

# CSC411 Fall 2017

## Assignment 2 Report

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### 1 Q1: Class-conditional gaussians

Given:

$$p(y = k) = a_k \quad (1)$$

$$p(\mathbf{x}|y = k, \mu, \sigma) = \left(\prod_{i=1}^d 2\pi\sigma_i^2\right)^{-1/2} \exp\left\{-\sum_{i=1}^d \frac{1}{2\sigma_i^2} (x_i - \mu_{ki})^2\right\} \quad (2)$$

#### 1.1 Bayes' rule derivation

$$\begin{aligned} p(y = k|\mathbf{x}, \mu, \sigma) &= \frac{p(\mathbf{x}|y = k, \mu, \sigma)p(y = k)}{p(\mathbf{x}|\mu, \sigma)} \\ &= \frac{p(\mathbf{x}|y = k, \mu, \sigma)p(y = k)}{\sum_{j=1}^K p(\mathbf{x}|y = j, \mu, \sigma)} \\ &= \frac{(\prod_{i=1}^d 2\pi\sigma_i^2)^{-1/2} \exp\{-\sum_{i=1}^d \frac{1}{2\sigma_i^2} (x_i - \mu_{ki})^2\} a_k}{\sum_{j=1}^K (\prod_{i=1}^d 2\pi\sigma_i^2)^{-1/2} \exp\{-\sum_{i=1}^d \frac{1}{2\sigma_i^2} (x_i - \mu_{ji})^2\}} \end{aligned}$$

#### 1.2 Negative likelihood function (NLL)

$$\begin{aligned} \ell(\theta; D) &= -\log p(y^{(1)}, \mathbf{x}^{(1)}, y^{(2)}, \mathbf{x}^{(2)}, \dots, y^{(N)}, \mathbf{x}^{(N)}|\theta) \\ &= -\log\left(\prod_{n=1}^N p(\mathbf{x}^{(n)}|y^{(n)}, \theta)p(y^{(n)})\right) \\ &= -\sum_{n=1}^N \log\left(\left(\prod_{i=1}^d 2\pi\sigma_i^2\right)^{-1/2} \exp\left\{-\sum_{i=1}^d \frac{1}{2\sigma_i^2} (x_i^{(n)} - \mu_{ki})^2\right\} a_k\right) \end{aligned}$$