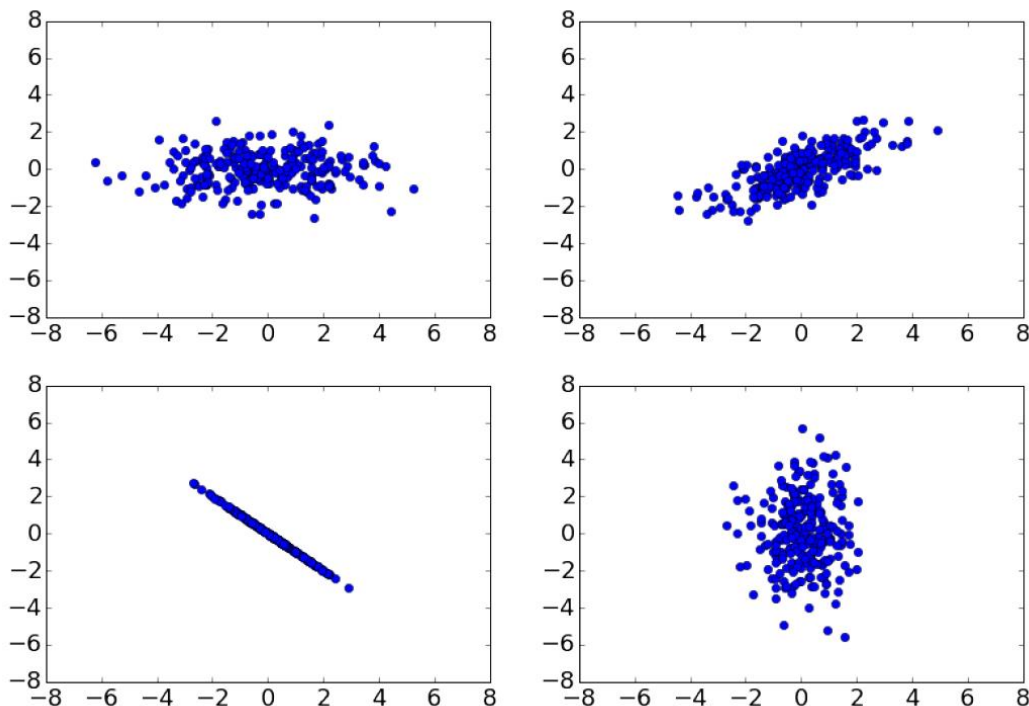


Q1: Is the following set of vectors an orthonormal basis of \mathbb{R}^3 ? Explain why or why not.

$$\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Q2: The following four figures show different 2-dimensional data sets. In each case, make a rough sketch of an ellipsoidal contour of the covariance matrix and indicate the directions of the first and the second eigenvectors (mark which is which).



Q3: Let $u_1, u_2 \in \mathbb{R}^p$ be two vectors with $\|u_1\| = \|u_2\| = 1$ and $u_1 \cdot u_2 = 0$. Define

$$U = \begin{pmatrix} \uparrow & \uparrow \\ u_1 & u_2 \\ \downarrow & \downarrow \end{pmatrix}$$

a) What are the dimensions of each of the following?

- U
- U^T
- UU^T
- $u_1 u_1^T$

b) What are the differences, if any, between the following four projections?

- $x \mapsto (u_1 \cdot x, u_2 \cdot x)$
- $x \mapsto (u_1 \cdot x)u_1 + (u_2 \cdot x)u_2$
- $x \mapsto U^T x$
- $x \mapsto UU^T x$

Q4: For the 2-D data file “**data2.xlsx**”. Determine the PCA.