# Axial form factor, axial sum-rule and polarized quark distributions

(LFHS Collaboration)

## Polarized GPDs

$$F_A^{\tau}(t) = (\tau - 1)\int_0^1 d\,x\, w'(x)\, w\,(x)^{-\frac{t}{4\lambda}}\,[1 - w(x)]^{\tau-2}$$

$$w(0) = 0,\;\; w(1) = 1,\;\; w'(x) \geq 0,\;\; w''(1) \neq 0$$

$$F_A^{q}(t) = \int_0^1 dx\, \tilde{H}^{q}(x, t)$$

$$\tilde{H}^{q}(x, t) = (\tau - 1)\,[1 - w(x)]^{\tau-2}\, w'(x)\, e^{\frac{t}{4\lambda}\log\left(\frac{1}{w(x)}\right)}$$

$$= \Delta\, q_{\tau}(x)\, \exp[t\, f(x)]$$

$$\int_0^1 dx\, \Delta q_{\tau}(x) = 1$$

$$\Delta\, q_{\tau}(x) = (\tau - 1)\,(1 - w(x))^{\tau-2}\, w'(x),$$

$$f(x) = \frac{1}{4\,\lambda}\log\left(\frac{1}{w(x)}\right)$$

$$w(x) = x^{1-x}\, e^{-a\,(1-x)^2}$$

$$|\,\Delta\, q(x)\,| \;\leq q(x)$$

$$\Delta\, u(x) = \;\; g_A\, C_3\, q_A^{\tau=3}(x)$$

$$\Delta\, d(x) = -g_A\, C_4\; q_A^{\tau=4}(x)$$

$$\Delta\, \overline{u}(x) = \;\; g_A\, C_5\; q_A^{\tau=5}(x)$$

$$\Delta\, \overline{d}(x) = -g_A\, C_6\; q_A^{\tau=6}(x)$$

$$C_3 \;= 1 - C_4 - \;C_5 - C_6$$

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gA := 1.2723
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w[x_, a_] := x^(1 - x) Exp[-a (1 - x)^2]
Dq[x_, tau_, a_] = (tau - 1) (1 - w[x, a])^(tau - 2) ∂x w[x, a]

Du[x_, C4_, C5_, C6_, a_] := gA (1 - C4 - C5 - C6) Dq[x, 3, a]
Dd[x_, C4_, a_] := -gA C4 Dq[x, 4, a]
Dub[x_, C5_, a_] := gA C5 Dq[x, 5, a]
Ddb[x_, C6_, a_] := -gA C6 Dq[x, 6, a]

gDu := Plot[ {x Du[x, 0.25, 0.03, 0.06, -0.08], x Du[x, 0.25, 0.03, 0.06, 0],
    x Du[x, 0.25, 0.03, 0.06, 0.08]}, {x, 0, 1}, PlotRange → {{0, 1}, {-0.25, 0.6}},
  Frame → True, FrameLabel → {x, x Δq(x)}, AspectRatio → 1, Axes → False,
  PlotStyle → Darker[Red], {Filling → {1 → {{3}, {Lighter[Red]}}}}]

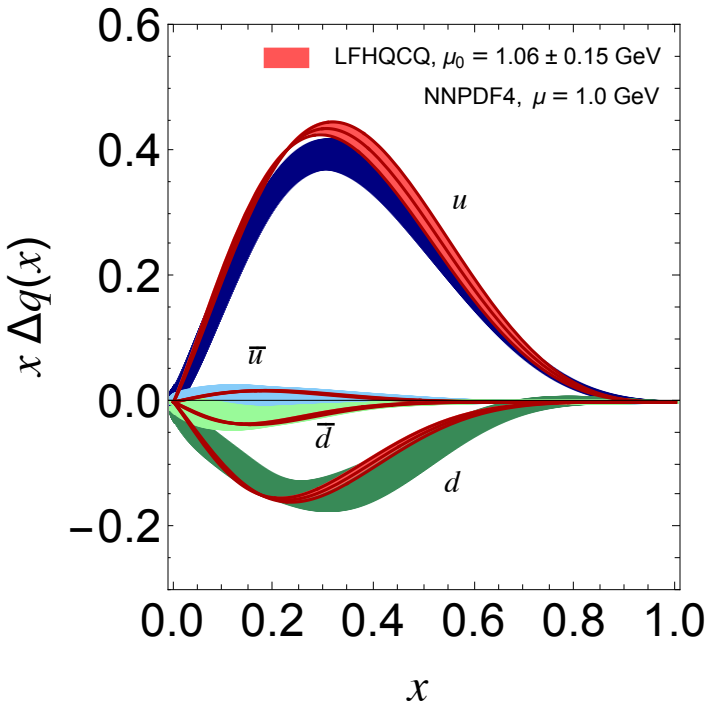
gDd := Plot[ {x Dd[x, 0.25, -0.08], x Dd[x, 0.25, 0], x Dd[x, 0.25, 0.08]},
  {x, 0, 1}, PlotRange → {{0, 1}, {-0.25, 0.6}}, Frame → True,
  FrameLabel → {x, x Δq(x)}, AspectRatio → 1, Axes → False,
  PlotStyle → Darker[Red], {Filling → {1 → {{3}, {Lighter[Red]}}}}]

gDub := Plot[ {x Dub[x, 0.03, -0.08], x Dub[x, 0.03, 0], x Dub[x, 0.03, 0.08]},
  {x, 0, 1}, PlotRange → {{0, 1}, {-0.25, 0.6}}, Frame → True,
  FrameLabel → {x, x Δq(x)}, AspectRatio → 1, Axes → False,
  PlotStyle → Darker[Red], {Filling → {1 → {{3}, {Lighter[Red]}}}}]

gDdb := Plot[ {x Ddb[x, 0.06, -0.08], x Ddb[x, 0.06, 0], x Ddb[x, 0.06, 0.08]},
  {x, 0, 1}, PlotRange → {{0, 1}, {-0.25, 0.6}}, Frame → True,
  FrameLabel → {x, x Δq(x)}, AspectRatio → 0.7, Axes → False,
  PlotStyle → Darker[Red], {Filling → {1 → {{3}, {Lighter[Red]}}}}]

gLab3 := Plot[0.55, {Q2, 0.2, 0.25}, PlotStyle → {Thickness[0.04], Lighter[Red]}]

Show[gDqdata, gDu, gDd, gDub, gDdb, gLab3]
```




## Axial sum rule

$$F_A(t) = \int_0^1 dx \left[\left(\tilde{H}^u(x,t) + \tilde{H}^{\overline{u}}(x,t)\right) - \left(\tilde{H}^d(x,t) + \tilde{H}^{\overline{d}}(x,t)\right)\right]$$

$$\Delta u(x) - \Delta d(x) + \Delta \overline{u}(x) - \Delta \overline{d}(x) =$$

$$g_A\left[C_3\,\Delta q^{\tau=3}(x) + C_4\,\Delta q^{\tau=4}(x) + C_5\,\Delta q^{\tau=5}(x) + C_6\,\Delta q^{\tau=6}(x)\right]$$

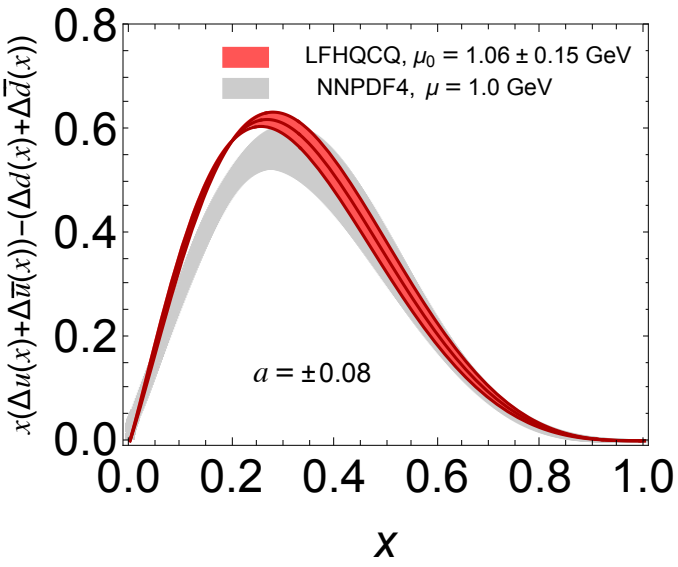
$$C_3 = 1 - C_4 - C_5 - C_6$$

```
DqASR[x_, C4_, C5_, C6_, a_] :=
 gA (1 - C4 - C5 - C6) Dq[x, 3, a] + gA C4 Dq[x, 4, a] + gA C5 Dq[x, 5, a] + gA C6 Dq[x, 6, a]

gqASR := Plot[ {x DqASR[x, 0.25, 0.03, 0.06, -0.08],
    x DqASR[x, 0.25, 0.03, 0.06, 0], x DqASR[x, 0.25, 0.03, 0.06, 0.08]},
  {x, 0, 1}, PlotStyle → Darker[Red], {Filling → {1 → {{3}, {Lighter[Red]}}}}]

gLab2 := Plot[0.74, {Q2, 0.2, 0.25}, PlotStyle → {Thickness[0.04], Lighter[Red]}]
gLab3 := Plot[0.68, {Q2, 0.2, 0.25},
  PlotStyle → {Thickness[0.04], RGBColor[0.8, 0.8, 0.8]}]

Show[gqASRdata, gqASR, gLab2, gLab3]
```




## Axial form factor

$$F_A^{\tau}(t) = (\tau - 1)\, B\Big(\tau - 1, 1 - \frac{t}{4\lambda}\Big), \quad F_A^{\tau}(0) = 1$$

$$F_A(t) = g_A \sum_{\tau} C_{\tau} F_{\tau}^{A}(t), \quad F_A(0) = g_A, \quad \sum_{\tau} C_{\tau} = 1$$

$$F_A(t) = g_A\Big[C_3 F_A^{\tau=3}(t) + C_4 F_A^{\tau=4}(t) + C_5 F_A^{\tau=5}(t) + C_6 F_A^{\tau=6}(t)\Big]$$

$$C_3 = 1 - C_4 - C_5 - C_6$$

```
FA[Q2_, kappa_, tau_] := (tau - 1) Beta[tau - 1, 1 + Q2 / (4 kappa^2)]
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```
FFA[Q2_, kappa_, C4_, C5_, C6_] := (1 - C4 - C5 - C6) FA[Q2, kappa, 3] +
  C4 FA[Q2, kappa, 4] + C5 FA[Q2, kappa, 5] + C6 FA[Q2, kappa, 6]
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```
gFA := Plot[{FFA[Q2, 0.499, 0.23, 0.03, 0.06],
    FFA[Q2, 0.523, 0.23, 0.03, 0.06], FFA[Q2, 0.547, 0.23, 0.03, 0.06]},
   {Q2, 0, 2.62}, PlotRange → {{0, 2.62}, {-0.02, 1.08}}, Frame → True,
   FrameLabel → {"Q² (GeV²)", G_A(Q²) / g_A}, AspectRatio → 1,
   Axes → False, PlotStyle → RGBColor[0.3984, 0.7, 0.664],
   {Filling → {1 → {{3}, RGBColor[0.3984, 0.8, 0.664]}}},
   LabelStyle → Directive[Large]]

gLab1 := Plot[0.95, {Q2, 1.5, 1.65},
   PlotStyle → {Thickness[0.04], RGBColor[0.3984, 0.7, 0.664]}]

Show[gFA, gAFFData, gLab1]
```