Robot Learning and Estimation
Homework 3

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May 10, 2023

1 SLAM as a Least Squares Problem

Part (a)

$$\begin{aligned}
x_{0:t}^*, M^* &= \underset{x_{0:t}^*, M^*}{\operatorname{argmax}}\, p(x_{0:t}, M, u_{0:t}, z_{1:t}) \\
&= \underset{x_{0:t}^*, M^*}{\operatorname{argmax}} \prod_{i=1}^{n} p(x_i | u_{0:i-1}, z_{1:i-1}) p(x_0) \prod_{k=1}^{K} p(z_k | x_{tk}, m_{jk}) \\
&= \underset{x_{0:t}^*, M^*}{\operatorname{argmin}} - \log \left(p(x_0) \prod_{i=1}^{n} \mathcal{N}(f(x_{i-1}, u_i), R) \prod_{k=1}^{K} \mathcal{N}(h(x_{tk}, m_{jk}), Q) \right) \\
&= \underset{x_{0:t}^*, M^*}{\operatorname{argmin}} \sum_{i=1}^{n} |f(x_{i-1}, u_i) - x_i|_{R^{-1}}^2 + \sum_{k=1}^{K} |h(x_{tk}, m_{jk}) - z_k|_{Q^{-1}}^2
\end{aligned}$$

Part (b)

By first order Taylor expansion at x_i^0, we have

$$\begin{aligned}
f(x_{i-1}, u_i) - x_i &\approx f(x_{i-1}^0, u_i) + F_i^{i-1}(x_{i-1} - x_{i-1}^0) - x_i \\
&= F_i^{i-1} \delta_{x_{i-1}} + f(x_{i-1}^0, u_i) - x_i^0 + x_i^0 - x_i \\
&= F_i^{i-1} \delta_{x_{i-1}} - \delta_{x_i} - a_i
\end{aligned}$$

Similarly, we have

$$\begin{aligned} h(x_{tk}, m_{jk}) - z_k &= h(x_{tk}^0, m_{jk}^0) + H_k^{ik}(x_{tk} - x_{tk}^0) + J_k^{ik}(m_{jk} - m_{jk}^0) - z_k \\ &= H_k^{ik}\delta_{x_{tk}} + J_k^{ik}\delta_{m_{jk}} + c_k \end{aligned}$$

where $H_k^{ik} = \frac{\delta h(x_{tk}^0, m_{jk}^0)}{\delta x_{tk}}$ and $J_k^{ik} = \frac{\delta h(x_{tk}^0, m_{jk}^0)}{\delta m_{jk}}$.

Then, the problem is

$$\begin{aligned} &\underset{x_{0:t}^*, M^*}{\operatorname{argmin}} \sum_{i=1}^{n} |f(x_{i-1}, u_i) - x_i|_{R^{-1}}^2 + \sum_{k=1}^{K} |h(x_{tk}, m_{jk}) - z_k|_{Q^{-1}}^2 \\ =&\underset{x_{0:t}^*, M^*}{\operatorname{argmin}} \sum_{i=1}^{n} |F_i^{i-1}\delta_{x_{i-1}} - I\delta_{x_i} - a_i|_{R^{-1}}^2 + \sum_{k=1}^{K} |H_k^{ik}\delta_{x_{tk}} + J_k^{ik}\delta_{m_{jk}} + c_k|_{Q^{-1}}^2 \\ =&\underset{x_{0:t}^*, M^*}{\operatorname{argmin}} \sum_{i=1}^{n} |R^{-\frac{1}{2}}(F_i^{i-1}\delta_{x_{i-1}} - I\delta_{x_i} - a_i)|^2 + \sum_{k=1}^{K} |Q^{-\frac{1}{2}}(H_k^{ik}\delta_{x_{tk}} + J_k^{ik}\delta_{m_{jk}} + c_k)|^2 \\ =&\underset{x_{0:t}^*, M^*}{\operatorname{argmin}} \sum_{i=1}^{n} |\bar{F}_i^{i-1}\delta_{x_{i-1}} - \bar{I}\delta_{x_i} - \bar{a}_i|^2 + \sum_{k=1}^{K} |\bar{H}_k^{ik}\delta_{x_{tk}} + \bar{J}_k^{ik}\delta_{m_{jk}} + \bar{c}_k|^2 \end{aligned}$$

This can be written as

$$\theta^* = \underset{\delta\theta}{\operatorname{argmin}} |A\delta\theta - b|^2$$

where

$$\begin{aligned} A &= \begin{bmatrix} \bar{F}_0 & \bar{I} & 0 & 0 & \dots & 0 \\ 0 & \bar{F}_1 & \bar{I} & 0 & \dots & 0 \\ 0 & 0 & \bar{F}_2 & \bar{I} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \bar{H}_k^{ik} & \bar{J}_k^{ik} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \\ \delta\theta &= \begin{bmatrix} \delta x_0 & \dots & \delta x_t & \delta m_1 & \dots & \delta m_k \end{bmatrix}^\top \\ A &= \begin{bmatrix} \bar{a}_1 & \dots & \bar{a}_t & \bar{c}_0 & \dots & \bar{c}_k \end{bmatrix}^\top \end{aligned}$$

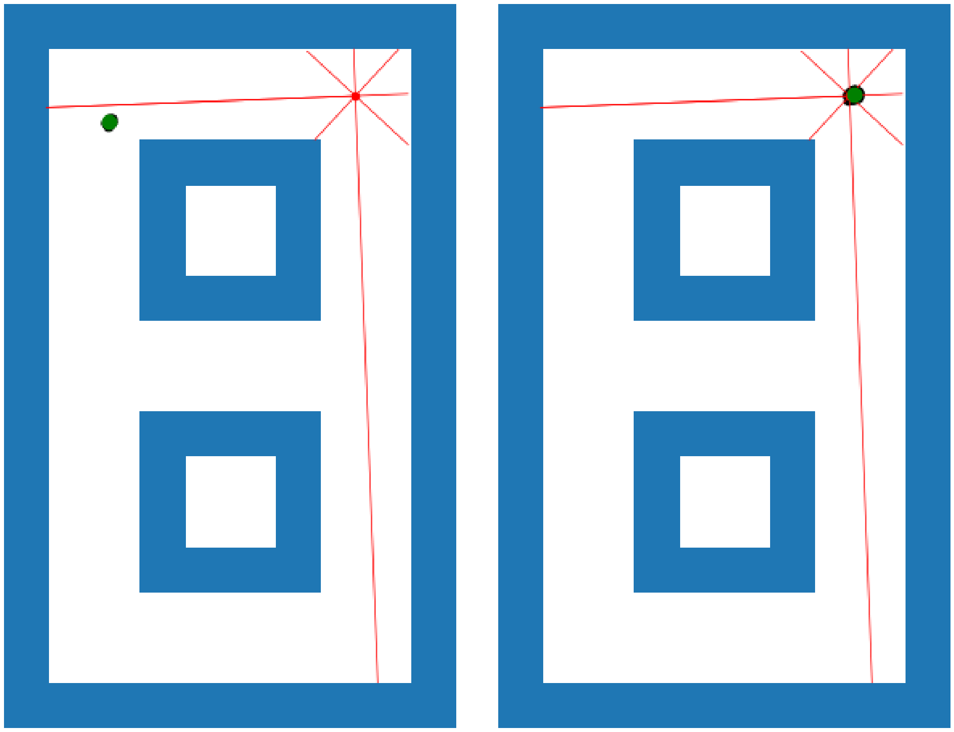

Figure 1: Loops world path 0: fails with 100 particles (left) and runs well with 1000 particles (right).

Part (c)

We could utilize the fact that A is a sparse matrix.

2 Particle Filter Localization

Part (d)

Please see fig. 1 to fig. 6.

Part (e)

We can see that the for simple world path 2 and rooms world path 2, 1000 particles are enough to get a good result. Since some part of the map is symmetric and the measurements and motion models are also symmetric, the particles often get into two clusters when the initialization is uniform. Therefore, we need more particles for them to eventually converge to the correct cluster.

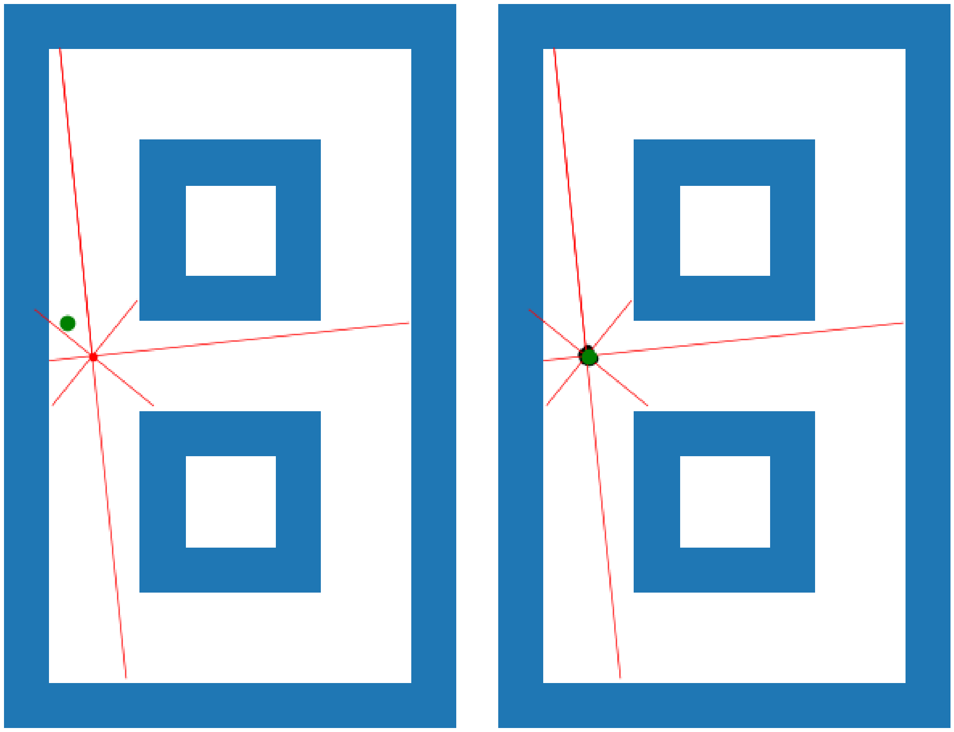

Figure 2: Loops world path 1: fails with 10 particles (left) and runs well with 100 particles (right).

Part (f)

For this question, the figures are fig. 8 and fig.9.

When we never resample, the particles do not converge no matter what N we use. This is because the particles are not resampled based on how likely the observations are so when we add Gaussian noise at each step the particles eventually become completely noisy.

When we only resample when the number of effective particles is less than $\frac{3N}{2}$, the number of particles required for the algorithm to converge is the same as in the last question when we always resample.

3 EKF SLAM

Part (a)

Denote z_t by $[z_1\, z_2]^\top$, then we can expand eq.6 as

$$z_{t,1} = \cos\theta_t(x_m - x_t) + \sin\theta_t(y_m - y_t) + w_t$$

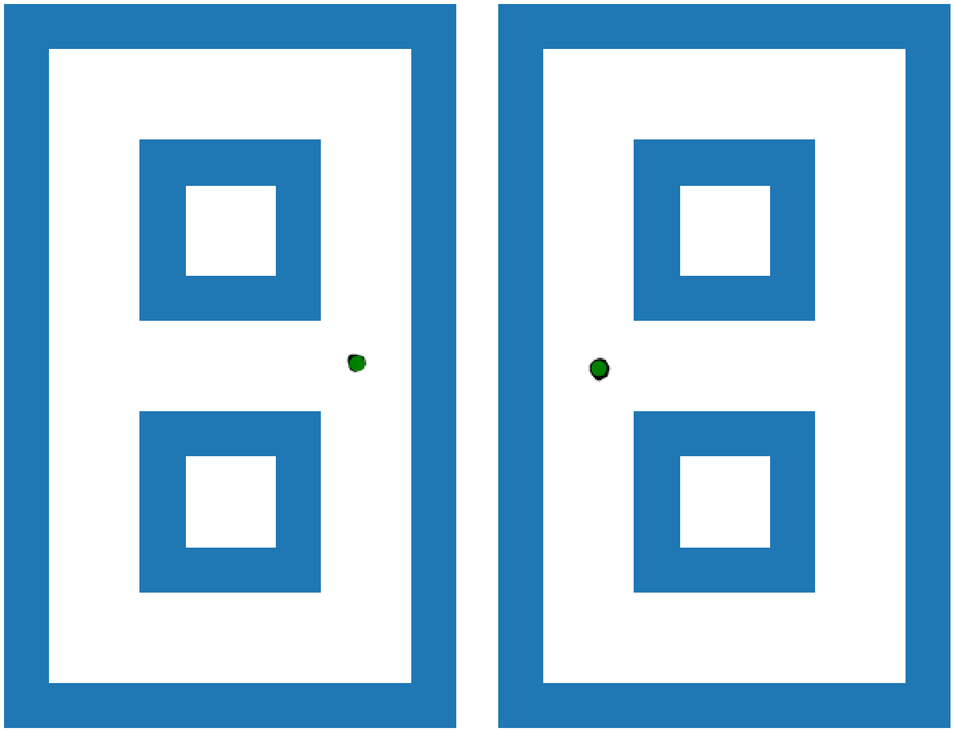

Figure 3: Loops world path 2: fails with 10 particles (left) and runs well with 100 particles (right).

$$z_{t,2} = -\sin\theta_t(x_m - x_t) + \cos\theta_t(y_m - y_t) + w_t$$

Multiplying the first equation by $\sin\theta_t$ and the second by $\cos\theta_t$ and summing them together, we have

$$\begin{aligned} z_{t,1}\sin\theta_t + z_{t,2}\cos\theta_t = y_m - y_t + (\sin\theta_t + \cos\theta_t)w_t \\ y_m = y_t - (\sin\theta_t + \cos\theta_t)w_t + z_{t,1}\sin\theta_t + z_{t,2}\cos\theta_t \end{aligned}$$

Multiplying the first equation by $\cos\theta_t$ and the second by $\sin\theta_t$ and subtracting the first from the second, we have

$$\begin{aligned} z_{t,1}\cos\theta_t - z_{t,2}\sin\theta_t = x_m - x_t + (\cos\theta_t - \sin\theta_t)w_t \\ x_m = x_t - (\cos\theta_t - \sin\theta_t)w_t + z_{t,1}\cos\theta_t - z_{t,2}\sin\theta_t \end{aligned}$$

Taking away the noise, the Jacobian is

$$\begin{bmatrix} 1 & 0 & \sin -\theta_t z_{t,1} - \cos\theta_t z_{t,2} \\ 0 & 1 & -\sin\theta_t z_{t,2} + \cos\theta_t z_{t,1} \end{bmatrix}$$

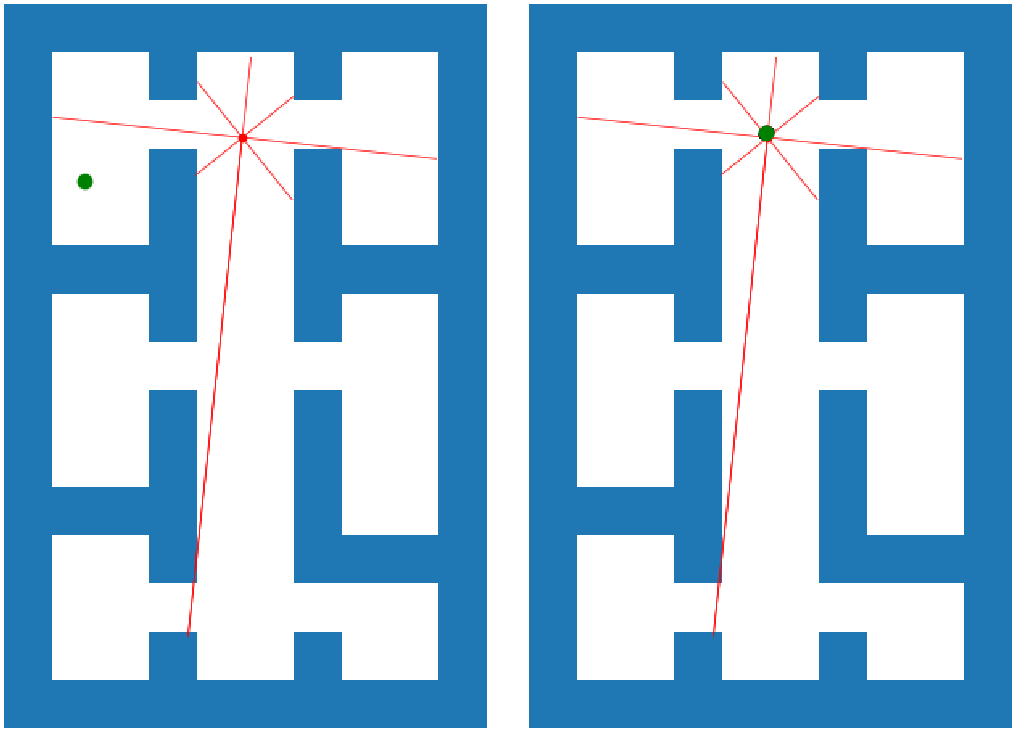

Figure 4: Rooms world path 1: fails with 10 particles (left) and runs well with 100 particles (right).

Part (b)

4 Time and Collaboration Accounting

Part (a)

I worked with Ziteng.

Part (b)

I spend 2 hours on theory and 10 hours on coding.

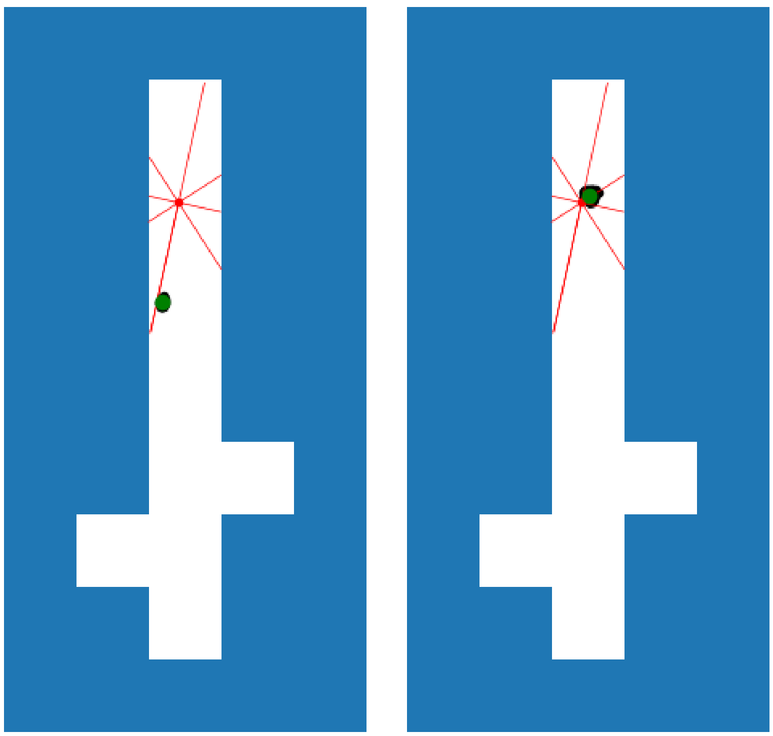

Figure 5: Simple world path 1: fails with 100 particles (left) and runs well with 1000 particles (right).

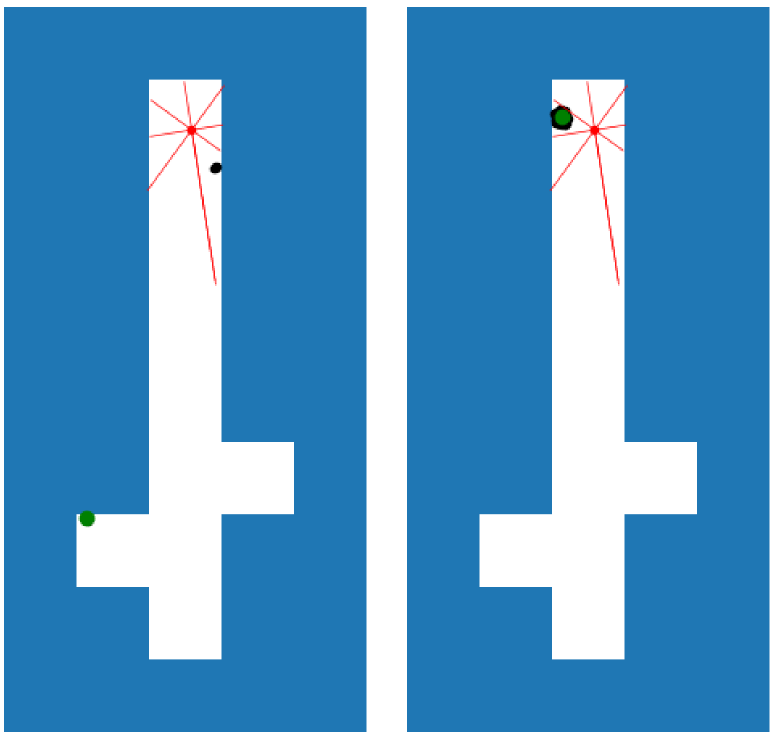

Figure 6: Simple world path 3: fails with 100 particles (left) and runs well with 1000 particles (right).

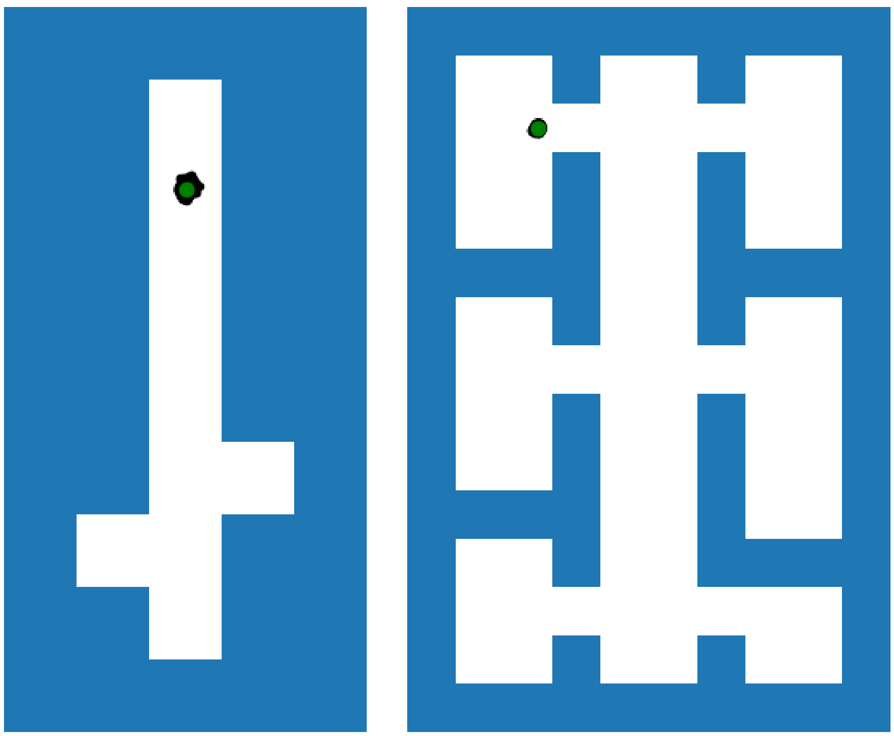

Figure 7: Simple world path 2 works with 1000 particles (left); rooms world path 2 with 1000 particles (right).

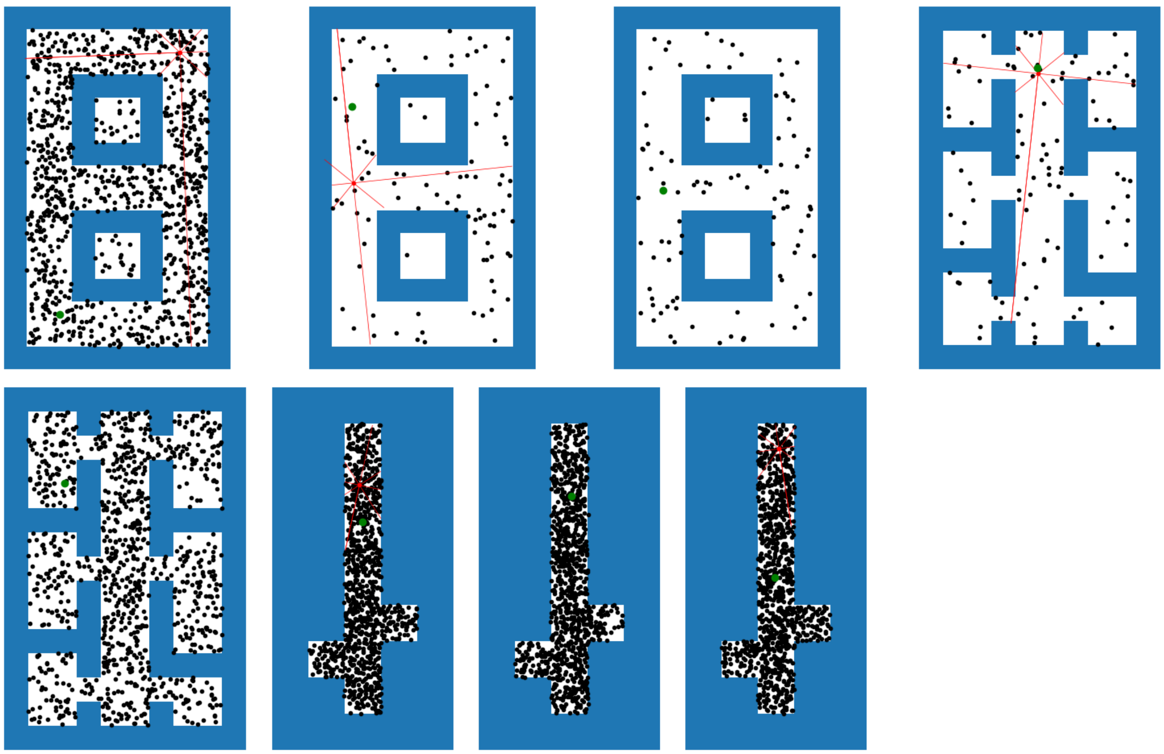

Figure 8: Here are the results when we never resample if the effective number of particles are smaller than $\frac{3N}{2}$. From left to right, top to bottom are loops world path 0,1,2, rooms world path 1,2, and simple world path 1,2,3. The number of particles for each path is the same as in the last question.

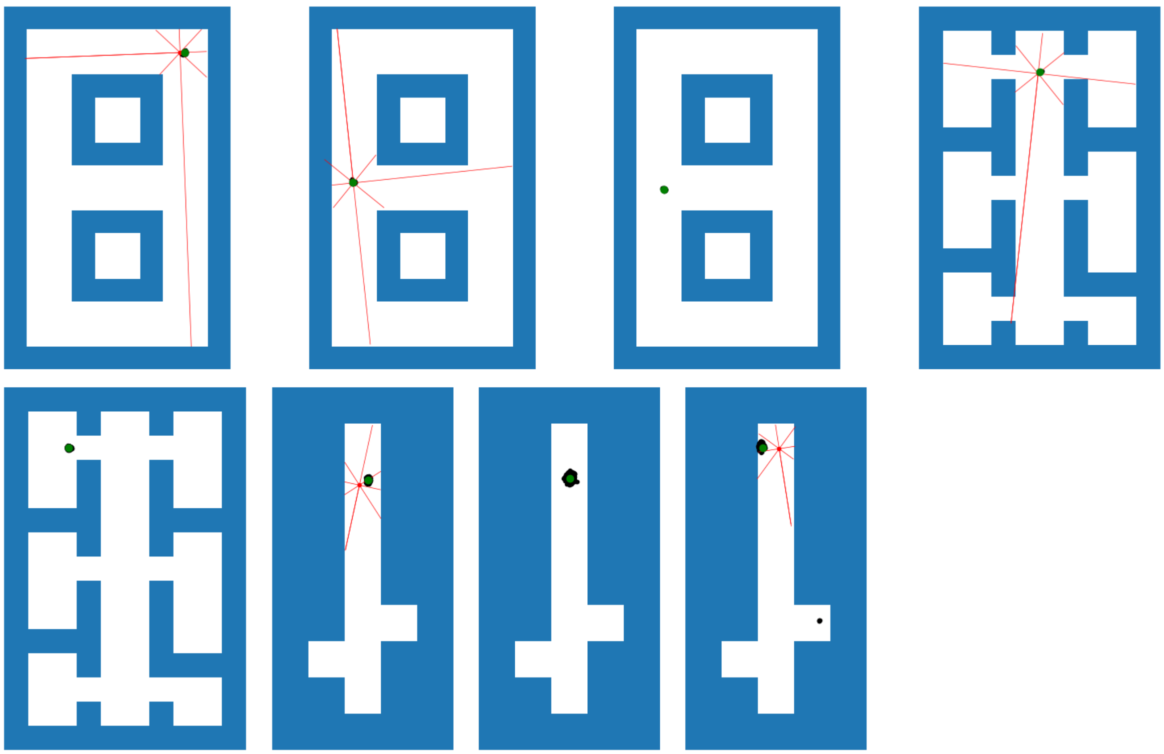

Figure 9: Here are the results when we never resample. (However, when a particle hit the wall, we still place it to a random location as usual.) From left to right, top to bottom are loops world path 0,1,2, rooms world path 1,2, and simple world path 1,2,3. The number of particles for each path is the same as in the last question.

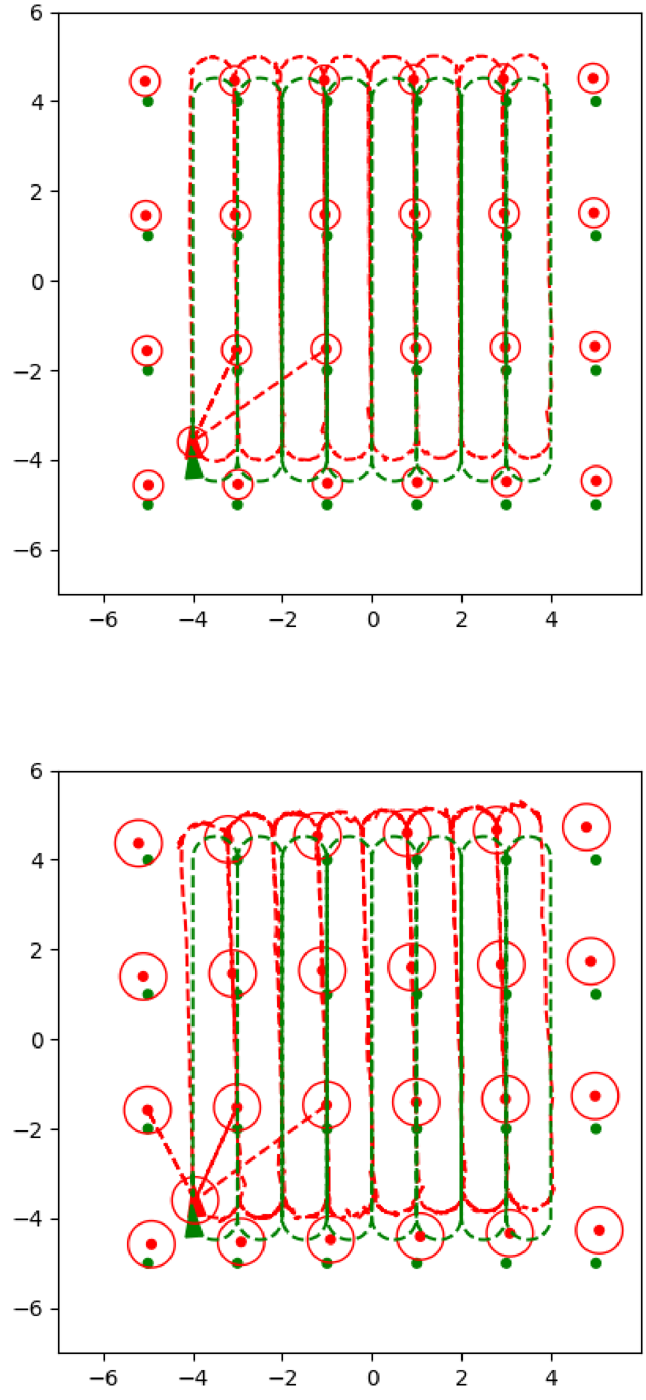

Figure 10: Final trajectory, ellipse, and ground truth with small (upper) and large (lower) noise

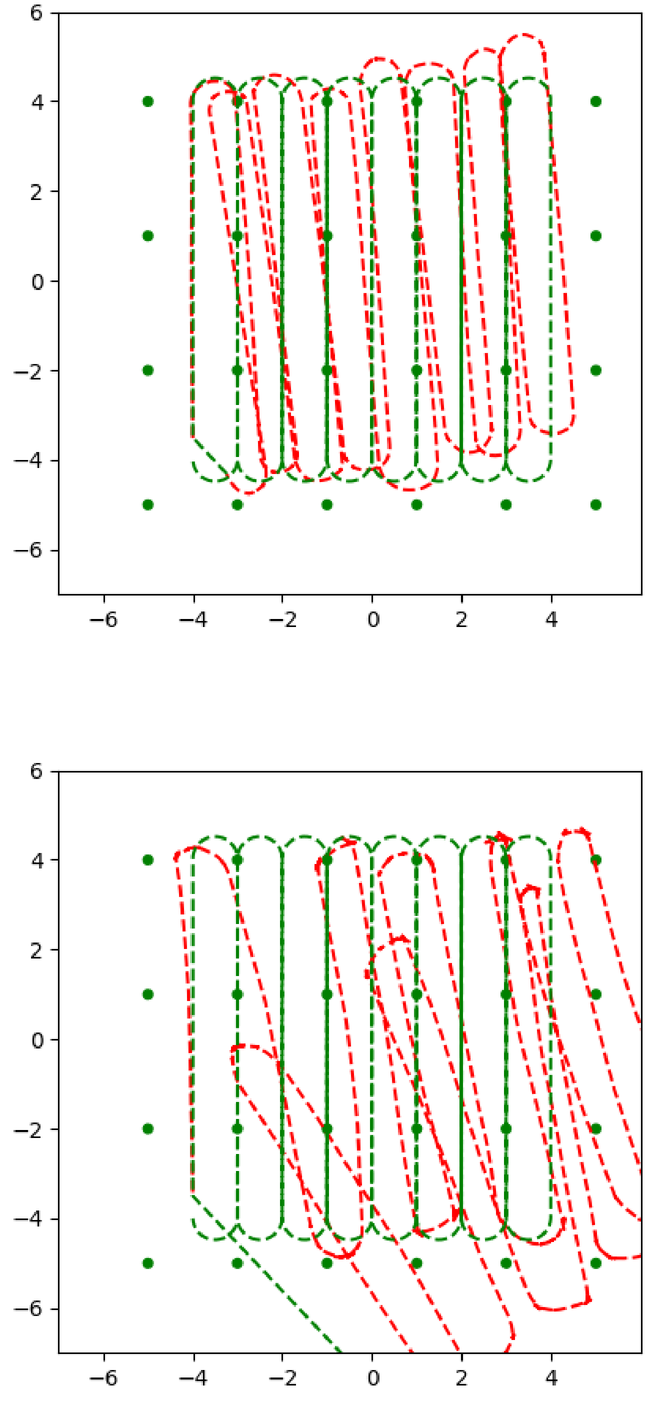

Figure 11: Final trajectory and ground truth with small (upper) and large (lower) noise with dead reckoning