Flavor-Spin Couplings

HLFHS Collaboration

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Vector couplings

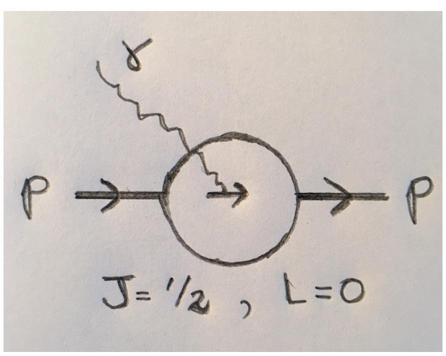
SU(6) result

EM form factor

Recall the general expression for a spin-non flip FF [1]

$$G_\pm(Q^2) = g_\pm R^4 \int \frac{dz}{z^4} V(Q^2, z)\, \Psi_\pm^2(z), \tag{1}$$

for the components Ψ_+ and Ψ_- with angular momentum $L^z = 0$ and $L^z = +1$ respectively. The effective charges g_+ and g_- are determined from the spin-flavor structure of the theory which is not specified in LF holography.



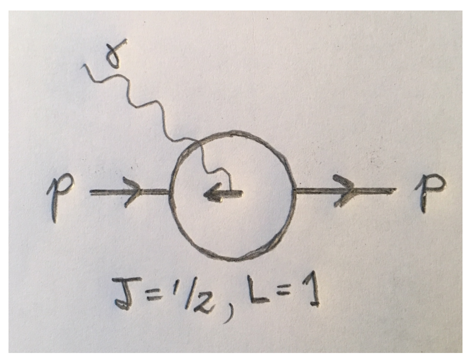


Figure 1: EM current coupling to struck constituent quark. Left: quark spin aligned with proton spin. Right: quark spin anti-aligned.

Using the $SU(6)$ spin-flavor symmetry the probability to find a constituent q in a nucleon with spin up or down is

$$P_{u\uparrow} = \frac{5}{3}, \qquad P_{u\downarrow} = \frac{1}{3}, \qquad P_{d\uparrow} = \frac{1}{3}, \qquad P_{d\downarrow} = \frac{2}{3}, \tag{2}$$

for the proton and

$$P_{u\uparrow} = \frac{1}{3}, \qquad P_{u\downarrow} = \frac{2}{3}, \qquad P_{d\uparrow} = \frac{5}{3}, \qquad P_{d\downarrow} = \frac{1}{3}, \tag{3}$$

for the neutron. The effective charges g_+ and g_- in (1) are computed by the sum of the charges of the struck quark composed by the corresponding probability for quark spin aligned or anti-aligned with the proton spin:

$$g_+^p = q_u P_{u\uparrow}^p + q_d P_{d\uparrow}^p = 1 \tag{4}$$
$$g_-^p = q_u P_{u\downarrow}^p + q_d P_{d\downarrow}^p = 0 \tag{5}$$
$$g_+^n = q_u P_{u\uparrow}^n + q_d P_{d\uparrow}^n = -1/3 \tag{6}$$
$$g_-^n = q_u P_{u\downarrow}^n + q_d P_{d\downarrow}^n = 1/3 \tag{7}$$

The nucleon Dirac form factors in the $SU(6)$ limit are thus given by

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q^2, z)\, \Psi_+^2(z), \tag{8}$$
$$F_1^n(Q^2) = -\frac{1}{3} R^4 \int \frac{dz}{z^4} V(Q^2, z) \left[\Psi_+^2(z) - \Psi_-^2(z)\right], \tag{9}$$

where $F_1^p(0) = 1$ and $F_1^n(0) = 0$.

Quark distribution amplitudes

The unpolarized distribution amplitudes $q(x) = q^\uparrow(x) + q^\downarrow$ are also expressed in terms of the probability for quark spin aligned or antialigned:

$$q(x) = P_{q\uparrow}\, q_{\tau=3}(x) + P_{q\downarrow}\, q_{\tau=4}(x), \tag{10}$$

normalized to

$$\int_0^1 dx\, q(x) = P_\uparrow^q + P_\downarrow^q = N_q, \tag{11}$$

with $N_u = 2, N_d = 1$ in the proton. Thus the SU(6) result for the longitudinal quark distribution in the proton.

$$u(x) = \frac{5}{3}\, q_{\tau=3}(x) + \frac{1}{3}\, q_{\tau=4}(x), \tag{12}$$
$$d(x) = \frac{1}{3}\, q_{\tau=3}(x) + \frac{2}{3}\, q_{\tau=4}(x), \tag{13}$$

SU(6) Breaking

The SU(6) weights in (2) and (3) are computed from the SU(6) wave function which only includes the internal spin configurations in the nucleon, it does not include orbital angular momentum (See Fig. 1 right). To include the ortial angular momentum weight we include a factor r, thus we modify the twist-4 contribution to the distribution amplitudes by this factor. Once this is done, we must modify accodingly the weight of the twist-3 contribution in order to conserve probability. The result is given below and is identical to the result found in [3] from the modification of the neutron form factor in Ref. [3].

$$u(x) = \left(2 - \frac{r}{3}\right) q_{\tau=3}(x) + \frac{r}{3}\, q_{\tau=4}(x), \tag{14}$$

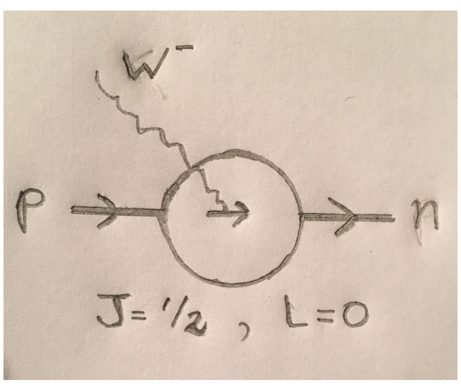
$$d(x) = \left(1 - \frac{2r}{3}\right) q_{\tau=3}(x) + \frac{2r}{3}\, q_{\tau=4}(x), \tag{15}$$

As it is well know, the vector coupling from the EM current is identical to the spin aligned ($L = 0$) or spin anti-aligned ($L = 1$) since $\bar{u}^{\uparrow}\gamma^{+}u^{\uparrow} = \bar{u}^{\downarrow}\gamma^{+}u^{\downarrow}$ [4].

Axial couplings

Axial form factor

The axial vector couplings for the flavor changing weak current in a proton are depicted in Fig. 2.



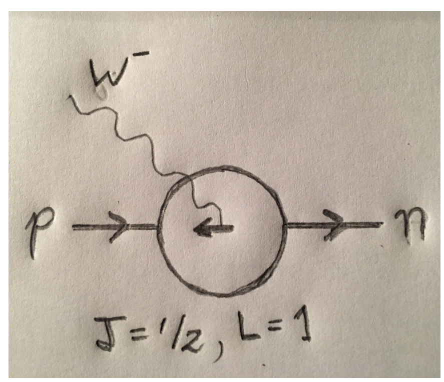


Figure 2: Axial vector current coupling to struck constituent quark (right handed). Left: quark spin aligned with proton spin. Right: quark spin anti-aligned (left handed).

Since the flavor changing weak current

$$J_W^- = \bar{\psi}_d \tfrac{1}{2}\left(1 - \gamma_5\right) \psi_u, \tag{16}$$

couples only the left handed spinor, the contribution from the $L = 1$ is dominant; indeed the contribution from the $L = 0$ state is zero in the limit of zero quark masses where helicity is equivalent to chirality (See Appendix A in https://arxiv.org/pdf/0707.3859.pdf) for the LF spinors)

$$(1-\gamma_5)\, u^{\uparrow}(p) = 0, \quad (1-\gamma_5)\, u^{\downarrow}(p) = u^{\downarrow}(p). \tag{17}$$

Therefore in the chiral limit the axial FF is twist 4 and given by

$$F_\tau^A(t) = \frac{1}{N_\tau} g_A B\left(\tau - 1, 1 - \frac{t}{4\lambda}\right), \tag{18}$$

with normalization $N_\tau = 1/(\tau - 1)$. For $\tau = 4$ we express the axial form factor through the axial sum rule:

$$\begin{aligned} F_A(t) &= 3 g_A B\left(3, 1 - \frac{t}{4\lambda}\right) && (19)\\ &= \int_0^1 dx \left[[\tilde{H}^u(x,t) + \tilde{H}^{\overline{u}}(x,t)] - [\tilde{H}^d(x,t) + \tilde{H}^{\overline{d}}(x,t)] \right], && (20) \end{aligned}$$

and therefore

$$\Delta[u(x) + \overline{u}(x)] - \Delta[d(x) + \overline{d}(x)] = 3\, g_A w'(x)[1 - w(x)]^2. \tag{21}$$

In Fig. 3 and Fig. 4 we compare the $\tau = 4$ results with our previous $\tau = 3$ results for the axial FF and the combination of quark polarized distributions $\Delta u_+ - \Delta d_+$at the initial scale

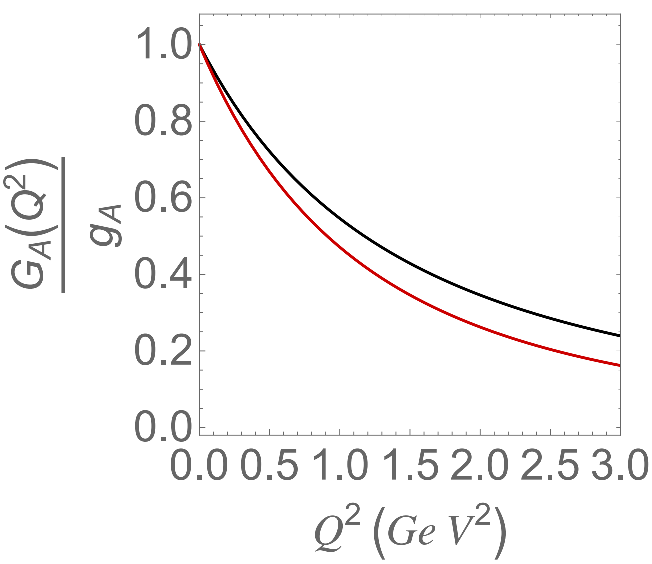


Figure 3: Black curve $\tau = 3$, red curve $\tau = 4$.

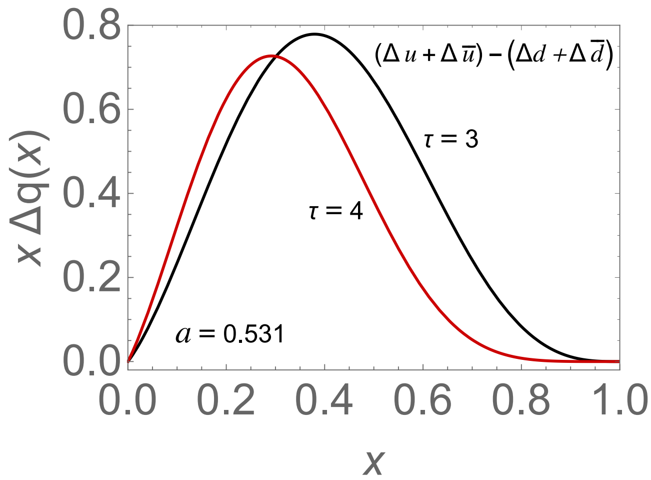


Figure 4: Black curve $\tau = 3$, red curve $\tau = 4$.

References

[1] For a review of LFHQCD see, S. J. Brodsky, G. F. de Teramond, H. G. Dosch and J. Erlich, Light-front holographic QCD and emerging confinement, Phys. Rep. **584**, 1 (2015) [arXiv:1407.8131 [hep-ph]].

[2] G. F. de Teramond, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, A. Deur, Universality of generalized parton distributions in light-front holographic QCD, arXiv:1801.09154 [hep-ph].

[3] G. F. de Teramond, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky and A. Deur [HLFHS Collaboration], Universality of generalized parton distributions in light-front holographic QCD, Phys. Rev. Lett. **120**, 182001 (2018) [arXiv:1801.09154 [hep-ph]].

[4] G. P. Lepage and S. J. Brodsky, Exclusive processes in perturbative quantum chromodynamics, Phys. Rev. D **22**, 2157 (1980).