Note HGD-20-10-18

The general e-m interaction in HQCD is:

$$\mathcal{S} = \int d^4x\, dz\, e_5(z) e^{\lambda z^2} \sqrt{g} g^{MM} \bar{\Psi}(x,z) \left(\mathcal{O}^1_M \, + \chi\, \mathcal{O}^2_M \right) \Psi(x,z)\, A_{M'}(x,z) \quad (1)$$

In 4-dim momentum space we have for the interaction term in HQCD ($Q^2 = -(p-p')^2 = -t$):

$$\frac{\bar{dz}}{z^4}\, \Psi(p,z) \left(\mathcal{O}^1_\mu \, + \mathcal{O}^2_\mu \right) \Psi(p,z)\, V(z,Q^2)\, \epsilon^\mu \quad (2)$$

where

$$\Psi(p,z) = \Psi_\tau(x) \frac{1+\gamma_5}{2} u(p) + \Psi_{\tau+1}(x) \frac{1-\gamma_5}{2} u(p) \quad (3)$$

where τ is the twist. and

$$V(z,Q^2)\, \epsilon^\mu = |\lambda| z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{Q^2/(4|\lambda|)} e^{-|\lambda| z^2 x/(1-x)} \quad (4)$$

$F_1(Q^2)$ is the FF with the interaction operator $\mathcal{O}^1_\mu$, F_2 th one with $\mathcal{O}^2_\mu$

For the ground state the wave functions are, see Guy's note (GN) (1):

$$\Psi_\tau = 2\sqrt{\frac{2\,\lambda^{\tau-1}}{\Gamma(\tau-1)}}\; z^{\tau+1/2}\, e^{-\lambda z^2/2} \quad (5)$$

and for the first radial excitation:

$$\Psi^*_\tau = \sqrt{\frac{2\,\lambda^{\tau-1}}{\Gamma(\tau)}}\; z^{\tau+1/2}\, e^{-\lambda z^2/2}\, (\tau - 1 - \lambda z^2) \quad (6)$$

The interaction operators are: For the elastic FF:

$$\mathcal{O}^1_\mu = \gamma_\mu; \qquad \mathcal{O}^2_\mu \, = i\, \sigma_{\mu\nu} q^\nu/(2M) \quad (7)$$

and for the transition FF

$$\mathcal{O}^1_\mu = \gamma_\mu - \gamma \cdot q \frac{q_\mu}{q^2}; \qquad \mathcal{O}^2_\mu \, = i\, \sigma_{\mu\nu} q^\nu/(m^* + M) \quad (8)$$

From (2) and reqdec follows from LFHQC for the vector FF:

$$F_1 = \tfrac{1}{2}(F_\tau + F_{\tau+1}); \quad F^*_1 = \tfrac{1}{2}(F^*_\tau + F^*_{\tau+1}) \quad (9)$$

with

$$F_\tau = \int \frac{dz}{z^4} (\Psi_\tau(z))^2\, V(Q^2,z); \quad F^*_\tau = \int \frac{dz}{z^4} \Psi^*_\tau(z)\, \Psi_\tau(z)\, V(Q^2,z) \quad (10)$$

The integral representation of the B function $B[u,w] = \int_0^1 dx (1-x)^{u-1}\, x^{W-1}$ allows easily to perform the z intrgration.

The The LFHQCD result eq. 9 has to be modified :

1. Twist: The original LFHQCD twist $\tau = 2$ has to be changed to the physical twist $\tau \geq 3$

2. Pole shift: For the Vector FF the poles have to be shifted to the physical positions:
   $1 + \frac{Q^2}{4\lambda} \to 1 - \alpha_{VM}(-Q^2)$

3. Twist decomposition: The sum in (9) has to be modified to

$$F_1 = \tfrac{1}{2}(\alpha F_\tau + (1-\alpha) F_{\tau+1}) \tag{11}$$

where constraints on α arise if we calculate PDFs from the FF. In GN $\alpha = 1$.

For the Pauli FF one obtains:

a) Elastic:

$$F_2 = \chi \int \frac{dz}{z^3} \Psi_{\tau+1}(z)\, \Psi_\tau(z)\, V(Q^2, z) \tag{12}$$

b) inelastic:

$$F_2^* = \frac{\chi}{2} \int \frac{dz}{z^3} \left(\Psi^*_{\tau+1)}(z)\, \Psi_\tau(z) + \Psi^*_\tau(z)\, \Psi_{\tau+1}(z) \right) V(Q^2, z) \tag{13}$$

Results:

Elastic:

$$\begin{aligned} F_1 &= (\tau - 1)\, B[\tau - 1, 1 + Q^2/(4\lambda)] & (14) \\ F_2 &= \chi \sqrt{\frac{\tau - 1}{\lambda}}\, \tau\, B[\tau, 1 + Q^2/(4\lambda)] & (15) \end{aligned}$$

The dimensionful λ occurs since the curvature radius R of ADS_5 has been set to 1. It must be absorbed in the χ.

Inelastic:

$$\begin{aligned} F_1^* &= \frac{1}{\sqrt{\Gamma(\tau-1)\,\Gamma(\tau)}} \Big\{ (\tau-1)\Gamma(\tau) B[\tau - 1, 1 + Q^2/(4\lambda)] & (16) \\ & \qquad - \Gamma(\tau+1) B[\tau, 1 + Q^2/(4\lambda)] \Big\} & (17) \\ &= \sqrt{\tau - 1}\, \frac{Q^2}{4\lambda}\, B[\tau, 1 + Q^2/(4\lambda)]; & (18) \end{aligned}$$

This expression agrees with GN (8). It has the form as expected from duality (Veneziano-type) considerationes. The factor Q^2 is necessary to cancel the $1/Q^2$ in $\mathcal{O}^2_\mu$, the rest is an analytic B-function. Here the pole shift can be performed safely and the proposed final result is

$$F_1^*(t) = -\sqrt{\tau - 1} \frac{t}{4\lambda} \frac{B[\tau, 1/2 - \alpha_{VM}(t)]}{B[\tau, 1/2 - \alpha_{VM}(0)]}. \tag{19}$$

There is, however a certain ambiguity in the normalization, since the factor $\frac{1}{B[\tau,1/2-\alpha_{VM}(0)]}$ is only introduced by analogy to the elastic FF.

The Pauli form factor is even less unique:

$$
\begin{aligned}
F_2^* &= \frac{\chi}{\sqrt{\lambda\Gamma(\tau-1)\,\Gamma(\tau+1)}}\Big\{(\tau-\tfrac{1}{2})\Gamma(\tau+1)B[\tau,1+Q^2/(4\lambda)] \\
&\qquad -\Gamma(\tau+2)B[\tau+1,1+Q^2/(4\lambda)]\Big\} && (20)\\
&= \chi\sqrt{\frac{\tau(\tau-1)}{\lambda}}\left(\left(1-\frac{1}{2\tau}\right)\frac{Q^2}{4\lambda}-\frac{\tau+1}{2\tau}\right)B[\tau+1,1+\frac{Q^2}{4\lambda}] && (21)
\end{aligned}
$$

Here there is no compelling reason to perform the pole shift "$1+\frac{Q^2}{4\lambda}\to 1-\alpha_{VM}(-Q^2)$ " in (21) and not in (20).