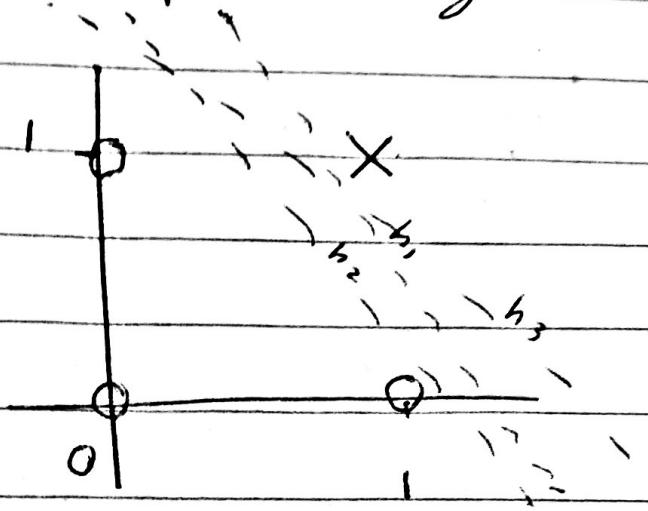
AND:

Since AND problem is linearly seperable and since a perceptron splits data based on a hyperplane parametrized by θ and b.



(here I assume input $x_1, x_2 \in \{0,1\}$

There are infinite solutions.

~~as one solut.~~

The general way of finding a solution is ~~to~~ as follows:

In order to construct a decision boundary we need:

condition 1): $\theta^T x + b > 0$ when $x = [1\,1\,1 \ldots 1]$

condition 2): $\theta^T x + b < 0$ otherwise

let $\theta = [1\,1\,1 \ldots 1]^T$

condition (1):

$$\theta^T x + b = [1\,1 \ldots 1][1\,1\,1 \ldots 1]^T + b > 0$$

$$N + b > 0 \Rightarrow b > -N \quad -(1)$$

where N is the dimension

for condition ②, we have many choices for x. Let $x \in \{[0\,1\,1\,\ldots\,1], [1\,0\,1\,1\,\ldots\,1], \ldots [1\,1\,1\,\ldots\,0]\}$

then $\theta^T x + b = N-1+b < 0$

$$b < -N+1 \quad \text{—(2)}$$

when taking other choices for x it will yield $b < -N+m$, $m > 1$ which is implied by (2) [(2) is the smallest upper bound]

~~(1)~~

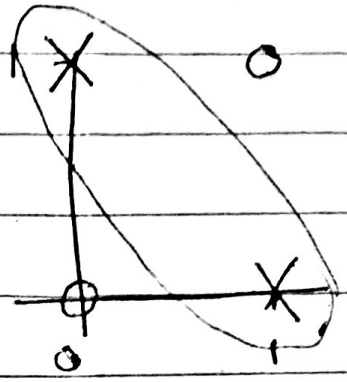
combine (1) and (2)

$$-N < b < -N+1$$

[we can construct other solutions when selecting θ other than all-one vector

XOR:

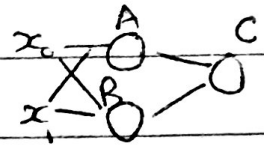
XOR depends on the parity of the input



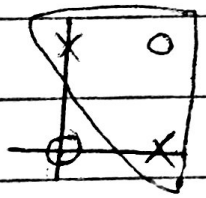
There is no way to split X and O using a hyperplane

$\Rightarrow$ a perceptron can't solve the XOR problem

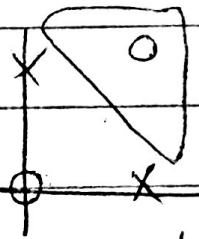
We need multi-stages



stage1

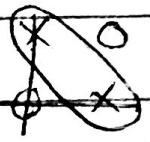


perceptron A



perceptron B

stage2



perceptron C

perceptron A: $\Theta = [1\ 1]$ $b = -\frac{1}{2}$ (OR gate)

perceptron B: $\Theta = [1\ 1]$ $b = \frac{-3}{2}$ (AND gate)

using sign activation, i.e output = -1 or 1

perceptron C: $\Theta = [-1\ +1]$ $b = +\frac{1}{2}$

$\Theta = [-1\ +1]$ $b = +\frac{1}{2}$