ECE 407 HW1-Solution

1-Monitor three consecutive packets going through an Internet router. Based on the packet header, each packet can be classified as either video (v) if it was sent from a Youtube server or as ordinary data (d). Your observation is a sequence of three letters (each letter is either v or d). For example, two video packets followed by one data packet corresponds to vvd. Write the elements of the following sets:

A1 = second packet is video; B1 = second packet is data;

A2 = all packets are the same; B2 = video and data alternate;

A3 = one or more video packets; B3 = two or more data packets:

For each pair of events A1 and B1, A2 and B2, and so on, identify whether

the pair of events is mutually exclusive or not.

A1 = {vvv, vvd, dvv, dvd}

B1 = {vdv, vdd, ddv, ddd}

A2 = {vvv, ddd}

B2 = {vdv, dvd}

A3 = {vvv, vvd, vdv, dvv, vdd, dvd, ddv}

B3 = {ddd, ddv, dvd, vdd}

Recall that Ai and Bi are collectively exhaustive if Ai ∪Bi = S. Also, Ai and Bi are mutually exclusive if Ai∩Bi = ∅. Since we have written down each pair Ai and Bi above, we can simply check for these properties.

The pair A1 and B1 are mutually exclusive and collectively exhaustive. The pair A2 and B2 are mutually exclusive but not collectively exhaustive. The pair A3 and B3 are not mutually exclusive since dvd belongs to A3 and B3. However, A3 and B3 are collectively exhaustive.

2- A student's test score T is an integer between 0 and 100 corresponding to the experimental outcomes s0; s1, … , s100. A score of 90 to 100 is an A, 80 to 89 is a B, 70 to 79 is a C, 60 to 69 is a D, and below 60 is a failing grade of F. If all scores between 51 and 100 are equally likely and a score of 50 or less never occurs, find the following probabilities:

(a) P[s100]

(b) P[A]

(c) P[F]

(d) P[T < 90]

(e) P[a C grade or better]

(f) P[student passes]

There are exactly 50 equally likely outcomes: s51 through s100. Each of these outcomes has probability 1=50. It follows that

(a) P[{s100}] = 1/50 = 0.02:

(b) P[A] = P[{s90, s91, … , s100}] = 11/50 = 0.22:

(c) P[F] = P[{s51, … , s59}] = 9/50 = 0.18:

(d) P[T < 90] = P[{s51, … , s89}] = 39/50: = 0.78:

(e) P[C or better] = P[{s70, … , s100}] = 31 × 0:02 = 0.62:

(f) P[student passes] = P[fs60, : : : , s100}] = 41 × 0.02 = 0.82:

3- Monitor three consecutive packets going through an Internet router. Classify each one as either video (v) or data (d). Your observation is a sequence of three letters (each one is either v or d). For example, three video packets correspond to vvv. The outcomes vvv and ddd each have probability 0.2 whereas each of the other outcomes vvd, vdv,vdd, dvv, dvd, and ddv has probability 0.1. Count the number of video packets $NNVV$ in the three packets you have observed. Describe in words and also calculate the following probabilities:

(a) P[N_V= 2]

(b) P[N_V ≥1]

(c) P[{vvd}| N_V = 2]

(d) P[{ddv}| N_V = 2]

(e) P[N_V = 2 | N_V ≥1]

(f) P[N_V ≥1| N_V = 2]

(a) The probability of exactly two voice packets is P[N_V = 2] = P[{vvd, vdv, dvvg] = 0.3

(b) The probability of at least one voice packet is P[N_V ≥1] = 1 − P[N_V = 0]

= 1 − P[ddd] = 0.8

(c) The conditional probability of two voice packets followed by a data packet given that there were two voice packets is

$$\text{P}[\{\text{vvd}\}|\ N_V = 2] = \frac{\text{P}[\{\text{vvd}\}|\ \text{NV} = 2]}{\text{P}[\text{NV} = 2]}$$

$$= \frac{\text{P}[\{\text{vvd}\}]}{\text{P}[\text{NV} = 2]} = \frac{0.1}{0.3} = \frac{1}{3}$$

(d) The conditional probability of two data packets followed by a voice packet given there were two voice packets is

$$P[\{ddv\}|\ N_V = 2] = \frac{P[\{ddv\}|\ NV = 2]}{P[NV = 2]} = 0$$

The joint event of the outcome ddv and exactly two voice packets has probability zero since there is only one voice packet in the outcome ddv.

(e) The conditional probability of exactly two voice packets given at least one voice packet is

$$P[N_V = 2 \mid N_V \geq 1] = \frac{P[NV = 2 \mid NV \geq 1]}{P[NV \geq 1]} = \frac{P[NV = 2]}{P[NV \geq 1]} = \frac{0.3}{0.8} = \frac{3}{8}$$

(f) The conditional probability of at least one voice packet given there were exactly two voice packets is

$$P[N_V \geq 1|\ N_V = 2\ ] = \frac{P[NV \geq 1|\ NV = 2\ ]}{P[NV=2]} = \frac{P[NV=2]}{P[NV=2]} = 1$$

Given two voice packets, there must have been at least one voice packet.

4- Monitor customer behavior in the Phone smart store. Classify the behavior as buying (B) if a customer purchases a smartphone. Otherwise the behavior is no purchase (N). Classify the time a customer is in the store as long (L) if the customer stays more than three minutes; otherwise classify the amount of time as rapid (R). Based on experience with many customers, we use the probability model P[N] = 0.7, P[L] = 0.6, P[NL] = 0.35. Find the following probabilities:

(a) P[B ∪ L]

(b) P[N ∪ L]

(c) P[N ∪ B]

(d) P[LR]

We can describe this experiment by the event space consisting of the four-possible events NL, NR, BL, and BR. We represent these events in the table:

	N	B
L	0.35	?
R	?	?

First, we try to fill the table before we find the various probabilities.

P[N] = 0.7 = P[NL]+P[NR]

P[L] = 0.6 = P[NL]+P[BL]

Since P[NL] = 0.35, we can conclude that P[NR] = 0.7−0:35 = 0.35 and that P[BL] = 0.6 – 0.35 = 0.25. This allows us to fill in two more table entries:

	N	B
L	0.35	0.25
R	0.35	?

The remaining table entry is filled in by observing that the probabilities must sum to 1.

This implies P[BR] = 0.05 and the complete table is

	N	B
L	0.35	0.25
R	0.35	0.05

The various probabilities are now simple:

(a) $P[B \cup L] = P[NL]+P[BL]+P[BR]$

$=0.35+0.25+0.05 = 0.65.$

(b) $P[N \cup L] = P[N]+P[L] - P[NL]$

$= 0.7+0.6 - 0{:}35 = 0.95.$

(c) $P[N \cup B] = P[S] = 1.$

(d) $P[LR] = P[LL^c] = 0.$

5- We have 6 red balls and 6 blue balls. We store them in two urns: U1 and U2. U1 has 2 red balls and 4 blue balls. U2 has 2 blue balls and 4 red balls. Prior probabilities or urns are equal to each other: P(U1)=P(U2)=0.5.

We pick a ball and it is red. Determine the probabilities P(R | U1), P(U1|R) and P(U2|R).

Since U1 and U2 are not biased, then

$P(U1) = P(U2) = \frac{1}{2}$

$P(R \mid U1) = \frac{2}{6} = \frac{1}{3}$

$P(R \mid U2) = \frac{4}{6} = \frac{2}{3}$

$$P(U1|R) = \frac{P(U1)P(R \mid U1)}{P(U1)\,P(R \mid U1)+P(U2)P(R \mid U2)}$$

$$= \frac{\frac{1}{2}*\frac{1}{3}}{\frac{1}{2}*\frac{1}{3}+\frac{1}{2}*\frac{2}{3}} = \frac{1}{3}$$

$$P(U2|R) = \frac{P(U2)P(R \mid U2)}{P(U1)\ P(R \mid U1)+P(U2)P(R \mid U2)}$$

$$= \frac{\frac{1}{2}*\frac{2}{3}}{\frac{1}{2}*\frac{1}{3}+\frac{1}{2}*\frac{2}{3}} = \frac{2}{3}$$