Nucleon Transition Form Factor

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Twist-τ effective AdS WFs:

$$\Psi_+^{n,\tau}(z) = \kappa^{\tau-1}\sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}}\, z^{1/2+\tau} e^{-\kappa^2 z^2/2} L_n^{\tau-2}(\kappa^2 z^2), \tag{1}$$

$$\Psi_-^{n,\tau}(z) = \kappa^{\tau}\sqrt{\frac{1}{n+\tau-1}}\sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}}\, z^{3/2+\tau} e^{-\kappa^2 z^2/2} L_n^{\tau-1}(\kappa^2 z^2). \tag{2}$$

The effective WFs $\Psi_\pm$ are orthonormal

$$\int \frac{dz}{z^4}\, \Psi_\pm^{n',\tau}(z)\Psi_\pm^{n,\tau}(z) = \delta_{n',n}. \tag{3}$$

Dirac Transition Form Factor

To compute the Dirac transition form factor

$$F_1(Q^2)_{N\to N^*} = \int \frac{dz}{z^4}\Psi_+^{N^*}(z)V(Q^2,z)\Psi_+^{N}(z), \tag{4}$$

we use the integral representation of the bulk-to-boundary propagator [1]

$$V(Q^2,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{Q^2/4\kappa^2} e^{-\kappa^2 z^2 x/(1-x)}. \tag{5}$$

Thus

$$F_1(Q^2)_{N\to N^*} = \kappa^2 \int_0^1 \frac{dx}{(1-x)^2} x^{Q^2/4\kappa^2} \int \frac{dz}{z^2}\, e^{-\kappa^2 z^2 x/(1-x)} \Psi_+^{N^*}(z)\Psi_+^{N}(z), \tag{6}$$

Integrating (21) over the variable z using (1) we find

$$F_1^\tau(Q^2)_{N\to N^*} = \sqrt{\tau-1}\int_0^1 dx\,(\tau x-1)(1-x)^{\tau-2}\, x^{Q^2/4\kappa^2}. \tag{7}$$

Finally, integrating over x we find

$$F_1^\tau(Q^2)_{N\to N^*} = \sqrt{\tau-1}\,\frac{\frac{Q^2}{4\kappa^2}}{\tau+\frac{Q^2}{4\kappa^2}}\,\frac{\Gamma(\tau)\Gamma\left(1+\frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau+\frac{Q^2}{4\kappa^2}\right)}. \tag{8}$$

Recall that the elastic form factor for twist-τ is given by [2]

$$F_\tau(Q^2) = \Gamma(\tau)\frac{\Gamma\left(1+\frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau+\frac{Q^2}{4\kappa^2}\right)}. \tag{9}$$

For integer twist $\tau = N$, with N the number of constituents for a given Fock component, we can simplify (9) by using the recurrence formula

$$\Gamma(N+z) = (N-1+z)(N-2+z)\dots(1+z)\Gamma(1+z). \tag{10}$$

We find

$$F_\tau(Q^2) = \frac{(\tau-1)!}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)\cdots\left(\tau-1+\frac{Q^2}{4\kappa^2}\right)}, \tag{11}$$

Therefore, for integer twist, we can rewrite (8) as

$$F_1^\tau(Q^2)_{N\to N^*} = \frac{\sqrt{\tau-1}}{\tau}\,\frac{Q^2}{4\kappa^2}\,F_{\tau+1}(Q^2). \tag{12}$$

We can express the transition form factor in a universal form valid for axial or vector currents. For doing this, we recall that the form factor in LFHQCD can also be expressed in the Veneziano form [3]

$$F_\tau(t) = \frac{1}{N_\tau}B\left(\tau-1, 1-\alpha(t)\right), \tag{13}$$

where $t = -Q^2$, $N_\tau = B\left(\tau-1, 1-\alpha(0)\right)$ and $\alpha(t)$ is a linear Regge trajectory

$$\alpha(t) = \alpha(0) + \alpha' t. \tag{14}$$

For integer twist $N = \tau$, (13) can be expressed as [4]

$$F_\tau(Q^2) = \frac{1}{\left(1+\frac{Q^2}{M^2_{n=0}}\right)\left(1+\frac{Q^2}{M^2_{n=1}}\right)\cdots\left(1+\frac{Q^2}{M^2_{n=\tau-2}}\right)}, \tag{15}$$

which is a product of $\tau - 1$ poles located at

$$-Q^2 = M_n^2 = \frac{1}{\alpha'}\Big(n+1-\alpha(0)\Big), \tag{16}$$

the radial excitation spectrum for the exchanged particles in the t-channel.

Comparing (11) and (15) it is clear that the factor $\frac{Q^2}{4\kappa^2}$ in (12) corresponds to the lowest pole for $n = 0$ located at

$$-Q^2 = t = \frac{1}{\alpha'}\Big(1 - \alpha(0)\Big). \tag{17}$$

Therefore the Veneziano-like form

$$F_1^\tau(Q^2)_{N\to N^*} = -\frac{1}{\tau}\sqrt{\tau - 1}\,\frac{\alpha' t}{1-\alpha(0)}\,\frac{B(\tau, 1-\alpha(t))}{B(\tau, 1-\alpha(0))}, \tag{18}$$

valid for axial or vector currents. In particular, the expression for the EM transition form factor for $\tau = 3$ which follows from (18)

$$F_1^\tau(Q^2)_{N\to N^*} = \frac{\sqrt{2}}{3}\frac{Q^2}{M_\rho^2}\frac{1}{\left(1+\frac{Q^2}{M_\rho^2}\right)\left(1+\frac{Q^2}{M_{\rho'}^2}\right)\left(1+\frac{Q^2}{M_{\rho''}^2}\right)}, \tag{19}$$

is identical to the expression used in [5] to compute the Dirac nucleon to Roper transition form factor in the valence approximation.

Pauli Transition Form Factor

To compute the Pauli transition form factor from

$$F_2(Q^2)_{N\to N^*} = \frac{1}{2}\chi_p\, N_\chi \int \frac{dz}{z^3}\left(\Psi_-^{N^*}(z) V(Q^2, z)\Psi_+^N(z) + \Psi_+^{N^*}(z) V(Q^2, z)\Psi_-^N(z)\right), \tag{20}$$

we use the AdS WF Ψ_+ and Ψ_- given by (1) and (2) and the integral representation of $V(Q^2, z)$ Eq. (5). Thus

$$F_2(Q^2)_{N\to N^*} = \frac{1}{2}\chi_p\, N_\chi \kappa^2 \int_0^1 \frac{dx}{(1-x)^2} x^{Q^2/4\kappa^2} \int \frac{dz}{z}\, e^{-\kappa^2 z^2 x/(1-x)} \left(\Psi_-^{N^*}(z)\Psi_+^N(z) + \Psi_+^{N^*}(z)\Psi_-^N(z)\right). \tag{21}$$

The normallization factor N_χ for the $N \to N$ elastic transition is determined by the condition

$$F_2^\tau(0) = \chi_N. \tag{22}$$

It is given by

$$N_\chi = \frac{M_N}{4\sqrt{\tau-1}} = \frac{\kappa}{2\sqrt{\tau-1}}. \tag{23}$$

To find the corresponding normalization for the $N \to N^*$ transition we make the replacement $2M \to M + M^*$, therefore there is an additional multiplying factor $\frac{1+\sqrt{2}}{2}$. Thus

$$N_\chi^* = \frac{M_N + M_N^*}{8\sqrt{\tau - 1}} = \frac{1+\sqrt{2}}{2} N_\chi. \tag{24}$$

Note: The normalization of the AdDS WF $\Psi_\pm$ with $\Psi_\mp$ is given by

$$\frac{\kappa}{4\sqrt{\tau-1}} \int \frac{dz}{z^3} \left(\Psi_-^{n'=1,\tau}(z) \Psi_+^{n=0,\tau}(z) + \Psi_+^{n'=1,\tau}(z) \Psi_-^{n=0,\tau}(z) \right) = \begin{cases} 1, & \text{if } n' = 0, \\ -\frac{1}{2\sqrt{\tau-1}}, & \text{if } n' = 1, \\ 0, & \text{if } n' > 1. \end{cases}$$

Integrating (21) over the variable z we find

$$F_2^\tau(Q^2)_{N \to N^*} = \frac{1}{4}(1+\sqrt{2})\chi_p \sqrt{\frac{\tau}{\tau-1}} \int_0^1 dx \, \left(-\sqrt{\tau}(2-x) - \sqrt{\tau-1}(1-x) + \tau\sqrt{\tau-1}x + \tau^{3/2}x \right) (1-x)^{\tau-1} \, x^{Q^2/4\kappa^2}. \tag{25}$$

Finally, integrating over x we find

$$F_2^\tau(Q^2)_{N \to N^*} = \frac{\chi_p}{4}(1+\sqrt{2}) \frac{\sqrt{\tau-1}}{\tau^2-1} \left(\frac{Q^2}{4\kappa^2} (\sqrt{\tau(\tau-1)} + \tau - 1) - 1 - \tau \right) F^{\tau+2}(Q^2). \tag{26}$$

As for the Dirac transition form factor, we can express the Pauli transition form factor (26) in a universal Veneziano form

$$F_2^\tau(t)_{N \to N^*} = -\frac{\chi_p}{4}(1+\sqrt{2}) \frac{\sqrt{\tau-1}}{\tau^2-1} \left((\tau - 1 + \sqrt{\tau(\tau-1)}) \frac{\alpha' t}{1-\alpha(0)} + 1 + \tau \right) \frac{B(\tau+1, 1-\alpha(t))}{B(\tau+1, 1-\alpha(0))}, \tag{27}$$

valid for axial or vector currents.

References

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