1. Fit to trajectories

We take the trajectory formula

$$\alpha(t) = \alpha(0) + \frac{t}{4\lambda}. \tag{1}$$

At the massless quark limit, the intercept for vector mesons $\alpha(0) = \frac{1}{2}$.

A simultaneous fit to the trajectories: $\rho - a$, $\omega - f$, and $\phi - f'$, gives the slope and intercept values

$$\lambda = 0.2873\,\text{GeV}^2 = (0.5360\,\text{GeV})^2,$$
$$\alpha_\rho(0) = 0.5049,$$
$$\alpha_\omega(0) = 0.5223,$$
$$\alpha_\phi(0) = 0.0269.$$

The results are shown in Figure 1. The three points: $\rho_5(2350)$, $a_6(2450)$, and $f_6(2510)$ (plotted in figure), are not included in the fit, because they are not experimentally established.

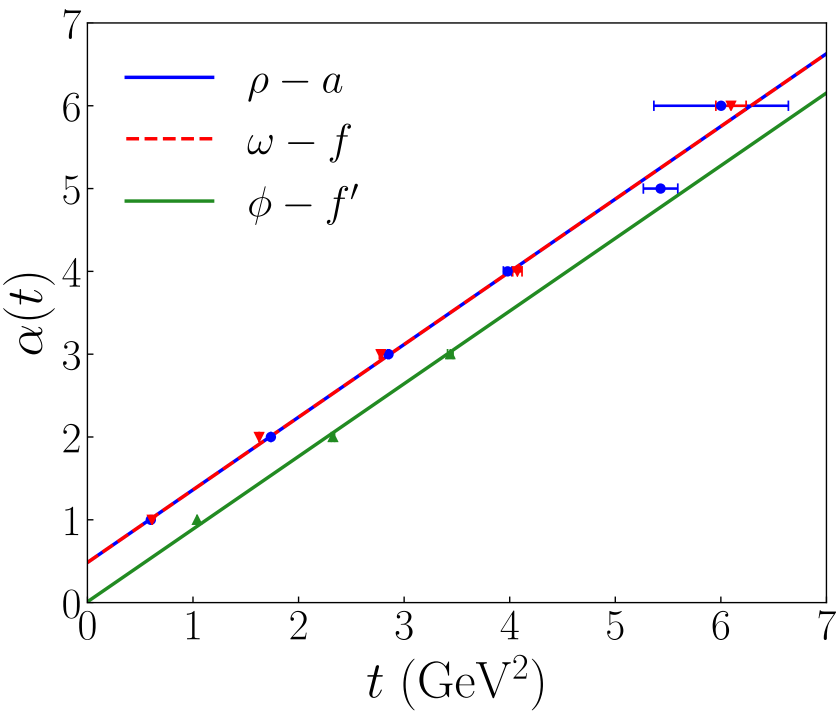

Figure 1: Chew-Frantschi plot.

The ρ and ω trajectories are almost degenerate. If not differentiating them, we take the average intercept value

$$\alpha_{\rho/\omega}(0) = \frac{1}{2}[\alpha_\rho(0) + \alpha_\omega(0)] = 0.5136. \tag{2}$$

2. Nucleon electromagnetic form factors

Here we use the Veneziano-type formula:

$$F_\tau(t) = \frac{B(\tau - 1, 1 - \alpha(t))}{B(\tau - 1, 1 - \alpha(0))}. \tag{3}$$

For the Dirac form factor, we truncate at twist-4, which means only considering the valence contribution:

$$F_1^p(Q^2) = a_3 F_{\tau=3}(Q^2) + a_4 F_{\tau=4}(Q^2) = (1-a_4)F_{\tau=3}(Q^2) + a_4 F_{\tau=4}(Q^2), \tag{4}$$

$$F_1^n(Q^2) = b_3 F_{\tau=3}(Q^2) + b_4 F_{\tau=4}(Q^2) = b_3 F_{\tau=3}(Q^2) - b_3 F_{\tau=4}(Q^2), \tag{5}$$

where charge normalizations $F_1^p(0) = 1$ and $F_1^n(0) = 0$ are applied.

For the Pauli form factor, we include the higher Fock state contribution at twist-6,

$$F_2^p(Q^2) = \chi_p[c_4 F_{\tau=4}(Q^2) + c_6 F_{\tau=6}(Q^2)] = \chi_p[(1-c_6)F_{\tau=4}(Q^2) + c_6 F_{\tau=6}(Q^2)], \tag{6}$$

$$F_2^p(Q^2) = \chi_n[d_4 F_{\tau=4}(Q^2) + d_6 F_{\tau=6}(Q^2)] = \chi_n[(1-d_6)F_{\tau=4}(Q^2) + d_6 F_{\tau=6}(Q^2)], \tag{7}$$

$$\tag{8}$$

where χ_p and χ_n are anomalous magnetic moments, and normalizations $F_2^p(0) = \chi_p$ and $F_2^n = \chi_n$ are applied.

Determine the coefficients

Now we determine the coefficients introduced above. Here we use the degenerate ρ/ω trajectory

$$\alpha(t) = \alpha_{\rho/\omega}(0) + \frac{t}{4\lambda}. \tag{9}$$

Instead of fitting the form factor data as we did previously, we can fix the coefficients above with proton and neutron static properties.

The charge and magnetic form factors are

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2}F_2(Q^2), \tag{10}$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2). \tag{11}$$

The charge radius is

$$< r_E^2 > = -6\frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0} \tag{12}$$

$$= -6F_1'(0) + \frac{3}{2M_N^2}\chi_N, \tag{13}$$

and the magnetic radius is

$$< r_M^2 > = -6\frac{dG_M(Q^2)}{\mu_N\, dQ^2}\Big|_{Q^2=0} \tag{14}$$

$$= -\frac{6}{\mu_N}[F_1'(0) + F_2'(0)], \tag{15}$$

where $\mu_N = G_M(0)$, and $\mu_p = 1+\chi_p$, $\mu_n = \chi_n$.

Then we have the constraints:

$$F_1^{p\prime}(0) = \frac{\chi_p}{4M_p^2} - \frac{1}{6} < r_E^{p2} >, \tag{16}$$

$$F_1^{n\prime}(0) = \frac{\chi_p}{4M_n^2} - \frac{1}{6} < r_E^{n2} >, \tag{17}$$

$$F_2^{p\prime}(0) = -\frac{1+\chi_p}{6} < r_M^{p2} > - F_1^{p\prime}(0), \tag{18}$$

$$F_2^{n\prime}(0) = -\frac{\chi_n}{6} < r_M^{n2} > - F_1^{n\prime}(0). \tag{19}$$

Taking experimental values from PDG, see Mathematica notebook, we find

$$a_4 = 0.408, \tag{20}$$

$$b_3 = -1.268, \tag{21}$$

$$c_6 = -0.162, \tag{22}$$

$$d_6 = 1.584. \tag{23}$$