# Axial form factor and polarized quark distributions

Recap

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Matrix element of the local iso-vector axial current,

$$A^a_\mu(x) = \sum_{ij} \bar{q}_i(x)\gamma_\mu\gamma_5\tau^a_{ij}q_j(x), \tag{1}$$

between nucleon states defines the axial $G_A$ and the induced pseudoscalar form factor $G_P$ for $q = p' - p$,

$$\langle N(p')|A^a_\mu(x)|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5\frac{q_\mu}{2M_N}G_P(q^2)\right]\tau^a u(p), \tag{2}$$

for any isospin component $\tau^a$:

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{3}$$

which acts on the quark fields

$$q = \begin{pmatrix} u \\ d \end{pmatrix}. \tag{4}$$

In particular for

$$\tau^+ = \tfrac{1}{2}\left(\tau^1 + i\tau^2\right) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{5}$$

we have for $q = 0$

$$\langle P|\bar{u}\gamma_\mu\gamma_5 d|N\rangle = g_A\bar{u}_P\gamma_\mu\gamma_5\tau^+u_N, \tag{6}$$

for the matrix elements between neutron $N$ and proton $P$ states.

Likewise, we can compute the matrix elements of $\tau^3$ between proton states. We find:

$$\langle P|\left(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\right)|P\rangle = g_A\bar{u}_P\gamma_\mu\gamma_5\tau^3u_P. \tag{7}$$

The matrix elements of (6) and (7) have identical normalization, therefore, using isospin symmetry, we can express the axial form factor in terms of the $u - d$ polarized quark distributions.