2022-23-1 A

一. 3. $P\{X\le 1 \mid X\le 2\} = \dfrac{P\{X\le 1\}}{P\{X\le 2\}}$

$= \dfrac{e^{-\lambda}(1+\lambda)}{e^{-\lambda}(1+\lambda+\frac{\lambda^2}{2})} = \dfrac{1}{\lambda}$

$\lambda+\lambda^2 = 1+\lambda+\dfrac{\lambda^2}{2}$

$\dfrac{\lambda^2}{2} = 1 \Rightarrow \lambda = \pm\sqrt{2} \quad \lambda>0$

故 $\lambda=\sqrt{2}$

$E[(X-2)^2] = D(X-2) + E^2(X-2)$

$= D(X) + (EX-2)^2$

$= \sqrt{2} + (\sqrt{2}-2)^2$

$= 6-3\sqrt{2}$

4. $P\{X>a+1 \mid X<b-1\} = \dfrac{P\{a+1<X<b-1\}}{P\{X<b-1\}}$

$= \dfrac{b-1-a-1}{b-1-a} = \dfrac{1}{2} \Rightarrow 2b-2a-4 = b-1-a$

$\Rightarrow b-a=3$

$DX = \dfrac{1}{12}(b-a)^2 = \dfrac{1}{12}\cdot 9 = \dfrac{3}{4}$

6. $EZ = E(2X - Y + 1)$

$= 2EX - EY + 1$

$= 2\cdot 1 - 2 + 1 = 1$

$Cov(X, Z) = Cov(X, 2X - Y + 1)$

$= 2D(X) - Cov(X, Y)$

$= 2\cdot 2^2 - \frac{1}{3}\cdot 2\cdot 3$

$= 6$

9. $D(X) = \frac{1}{12}\theta^2$

所以 $E(S^2) = E\left(\frac{1}{2}\left[(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + (X_3 - \bar{X})^2\right]\right)$

~~是$\frac{1}{12}\theta^2$的无偏估计.~~ $= \frac{1}{12}\theta^2$

因此 $E\left(6\left[(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + (X_3 - \bar{X})^2\right]\right) = \theta^2$

故 $C = 6.$

此题利用了S^2是方差的无偏估计的结论!

2、1. $p(B|A)+p(\bar{B}|\bar{A})\geq 1$

$$p(B|A)\geq 1-p(\bar{B}|\bar{A})=p(B|\bar{A})$$

$$\frac{p(AB)}{p(A)}\geq\frac{p(B\bar{A})}{p(\bar{A})}=\frac{p(B)-p(AB)}{1-p(A)}$$

$$p(AB)\geq p(A)p(B)$$

$$p(B|A)\geq p(B)$$

故选A.

2. $\frac{1}{n}\left(\frac{X_2}{X_1}+\frac{X_4}{X_3}+\cdots+\frac{X_{2n}}{X_{2n-1}}\right)\xrightarrow{P}E\left(\frac{X_2}{X_1}\right)$

$$E(X_2)=\int_0^1 x\cdot 2x\,dx=\frac{2}{3}$$

$$E\left(\frac{1}{X_1}\right)=\int_0^1\frac{1}{x}\cdot 2x\,dx=2$$

$$E\left(\frac{X_2}{X_1}\right)=E(X_2)E\left(\frac{1}{X_1}\right)=\frac{4}{3}$$