

2023-20-1-48A

1. $A-(B \cup C)$ 表示 A 发生且 $(B \cup C)$ 不发生.

$\overline{B \cup C} = \bar{B} \bar{C}$ 即 A 发生, B, C 不发生.

故选 B.

2. $Y = X^2$ 说明 Y 与 X 有关, 但不独立.

故 X 与 Y 不独立, 不独立. 故选 D.

3. A. $F(x) = \frac{1}{1+x^2}$ $F(+\infty) = 0 \neq 1$ X

B $F(x) = \begin{cases} \frac{1}{2}(1-e^{-x}) & x > 0 \\ 0 & x \leq 0 \end{cases}$ $F(+\infty) = \frac{1}{2} \neq 1$ X

C. $F(x) = \begin{cases} \frac{x+1}{x+2} & x > 0 \\ -\frac{1}{x-1} & x \leq 0 \end{cases}$ $F(0^+) = \frac{1}{2} \neq F(0) = 1$
在 0 处不右连续. X

故选 D.

4. $P\{X+Y \leq \frac{1}{3}\} = P\{X=0, Y \leq \frac{1}{3}\} + P\{X=1, Y \leq -\frac{2}{3}\}$

$= P\{X=0\}P\{Y \leq \frac{1}{3}\} + P\{X=1\}P\{Y \leq -\frac{2}{3}\}$

$$= \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot 0 = \frac{1}{6} \quad \text{故选 A.}$$

$$5. \quad P\{0 < X < 2\} = P\{|X-1| < 1\} \geq 1 - \frac{D(X)}{\varepsilon^2} = 1 - 0.1 = 0.9$$

$$E(X) = E(X^2) - D(X) = 1.1 - 0.1 = 1 \Rightarrow EX = 1$$

故选 C.

$$6. \quad X+Y \sim N(0, \frac{2}{3}) \quad X-Y \sim N(0, \frac{2}{3}) \quad \text{容易!}$$

$$\text{独立} \Rightarrow \text{不相关.} \quad D(X+Y) = D(X) + D(Y) + 2\text{Cov}(X, Y) \\ = 1 + 1 + 2 \cdot (-\frac{1}{2}) \cdot 1 \cdot 2 = 0$$

$$\text{Cov}(aX+bY, X) = a \text{Cov}(X, X) + b \text{Cov}(Y, X)$$

$$= a \cdot 1 + b \cdot \rho \cdot \sqrt{D(X)} \sqrt{D(Y)}$$

$$= a \cdot 1 + a \cdot (-\frac{1}{2}) \cdot 1 \cdot 2 = 0$$

$$a = \frac{\sqrt{3}}{2} \Leftrightarrow aX+bY \sim N(0, 1)$$

故选 C.

$$7. \quad E(S^2) = \sigma^2 \quad E(\bar{X}) = 0 \quad E(\bar{X}^2) = D(\bar{X}) \\ = \frac{1}{n} \sigma^2$$

$$A. E(n\bar{x}^2 + S^2) = n \cdot \frac{1}{n} \sigma^2 + \sigma^2 = 2\sigma^2$$

$$B. E\left(\frac{1}{2}(n\bar{x}^2 + S^2)\right) = \sigma^2 \quad \checkmark$$

选 B.

$$8. \sigma^2 \text{ 未知, 估计 } \mu. \text{ 统计量 } \frac{\bar{x} - \mu}{S/\sqrt{n}} \sim t(n-1)$$

$$\text{置信区间: } \left(\bar{x} - \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}}(n-1), \bar{x} + \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}}(n-1) \right)$$

选 B.

$$= . \quad \text{例 10.2, } P(A\bar{B} \cup \bar{A}B) = 0.3$$

$$P(A) + P(B) = 0.5$$

$$\text{求: } P(\bar{A} \cup \bar{B})$$

$$A = A\bar{B} + AB \quad P(A) = P(A\bar{B}) + P(AB)$$

$$B = \bar{A}B + AB \quad P(B) = P(\bar{A}B) + P(AB)$$

$$\text{所以 } 2P(AB) = P(A) + P(B) - P(A\bar{B} \cup \bar{A}B) = 0.2$$

$$P(AB) = 0.1$$

$$P(\bar{A} \cup \bar{B}) = 1 - P(AB) = 1 - 0.1 = 0.9$$

$$12. Y \sim B(4, p),$$

$$\text{其中 } p = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2} \cos \frac{x}{2} dx = \sin \frac{x}{2} \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$EY = 4 \cdot p = 2$$

$$13. \frac{1}{n} \sum_{r=1}^n X_r^2 \xrightarrow{P} E(X_r^2)$$

$$X_r \sim p(2) \quad E(X_r) = 2 \quad D(X_r) = 2$$

$$E(X_r^2) = D(X_r) + E^2(X_r) = 2 + 4 = 6$$

15. 设 X_r 表示第 r 个灯泡的寿命.

$$X_1 + \dots + X_{300} \text{ 近似服从 } N(0, 300 \cdot \frac{1}{12})$$

$$p\{|X_1 + \dots + X_{300}| > 15\}$$

$$= 1 - p\{|X_1 + \dots + X_{300}| \leq 15\}$$

$$= 1 - p\left\{\left|\frac{X_1 + \dots + X_{300}}{\sqrt{\frac{100}{3}}}\right| \leq \frac{15}{\frac{10}{\sqrt{3}}}\right\}$$

$$= 1 - [\Phi(3) - \Phi(-3)] = 2 - 2\Phi(3) = 0.0026$$