The treatment of form factors in light front holographic QCD (LFHQCD) leads to particularly simple expressions for the latter![]. From a dressed conserved current in the 4-dimensional holographic space $AdS_5$ , that is a current propagating in the gravitational field, one obtains the general results for form factors of hadrons with $\tau$ constituents:

$$F_\tau(t) = \frac{1}{\mathcal{N}_\tau} B\left[\tau - 1, 1 - \frac{t}{4\lambda}\right] \tag{1}$$

The current, which is conserved in $AdS_5$ that is with an AdS mass $\mu = 0$, has in LFHQCD the quantum numbers $J^P = 1^-$, since $\mu = 0$ corresponds to $J = L = 1$ []. These are the quantum numbers of an axial current and not of a vector current. For Form fasctors vector currents one must expects poles at the $\rho - omega$ meson and its radial excitations and not at the axial vector meson poles. Therefore we have shifted the second argument of the Beta function to $\frac{1}{2} - \frac{t}{4\lambda}$ in order to have the poles at the right positions []

$$F_\tau(t)^v = \frac{1}{\mathcal{N}_\tau} B\left[\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}\right] \tag{2}$$

The quantity $\lambda = \kappa^2$ is the only scale in LFHQCD with massless quarks and can be determined in this frame from hadron spectroscopy. From there the value $0.523 \pm 0.024$ GeV$^2$ has been obtained []. With this general result and without any additional parameter good agreement with the experimental data for the proton Dirac FF has been obtained [].

An expression like (1) or (2), where the FF is expressed in terms of the Euler Beta function has also been derived By Ademollo and del Giudice [**?**] in 1969. They extended the Veneziano model for hadron scattering amplitudes, which was based on the peculiar properties of the Beta function, to current induced reactions and arrived at an expression for the vector FF (without normalization): the result (without normalization):

$$F(t) = B[1 - \alpha(t), \gamma - 1] \tag{3}$$

where $\alpha(t)$ is a linear Regge trajectory ( the rho trajectory).

Since the expression (1) describes in LFHQCD the FF of an axial vector current, it is natural to use this expression the calculation of the axial form factor. In this case we have no free parameter, since the value of $\sqrt{\lambda} = \kappa$ is determined from hadron spectroscopy. In Fig. **??** we show the result of the prediction of the axial FF according to (1).

The discussion should cover the following points:

Reasonable, but not perfect fit, especially to small axial radius when z-data are taken. Higher Fock state admixture can in the range be compensated by small variation of $\lambda$ within the uncertainties.

Inserting the "observed" masses together with the counting-rule constraint gives no good description of the data, only if we relax the counting rule, we obtain a reasonable fit, comparable to ours.

Comparison with a free dipole fit

Possible appendix:

The Euler Beta function $B[x, y] = \frac{\Gamma[x]\,\Gamma[y]}{\Gamma[x+y]}$ has two remarkable properties:

$$\lim_{x\to\infty,\, y\,\mathrm{fixed}} B[x, y] = \Gamma[y]\, x^{-y} \tag{4}$$

$$B[x,y] = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{\Gamma[y]}{\Gamma[y-n]} \frac{1}{y+n} \tag{5}$$

Because of the symmetry in $x$ and $y$ an analogous formula holds for an expansion of poles in $y$.

These two properties induced Veneziano [**?**] in 1968 to propose the Beta function as a model for hadron-hadron scattering amplitudes. He made the following ansatz for a scattering amplitude depending on the two invariants $s$ and $t$ [1] :

$$\mathcal{T}(s,t) = \beta B[1-\alpha(s), 1-\alpha(t)] \text{ with } \alpha(s) = \alpha_0 + \alpha' s; \; \alpha(t) = \alpha_0 + \alpha' t \tag{6}$$

The intercepts $\alpha_0$ and the slopes $\alpha'$ for the two trajectories can be different, essential is only the linear dependence.

Because of the properties (4) and (5) this model fulfilled two important structural constraints which were important guidelines for strong interaction theory in the pre-QCD time. 1)The amplitude can be either expressed as the sum over the singularities in the $s$ **or** the $t$-channel. 2) It fulfills the so called Dolen-Horn-Schmidt duality, that is the sum over the singularities in obe channel leads to Regge behaviour with the Regge trajectory of the other channel. These properties are not destroyed if one takes the sum of Beta functions [**?**]

$$\mathcal{T}(s,t) = \sum_{n,m=0} \beta_n B[1+n-\alpha(s), 1+m-\alpha(t)] \tag{7}$$

A peculiar property of the Veneziano model is the occurrence of daughter trajectories, that is trajectories with an intercept $\alpha_{0d}$ which is by a positive integer smaller than $\alpha_0$, the intercept of the parent trajectory, but the slopes are equal. This is in accordance with the generalized form (7).

Though the analytical duality and the Regge behaviour were the main *raison d'être* for the Veneziano model, the increasing importance of sum rules derived from current algebra [] and the occurrence of fixed "Regge" poles in current induced reacyions [] induced the investigation of Veneziano-like amplitudes [**?**, **?**, **?**] where one of the arguments was not a variable trajectory (say $\alpha_0 + \alpha' s$ with $\alpha' \neq 0$, but a constant integer value [2]. These could be viewed as scattering amplitudes involving leptons, which do not lead to resonance poles. Indeed, Ademollo and Del Giudice [**?**] derived from from lepton-hadron elastic scattering for the electromagnetic form factor the result (without normalization):

$$F(t) = B[1-\alpha(t), \gamma - 1] \tag{8}$$

Though the structure is exactly the same as that of our result, there is a subtle difference: In our expression the argument $\frac{1}{2} - \frac{t}{4\lambda^2}$ is not a Regge trajectory in the sense that it describes the resonances with increasing angular momentum, but is describes the radial excitation. This is necessary, since all the intermediate states in the $t$ channel must have the same quantum numbers $J^P = 1^-$. In our model the slope of the radial excitations is the same as that of the orbital excitations and therefore this expression coincides with the Regge trajectory. The result (9) is also valid, because in the Veneziano the daughters correspond exactly the radial excitations which are Kaluza-Klein towers in $AdS_5$.

[1] possible crossing symmetries are not taken into account here though thy played an important role in the establishment and the discussion of the Veneziano model

[2] an integer value since the current algebra relations lead to fix poles at integer values

The ambiguity of the Venziano amplitude expressed in (7) also applies to the derived form factor (9) which then has the form:

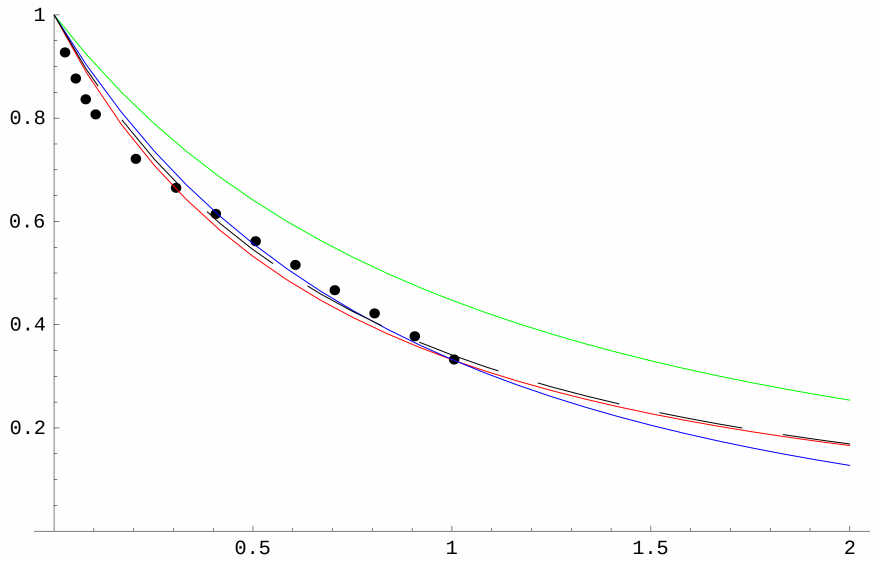
$$F(t) = \sum_{n,m=0} \beta_m \, B[1 - \alpha(t) + n, \gamma - 1 + m] \tag{9}$$

The contributions with $m > 0$ correspond the higher twist contributions of LFHQCD.

| | P | $\sqrt{\lambda} = \kappa$ | 1. pole | 2. pole | $\chi^2$ |
|---|---|---|---|---|---|
| | | GeV | GeV | GeV | |
| LFHQCD | 0 | 0.510 | 1.020 | 1.442 | 0.019 |
| LFHQCD | 0.3 | 0.545 | 1.090 | 1.541 | 0.020 |
| dipole,free | - | - | 1.230 | 1.640 | 0.25 |
| dipole,constr | - | - | 1.230 | 1.640 | 1.20 |

Table 1: Different approximations for the axial FF. $\sqrt{\lambda} = \kappa$ is result of a fit to the model independent z-data, 1. and 2. pole denotes the first and second pole mass; $\chi^2 = \sum_i (\text{data}_i - \text{fit}_i)^2/\text{data}_i$ . First two lines: LFHQCD with no higher twist admixture ($P = 0$) and with admixture of 30%. last two lines: Fit with poles at observed $a_1(1260)$ mass and the unconfirmed $a_1(1640)$ mass, free: without constraints on asymptotic behaviour, constr. : asymptotic behaviour $\sim 1/t^2$.

The following figure is for our information and discussion:




**red: LFHQCD,twist 3; black dahes: LFHQCD, 30 % twist 5; green: ''observed"masses+ counting rule**

**blue: observed masses, no counting rule imposed**

This table might be given in a modified form or not, but is a good basis for the discussion,