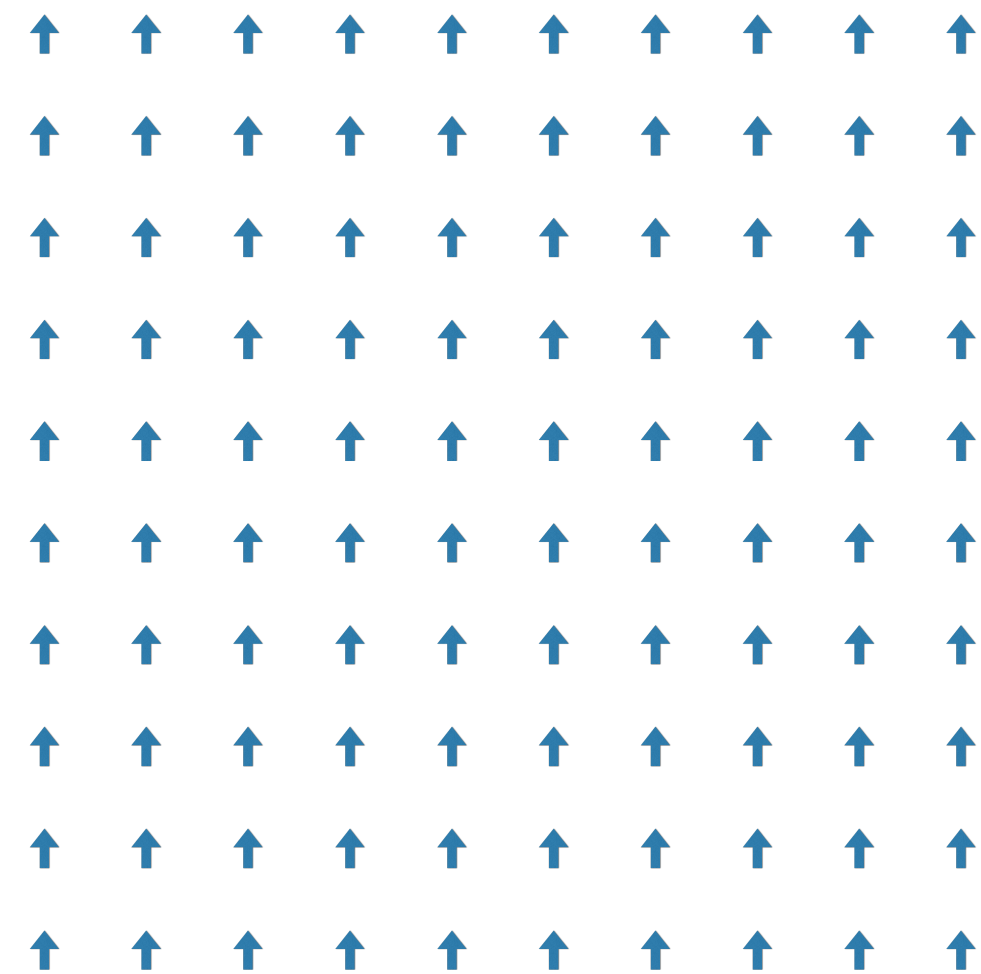
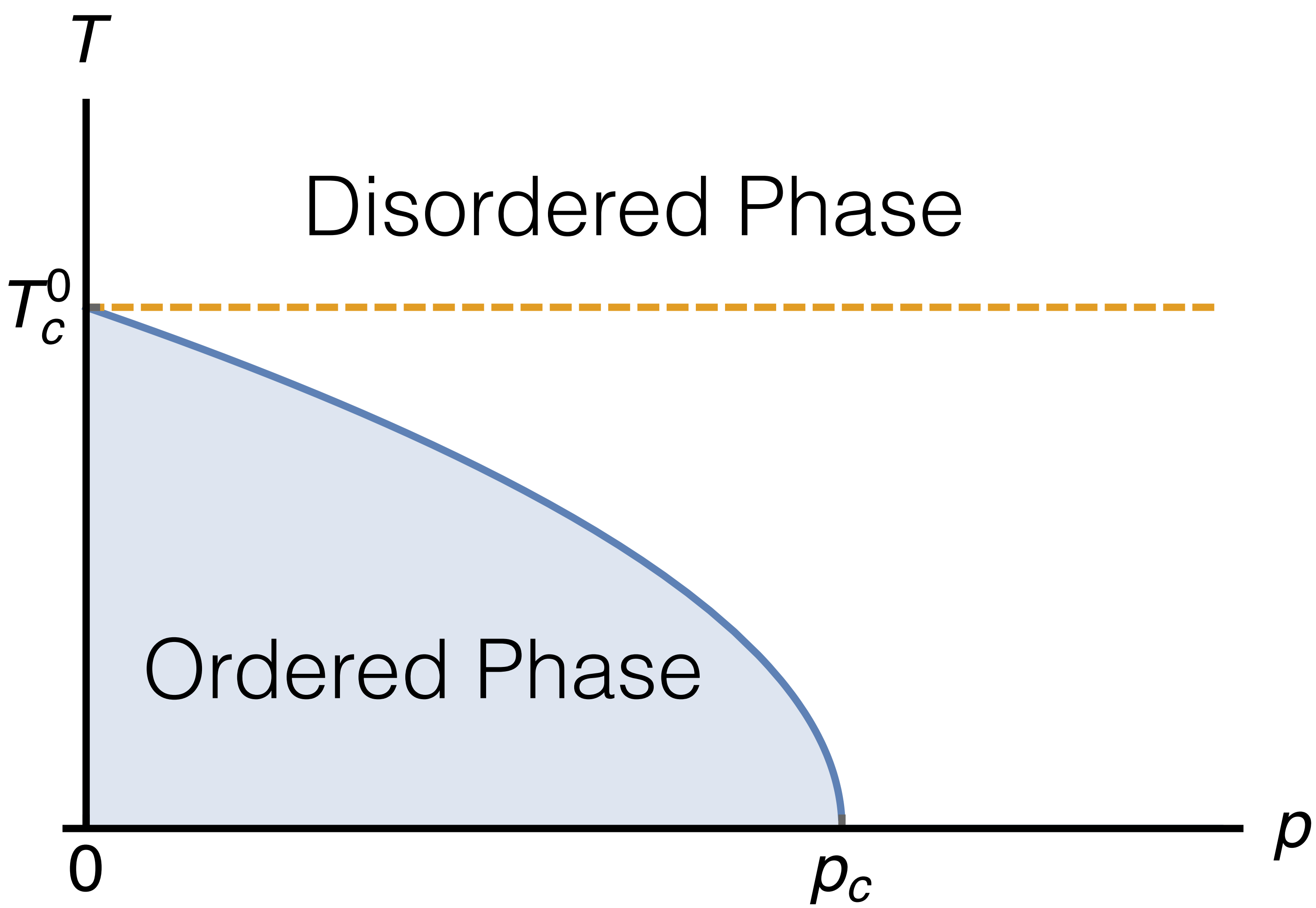
Griffiths Physics

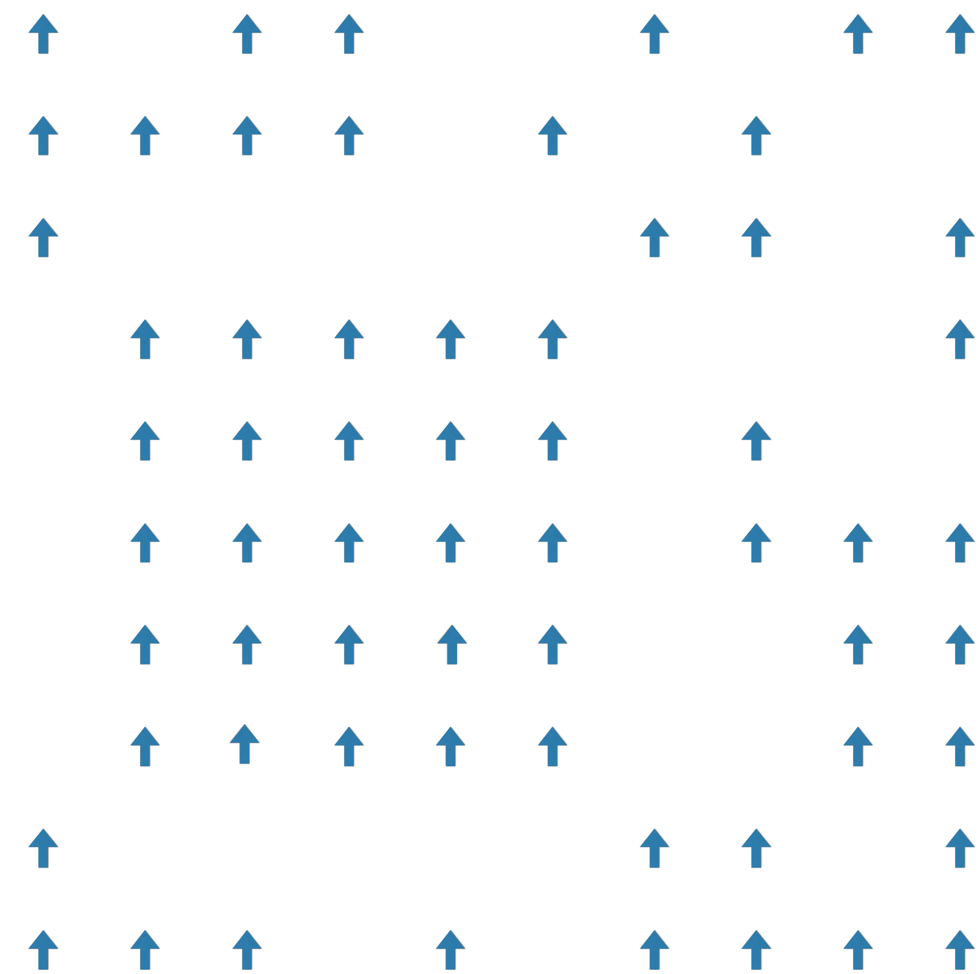




$$\mathcal{H} = -\mathcal{J} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$\mathcal{H} = -\mathcal{J}\sum_{\langle i,j\rangle} \varepsilon_i \varepsilon_j \mathbf{S}_i \cdot \mathbf{S}_j \qquad \varepsilon_i = \begin{cases} 1 & \text{Occupied Site} \\ 0 & \text{Vacancy} \end{cases}$$

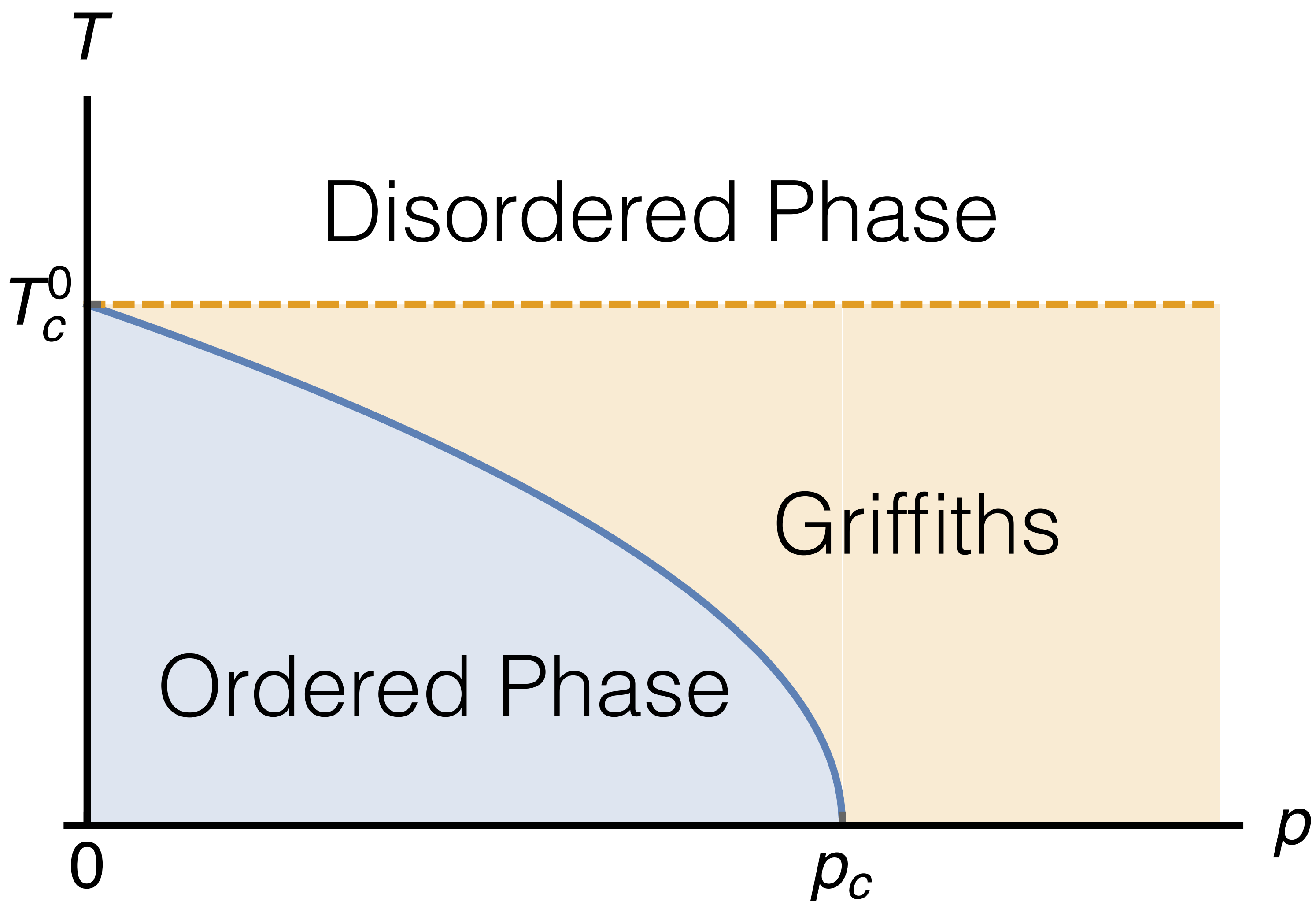


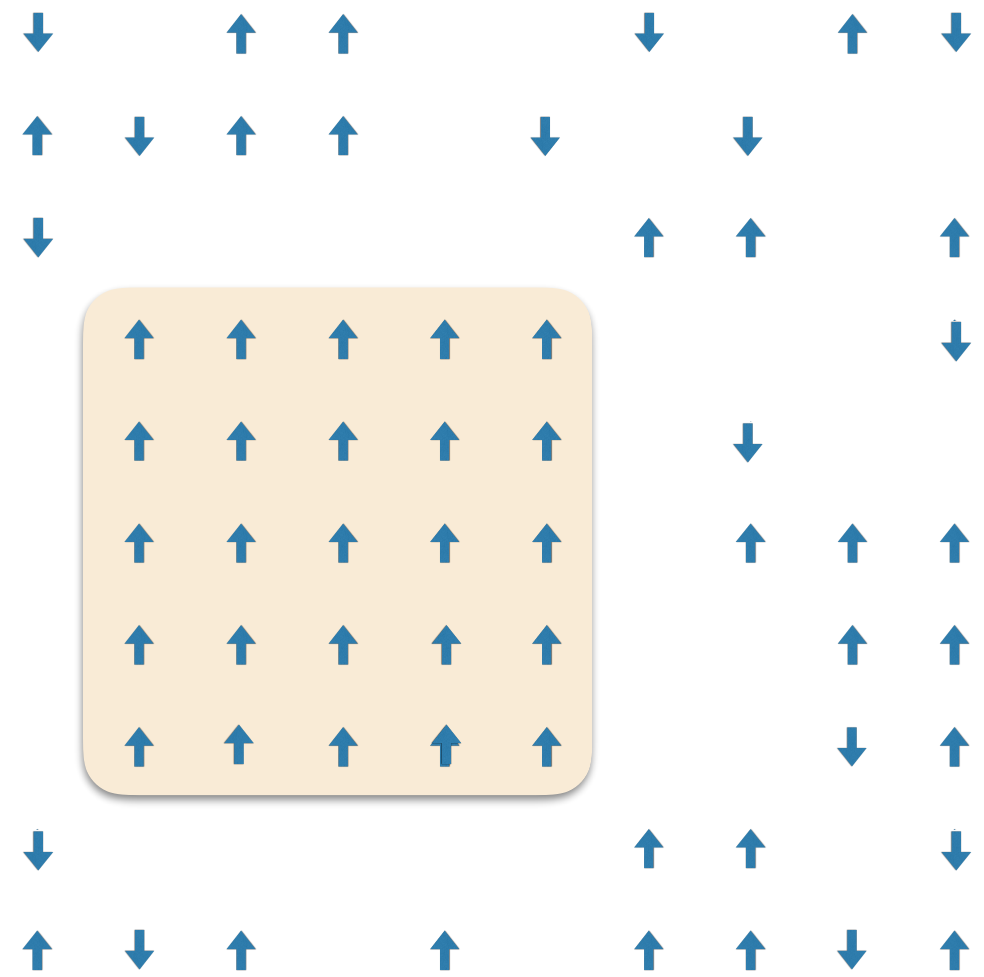


$$P(\varepsilon_i) = (1-p)\,\delta(1-\varepsilon_i) + p\,\delta(\varepsilon_i)$$

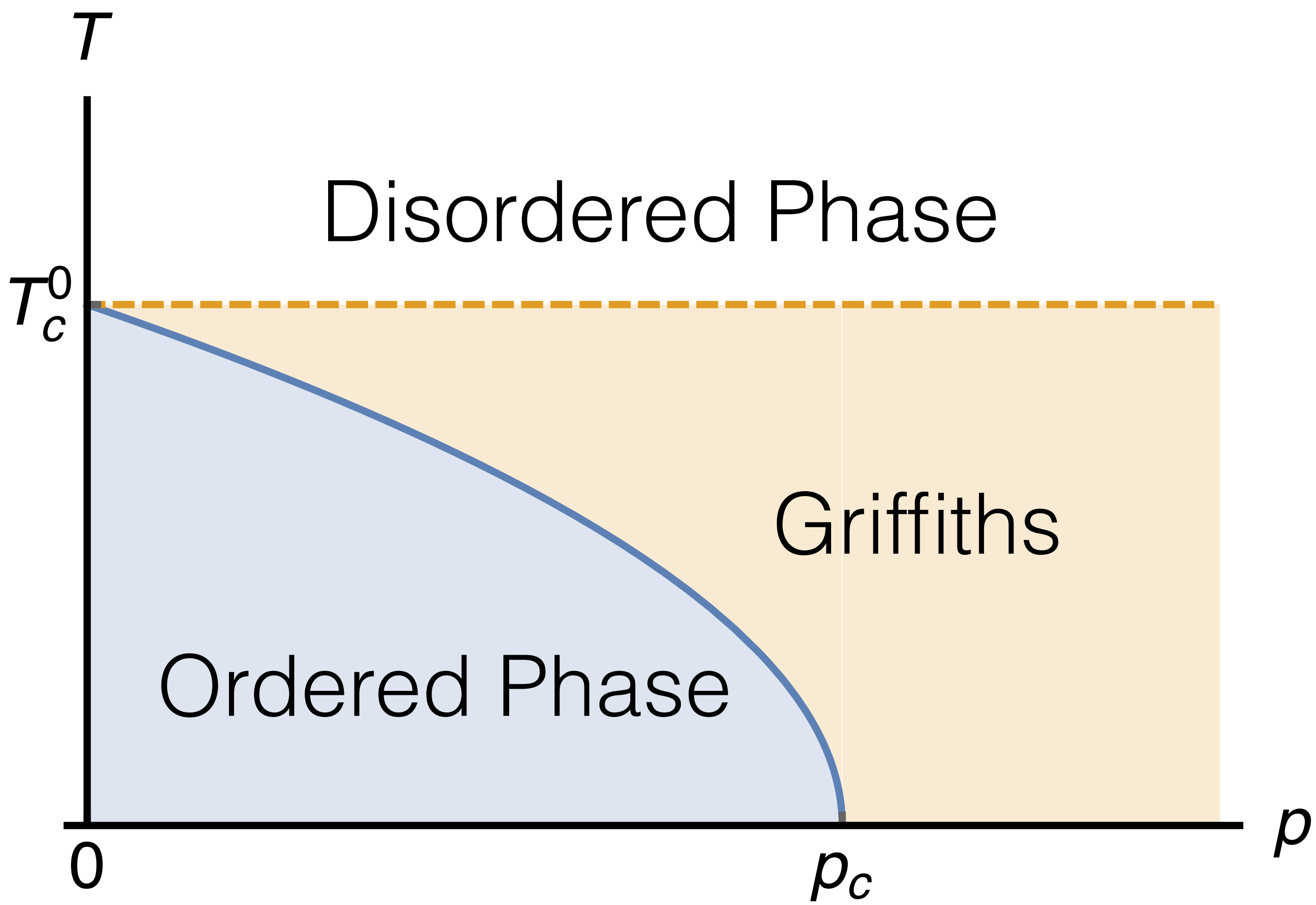
[1] B. I. Shklovskii and A. L. Efros, Electronic Properties of Doped Semiconductors

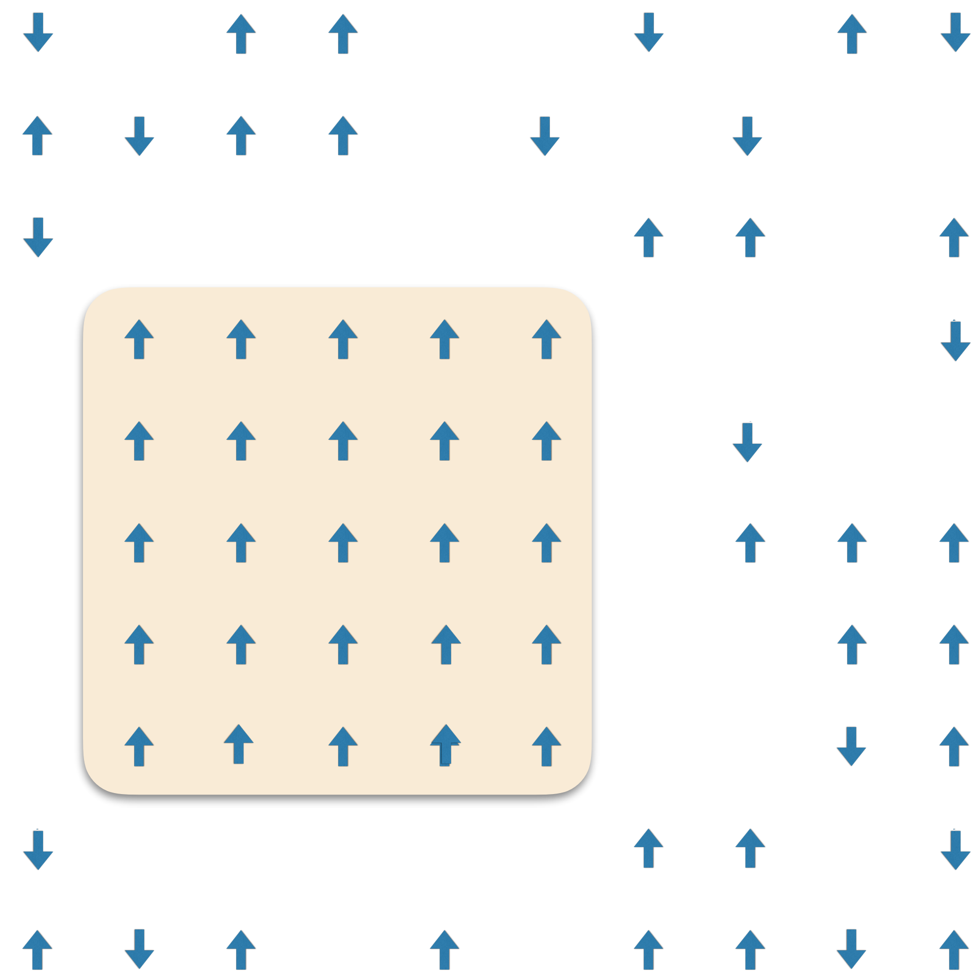
Griffiths Physics





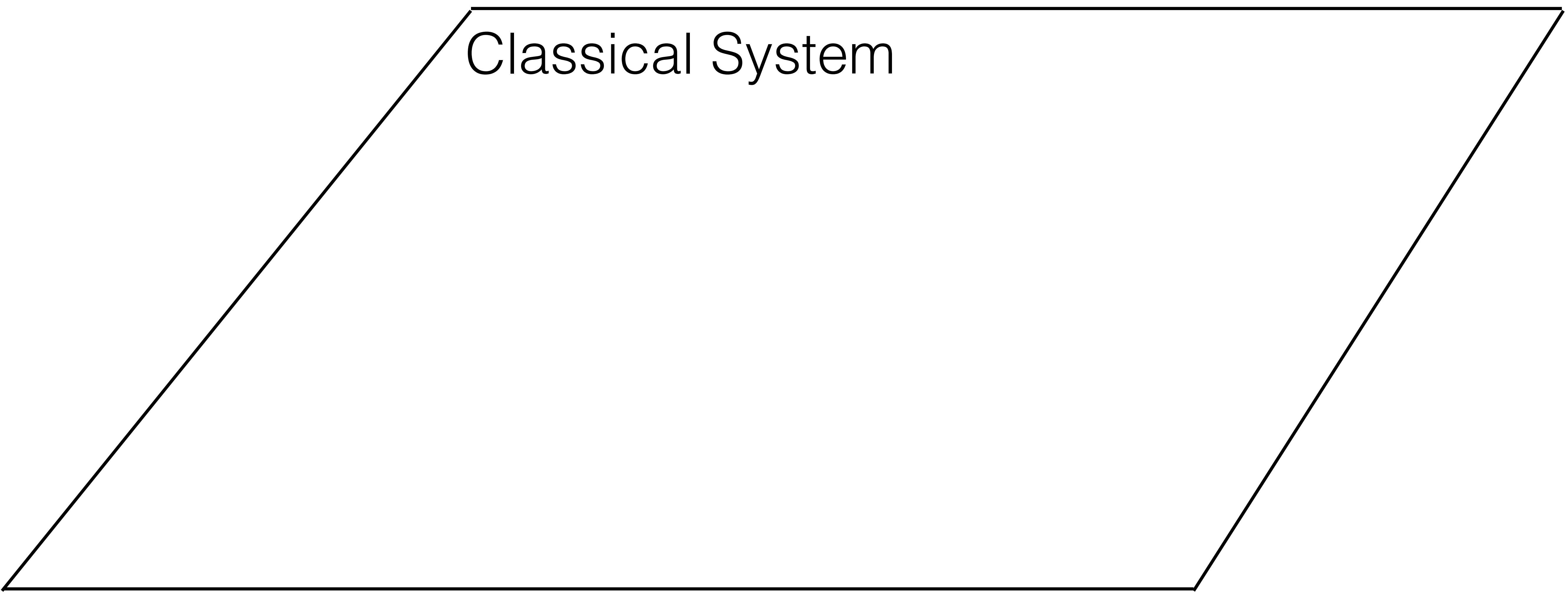
Griffiths Physics





$$w(L) \sim (1-p)^{(L/a_0)^d} = \exp\left(-cL^d\right)$$

Dimensionality of Rare Region



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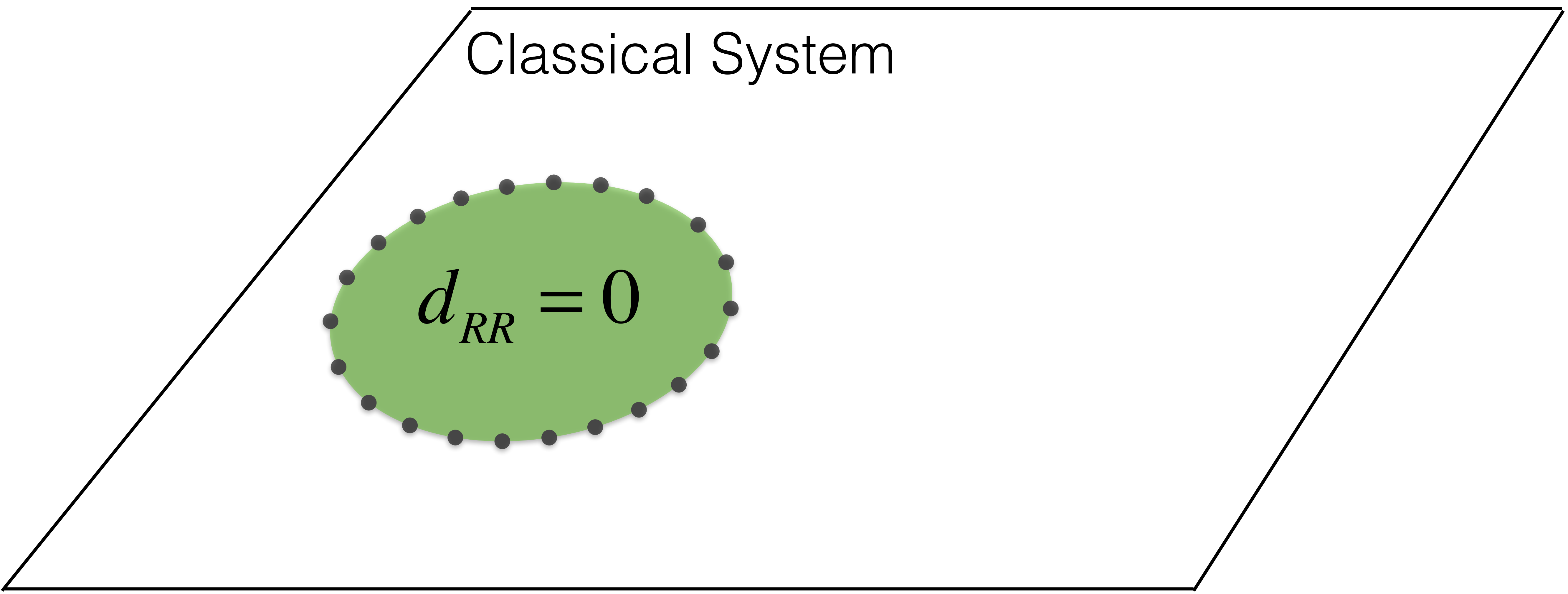

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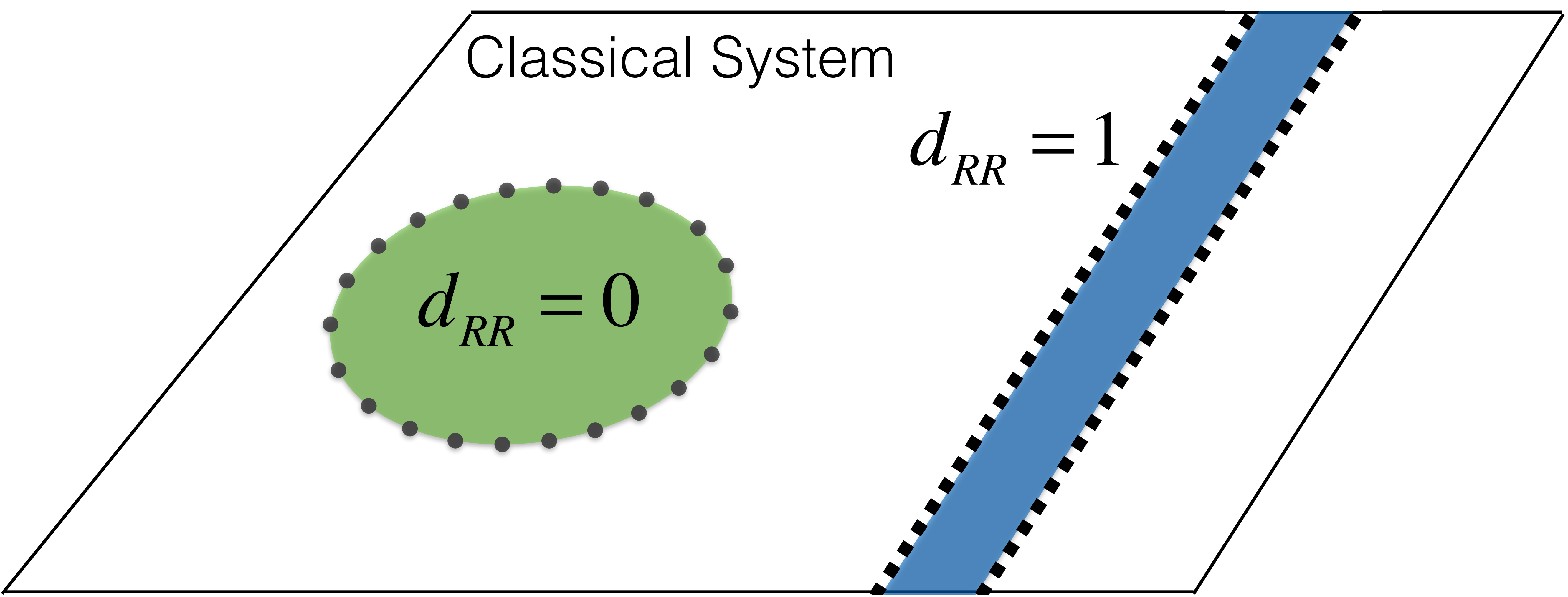

Griffiths Physics

$$w(V) \sim \exp(-cV)$$

Griffiths Physics

$$\boxed{d_{RR} < d_c^-}$$

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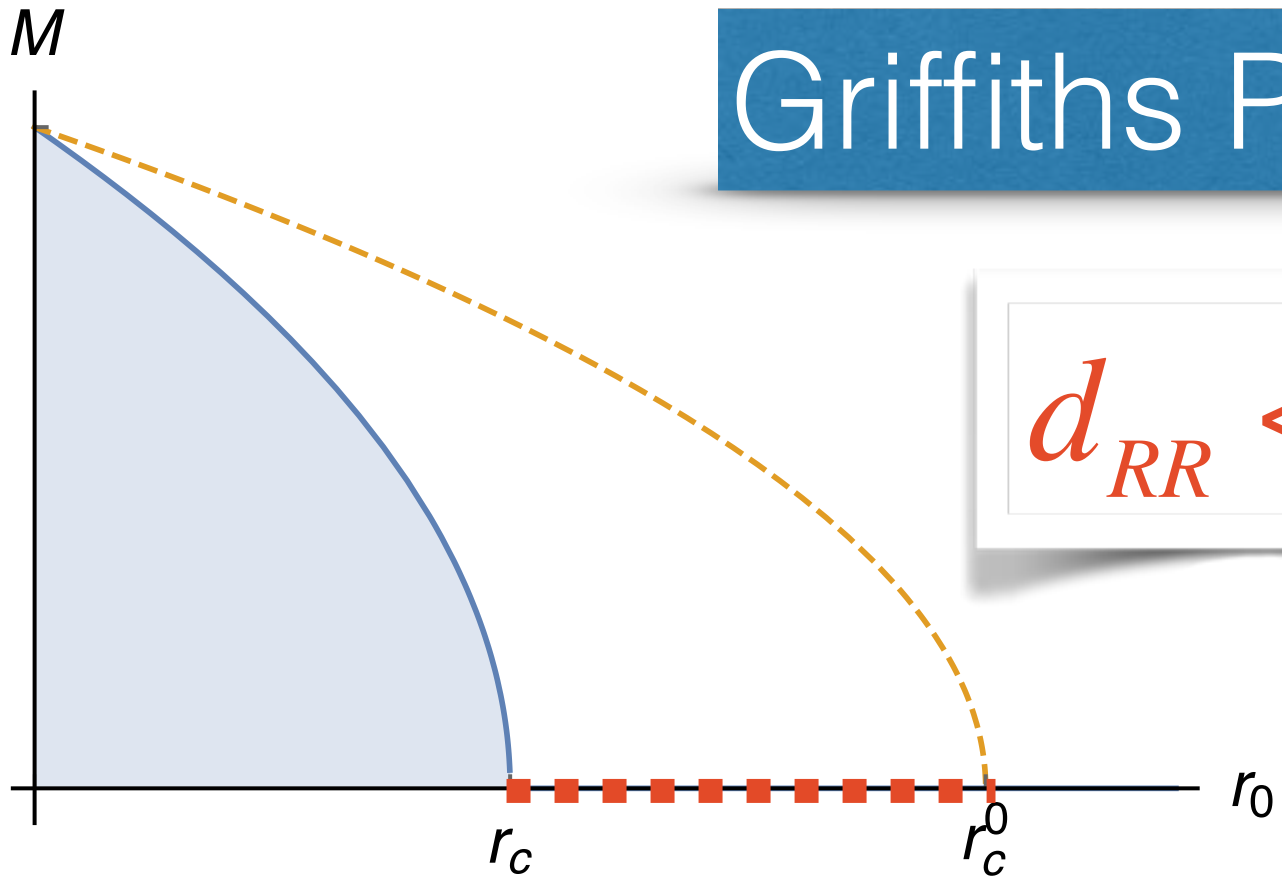

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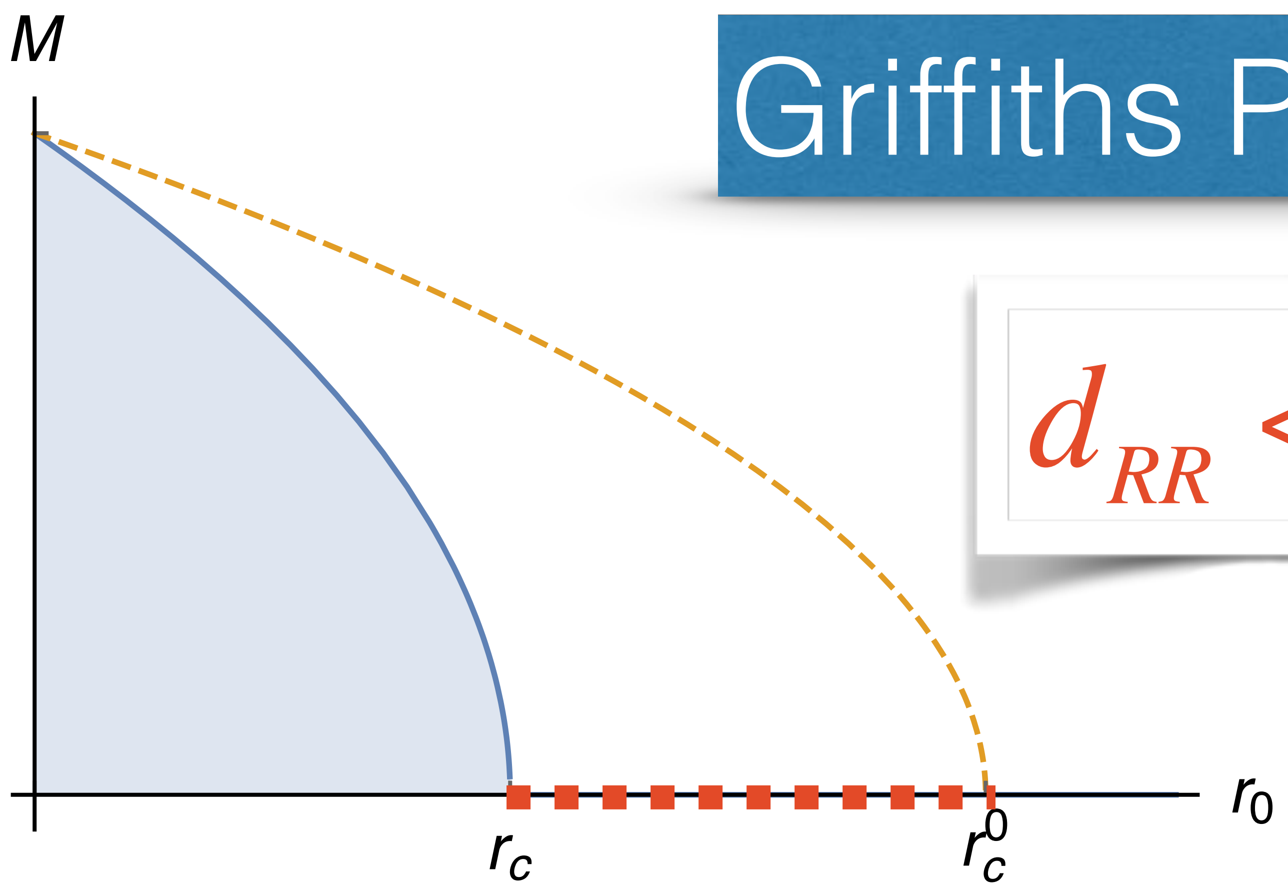

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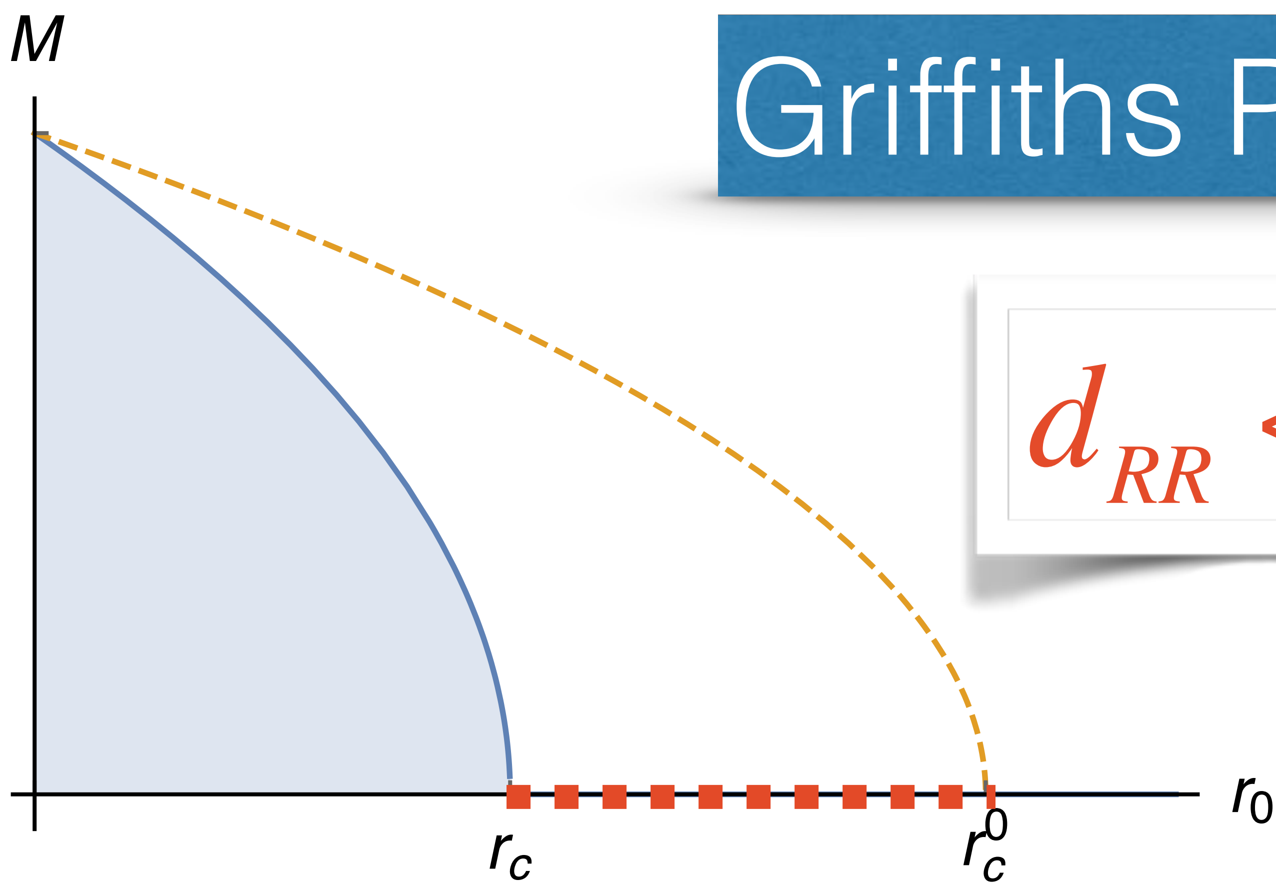
Energy of the lowest excitation

Griffiths Physics



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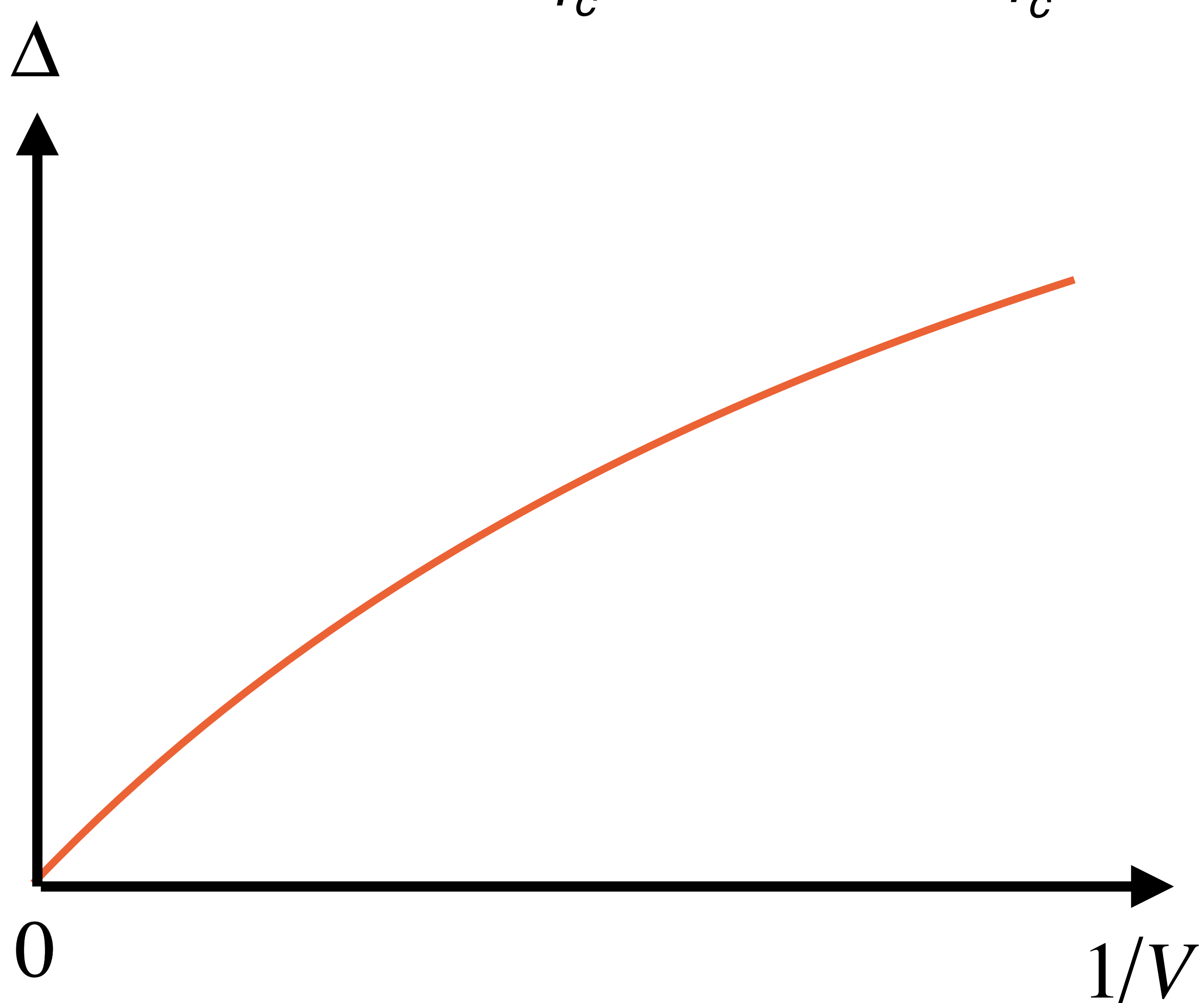

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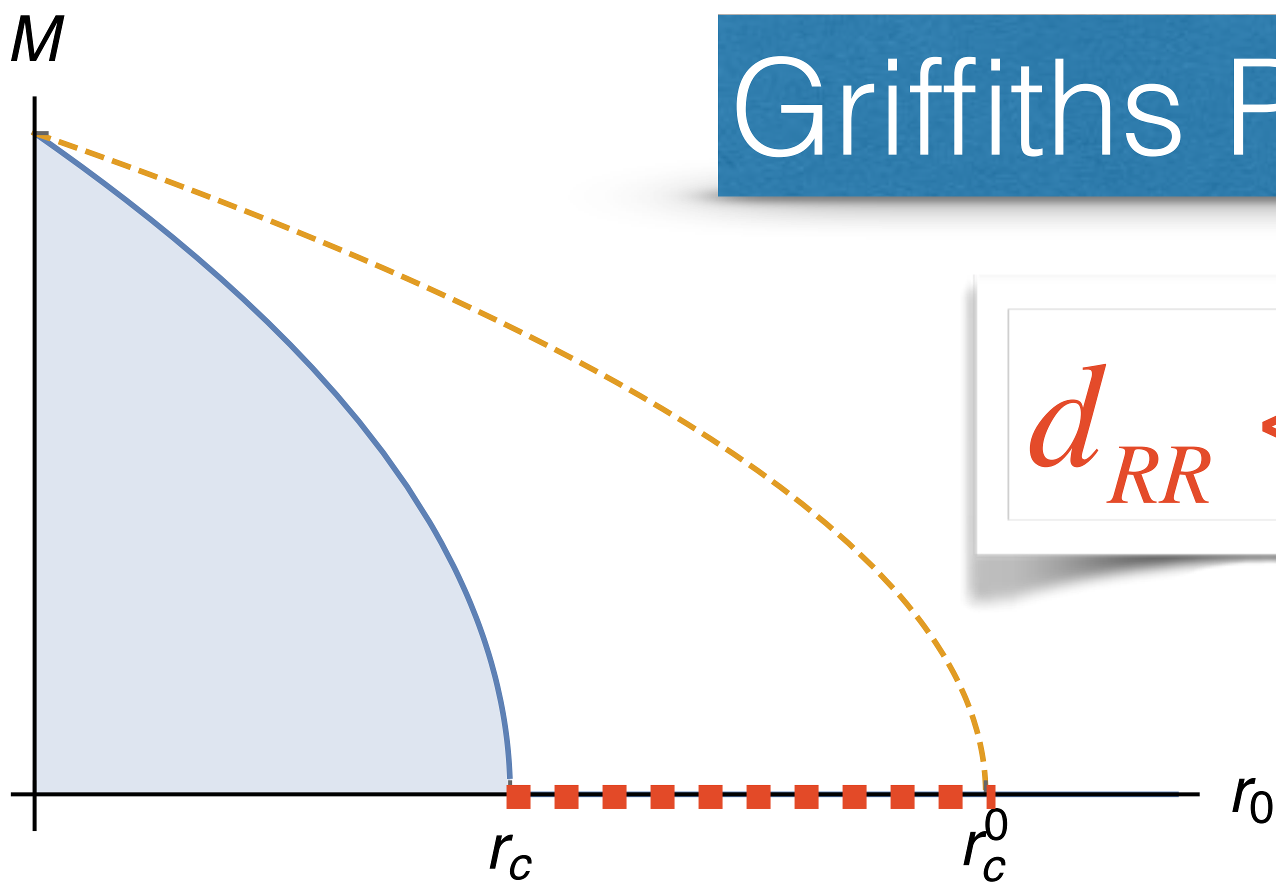

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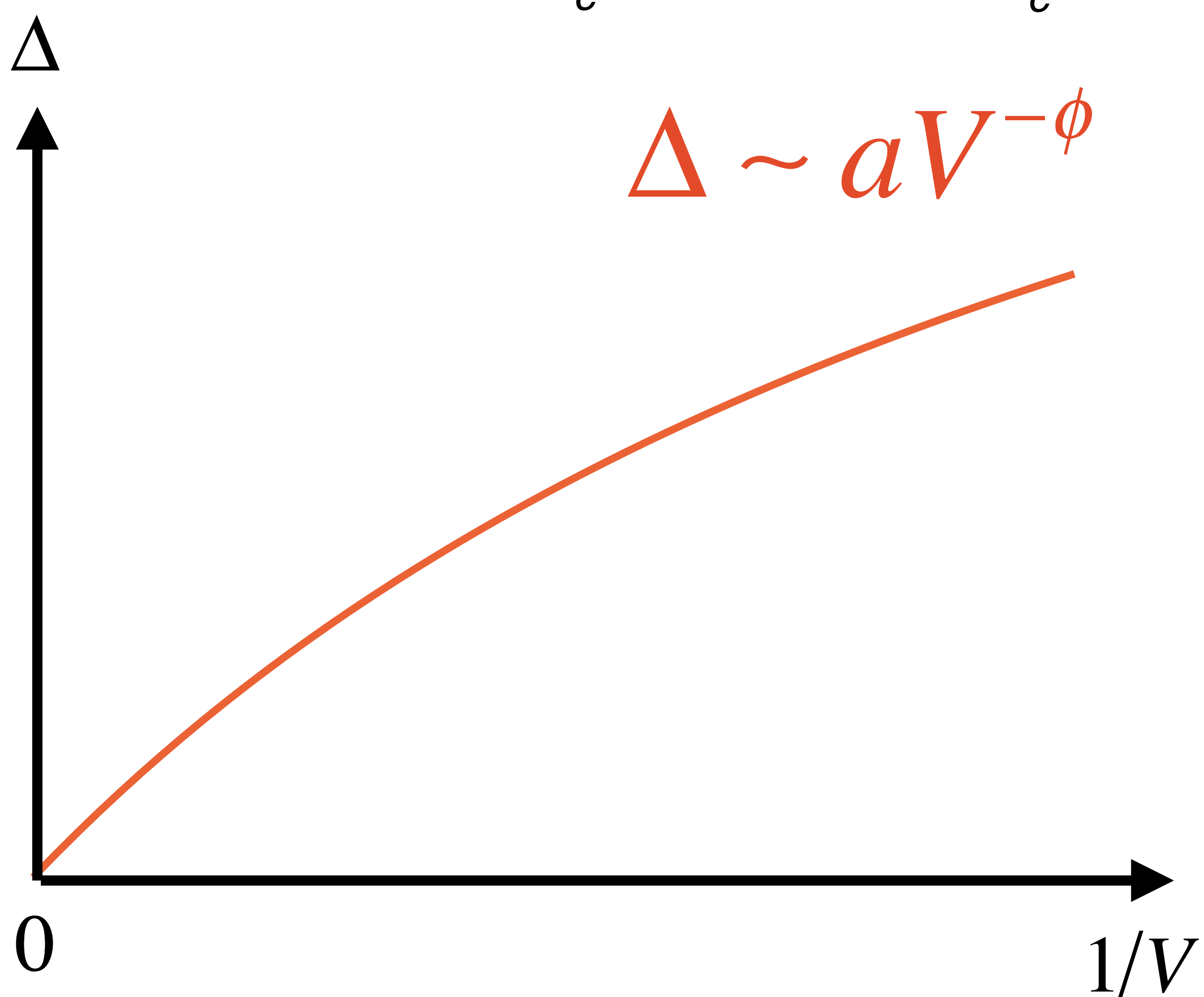
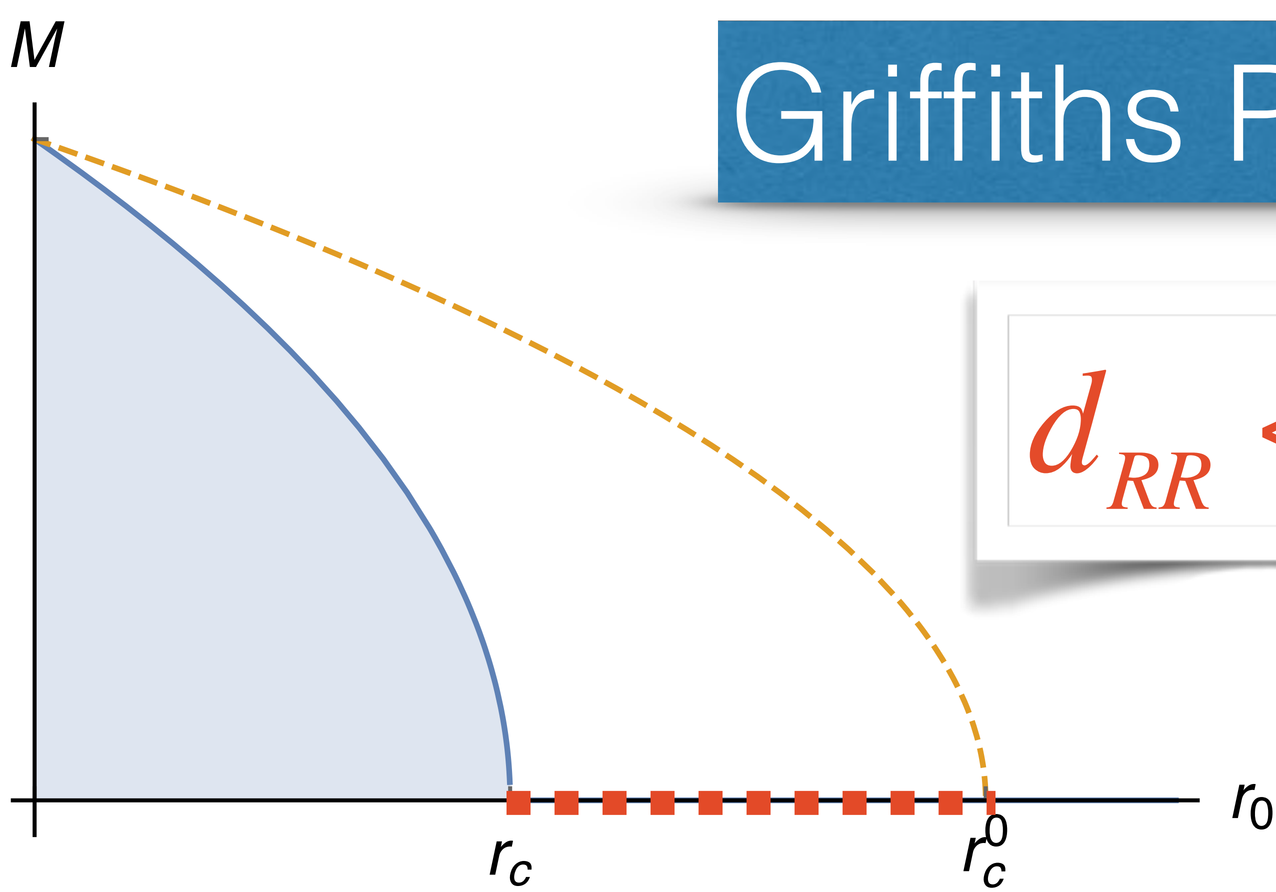


$$\Delta \sim aV^{-\phi}$$

Energy of the lowest excitation

Griffiths Physics



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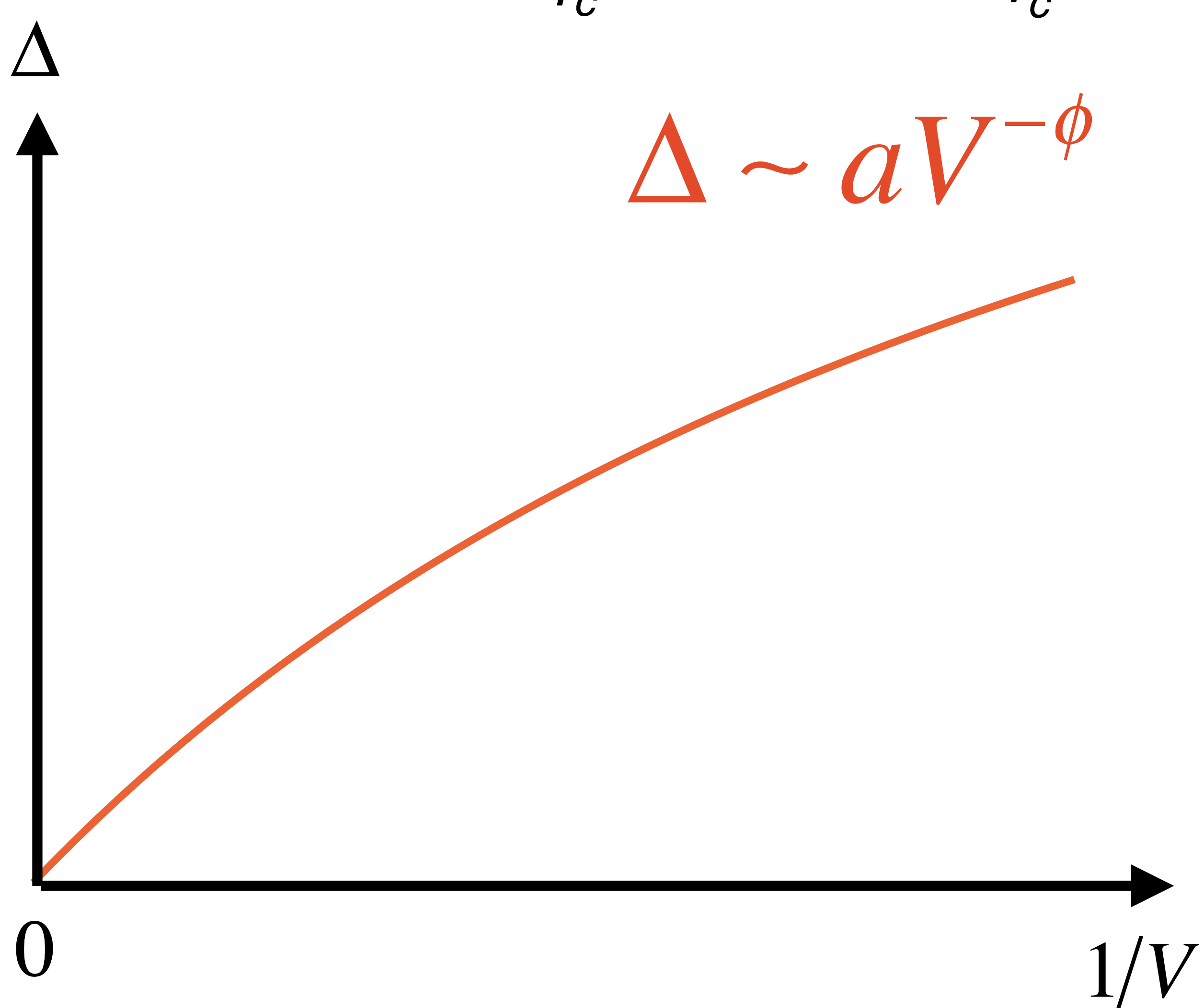
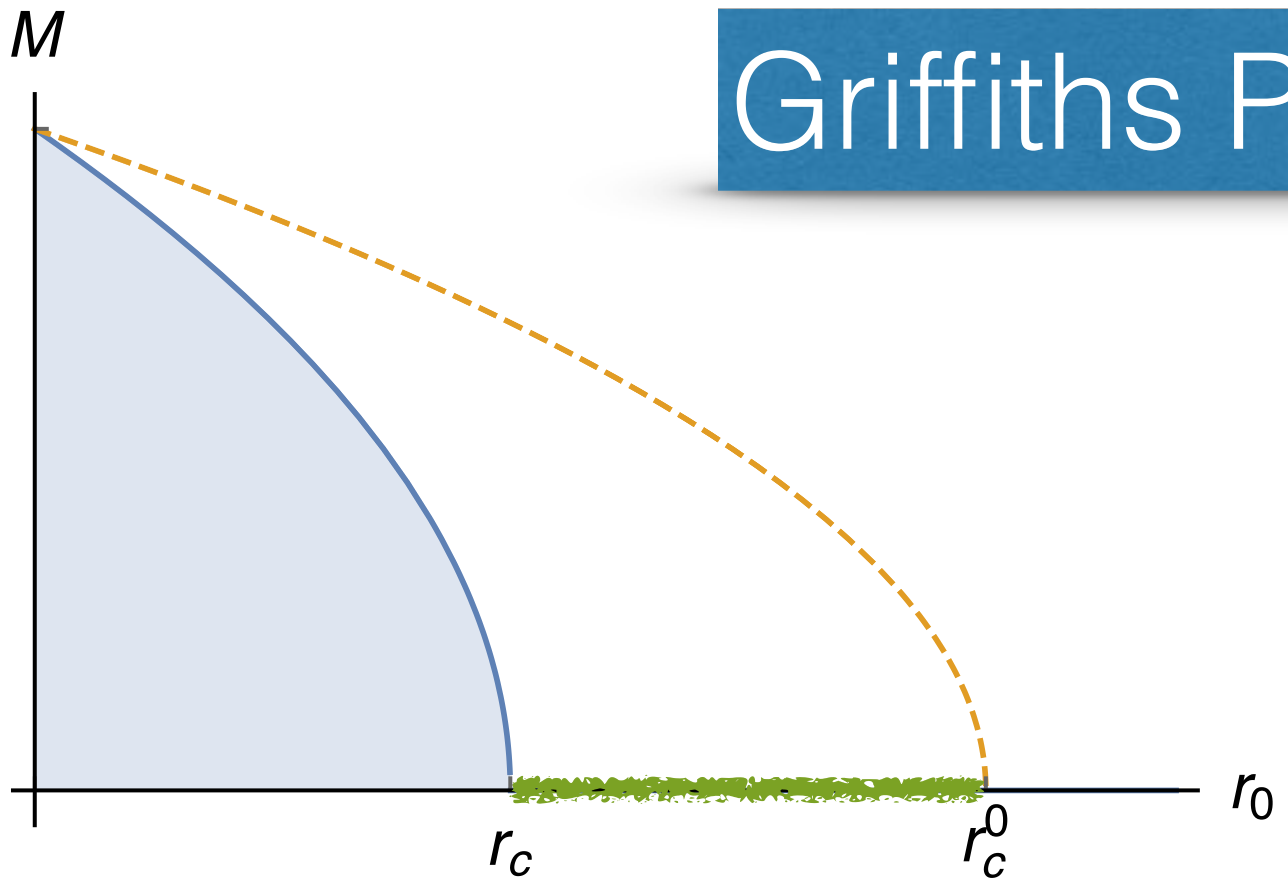


$$\rho(\Delta) \sim e^{-c(a/\Delta)^{1/\phi}}$$

Griffiths Physics



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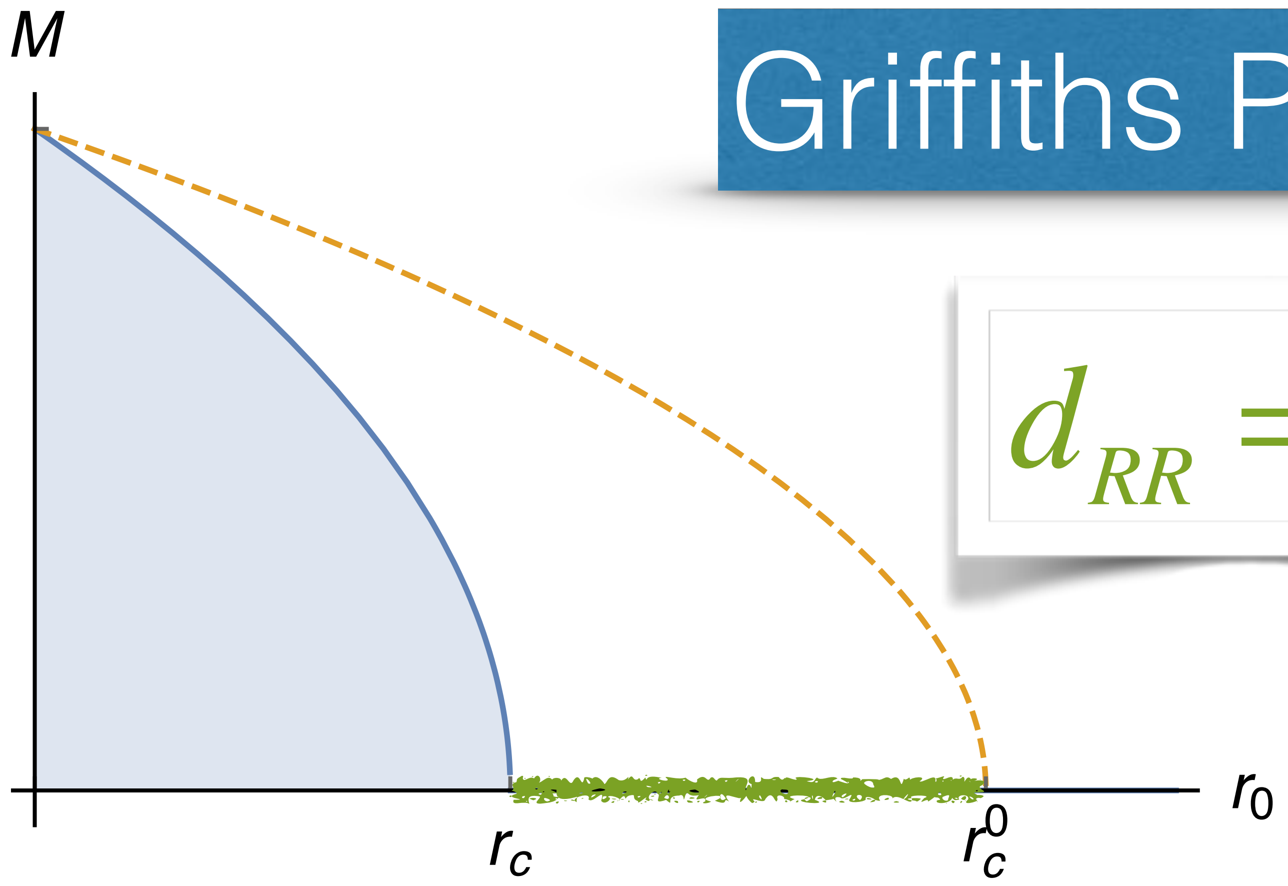

$$\boxed{d_{RR} = d_c^-}$$

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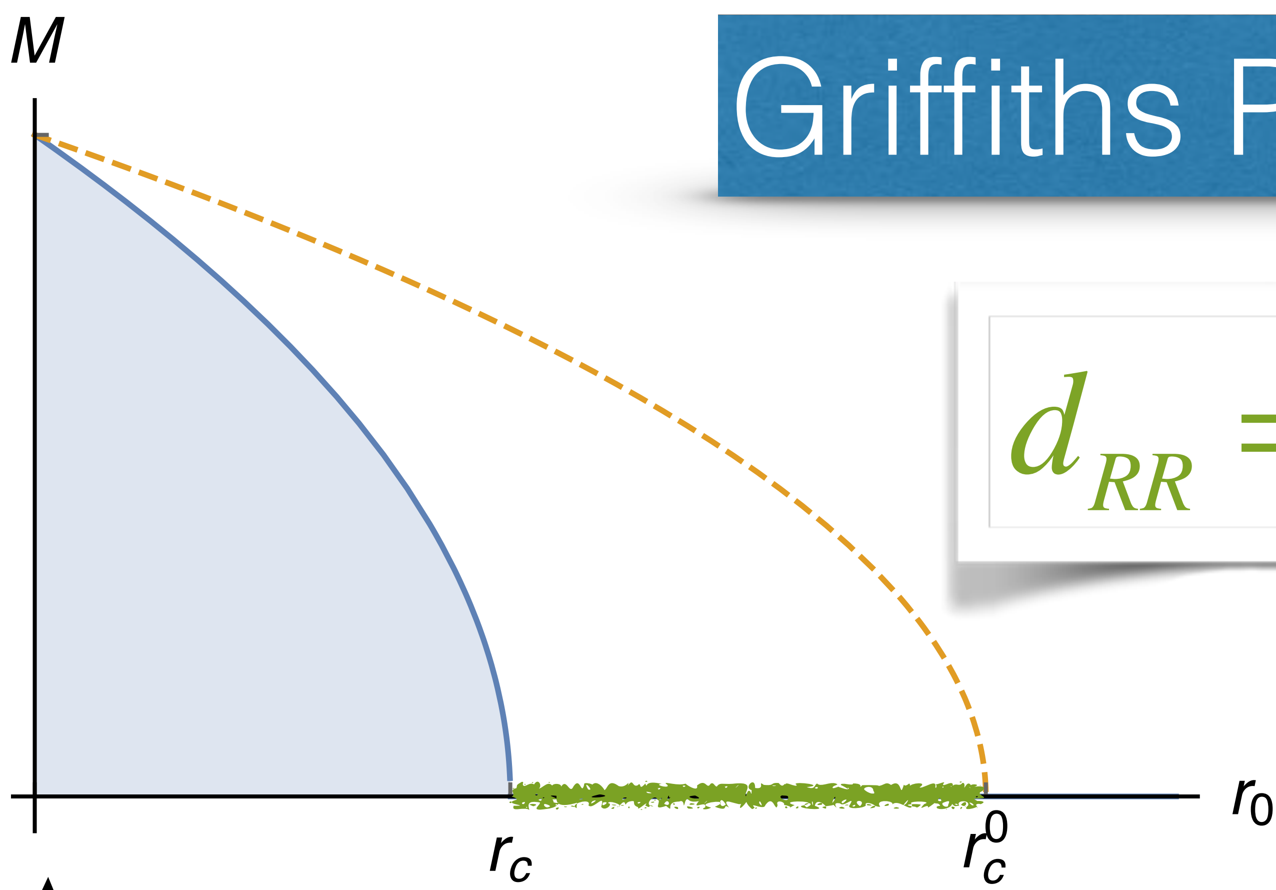
Griffiths Physics



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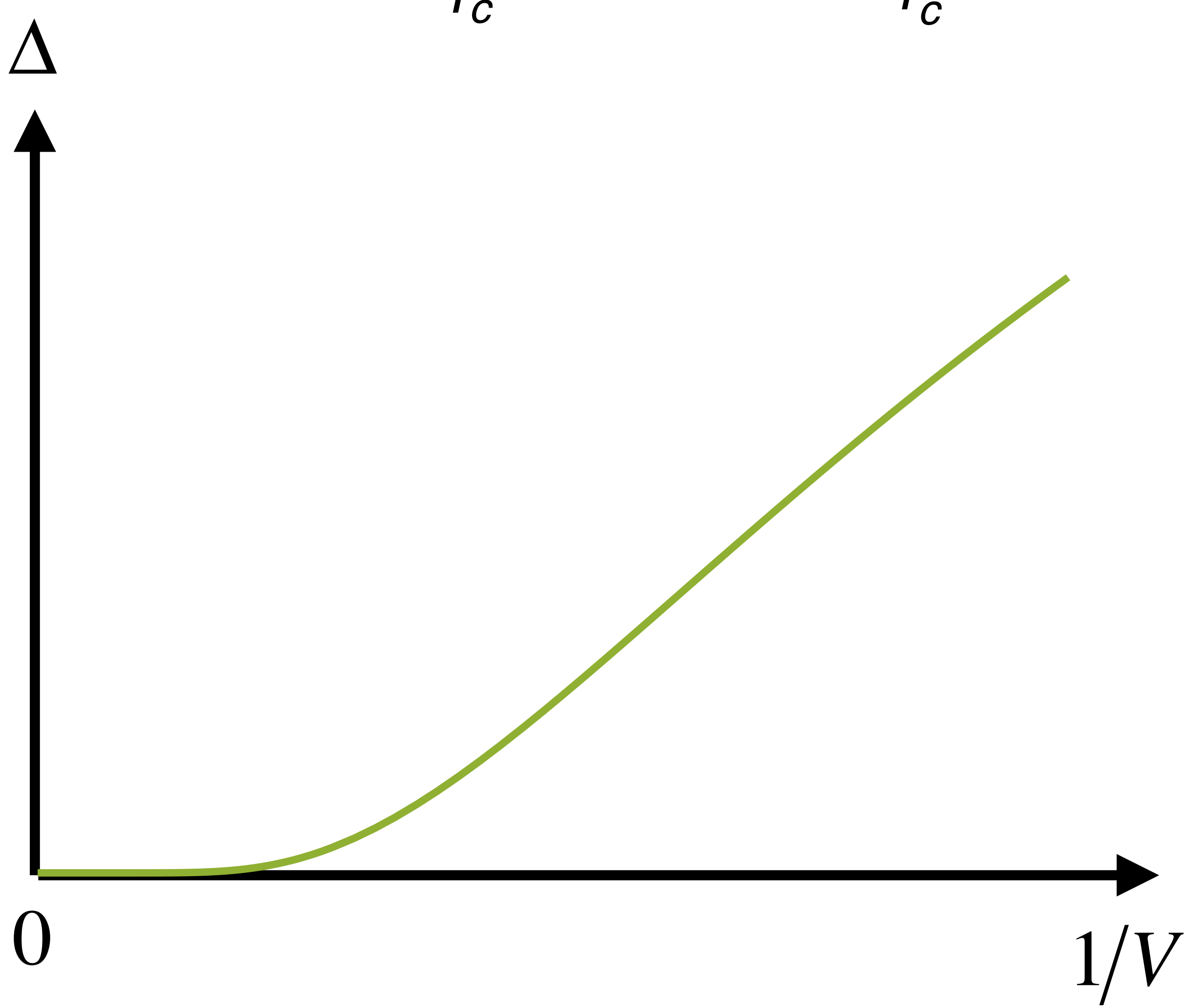

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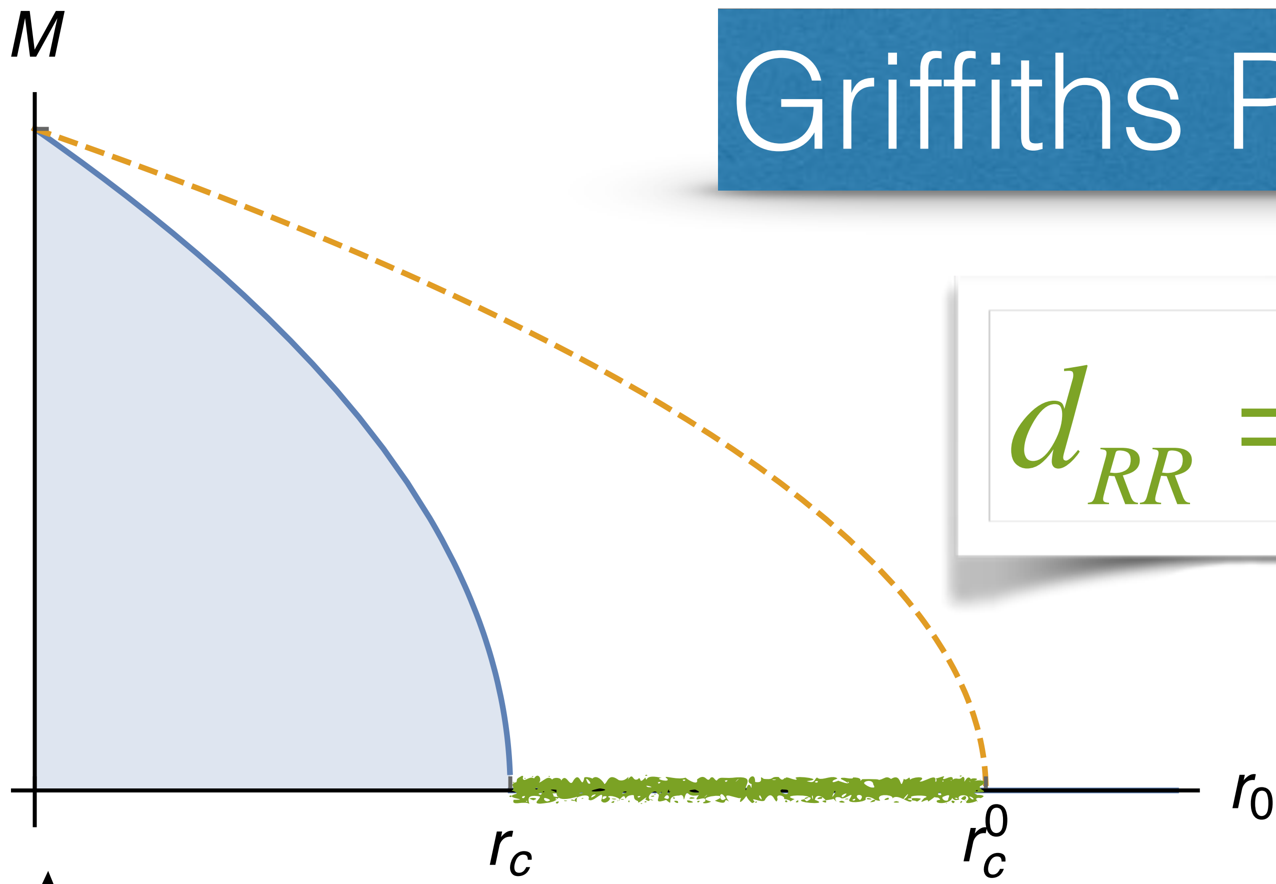

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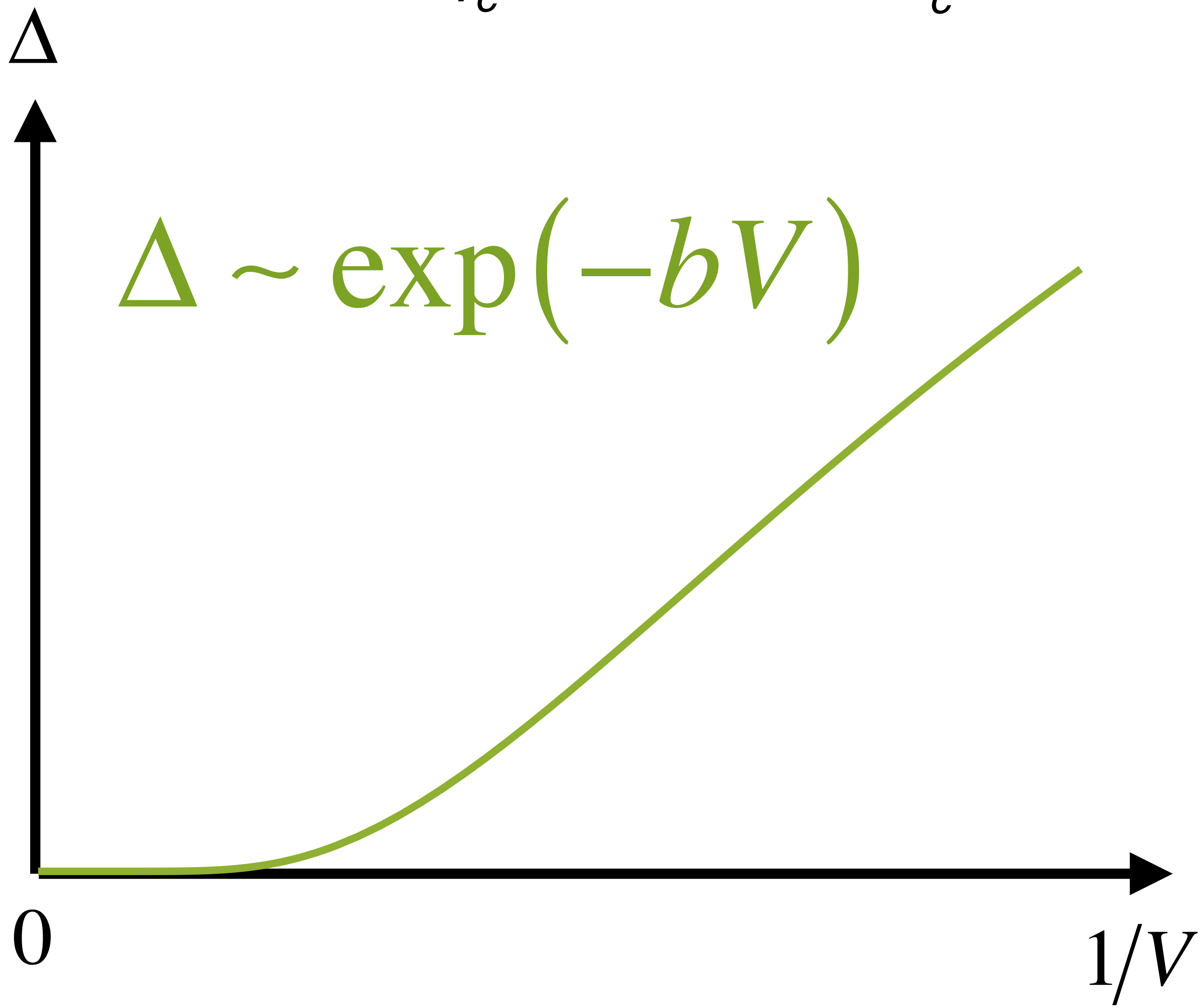

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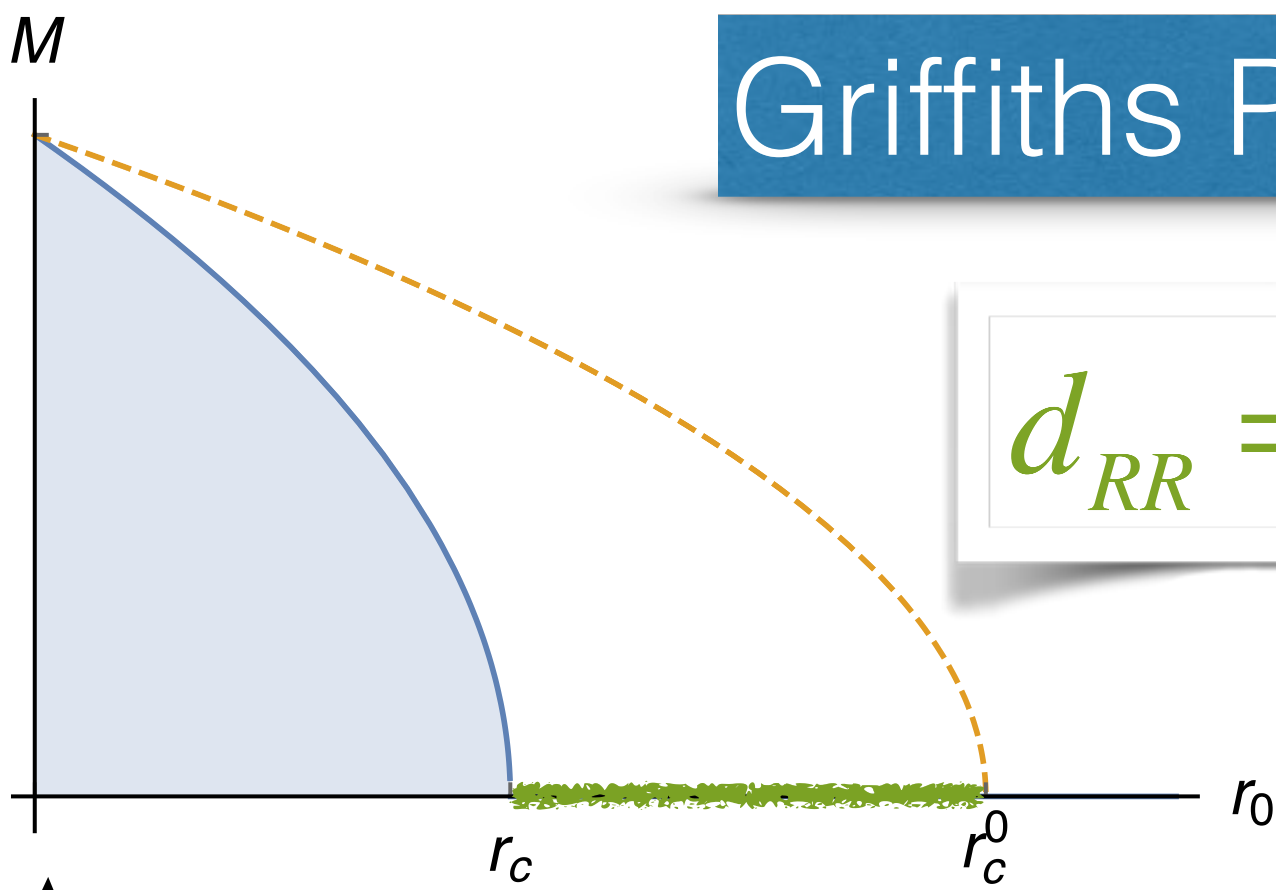

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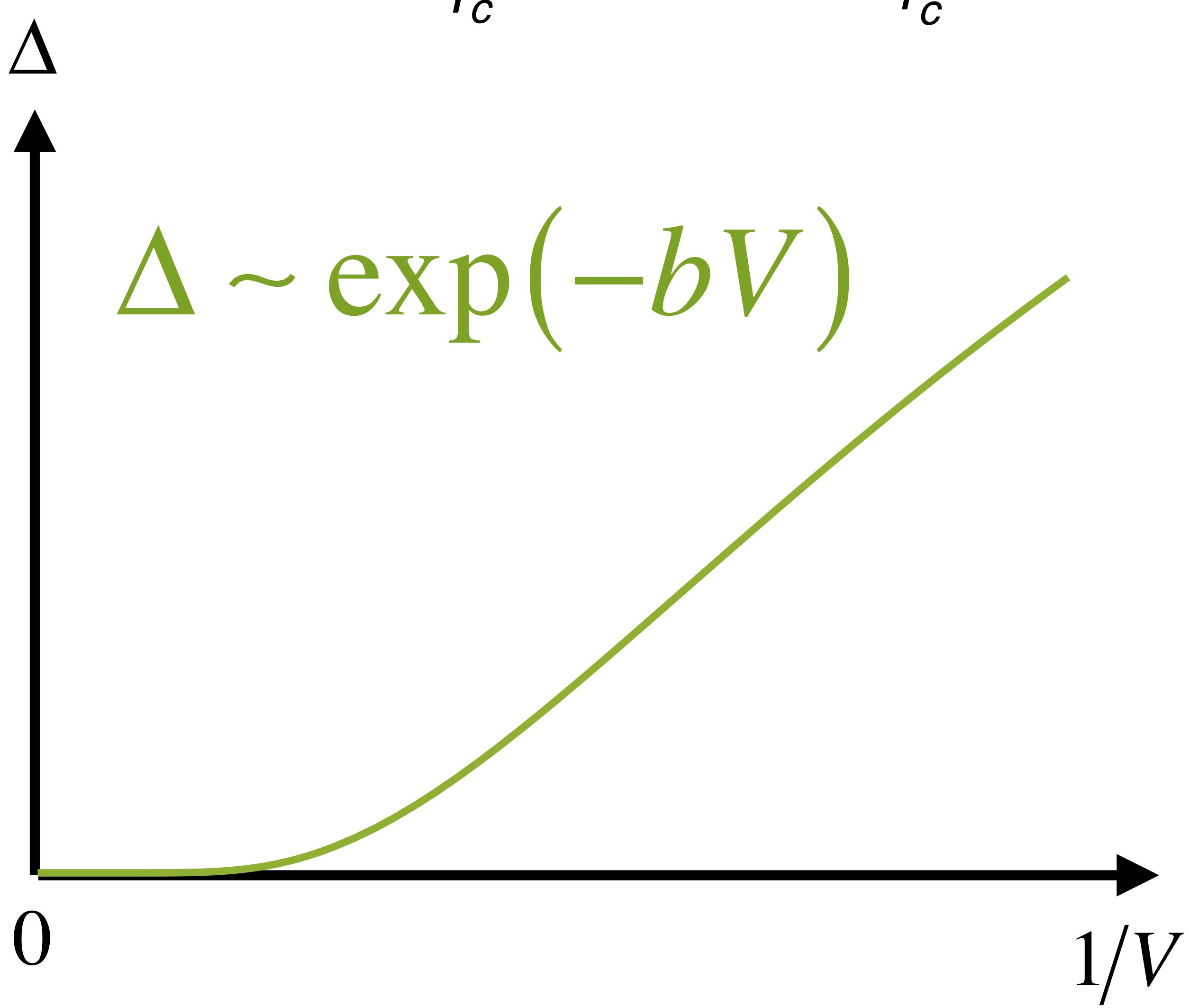
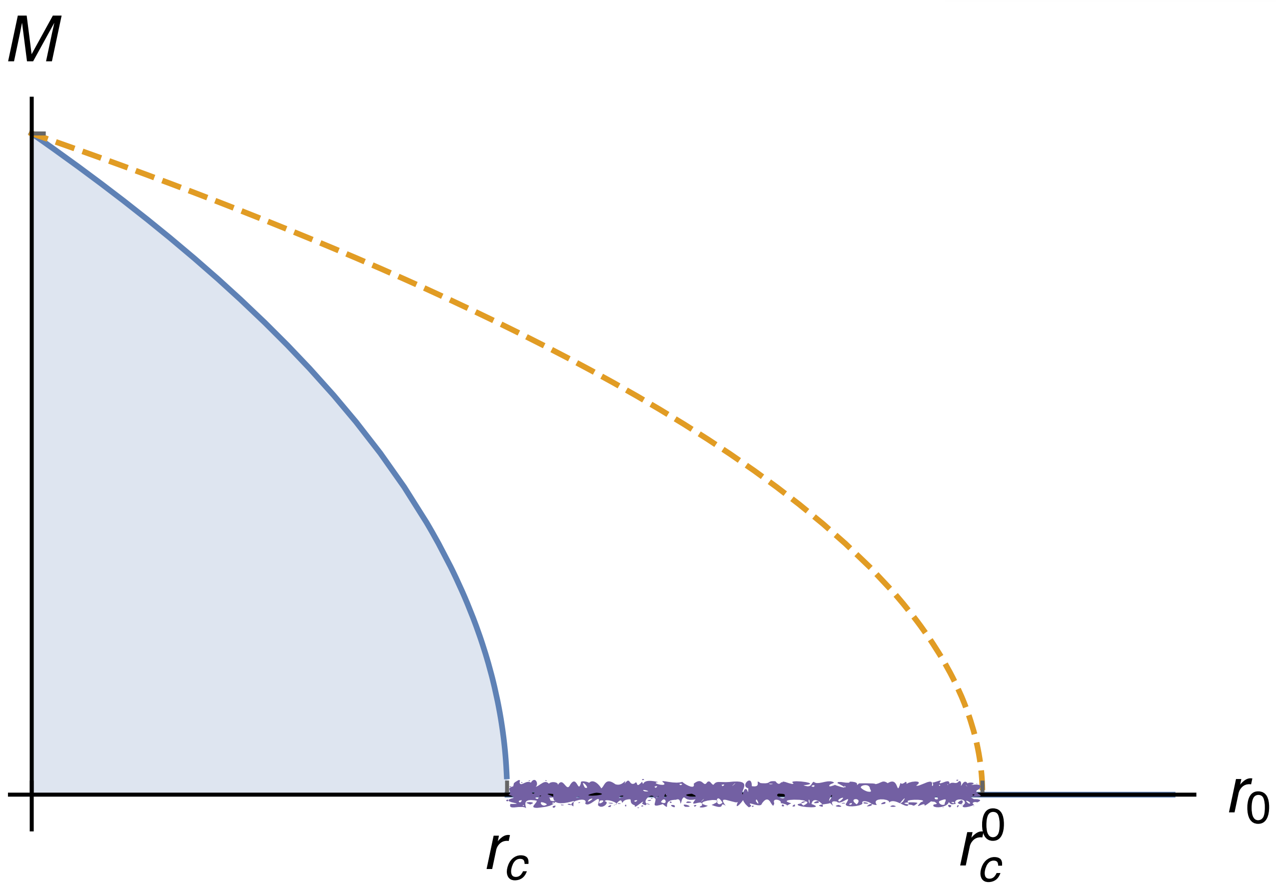


$$\rho(\Delta) \sim \Delta^{c/b-1}$$

Griffiths Physics



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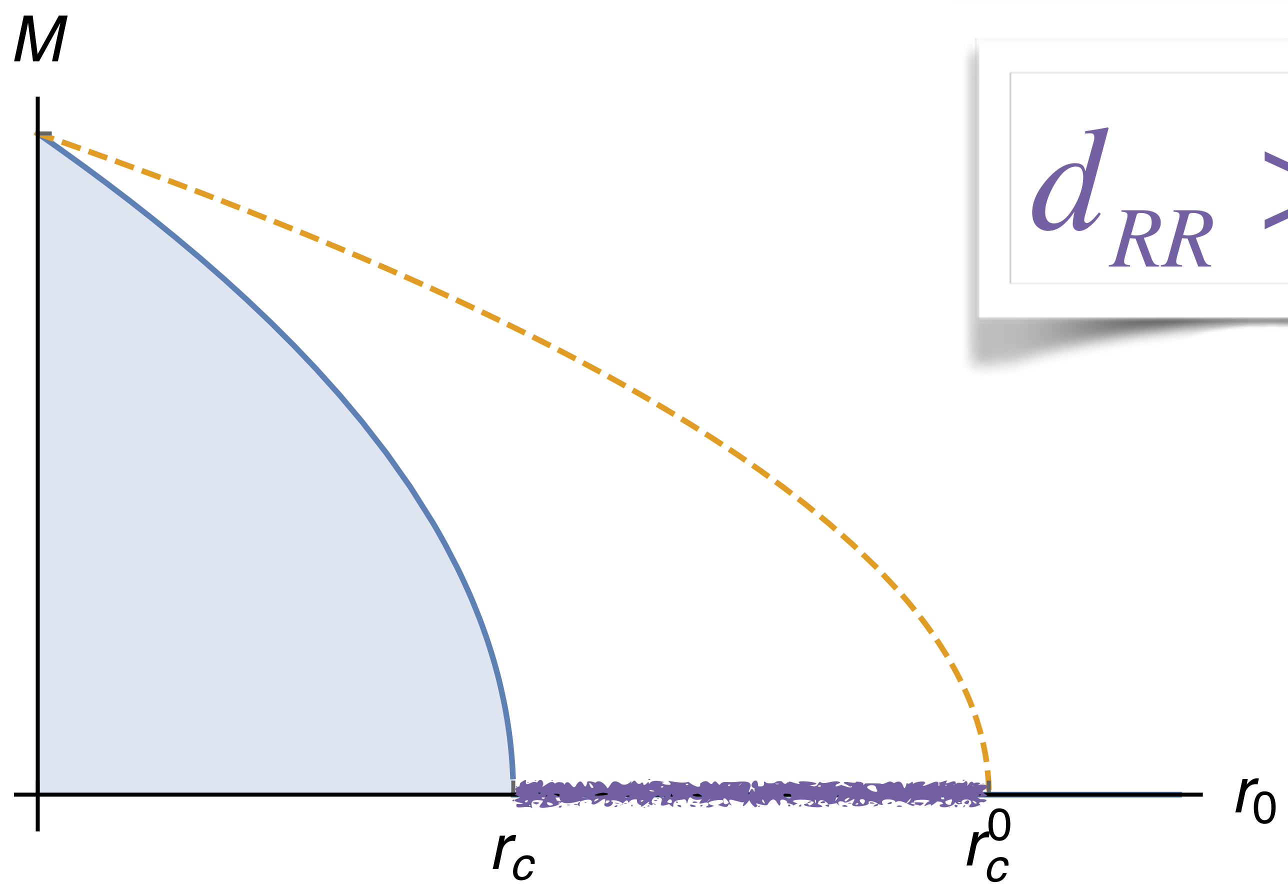

$$d_{RR} > d_c^-$$

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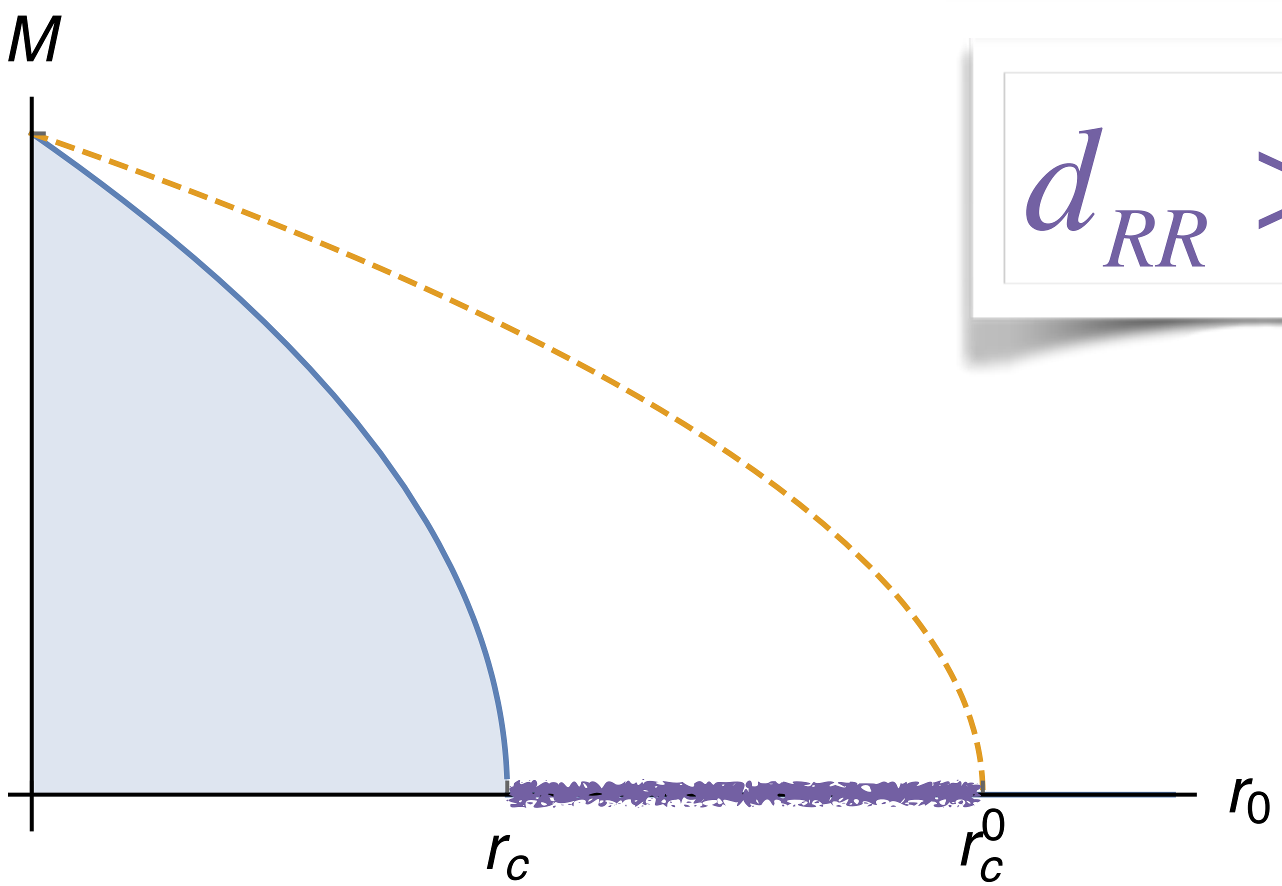
Griffiths Physics



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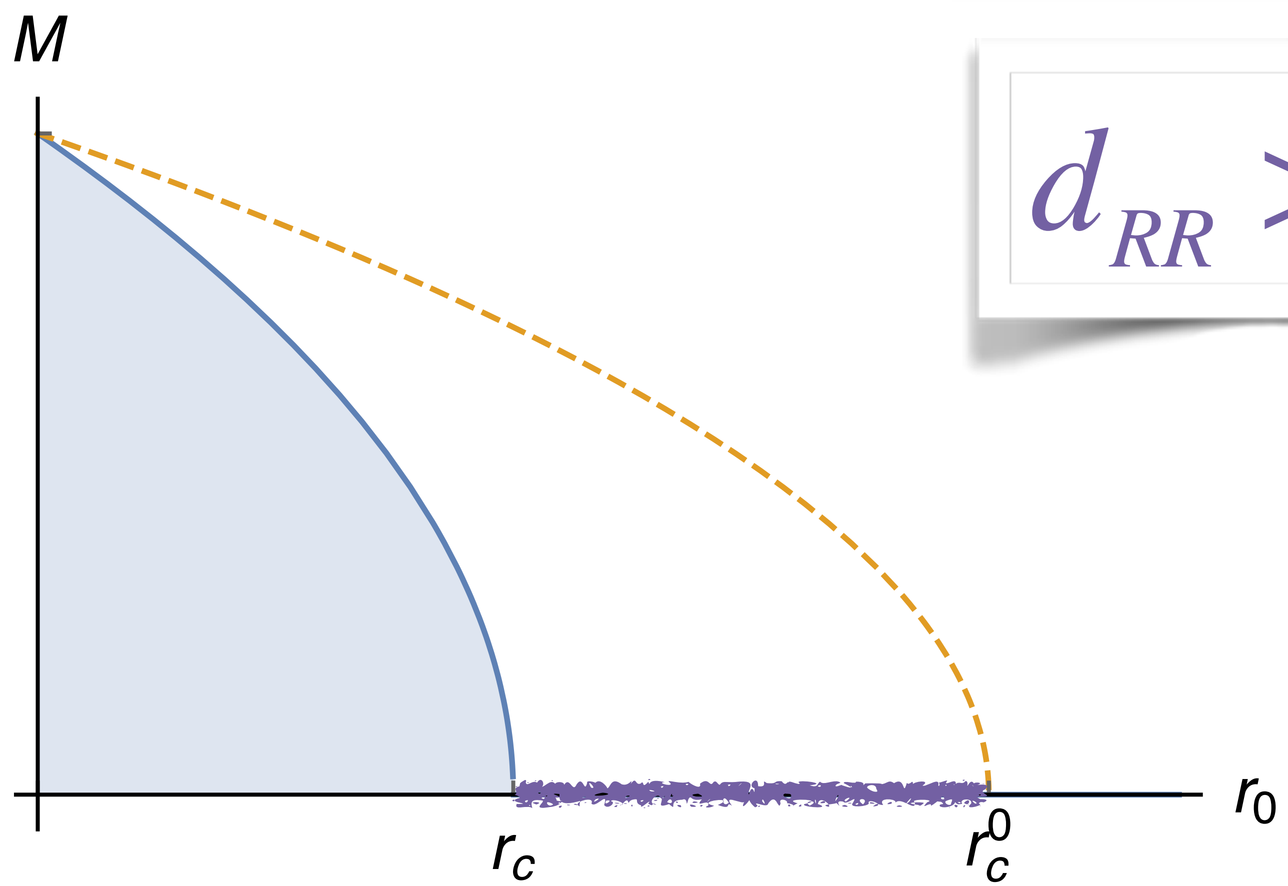
$$\Delta = \Delta_0 - \frac{\alpha}{V^\gamma}$$

Griffiths Physics



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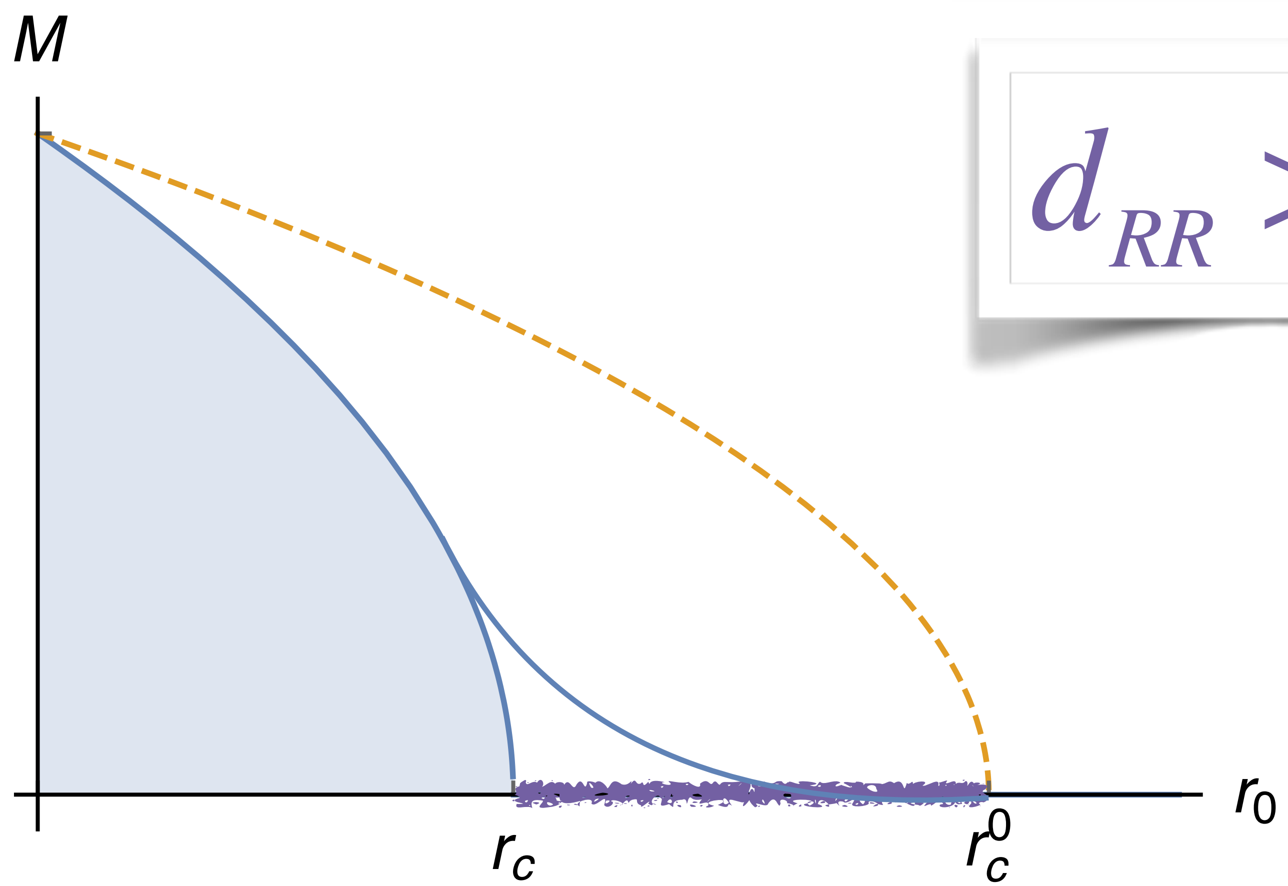
Griffiths Physics



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Griffiths Physics: Classification

Class	Rare Region Dimension	Rare Region Effects
A	$d_{RR} < d_c^-$	Weak exponential
B	$d_{RR} = d_c^-$	Strong power law
C	$d_{RR} > d_c^-$	Rare regions is ordered

[1] T. Vojta, J. Phys. A. Math. Gen. 39, R143 (2006).

Classical vs Quantum Phase Transitions

Classical

Quantum

$$z = 2$$

[1] J. A. Hertz, Phys. Rev. B **14**, 1165 (1976).
[2] A. J. Millis, Phys. Rev. B **48**, 7183 (1993).

Classical vs Quantum Phase Transitions

Classical

Quantum

$$d_{RR} = 0$$

$$z = 2$$

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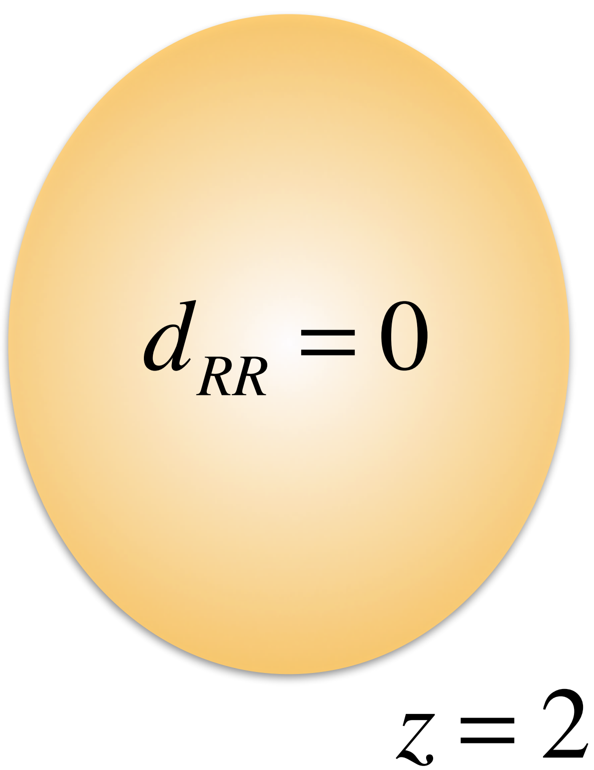

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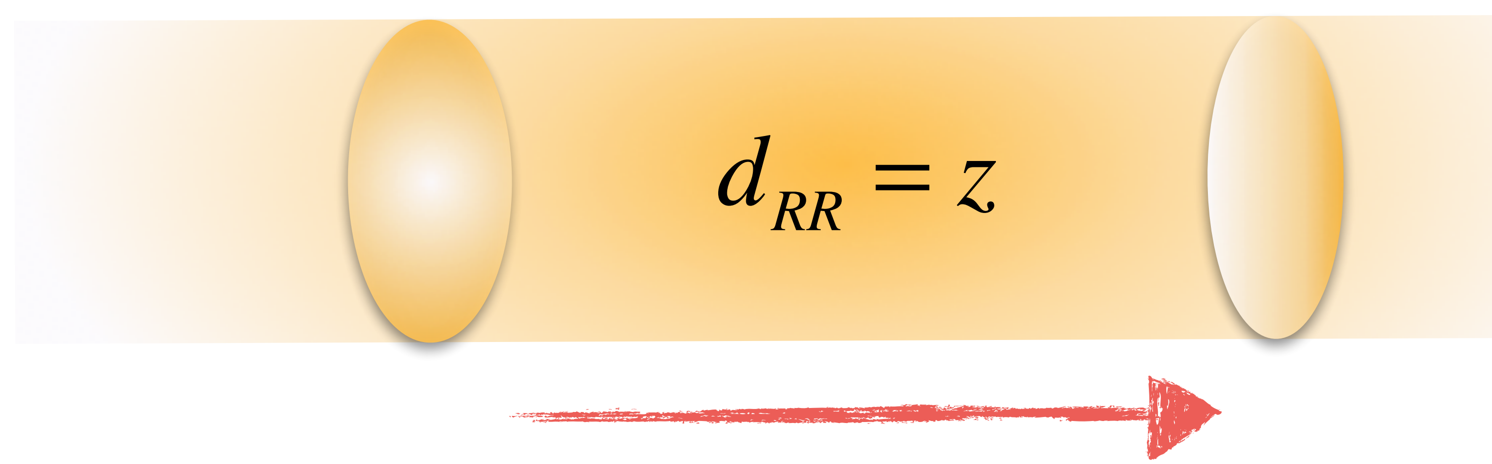

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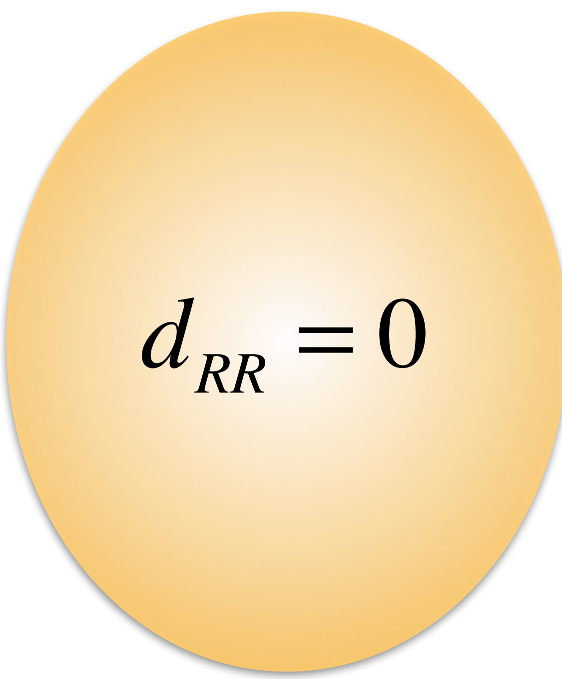
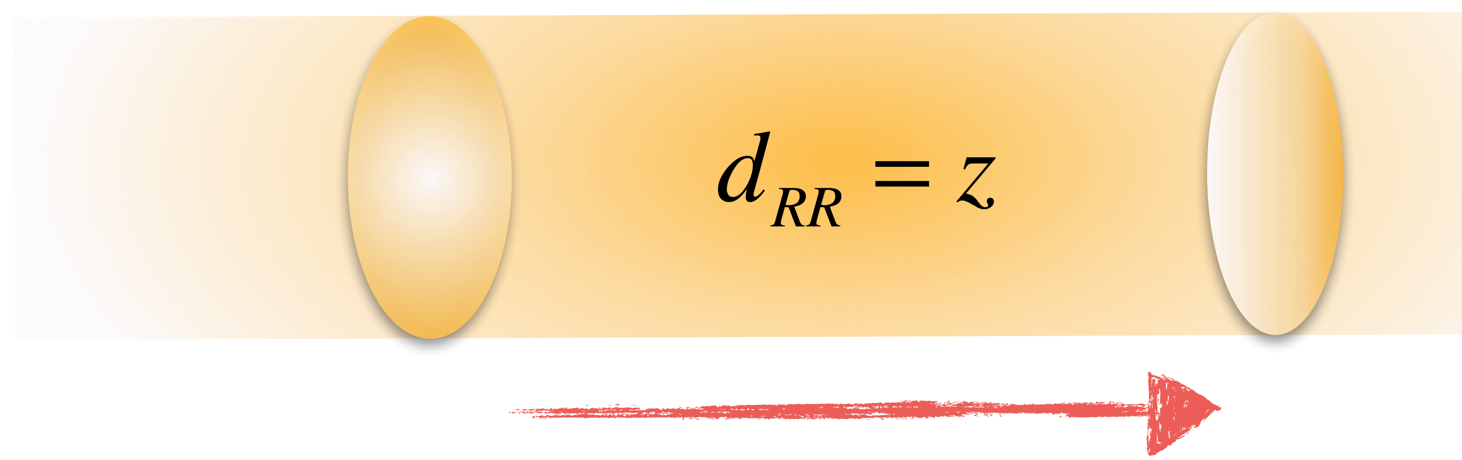




$z = 2$

$$z = \text{dynamical critical exponent} \begin{cases} z = 1 \\ z = 2 \\ z = 3 \end{cases}$$

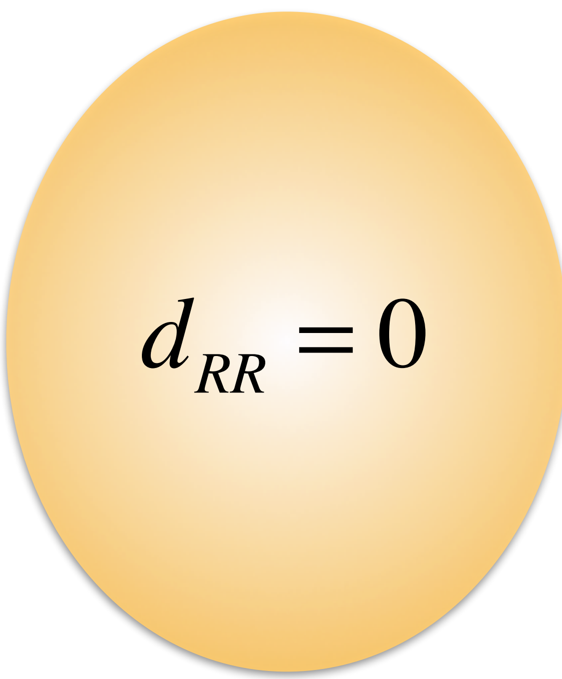
[1] J. A. Hertz, Phys. Rev. B **14**, 1165 (1976).

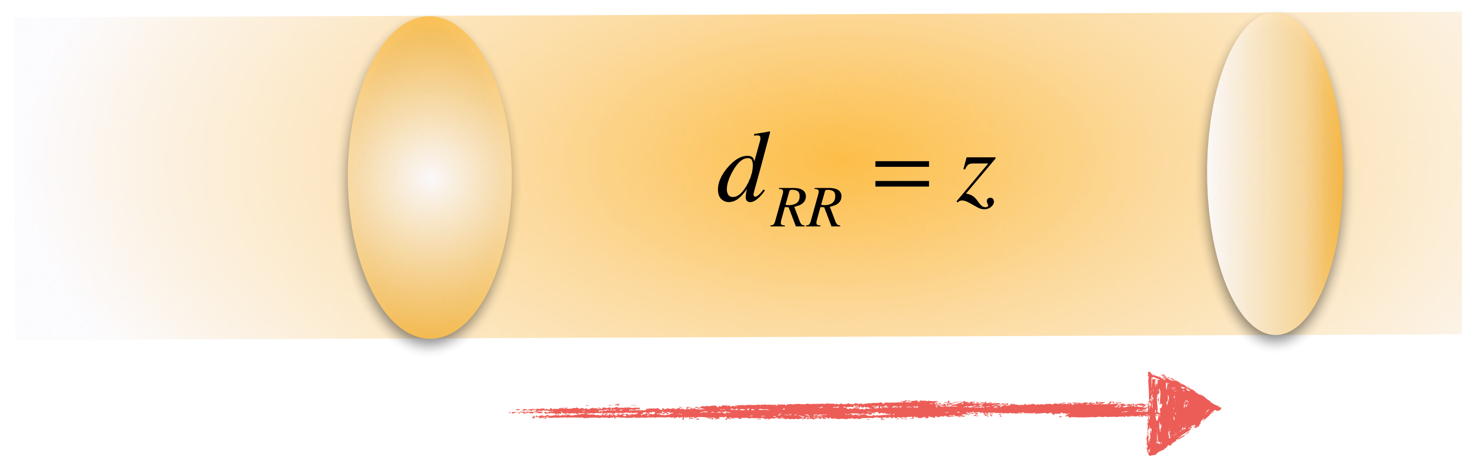
[2] A. J. Millis, Phys. Rev. B **48**, 7183 (1993).

Classical vs Quantum Phase Transitions

Classical

Quantum





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Outline

- Rare region effects on phase transitions: Griffiths physics
- Rare region effects on the Ising-nematic quantum phase transition

Motivation

	Lower-critical dimension	Dimension of rare-region	Order or not
Ising-nematic	$d_c^- = 1$	$d_{RR} = z = 2$	
Magnetic	$d_c^- = 2$	$d_{RR} = z = 2$	

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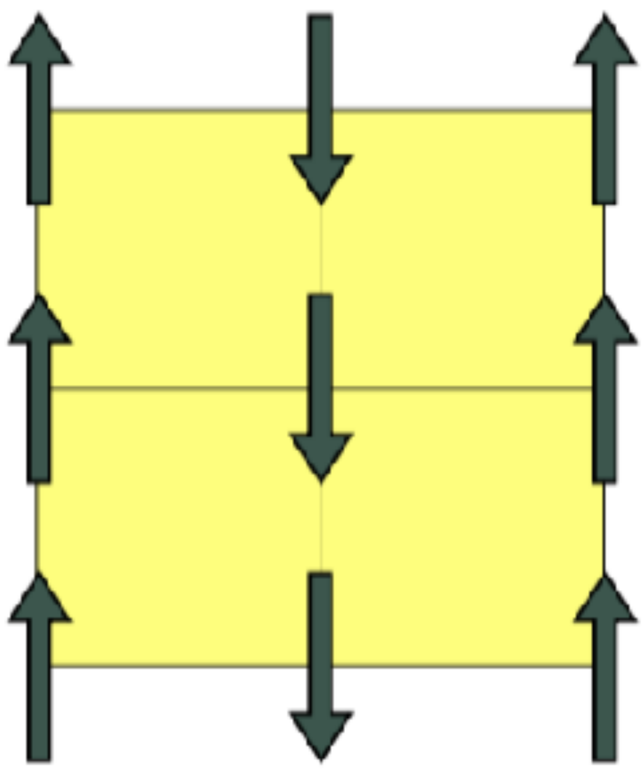
Split transitions

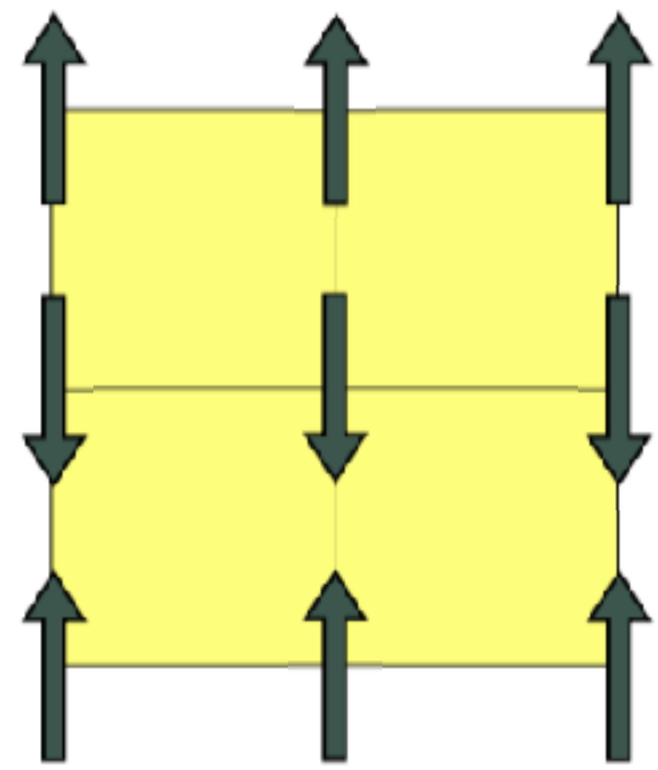
Landau-Ginzburg-Wilson Theory

$$S\left[\mathbf{M}_X^2, \mathbf{M}_Y^2\right] = \int_{q,\omega} \left\{ \chi_{q,\omega}^{-1} \left(\mathbf{M}_X^2 + \mathbf{M}_Y^2\right) + \frac{u}{2}\left(\mathbf{M}_X^2 + \mathbf{M}_Y^2\right)^2 - \frac{g}{2}\left(\mathbf{M}_X^2 - \mathbf{M}_Y^2\right)^2 \right\}$$

$$\chi_{q,\omega}^{-1} = r_0 + q^2 + \gamma|\omega| \qquad u > 0 \qquad g > 0$$



$\mathbf{M}_X$



$\mathbf{M}_Y$

Landau-Ginzburg-Wilson Theory

$$S\left[\mathbf{M}_X^2,\mathbf{M}_Y^2\right]=\int_{q,\omega}\left\{\chi_{q,\omega}^{-1}\left(\mathbf{M}_X^2+\mathbf{M}_Y^2\right)+\frac{u}{2}\left(\mathbf{M}_X^2+\mathbf{M}_Y^2\right)^2-\frac{g}{2}\left(\mathbf{M}_X^2-\mathbf{M}_Y^2\right)^2\right\}$$

$$\chi_{q,\omega}^{-1}=r_0+q^2+\gamma|\omega| \qquad u>0 \qquad g>0$$

Magnetic Fluctuation: $\psi \propto u\left\langle \mathbf{M}_X^2+\mathbf{M}_Y^2\right\rangle$

Ising-nematic order parameter: $\phi \propto g\left\langle \mathbf{M}_X^2-\mathbf{M}_Y^2\right\rangle$

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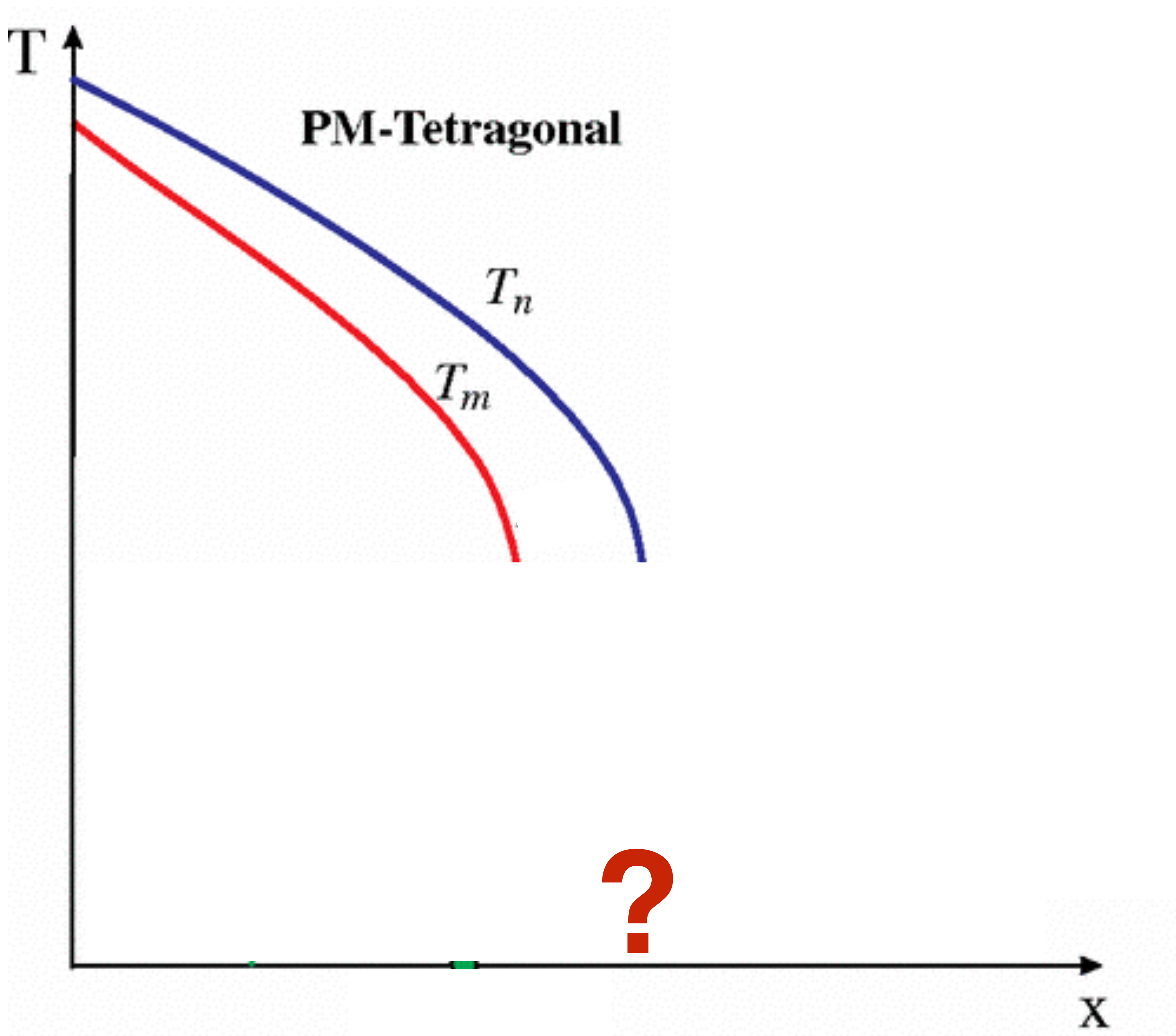
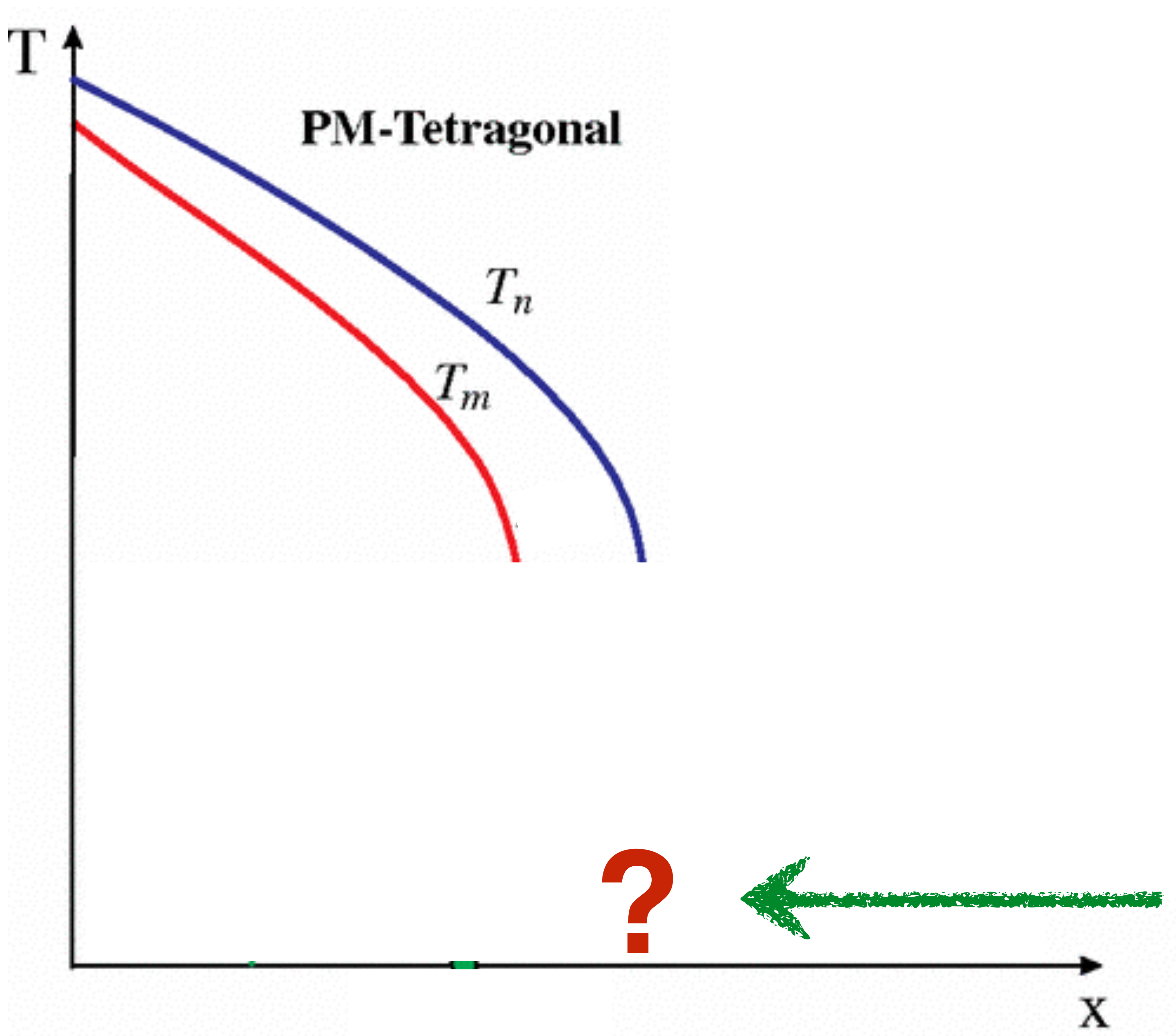
Ising-nematic order parameter: $\phi\propto g\left\langle\mathbf{M}_X^2-\mathbf{M}_Y^2\right\rangle$

$$S_{eff}[\psi,\phi]=\int_{q,\omega}\left\{\frac{\phi^2}{2g}-\frac{\psi^2}{2u}+\frac{1}{2}\ln\left[\left(\chi_{q,\omega}^{-1}+\psi\right)^2-\phi^2\right]\right\}$$

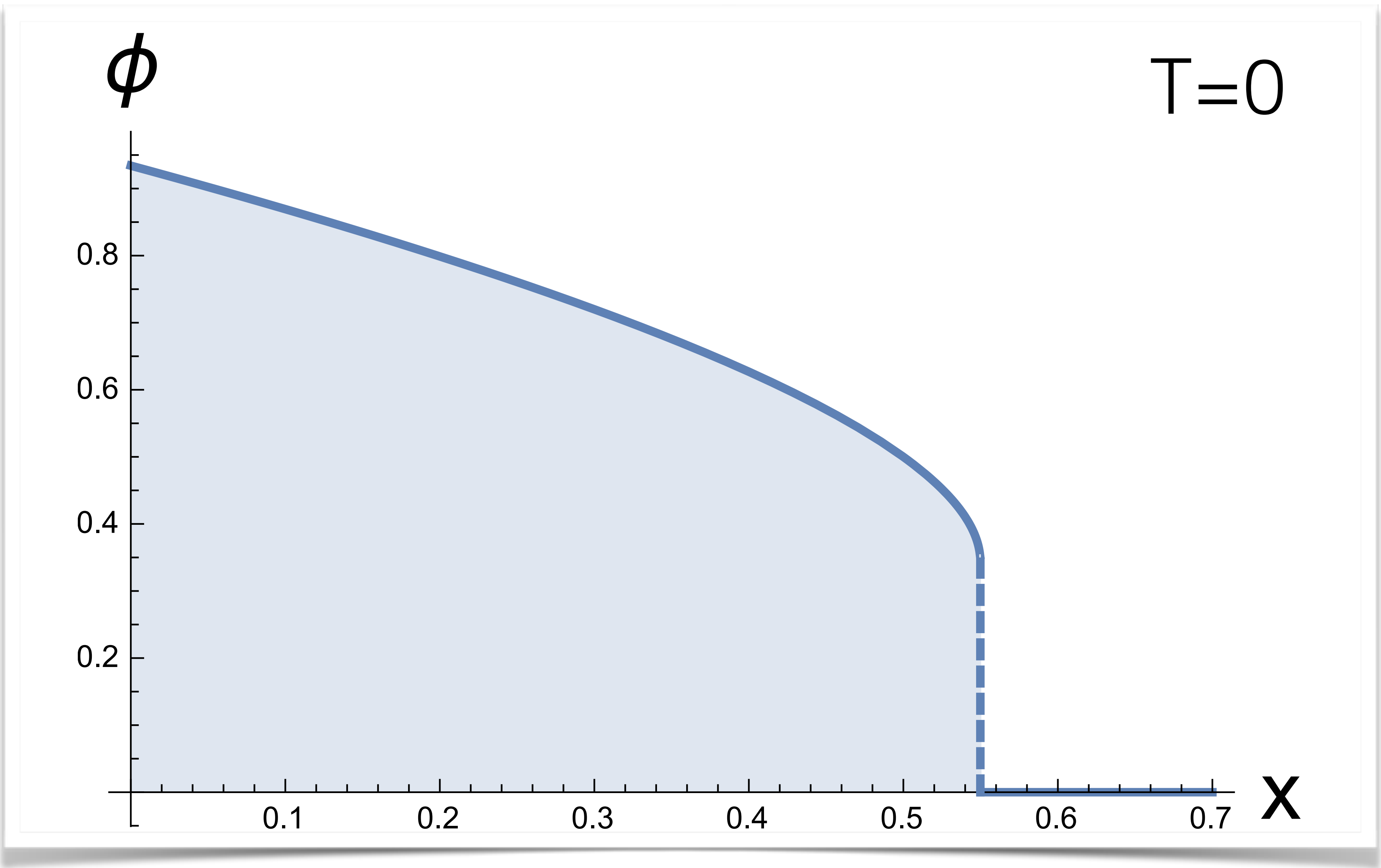
The Clean Limit at T=0



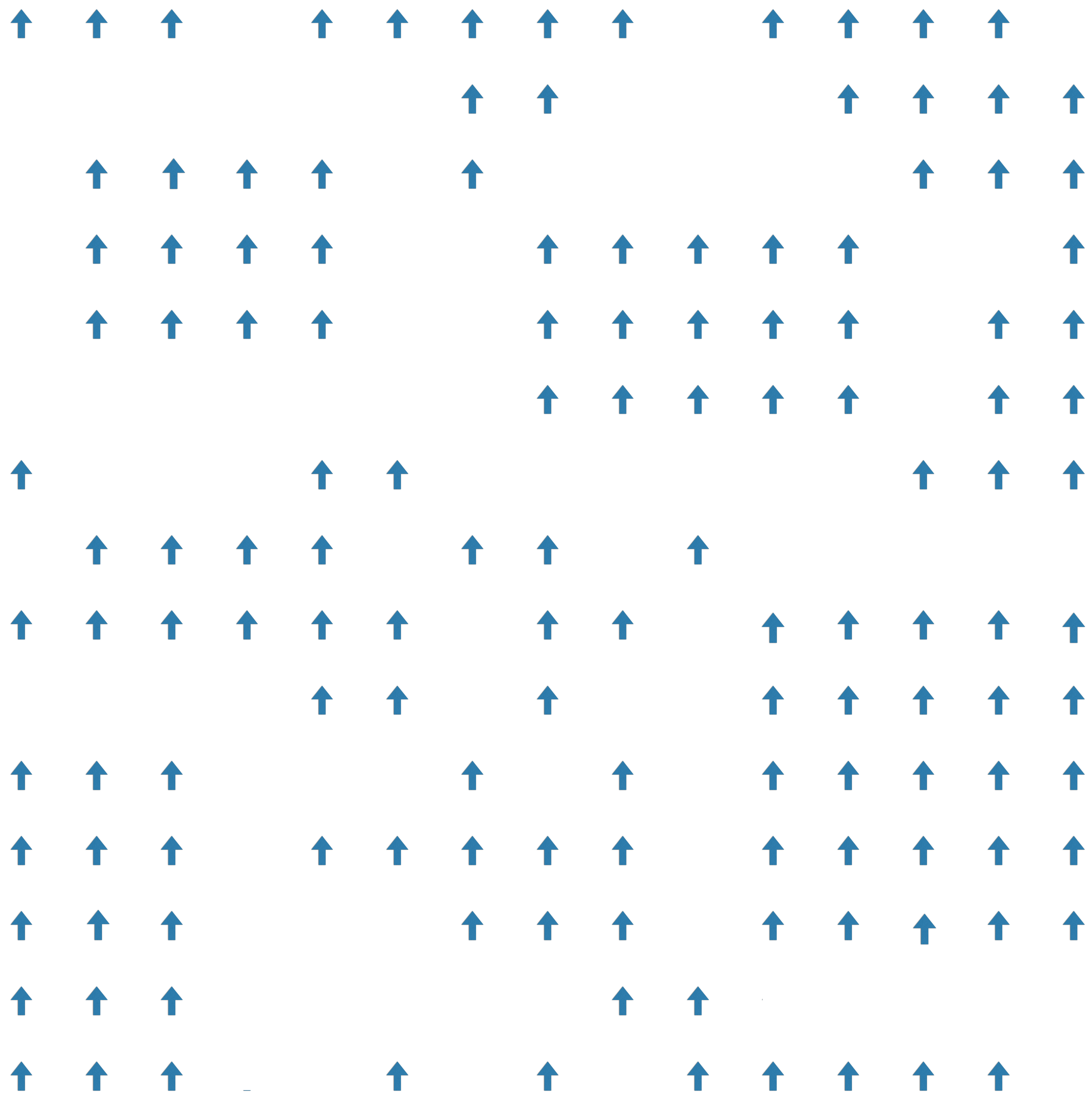
The Clean Limit at T=0



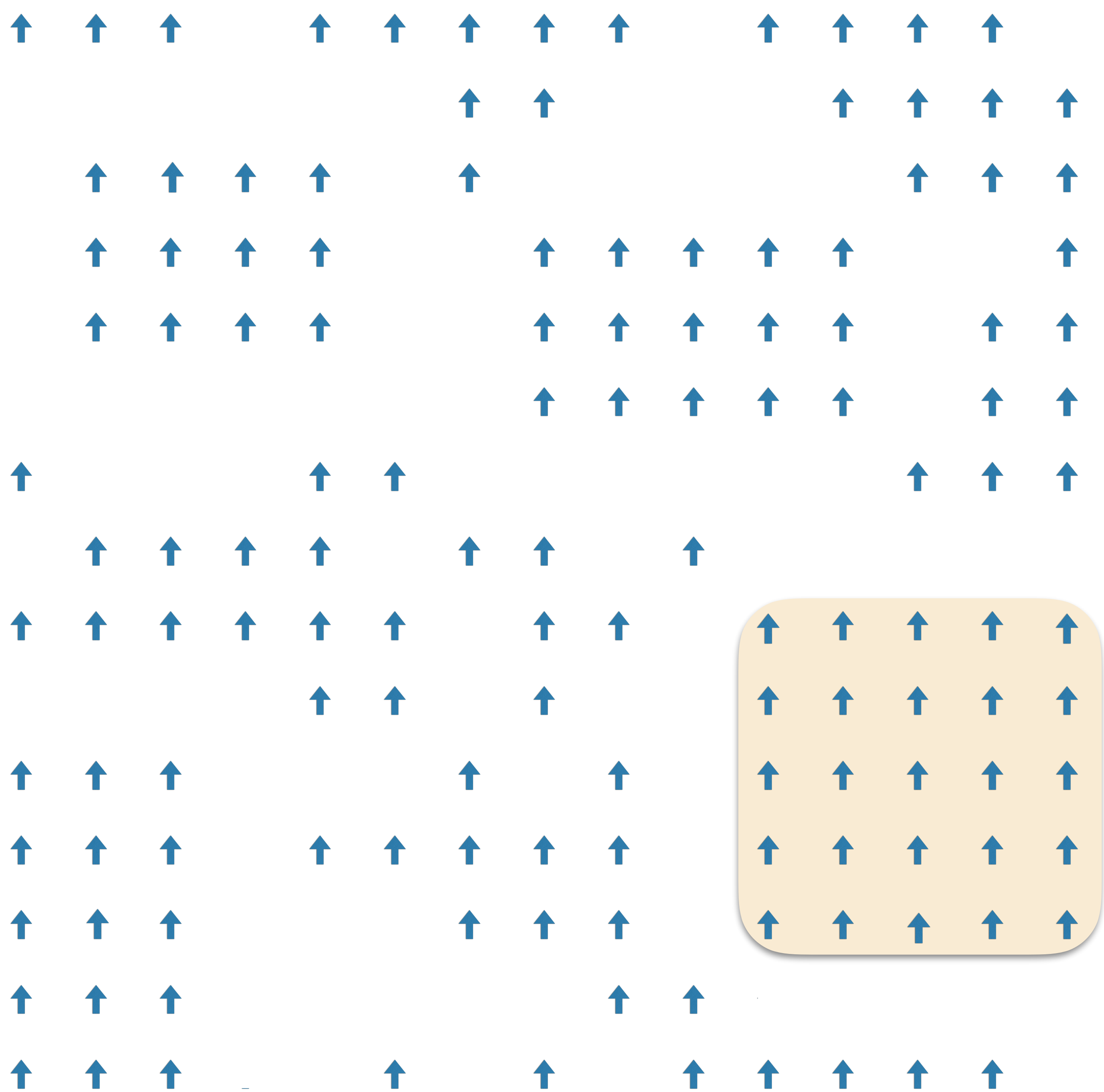
The Clean Limit: First Order Transition



Rare Region Effects — Strategy

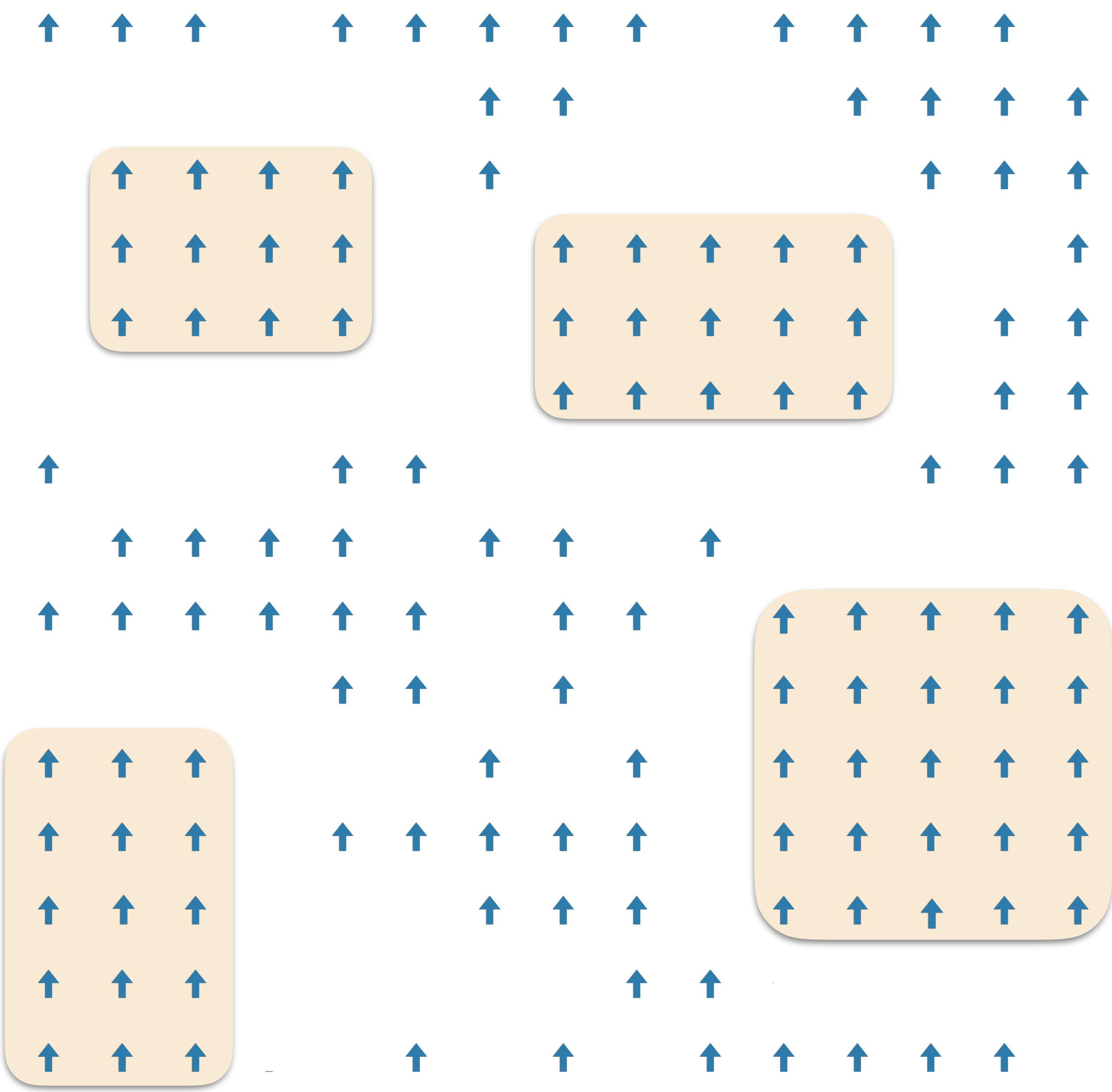


Rare Region Effects — Strategy



First, solve the Ising-nematic phase transition for a single droplet.

Rare Region Effects — Strategy



First, solve the Ising-nematic phase transition for a single droplet.

Then, sum over all the droplets with a probability weight:

$$w(V)$$

Landau-Ginzburg-Wilson Theory

$$S_{eff}[\psi,\phi]=\int_{q,\omega}\left\{\frac{\phi^2}{2g}-\frac{\psi^2}{2u}+\frac{1}{2}\ln\left[\left(\chi_{q,\omega}^{-1}+\psi\right)^2-\phi^2\right]\right\}$$

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$$\mathbf{q}=\frac{2\pi}{L}\mathbf{n}$$

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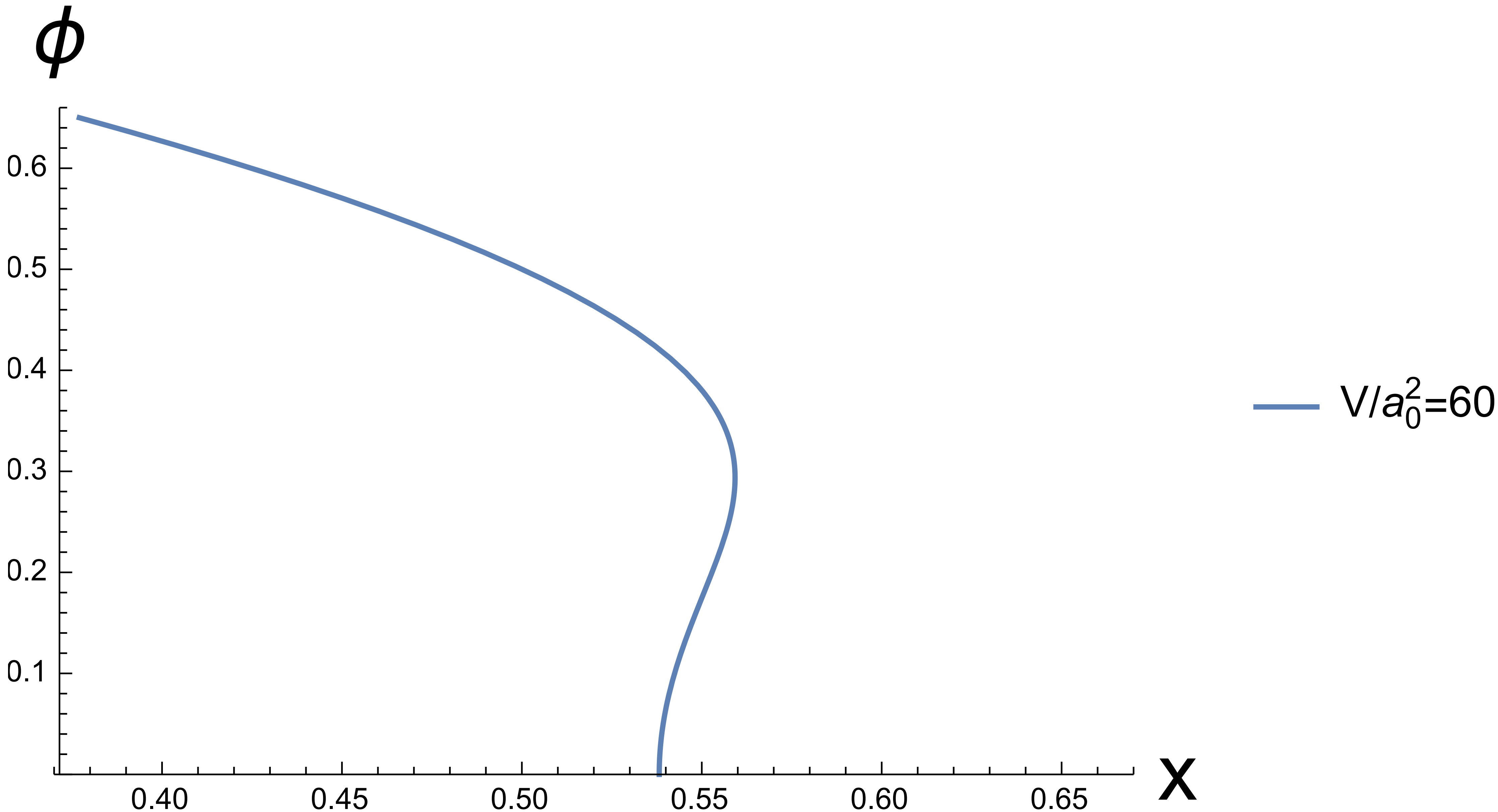
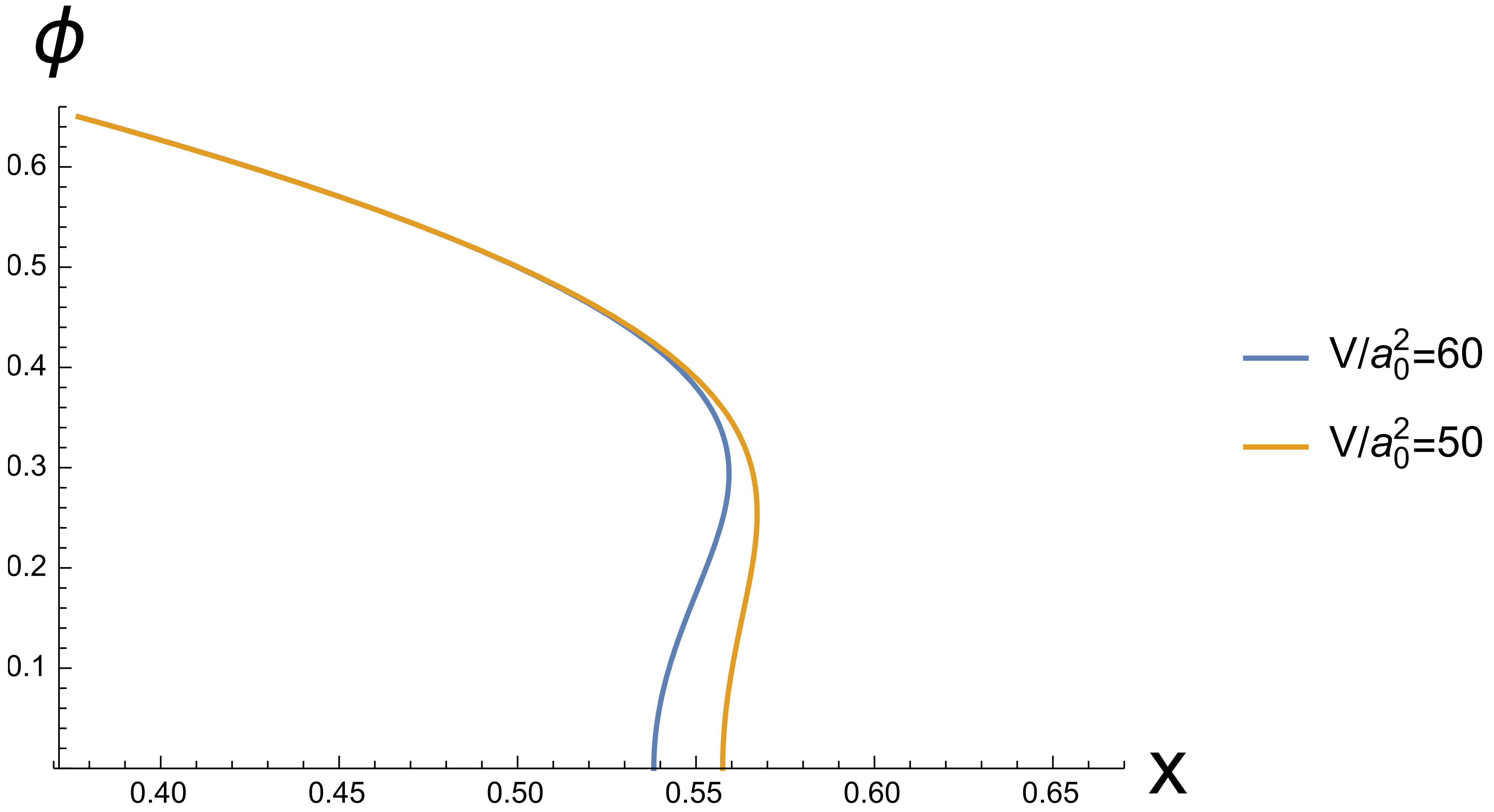
$$\mathbf{q}=\frac{2\pi}{L}\mathbf{n}$$

$$S_{eff}[\psi,\phi]=\frac{1}{V}\sum_{\mathbf{q}}\int\frac{d\omega}{2\pi}\left\{\frac{\phi^2}{2g}-\frac{\psi^2}{2u}+\frac{1}{2}\ln\left[\left(\chi_{q,\omega}^{-1}+\psi\right)^2-\phi^2\right]\right\}$$

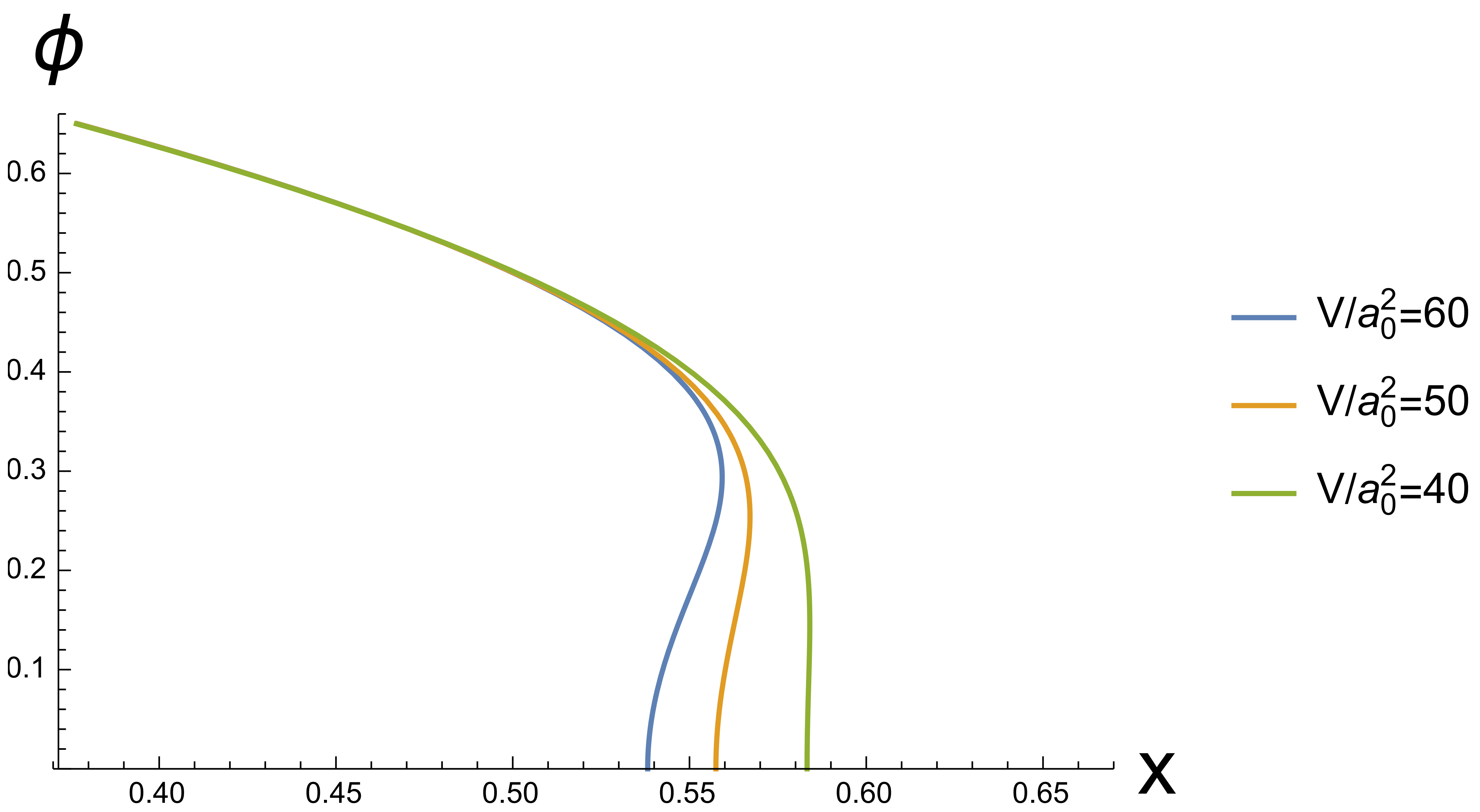
Results for finite size droplet



Results for finite size droplet



Results for finite size droplet



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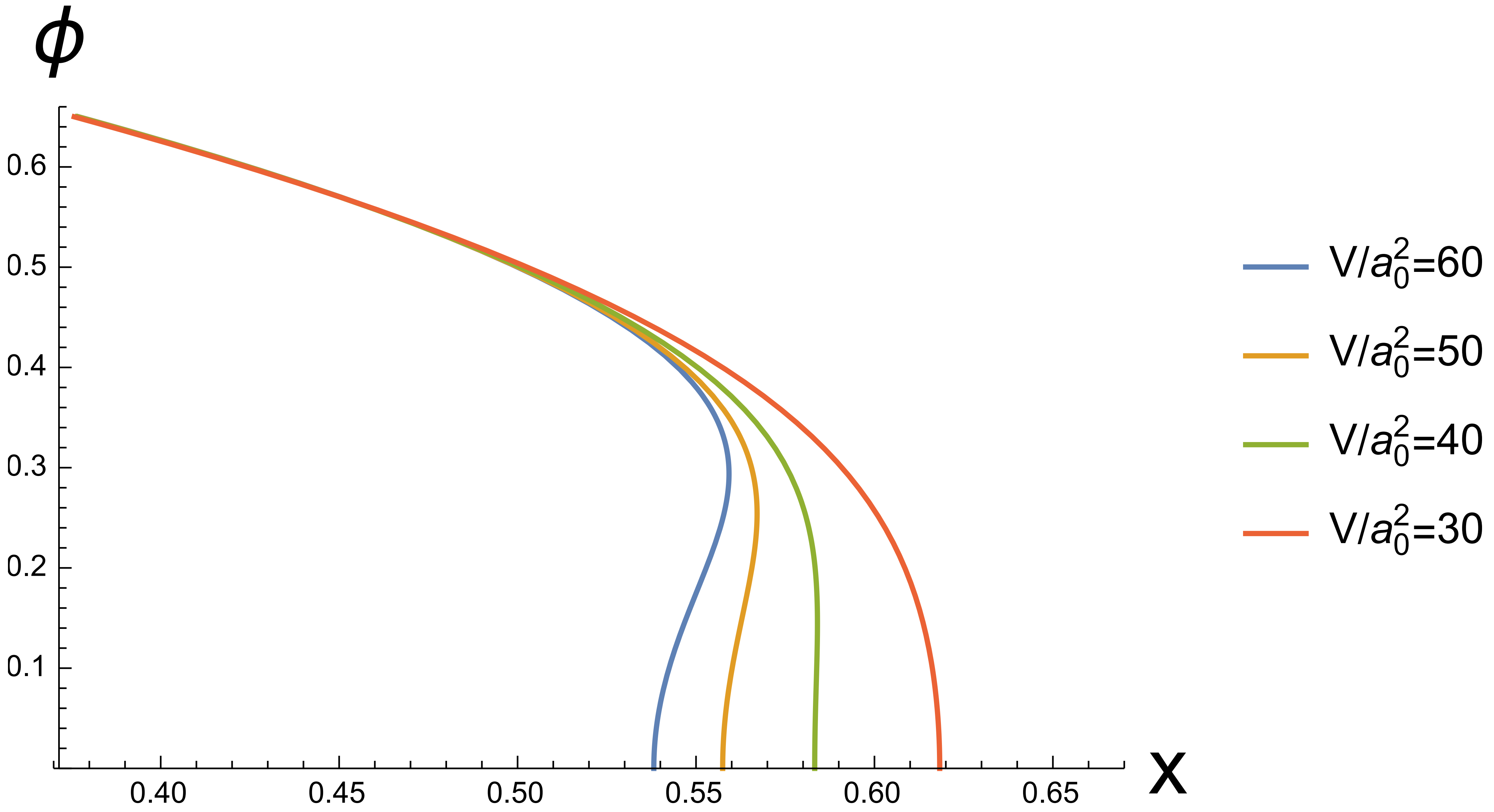
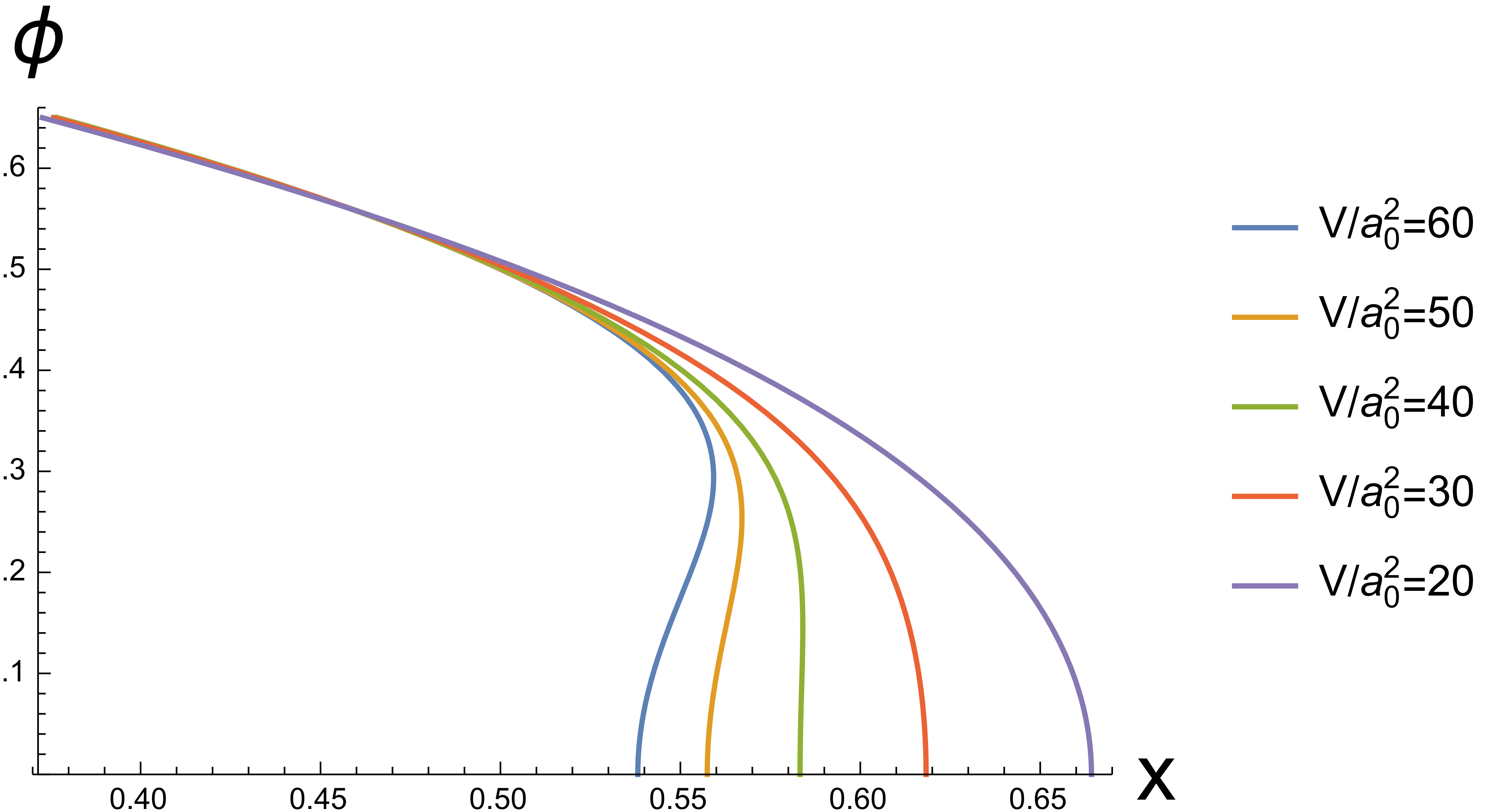


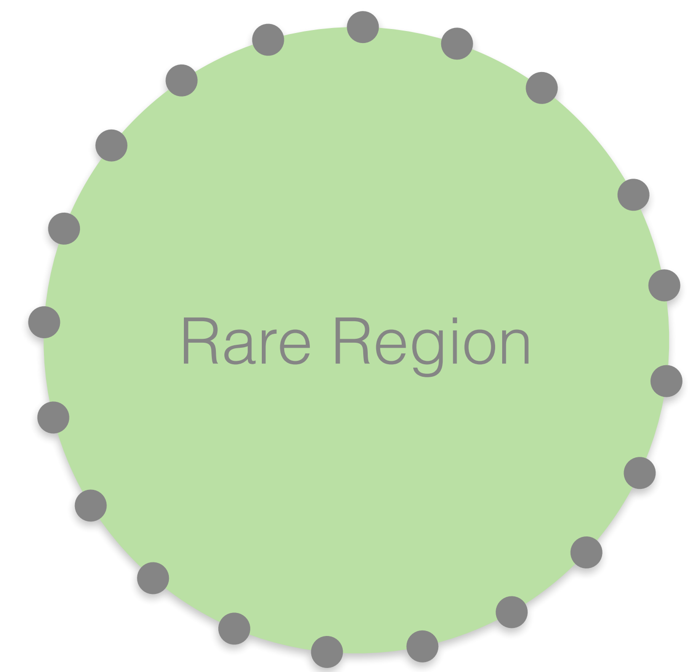
Results for finite size droplet



Sum over all droplets

Probability of finding a droplet of size V:

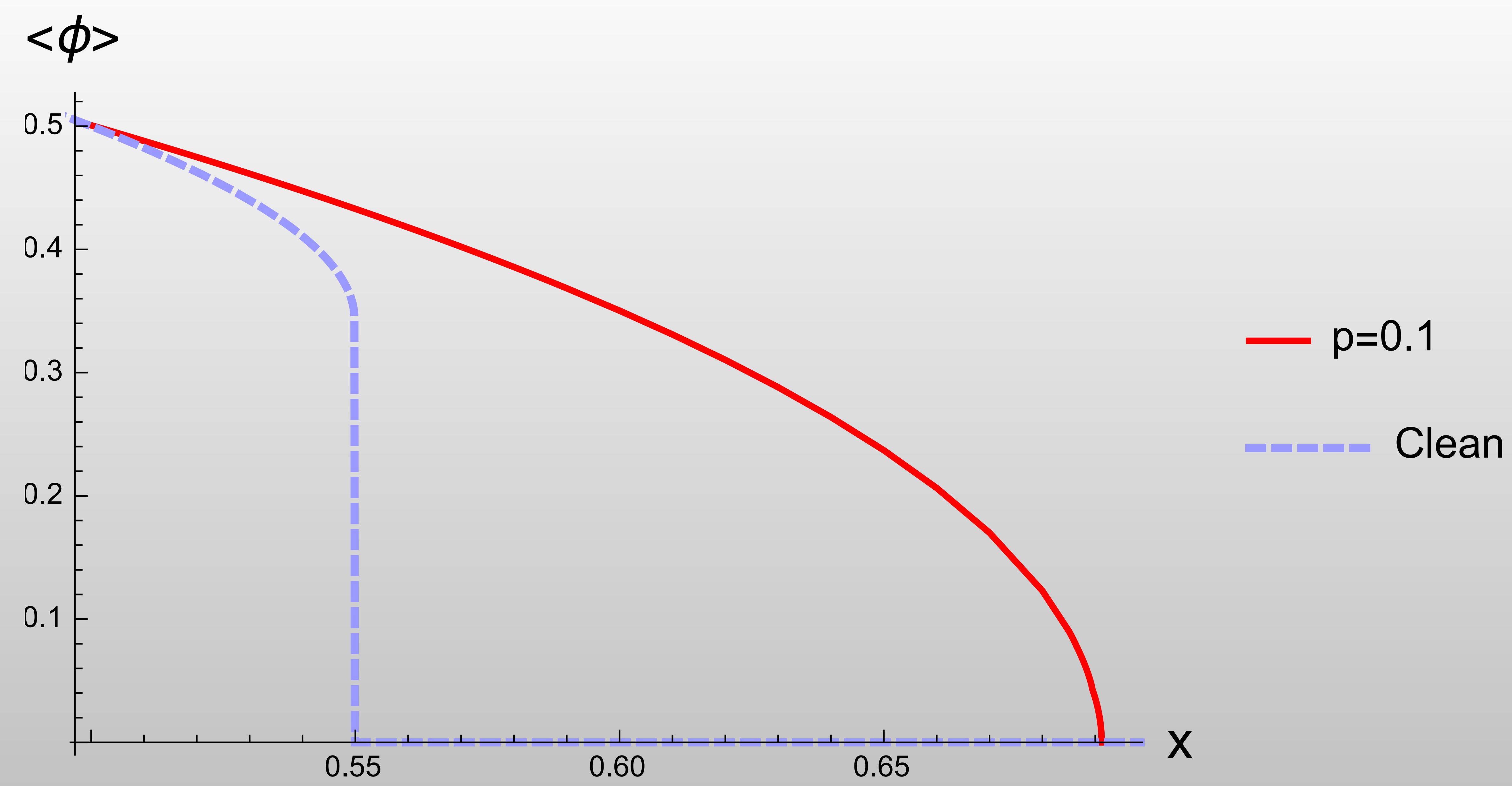
$$P(V) \sim p^{2\pi\sqrt{V}/a_0}(1-p)^{V/a_0^2}$$



Summing over all the droplets:

$$\langle \phi(\bar{r}_0) \rangle \sim \int P(V)\phi(\bar{r}_0, V)\,dV$$

Sum over all droplets



Conclusions

- Droplets with larger sizes behave like the clean bulk (first order nematic transition)

- Droplets with smaller sizes undergo continuous nematic phase transitions

- Ising-nematic quantum phase transition behaves as a continuous one due to rare region effects