Midterm Exam 2

ECE 407 Spring 2018 | UIN #
Firstname Lastname

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1. Question 1 (25pts)

   (a) Given the linear classifier $w^T x + b > 0$ where $w = [2 \; -1]^T$ and $x = [x_1 \;\; x_2]^T$. Sketch the decision boundary. Make sure to label precisely where the boundary intersects the coordinate axes, and also indicate which side of the boundary is the positive side.

   (b) We have two classes of data. Feature vectors of Class 1 and 2 are normally distributed with $N(\mu_1, \Sigma_1)$, $N(\mu_2, \Sigma_2)$, respectively. The mean vectors are $\mu_1 = [1 \; -1]^T$ and $\mu_2 = [1 \;\; 1]^T$, respectively. Covariance matrices are 2 by 2 identity matrices $\Sigma_1 = \Sigma_2 = I$. Determine the equation of the decision boundary based on the Maximum Likelihood principle and sketch it. Assume that class probabilities are equal $\pi_1 = \pi_2 = 0.5$.

   (c) (5pts) If $\pi_1 = 0.75, \pi_2 = 0.25$ what would be your decision rule.

2. Question 2 (25pts)

   (a) Given the observations (1, 2, -1.5, 1, -1, 0.5, -1, 1.5, -1,-2). Use K-means algorithm to cluster the data into K=2 clusters. Perform at least two iterations. Determine the cluster means.

   (b) What is your estimated decision rule.

3. Question 3 (25pts)

   (a) Given the observations $x_1 = [2 \; - .1]^T$, $x_2 = [1 \quad - 0.1]^T$, $x_3 = [1 \;\; 0.1]^T$, $x_4 = [1 \quad - 0.1]^T$, estimate the covariance matrix.

   (b) Perform the PCA.

   (c) Project the vector $x = [5 \; 5]^T$ onto the subspace generated by the largest eigenvector of the covariance matrix.

4. Question 4 (25pts)

   (a) (5pts) Given two Markov Models λ_1 and λ_2:

   Determine the missing transition probabilities.

   (b) Given the observation sequence $O = (S_1, S_1, S_2)$. Determine the model producing the observation sequence with the highest probability. Assume that model probabilities are equal. Also, assume that initial state probabilities are equal in both models.

   (c) (5pts) What is the difference between a Markov model and a Hidden Markov model?