

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

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The structure of generalized parton distributions is determined from light-front holographic QCD up to a universal reparametrization function $w(x)$ which incorporates Regge behavior at small x and inclusive counting rules at $x \rightarrow 1$. A simple ansatz for $w(x)$ which fulfills these physics constraints with a single-parameter results in precise descriptions of both the nucleon and the pion quark distribution functions, in contrast with global fits and models which require a large number of parameters. The analytic structure of the amplitudes leads to a connection with the Veneziano model.

INTRODUCTION

Generalized parton distributions (GPDs) [1–3] have emerged as a comprehensive tool to describe the nucleon structure as probed in hard scattering processes. GPDs link nucleon form factors (FFs) to longitudinal parton distributions (PDFs), and their first moment provide the angular momentum contribution of the nucleon constituents to its total spin through Ji’s sum rule [2]. The GPDs also encode information of the three-dimensional spatial structure of the hadrons: The Fourier transform of the GPDs gives the transverse spatial distribution of partons in correlation with their longitudinal momentum fraction x [4].

Since a precise knowledge of PDFs is required for the analysis and interpretation of the scattering experiments in the LHC era, considerable efforts have been made to determine PDFs and their uncertainties by global fitting collaborations such as MMHT [5], CT [6], NNPDF [7], and HERAPDF [8]. Lattice QCD calculations are using different methods, such as path-integral formulation of the deep-inelastic scattering hadronic tensor [9–11], inversion method [12, 13], *quasi*-PDFs [14–18], *pseudo*-PDFs [19, 20] and lattice cross-sections [21] to obtain the x -dependence of the PDFs. The current status and challenges for a meaningful comparison of lattice calculations with the global fit of PDFs can be found in [22].

There has been recent interest in the study of parton distributions using the framework of light-front holographic QCD (LFHQCD), an approach to hadron structure based on the holographic embedding of light-front dynamics in a higher dimensional gravity theory, with the constraints imposed by the underlying superconformal algebraic structure [23–29]. This effective semiclassical approach to relativistic bound-state equations in QCD captures essential aspects of the confinement dynamics which are not apparent from the QCD Lagrangian, such as the emergence of a mass scale $\lambda = \kappa^2$, a unique form

of the confinement potential, a zero mass state in the chiral limit: the pion, and universal Regge trajectories for mesons and baryons.

Various models of parton distributions based on LFHQCD [30–52] use as a starting point the analytic form of GPDs found in Ref. [53]. This simple analytic form incorporates the correct high-energy counting rules of FFs [54, 55] and the GPD’s t -momentum transfer dependence. One can also obtain effective light-front wave functions (LFWFs) [28, 56] which are relevant for the computation of FFs and PDFs, including polarization dependent distributions [44, 45, 48]. LFWFs are also used to study the skewness ξ -dependence of the GPDs [42, 46, 49, 51, 52], and other parton distributions such as the Wigner distribution functions [36, 39, 44]. The downside of the above phenomenological extensions of the holographic model is the large number of parameters required to describe simultaneously PDFs and FFs for each flavor.

Motivated by our recent analysis of the nucleon FFs in LFHQCD [57], we extend here our previous results for GPDs and LFWFs [53, 56]. Shifting the FF poles to their physical location [57] does not modify the exclusive counting rules but modifies the slope and intercept of the Regge trajectory, and hence the analytic structure of the GPDs which incorporates the Regge behavior. As a result, the x -dependence of PDFs and LFWFs is modified. Furthermore, the GPDs are defined in the present context up to a universal reparametrization function; therefore, imposing further physically motivated constraints is necessary.

GPDs IN LFHQCD

In LFHQCD the FF for arbitrary twist- τ is expressed in terms of Gamma functions [28, 53], an expression which can be recast in terms of the Euler Beta function $B(u, v)$

as [29]

$$F_\tau(t) = \frac{1}{N_\tau} B\left(\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}\right), \quad (1)$$

where

$$B(u, v) = \int_0^1 dy y^{u-1} (1-y)^{v-1}, \quad (2)$$

and $B(u, v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}$ with $N_\tau = \sqrt{\pi} \frac{\Gamma(\tau-1)}{\Gamma(\tau-\frac{1}{2})}$. For fixed u and large v we have $B(u, v) \sim \Gamma(u)v^{-u}$. We thus recover the hard-scattering scaling behavior [54, 55]

$$F_\tau(Q^2) \sim \left(\frac{1}{Q^2}\right)^{\tau-1}, \quad (3)$$

for large $Q^2 = -t$. For integer τ Eq. (1) generates the pole structure [53]

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_0^2}\right)\left(1 + \frac{Q^2}{M_1^2}\right) \cdots \left(1 + \frac{Q^2}{M_{\tau-2}^2}\right)}, \quad (4)$$

with $M_n^2 = 4\lambda\left(n + \frac{1}{2}\right)$, $n = 0, 1, 2, \dots, \tau - 2$, corresponding to the ρ vector meson and its radial excitations. Notice that the Beta function in (1) can be rewritten as $B(\tau - 1, 1 - \alpha(t))$ with Regge trajectory

$$\alpha(t) = \frac{t}{4\lambda} + \frac{1}{2}, \quad (5)$$

slope $\alpha' = \frac{1}{4\lambda}$ and intercept $\alpha(0) = \frac{1}{2}$. This expression is identical to the Veneziano amplitude [58]

$$(1 - \alpha(s), 1 - \alpha(t)), \quad (6)$$

in the t -channel. In the s -channel it leads to a fixed pole $1 - \alpha(s) \rightarrow \tau - 1$, since no resonances are formed [59]. The shift in the pole structure [28] incorporated in Eq. (1) thus yields the leading Regge trajectory for the ρ -meson (5).

Writing the flavor FF in terms of the valence GPD $H(x, \xi, t)$ at zero skewness $F^q(t) = \int_0^1 dx H^q(x, t)$, with

$$H^q(x, t) \equiv H^q(x, \xi = 0, t) = q(x) \exp[t f(x)], \quad (7)$$

Eqs. (1) and (2) imply that the twist- τ PDF, $q_\tau(x)$, and the profile function $f(x)$ are

$$q_\tau(x) = \frac{1}{N_\tau} (1 - w(x))^{\tau-2} w(x)^{-\frac{1}{2}} w'(x), \quad (8)$$

$$f(x) = \frac{1}{4\lambda} \log\left(\frac{1}{w(x)}\right). \quad (9)$$

Therefore, $q(x)$ and $f(x)$ in (7) are both determined from (8) and (9) in terms of the arbitrary reparametrization function $y = w(x)$, which satisfies

$$w(0) = 0, \quad w(1) = 1, \quad (10)$$

and is monotonically increasing in the interval $0 \leq x \leq 1$.

The simplest choice for $w(x)$, with conditions (10), is $w(x) = x$. It leads to the t -dependence $q(x, t) = x^{-\alpha' t} q(x)$, thus to

$$f(x) = \frac{1}{4\lambda} \log\left(\frac{1}{x}\right), \quad (11)$$

which is the Regge theory motivated ansatz for small- x given in Ref. [60]. We therefore impose the constraint

$$w(x) \sim x, \quad \text{for } x \sim 0, \quad (12)$$

to incorporate the small- x Regge behavior in the GPDs.

To study the behavior of $w(x)$ at large- x we perform a Taylor expansion near $x = 1$:

$$w(x) = 1 - (1-x)w'(1) + \frac{1}{2}(1-x)^2 w''(1) + \dots \quad (13)$$

Upon substitution of (13) in (8) we find that the leading term in the expansion, which behaves as $(1-x)^{\tau-2}$, vanishes if $w'(1) = 0$. Hence setting

$$w'(1) = 0 \quad \text{and} \quad w''(1) \neq 0, \quad (14)$$

we find

$$q_\tau(x) \sim (1-x)^{2\tau-3}, \quad (15)$$

which is precisely the perturbative QCD (pQCD) inclusive hard counting rule for large- x [61–63].

From Eq. (9) it follows that the conditions (14) are equivalent to $f'(1) = 0$ and $f''(1) \neq 0$. Since $\log(x) \sim 1 - x$ for $x \sim 1$, the simplest ansatz for $f(x)$ consistent with (10), (12) and (14) is

$$f(x) = \frac{1}{4\lambda} \left[(1-x) \log\left(\frac{1}{x}\right) + a(1-x)^2 \right], \quad (16)$$

with a being a flavor independent parameter. From (9)

$$w(x) = x^{1-x} e^{-a(1-x)^2}, \quad (17)$$

an expression which incorporates Regge behavior at small- x and inclusive counting rules at large- x .

Nucleon GPDs

The nucleon GPDs are extracted from nucleon FF data [64–68] choosing specific x - and t -dependences of the GPDs for each flavor. One then finds the best fit reproducing the measured FFs and the valence PDFs. In our analysis of nucleon FFs [57], three free parameters are required: These are r , interpreted as an SU(6) breaking effect for the Dirac neutron FF, and γ_p and γ_n , which account for the probabilities of higher Fock components (meson cloud), and are significant only for the Pauli FFs. The hadronic scale λ is fixed by the ρ -Regge trajectory [28], whereas the Pauli FFs are normalized to the experimental values of the anomalous magnetic moments.

Helicity Non-Flip Distributions

Using the results from [57] for the Dirac flavor FFs, we write the spin non-flip valence GPDs $H^q(x, t) = q(x) \exp[t f(x)]$ with

$$u_v(x) = \left(2 - \frac{r}{3}\right) q_{\tau=3}(x) + \frac{r}{3} q_{\tau=4}(x), \quad (18)$$

$$d_v(x) = \left(1 - \frac{2r}{3}\right) q_{\tau=3}(x) + \frac{2r}{3} q_{\tau=4}(x), \quad (19)$$

for the u and d PDFs normalized to the valence content of the proton: $\int_0^1 dx u_v(x) = 2$ and $\int_0^1 dx d_v(x) = 1$. The PDF $q_\tau(x)$ and the profile function $f(x)$ are given by (8) and (9), and $w(x)$ is given by (17). Positivity of the PDFs implies that $r \leq 3/2$, which is smaller than the value $r = 2.08$ found in [57]. We shall use the maximum value $r = 3/2$, which does not change significantly our results in [57].

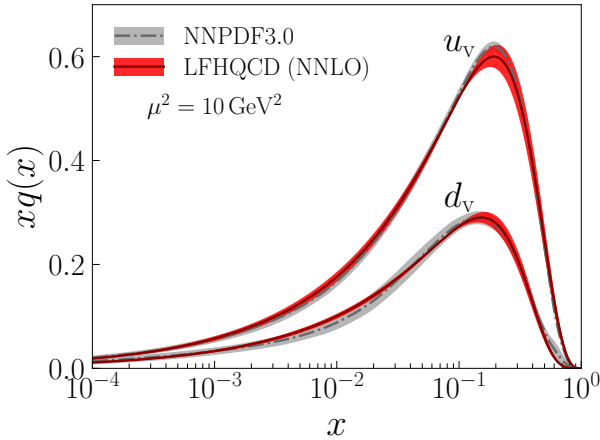


FIG. 1. Model comparison (red band) for $xq(x)$ in the proton with NNPDF3.0 (grey band) [75].

The PDFs (18) and (19) are evolved to a higher scale μ with the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [69–71] in the $\overline{\text{MS}}$ scheme using the HOPPET toolkit [72]. The initial scale is chosen at the matching scale between LFHQCD and pQCD as $\mu_0 = 1.06 \pm 0.15 \text{ GeV}$ [73] in the $\overline{\text{MS}}$ scheme at next-to-next-to-leading order (NNLO). The strong coupling constant α_s at the scale of the Z -boson mass is set to 0.1182 [74], and the heavy quark thresholds are set with $\overline{\text{MS}}$ quark masses as $m_c = 1.28 \text{ GeV}$ and $m_b = 4.18 \text{ GeV}$ [74]. The PDFs are evolved to $\mu^2 = 10 \text{ GeV}^2$ at NNLO to compare with the global fit by the NNPDF Collaboration [75] as shown in Fig. 1. The value $a = 0.507 \pm 0.034$ is determined from the first moment of the GPD, $\int_0^1 dx x H_v^q(x, t = 0) = A^q(0)$, from NNPDF3.0 [75]. The model uncertainty (red band) includes the uncertainties in a and μ_0 . The t -dependence of $H^q(x, t)$ is illustrated in Fig. 2. Since our PDFs scale as $q(x) \sim x^{-1/2}$ for small- x , the Kuti-Weisskopf behavior for the non-singlet structure func-

tions $F_{2p}(x) - F_{2n}(x) \sim x(u_v(x) - d_v(x)) \sim x^{1/2}$ is satisfied [76].

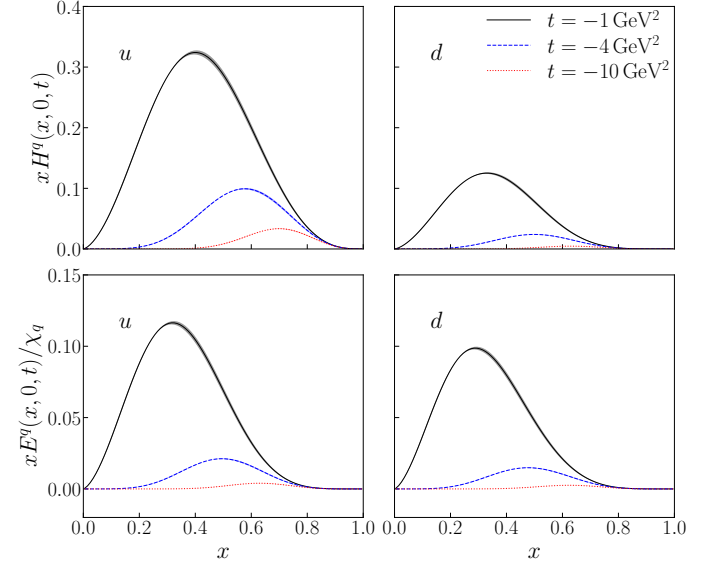


FIG. 2. Nucleon GPDs for different values of $-t = Q^2$. Top: spin non-flip $H^q(x, t)$. Bottom: spin-flip $E^q(x, t)$.

Helicity-Flip Distributions

The spin-flip GPDs $E^q(x, t) = e_q(x) \exp[t f(x)]$ follow from the flavor Pauli FFs in [57] given in terms of twist-4 and twist-6 contributions

$$e_q(x) = \chi_q [(1 - \gamma_q) q_{\tau=4}(x) + \gamma_q q_{\tau=6}(x)], \quad (20)$$

normalized to the flavor anomalous magnetic moment $\int_0^1 dx e_q(x) = \chi_q$, with $\chi_u = 2\chi_p + \chi_n = 1.673$ and $\chi_d = 2\chi_n + \chi_p = -2.033$. The factors γ_u and γ_d are

$$\gamma_u \equiv \frac{2\chi_p \gamma_p + \chi_n \gamma_n}{2\chi_p + \chi_n}, \quad \gamma_d \equiv \frac{2\chi_n \gamma_n + \chi_p \gamma_p}{2\chi_n + \chi_p},$$

where the higher Fock probabilities $\gamma_{p,n}$ represent the large distance pion contribution and have the values $\gamma_p = 0.27$ and $\gamma_n = 0.38$ [57]. Our results for $E^q(x, t)$ are displayed in Fig. 2.

We use Ji's sum rule [2] to compute the nonperturbative contribution to the total spin of the nucleon

$$J^q = \frac{1}{2} \int_0^1 dx x [H_v^q(x, t = 0) + E_v^q(x, t = 0)]. \quad (21)$$

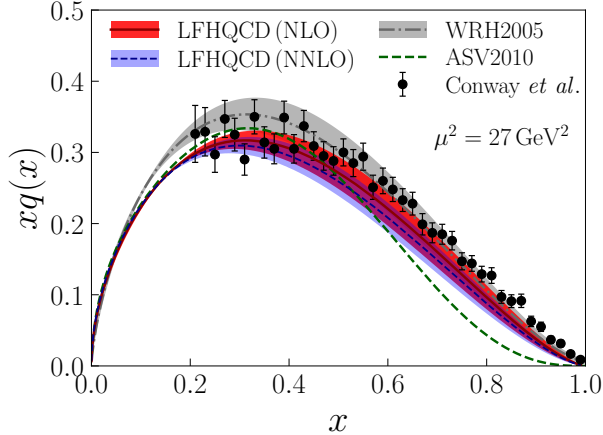
We compare our results for J^q in TABLE I, at the initial scale $\mu_0 = 1.06 \pm 0.15 \text{ GeV}$, with model fits constrained by nucleon FFs [66, 68] and lattice simulations [77–79].

Pion GPD

The expression for the pion GPD $H^{u,\bar{d}}(x, t) = q^{u,\bar{d}}(x) \exp[t f(x)]$ follows from the pion FF in [80], where

TABLE I. Results for the total angular momentum of quarks.

	$2J^u$	$2J^d$
This work	0.561 ± 0.008	-0.100 ± 0.002
[3]	0.58	-0.06
[66]	0.46, 0.56	-0.007, -0.019
[68]	0.572 ± 0.214	-0.098 ± 0.014
[77]	0.74 ± 0.12	0.08 ± 0.08
[78]	0.74 ± 0.12	-0.03 ± 0.08
[79]	0.62 ± 0.08	0.11 ± 0.08

FIG. 3. Model comparison (red band) for $xq(x)$ in the pion with the NLO fits [81, 82] (gray band and green curve) and the LO extraction of Fermilab E615 Drell-Yan data [83]. NNLO results are also included (light blue band).

the contribution from higher Fock components was determined from the analysis of the time-like region [80]. Up to twist-4

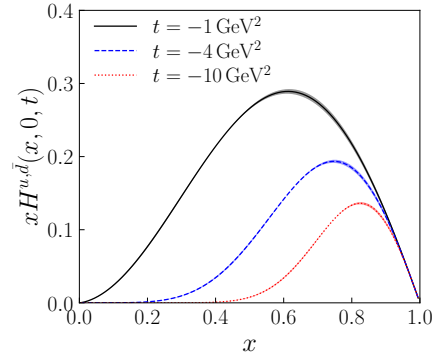
$$q_v^{u,\bar{d}}(x) = (1 - \gamma)q_{\tau=2}(x) + \gamma q_{\tau=4}(x), \quad (22)$$

where the PDFs are normalized to the valence quark content of the pion $\int_0^1 dx q_v^{u,\bar{d}}(x) = 1$, and $\gamma = 0.125$ represents the meson cloud contribution in [28].

The pion PDFs are evolved to $\mu^2 = 27 \text{ GeV}^2$ at next-to-leading order (NLO) to compare with the NLO global analysis in [81, 82] of the data [83]. The initial scale is set at $\mu_0 = 1.1 \pm 0.2 \text{ GeV}$ from the matching procedure in Ref. [73] at NLO. The result is shown in Fig. 3, and the t -dependence of $H^q(x, t)$ is illustrated in Fig. 4. We have also included the NNLO results in Fig. 3, to compare with future data analysis.

Our results are in good agreement with the data analysis in Ref. [81] and consistent with the NNPDF results through the GPD universality described here. There is however a tension with the data analysis in [82] for $x \geq 0.6$ and with the Dyson-Schwinger results in [84] with $(1 - x)^2$ falloff at large- x . Our nonperturbative results falloff as $1 - x$ from the leading twist-2 term in (22).

A softer falloff $\sim (1 - x)^{1.5}$ in Fig. 3 follows from DGLAP evolution.

FIG. 4. Pion GPD for different values of $-t = Q^2$.

CONCLUSION AND OUTLOOK

The results presented here for the GPDs provide a new structural framework for the exclusive-inclusive connection which is fully consistent with the LFHQCD results for the hadron spectrum. The PDFs are flavor-dependent and expressed as a superposition of PDFs $q_\tau(x)$ of different twist. In contrast, the GPD profile function $f(x)$ is universal. Both $q(x)$ and $f(x)$ can be expressed in terms of a universal reparametrization function $w(x)$, which incorporates Regge behavior at small- x and inclusive counting rules at large- x . A simple ansatz for $w(x)$, which satisfies all the physics constraints, leads to a precise description of parton distributions and form factors for the pion and nucleons in terms of a single physically constrained parameter. In contrast with the eigenfunctions of the holographic LF Hamiltonian [28], the effective LFWFs obtained here incorporate the non-perturbative pole structure of the amplitudes, Regge behavior and exclusive and inclusive counting rules. The analytic structure of FFs and GPDs leads to a connection with the Veneziano amplitude (6) which could give further insights into the quark-hadron duality and hadron structure. The falloff of the pion PDF at large- x is an unresolved issue [85] which requires a new generation of experiments.

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Appendix: Effective LFWFs

Form factors in light-front quantization can be written in terms of an effective single-particle density [86]

$$F(Q^2) = \int_0^1 dx \rho(x, Q), \quad (\text{A.23})$$

where $\rho(x, Q) = 2\pi \int_0^\infty db b J_0(bQ(1-x)) |\psi_{\text{eff}}(x, b)|^2$ with transverse separation $b = |\mathbf{b}_\perp|$. From (7) we find the effective LFWF

$$\psi_{\text{eff}}^\tau(x, \mathbf{b}_\perp) = \frac{1}{2\sqrt{\pi}} \sqrt{\frac{q_\tau(x)}{f(x)}} (1-x) \exp \left[-\frac{(1-x)^2}{8f(x)} \mathbf{b}_\perp^2 \right], \quad (\text{A.24})$$

in the transverse impact space representation with $q_\tau(x)$ and $f(x)$ given by (8) and (9). The normalization is $\int_0^1 dx \int d^2\mathbf{b}_\perp |\psi_{\text{eff}}(x, \mathbf{b}_\perp)|^2 = 1$, provided that $\int_0^1 dx q_\tau(x) = 1$. In the transverse momentum space

$$\psi_{\text{eff}}^\tau(x, \mathbf{k}_\perp) = 8\pi \frac{\sqrt{q_\tau(x)f(x)}}{1-x} \exp \left[-\frac{2f(x)}{(1-x)^2} \mathbf{k}_\perp^2 \right], \quad (\text{A.25})$$

with normalization $\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} |\psi_{\text{eff}}(x, \mathbf{k}_\perp)|^2 = 1$.

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