1. Given the following PMF

$$P_X(x)=\begin{cases}\frac{1}{4}, & x=0\\ \frac{1}{2}, & x=1.5\\ \frac{1}{4}, & x=2\end{cases}$$

Determine

a) $P(X \le 1.75)$

b) Plot the CDF of X.

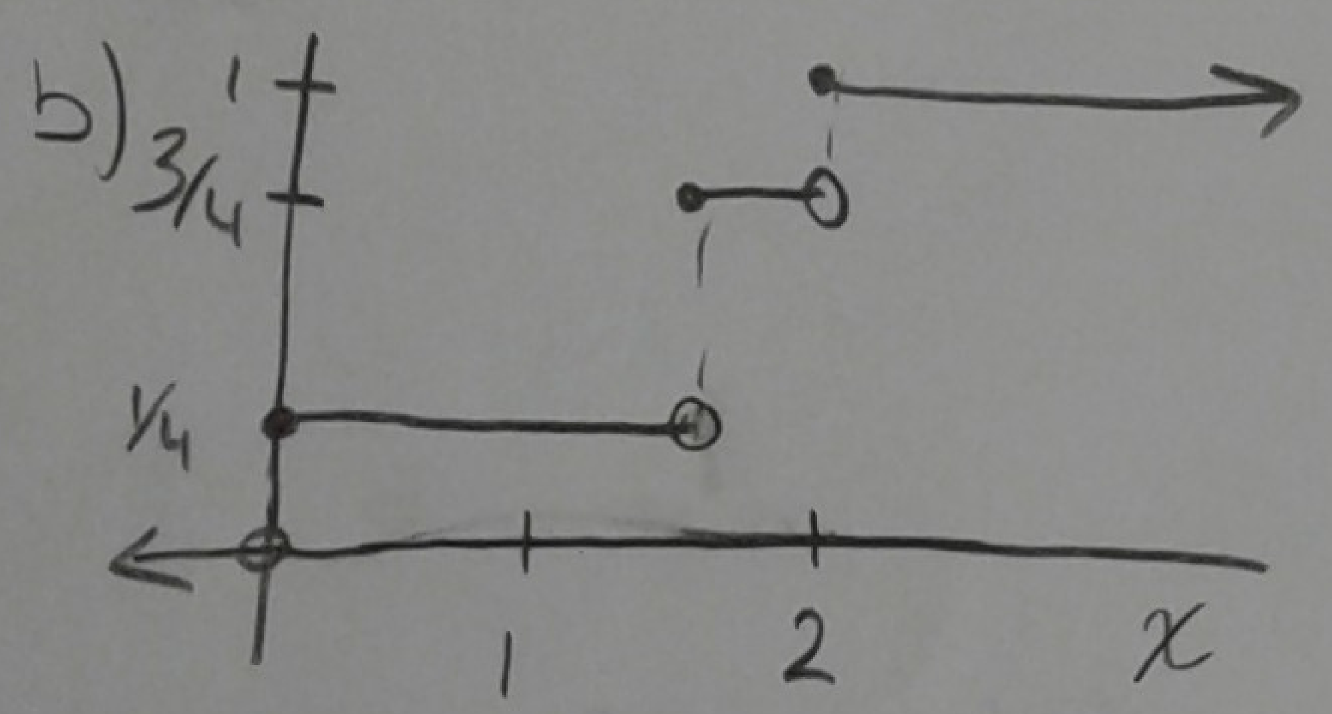
c) Determine E[X]

d) Determine Var[X]

a) $P(X\le 1.75) = P(x=0)+(x=1.5)$

$= 1/4 + 1/2 = 3/4$

b)



c) $E[x] = 1.5\cdot 1/2 + 1/4\cdot 2 + 1/4\cdot 0$

$\mu_x = 1.25$

d) $Var[x] = E[x^2] - (\mu_x)^2$

$= 2.125 - (1.25)^2$

$= .5625$

$E[x^2] = (1.5)^2\, 1/2 + (2)^2\cdot 1/4$

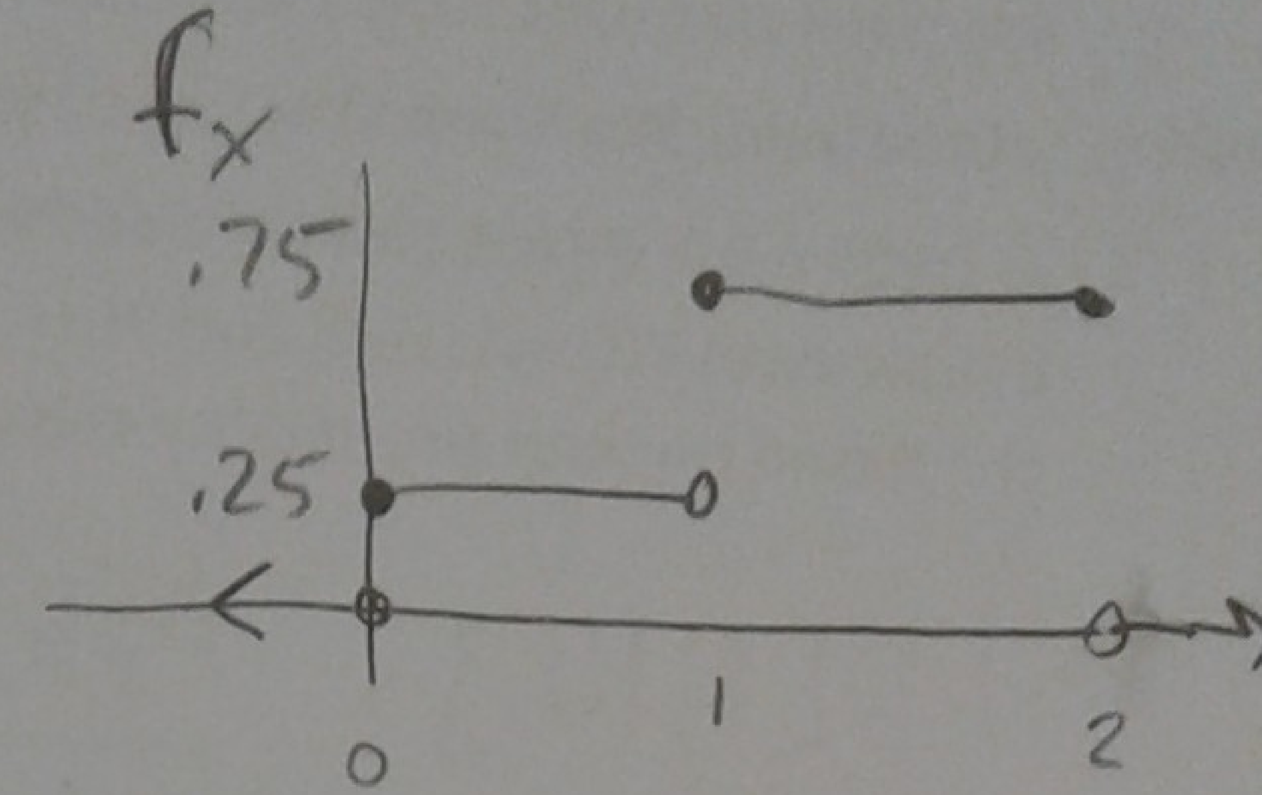
$= 2.125$

2. Given the following pdf

$$f_X(x) = \begin{cases} 0.25, & 0 \le x < 1 \\ 0.75, & 1 \le x \le 2 \end{cases}$$

a) Calculate $P(0.5 < X \le 1.5)$ and $P(X \le 1.5)$.

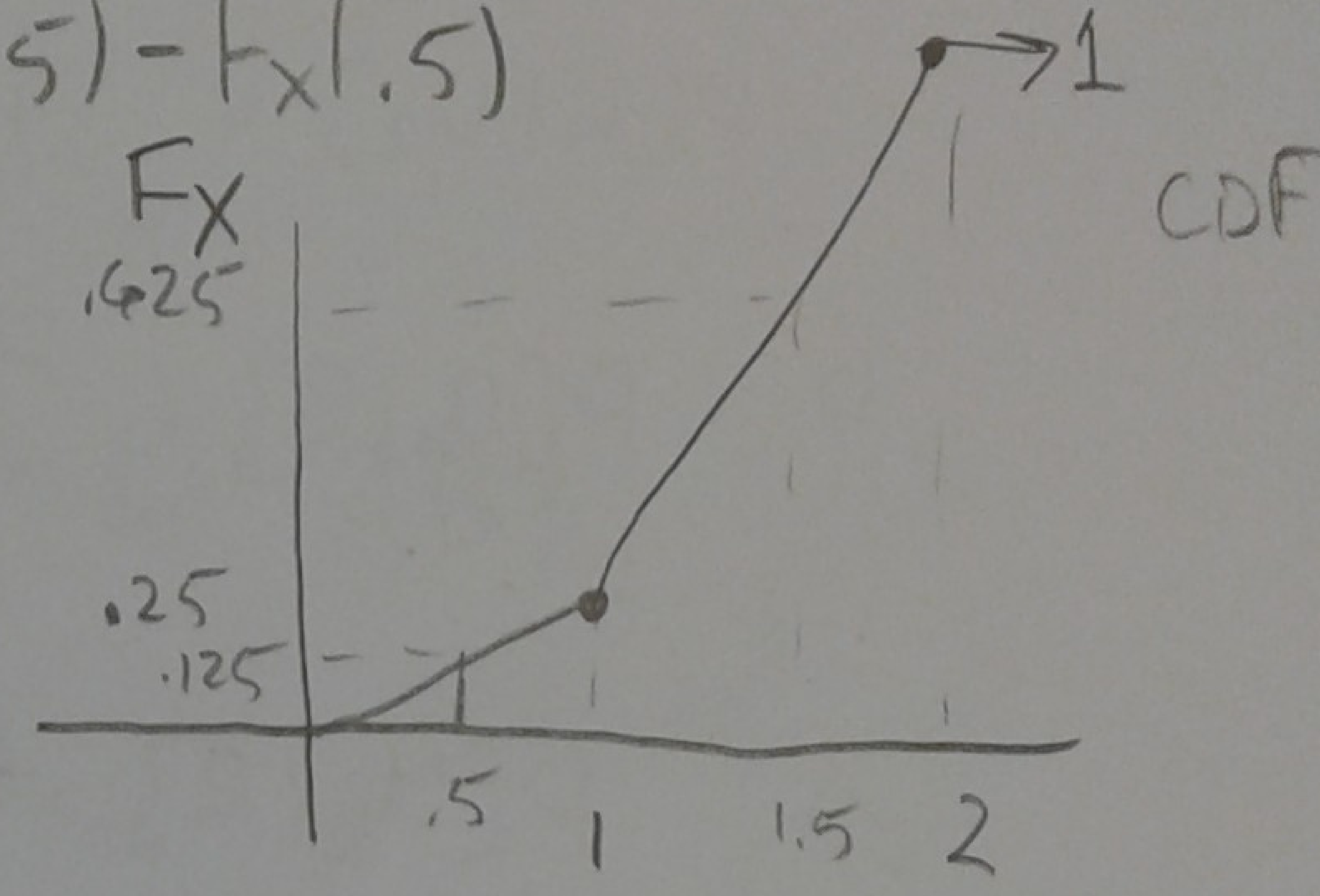
b) Calculate the mean (expected value) of X.



a) $P(.5 < X \le 1.5) = F_X(1.5) - F_X(.5)$

$= .625 - .125$

$= .5$



$P(X \le 1.5) = F_X(1.5)$

$= .625$

b) Bernoulli in nature 25% of being between 0 and 1 and 75% chance of being 1 to 2

$\mu_X = .25 \times .5 + .75 \times 1.5 = 1.25$

3. We have two urns. Urn 1 contains 2 red balls and 5 blue balls and Urn 2 contains 6 red balls and 3 blue balls. We know that $P(Urn1) = 0.4$, $P(Urn2) = 0.6$. We pick a ball and it is a red ball.

a) Determine P(red | Urn1) and P(red| Ur2).

b) Calculate P(Urn1|red) and P(Urn2|red).

c) Based on your probability calculations can you tell where the red ball comes from?

a) $P[R|U_1] = \frac{P[RU_1]}{P[U_1]} = 2/7$

$P[R|U_2] = 6/9 = 2/3$

b $P[U_1|R] = \frac{P[R|U_1] \cdot P[U_1]}{P[R]} = \frac{2/7 \cdot 4/10}{18/35} = 2/9$

$P[U_2|R] = \frac{P[R|U_2] \cdot P[U_2]}{P[R]} = \frac{2/3 \cdot 6/10}{18/35} = 7/9$

$P[R] = P[R|U_1] \cdot P[U_1] + P[R|U_2] \cdot P[U_2]$

$= 2/7 \cdot \frac{4}{10} + \frac{2}{3} \cdot \frac{6}{10}$

$= \frac{18}{35}$

c) There is a large chance that the red ball came from Urn 2, as $P[U_2|R] = 78\%$

4. A student gets an A with $P=0.8$, he/she gets a B with $P=0.2$. He does not get any other grades. He/she takes 10 courses each year. Determine the probability that he gets 8 A's and 2 B's in a given academic year.

$$P[2B] = \binom{10}{2}(.2)^2(1-.2)^{10-2} = .302$$

$$P[8A] = \binom{10}{8}(.8)^8(1-.8)^{10-8} = .302$$

Choose either 8 A's or 2 B's

same result.

5. The number of hits at a website in any time interval is a Poisson random variable. A particular site has on average $\lambda = 2\frac{hits}{\sec}$

a) What is the probability that there are no hits in an interval of 1 second?
b) What is the probability that there are no more than two hits in an interval of one second?
c) What is the expected value of hits per **minute**?

$\alpha = \text{Avg} \cdot \text{Int}$

$= 2 \cdot 1 = 2$

$\alpha^x e^{-\alpha}/x!$

a) $= 2^0 e^{-2}/0!$ $\quad x=0$, no hits

$= .135$

b) $P[x \leq 2] = P[1] + P[0] + P[2]$

$= 2^0 e^{-2}/0! + 2^1 e^{-2}/1! + 2^2 e^{-2}/2!$

$= .135 + .271 + .271$

$= .677$

c) $E[x]$ for Poisson r.v. $= \alpha$

$\frac{hit}{min} = 2 \frac{hit}{sec} \cdot \frac{60 \text{ sec}}{1 \text{ min}}$

$= 120 \frac{hit}{min}$

6-

a) Suppose there are ten objects, A, B, C, D,...,J and we define an experiment in which the procedure is to choose three objects without replacement, and observe the result, and observe the result. Determine the number of possibilities?

b) Suppose there are ten objects, A, B, C, D,...,J and we define an experiment in which the procedure is to choose three objects without replacement, and **arrange them in alphabetical order**, and observe the result. Determine the number of possibilities?

a) $\underset{A-J}{\underline{10}} \cdot \underset{A-I}{\underline{9}} \cdot \underset{A-H}{\underline{8}} = 720$

theory

b) Order doesn't matter

ABC = BAC = CBA = CAB = BCA = ACB = 3!

$\frac{720}{3!} = 120$