Q1: Let us consider the HMM shown in Fig. 1. This HMM generates symbol sequences that consist of 'a's and/or 'b's. The transition probabilities are shown along the edges, and the emission probabilities are shown below the states S1, S2, and S3. Now, assume that the observed symbol sequence is X = 'abbbaba'. What is the observation probability $P(X|\Theta)$ of sequence X based on the given HMM Θ?

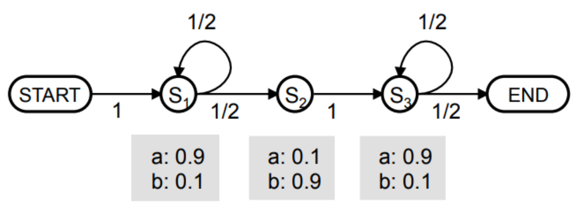


Figure 1: A hidden Markov model.

Q2: Let us consider the following experiment. We have two coins, where one of them is a fair coin with equal probabilities for head and tail, and the other one is a loaded coin with different probabilities for head and tail. We occasionally switch between the two coins according to the specified transition probabilities and continue the coin-toss experiment. This is illustrated in Fig. 2, where the state F represents the fair coin and state L represents the loaded coin. The transition probabilities are shown along the edges and the emission probabilities are shown next to the respective coins ('H' is for head and 'T' is for tail).

Let $X = x_1x_2 \ldots x_N$ be the observed sequence of 'H's and 'T's, $Y = y_1y_2 \ldots y_N$ be the underlying state sequence, and Θ the set of model parameters of the given HMM.

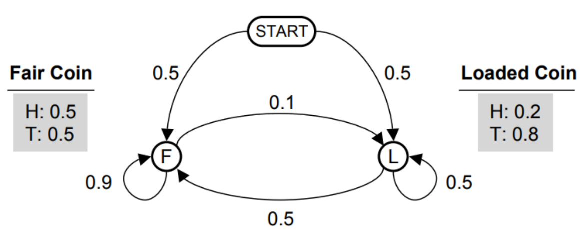


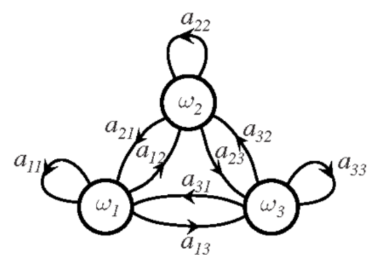
Figure 2: HMM for the coin-toss experiment.

Let X = 'TTHH'. Compute the $P(X|\Theta)$.

Q3: Consider the following 3-state first-order Markov model of the weather in Chicago:

- ω_1: rain / snow
- ω_2: cloudy
- ω_3: sunny

$$\Theta = \{a_{ij}\} = \begin{pmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{pmatrix}$$

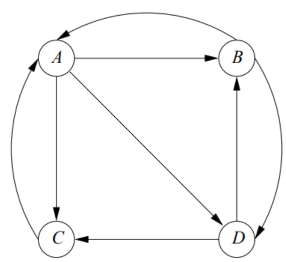


a) What is the probability that the weather for the next eight days will be "sunny-sunny-rainy- rainy-sunny-cloudy-sunny" ($W^8 = \{\omega_3, \omega_2, \omega_3, \omega_1, \omega_1, \omega_3, \omega_2, \omega_3\}$)? Calculate this probability using logarithms. Assume that the initial probabilities are equal.

b) Given that the model is in a known state, what is the probability that it stays in that state for exactly d days?

Q4: Suppose we apply the Markov model on the transition matrix of the web where the transition matrix $M^T = B$ is:

$$B = \begin{pmatrix} 0 & \frac{1}{2} & 1 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{pmatrix}$$



A hypothetical example of a Web

Find the probability of being at state i, i=1,2,3,4 at infinity (or steady state).

i) You can use the iteration $P_{k+1} = B\,P_k$ with initial state probabilities $P_0 = [0.25\ 0.25\ 0.25\ 0.25]^T$ to calculate the steady state probability values.

ii) Or, you can solve the equation: $P = BP$.

You should get the same answer in parts (i) and (ii). You can use MATLAB.