2022-2023-2 B—48 B

一. 1. $p(A)=\frac{1}{6}$ $p(B)=\frac{1}{6}$

$$p(C)=\frac{6}{36}=\frac{1}{6}$$

$p(AC)=\frac{1}{36}$ $p(BC)=\frac{1}{36}$ $p(AB)=\frac{1}{36}$

$p(ABC)=\frac{1}{36}$

所以 A, B, C 两两独立, 但不相互独立. 选 C.

2. $p\{X=2\}=\frac{C_3^1}{C_5^3}=\frac{3}{10}$

$$p\{X=3\}=\frac{C_2^1\cdot C_2^1}{C_5^3}=\frac{4}{10}$$

$$p\{X=4\}=\frac{C_3^1}{C_5^3}=\frac{3}{10}$$

$$EX=2\cdot\frac{3}{10}+3\cdot\frac{4}{10}+4\cdot\frac{3}{10}=3$$

故选 D.

5. 利用 $E(S^2)=\sigma^2$ 方差 结论.

$Var(X)=m(m-1)p.$

$E\left[\sum_{i=1}^{n}(X_i-\bar{X})^2\right]=E[(n-1)S^2]=(n-1)E(S^2)$

$=(n-1)m(m-1)p.$

故选B.

6. $X\sim Exp(2)$, $EX=\frac{1}{2}$ $Var(X)=\frac{1}{4}$

$E(X^2)=Var(X)+(EX)^2=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

$Cov(X^2,X-2)=E(X^2(X-2))-E(X^2)E(X-2)$

$=E(X^3-2X^2)-\cancel{E(X^2)}\frac{1}{2}\cdot(\frac{1}{2}-2)$

$=E(X^3)-2\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{3}{2}=\frac{3}{4}-1+\frac{3}{4}=\frac{1}{2}$

$E(X^3)=\int_0^{+\infty}x^3\cdot 2e^{-2x}dx=3\int_0^{+\infty}x^2e^{-2x}dx$

$=\frac{3}{2}E(X^2)=\frac{3}{2}\cdot\frac{1}{2}=\frac{3}{4}$

故选A.

7.

X	1	2
Y	-1	5
$Y-1$	-2	4
$\frac{Y-1}{2}$	-1	2

$Z_1 = \min\{X, Y\}$

$P\{Z_1 = 1\} = P\{X=1, Y=5\} = 0.2$

$P\{Z_1 = -1\} = P\{X=1, Y=-1\} + P\{X=2, Y=-1\} = 0.5$

$P\{Z_1 = 2\} = P\{X=2, Y=5\} = 0.3$

$Z_2 = \min\{X, Y-1\}$ $\quad Z_2: -2, 1, 2$ 与Z_1不同分布

$Z_3 = \min\{X, \frac{Y-1}{2}\}$

$P\{Z_3 = 1\} = P\{X=1, \frac{Y-1}{2} = 2\} = P\{X=1, Y=5\} = 0.2$

$P\{Z_3 = -1\} = P\{X=1, \frac{Y-1}{2} = -1\} + P\{X=2, \frac{Y-1}{2} = -1\}$

$= P\{X=1, Y=-1\} + P\{X=2, Y=-1\}$

$= 0.5$

$P\{Z_3 = 2\} = P\{X=2, Y=5\} = 0.3$ Z_3与Z_1同分布. 选B.

二、11. $Cov(3X-Y, X+Y)$

$= 3Cov(X, X) + 2Cov(X, Y) - Cov(Y, Y)$

$= 3DX + 0 - DY$

所以 $\rho(3X-Y, X+Y) = \cancel{\frac{3\cdot 2-7}{\sqrt{DX}\cdot\sqrt{DY}} = \frac{-1}{\sqrt{14}} = -\frac{\sqrt{14}}{14}}$

$= \frac{Cov(3X-Y, X+Y)}{\sqrt{D(3X-Y)}\sqrt{D(X+Y)}} = \frac{3\cdot 2-7}{\sqrt{9\cdot 2+7}\cdot\sqrt{2+7}}$

$= -\frac{1}{15}$

13. $(\frac{X_1}{2})^2 + (\frac{X_2}{2})^2 + (\frac{X_3}{\sigma})^2 \sim \chi^2(3)$

$= \frac{1}{4}(X_1^2 + X_2^2) + \frac{1}{\sigma^2}X_3^2$

$\frac{1}{\sigma^2} = 2 \Rightarrow \sigma^2 = \frac{1}{2}$