Appendix A

The complete derivation steps for transforming the bi-level problem in (10) into an equivalent MILP problem are as follows.

KKT conditions derivation of the lower-level problem

Here, the complete primal problem of the lower-level problem is expressed as follows:

$$\begin{aligned}
&\min_{\{P_{\text{sub},t}^{\phi},P_{j,t}^{\text{MT},\phi}\}} f_{\text{DSO}}=\sum_{t\in\Omega_{\text{T}}}\sum_{\phi\in\Phi}\lambda_t^{\phi}P_{\text{sub},t}^{\phi}+\sum_{j\in\Omega_{\text{MT}}}\sum_{t\in\Omega_{\text{T}}}\sum_{\phi\in\Phi}c_j^{\text{MT}}P_{j,t}^{\text{MT},\phi}\\
&s.t.\\
&P_{\text{sub},t}^{\phi}=\sum_{k\in C_1}P_{1k,t}^{\phi}:\pi_{1,t}^{p,\phi},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&Q_{\text{sub},t}^{\phi}=\sum_{k\in C_1}Q_{1k,t}^{\phi}:\pi_{1,t}^{q,\phi},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&\sum_{i\in A_j}P_{ij,t}^{\phi}+P_{j,t}^{\text{MT},\phi}-P_{j,t}^{\text{D},\phi}-P_{j,t}^{\text{Hub},\phi}=\sum_{k:j\to k}P_{jk,t}^{\phi}:\pi_{j,t}^{p,\phi},\forall j\in\Omega_{\text{N}},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&\sum_{i\in A_j}Q_{ij,t}^{\phi}+Q_{j,t}^{\text{MT},\phi}-Q_{j,t}^{\text{D},\phi}=\sum_{k:j\to k}Q_{jk,t}^{\phi}:\pi_{j,t}^{q,\phi},\forall j\in\Omega_{\text{N}},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&v_{j,t}^{\phi}=v_{i,t}^{\phi}-2\left(\tilde{\boldsymbol{r}}_{ij}^{\phi}\boldsymbol{P}_{ij,t}^{\Phi}+\tilde{\boldsymbol{x}}_{ij}^{\phi}\boldsymbol{Q}_{ij,t}^{\Phi}\right):\theta_{j,t}^{v,\phi},\forall j\in\Omega_{\text{N}},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&\left(V_j^{\min,\phi}\right)^2\le v_{j,t}^{\phi}\le\left(V_j^{\max,\phi}\right)^2:\gamma_{j,t}^{v,\phi,\min},\gamma_{j,t}^{v,\phi,\max},\forall j\in\Omega_{\text{N}},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&v_{1,t}^{\phi}=\left(V^{ref}\right)^2:\theta_{1,t}^{v,\phi},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&-P_{\max}\le P_{\text{sub},t}^{\phi}\le P_{\max}:\gamma_{1,t}^{p,\phi,\min},\gamma_{1,t}^{p,\phi,\max},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&-Q_{\max}\le Q_{\text{sub},t}^{\phi}\le Q_{\max}:\gamma_{1,t}^{q,\phi,\min},\gamma_{1,t}^{q,\phi,\max},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&0\le P_{j,t}^{\text{MT},\phi}\le P_j^{\text{MT},\phi,\max}:\gamma_{j,t}^{\text{MT},p,\phi,\min},\gamma_{j,t}^{\text{MT},p,\phi,\max},\forall j\in\Omega_{\text{N}},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&0\le Q_{j,t}^{\text{MT},\phi}\le P_{j,t}^{\text{MT},\phi}\tan(\arccos(\varphi^{\text{pf}})):\gamma_{j,t}^{\text{MT},q,\phi,\min},\gamma_{j,t}^{\text{MT},q,\phi,\max},\forall j\in\Omega_{\text{N}},\forall t\in\Omega_{\text{T}},\forall\phi\in\Phi\\
&-K_v\le v_{j,t}^{\phi_1}-v_{j,t}^{\phi_2}\le K_v:\gamma_{j,t}^{\phi_1\phi_2,\min},\gamma_{j,t}^{\phi_1\phi_2,\max},\forall j\in\Omega_{\text{N}},\forall t\in\Omega_{\text{T}}\\
&-K_v\le v_{j,t}^{\phi_2}-v_{j,t}^{\phi_3}\le K_v:\gamma_{j,t}^{\phi_2\phi_3,\min},\gamma_{j,t}^{\phi_2\phi_3,\max},\forall j\in\Omega_{\text{N}},\forall t\in\Omega_{\text{T}}\\
&-K_v\le v_{j,t}^{\phi_3}-v_{j,t}^{\phi_1}\le K_v:\gamma_{j,t}^{\phi_3\phi_1,\min},\gamma_{j,t}^{\phi_3\phi_1,\max},\forall j\in\Omega_{\text{N}},\forall t\in\Omega_{\text{T}}
\end{aligned}\tag{A1}$$

Then the primal problem with constraints can be converted to the equivalent optimization problem without constraints by Lagrangain relaxation. The Lagrangian function of (A1) is as

$$
\begin{aligned}
&L_{\mathrm{DSO}}\begin{pmatrix} P_{ij,t}^{\phi}, Q_{ij,t}^{\phi}, v_{j,t}^{\phi}, P_{\mathrm{sub},t}^{\phi}, Q_{\mathrm{sub},t}^{\phi}, P_{j,t}^{\mathrm{MT},\phi}, Q_{j,t}^{\mathrm{MT},\phi} \\ \pi_{j,t}^{p,\phi}, \pi_{j,t}^{q,\phi}, \theta_{j,t}^{v,\phi}, \gamma_{j,t}^{v,\phi,\min}, \gamma_{j,t}^{v,\phi,\max}, \gamma_{1,t}^{p,\phi,\min}, \gamma_{1,t}^{p,\phi,\max}, \gamma_{1,t}^{q,\phi,\min}, \gamma_{1,t}^{q,\phi,\max}, \gamma_{j,t}^{\mathrm{MT},p,\phi,\min}, \gamma_{j,t}^{\mathrm{MT},p,\phi,\max}, \\ \gamma_{j,t}^{\mathrm{MT},q,\phi,\min}, \gamma_{j,t}^{\mathrm{MT},q,\phi,\max}, \gamma_{j,t}^{\phi_1\phi_2,\min}, \gamma_{j,t}^{\phi_1\phi_2,\max}, \gamma_{j,t}^{\phi_2\phi_3,\min}, \gamma_{j,t}^{\phi_2\phi_3,\max}, \gamma_{j,t}^{\phi_3\phi_1,\min}, \gamma_{j,t}^{\phi_3\phi_1,\max} \end{pmatrix} \\
&= \sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\lambda_t^{\phi}P_{\mathrm{sub},t}^{\phi} + \sum_{j\in\Omega_{\mathrm{MT}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}c_j^{\mathrm{MT}}P_{j,t}^{\mathrm{MT},\phi} - \sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\pi_{1,t}^{p,\phi}\left(P_{\mathrm{sub},t}^{\phi} - \sum_{k\in C_1}P_{1k,t}^{\phi}\right) - \sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\pi_{1,t}^{q,\phi}\left(Q_{\mathrm{sub},t}^{\phi} - \sum_{k\in C_1}Q_{1k,t}^{\phi}\right) \\
&- \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\pi_{j,t}^{p,\phi}\left(\sum_{i\in A_j}P_{ij,t}^{\phi} + P_{j,t}^{\mathrm{MT},\phi} - P_{j,t}^{\mathrm{D},\phi} - P_{j,t}^{\mathrm{Hub},\phi} - \sum_{k\in C_j}P_{jk,t}^{\phi}\right) \\
&- \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\pi_{j,t}^{q,\phi}\left(\sum_{i\in A_j}Q_{ij,t}^{\phi} + Q_{j,t}^{\mathrm{MT},\phi} - Q_{j,t}^{\mathrm{D},\phi} - \sum_{k\in C_j}Q_{jk,t}^{\phi}\right) \\
&- \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\theta_{j,t}^{v,\phi}\left[v_{j,t}^{\phi} - v_{i,t}^{\phi} + 2\left(\tilde{\boldsymbol{r}}_{ij}^{\phi}\boldsymbol{P}_{ij,t}^{\Phi} + \tilde{\boldsymbol{x}}_{ij}^{\phi}\boldsymbol{Q}_{ij,t}^{\Phi}\right)\right] - \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\theta_{1,t}^{v,\phi}\left[v_{1,t}^{\phi} - \left(V^{ref}\right)^2\right] \\
&+ \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\min}\left[\left(V_j^{\min,\phi}\right)^2 - v_{j,t}^{\phi}\right] + \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\max}\left[v_{j,t}^{\phi} - \left(V_j^{\max,\phi}\right)^2\right] \\
&+ \sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\min}\left(-P_{\max} - P_{\mathrm{sub},t}^{\phi}\right) + \sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\max}\left(P_{\mathrm{sub},t}^{\phi} - P_{\max}\right) \\
&+ \sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\min}\left(-Q_{\max} - Q_{\mathrm{sub},t}^{\phi}\right) + \sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\max}\left(Q_{\mathrm{sub},t}^{\phi} - Q_{\max}\right) \\
&+ \sum_{j\in\Omega_{\mathrm{MT}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{j,t}^{\mathrm{MT},p,\phi,\min}\left(-P_{j,t}^{\mathrm{MT},\phi}\right) + \sum_{j\in\Omega_{\mathrm{MT}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{j,t}^{\mathrm{MT},p,\phi,\max}\left(P_{j,t}^{\mathrm{MT},\phi} - P_j^{\mathrm{MT},\phi,\max}\right) \\
&+ \sum_{j\in\Omega_{\mathrm{MT}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{j,t}^{\mathrm{MT},q,\phi,\min}\left(-Q_{j,t}^{\mathrm{MT},\phi}\right) + \sum_{j\in\Omega_{\mathrm{MT}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{j,t}^{\mathrm{MT},q,\phi,\max}\left(Q_{j,t}^{\mathrm{MT},\phi} - P_{j,t}^{\mathrm{MT},\phi}\tan(\arccos(\varphi^{\mathrm{pf}}))\right) \\
&+ \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_1\phi_2,\min}\left(-K_v - v_{j,t}^{\phi_1} + v_{j,t}^{\phi_2}\right) + \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_1\phi_2,\max}\left(v_{j,t}^{\phi_1} - v_{j,t}^{\phi_2} - K_v\right) \\
&+ \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_2\phi_3,\min}\left(-K_v - v_{j,t}^{\phi_2} + v_{j,t}^{\phi_3}\right) + \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_2\phi_3,\max}\left(v_{j,t}^{\phi_2} - v_{j,t}^{\phi_3} - K_v\right) \\
&+ \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_3\phi_1,\min}\left(-K_v - v_{j,t}^{\phi_3} + v_{j,t}^{\phi_1}\right) + \sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_3\phi_1,\max}\left(v_{j,t}^{\phi_3} - v_{j,t}^{\phi_1} - K_v\right)
\end{aligned} \tag{A2}
$$

1) KKT stationarity conditions

Base on the Lagrangian function L_{DSO} in (A2), the stationarity conditions can be obtained as (A3) by requiring that the gradient of L_{DSO} with respect to the decision variables is zero.

- *The gradient for $P_{ij,t}^{\phi}$*

$$\frac{\partial L_{\mathrm{DSO}}}{\partial P_{ij,t}^{\phi_1}} = \pi_{i,t}^{p,\phi_1} - \pi_{j,t}^{p,\phi_1} - 2\left(\tilde{\boldsymbol{r}}_{ij}^{\mathrm{A}_{11}}\theta_{j,t}^{v,\phi_1} + \tilde{\boldsymbol{r}}_{ij}^{\mathrm{A}_{21}}\theta_{j,t}^{v,\phi_2} + \tilde{\boldsymbol{r}}_{ij}^{\mathrm{A}_{31}}\theta_{j,t}^{v,\phi_3}\right) = 0, \forall(i,j)\in\Omega_{\mathrm{L}}, \forall t\in\Omega_{\mathrm{T}} \tag{A3a}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial P_{ij,t}^{\phi_2}} = \pi_{i,t}^{p,\phi_2} - \pi_{j,t}^{p,\phi_2} - 2\left(\tilde{\boldsymbol{r}}_{ij}^{\mathrm{A}_{12}}\theta_{j,t}^{v,\phi_1} + \tilde{\boldsymbol{r}}_{ij}^{\mathrm{A}_{22}}\theta_{j,t}^{v,\phi_2} + \tilde{\boldsymbol{r}}_{ij}^{\mathrm{A}_{32}}\theta_{j,t}^{v,\phi_3}\right) = 0, \forall(i,j)\in\Omega_{\mathrm{L}}, \forall t\in\Omega_{\mathrm{T}} \tag{A3b}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial P_{ij,t}^{\phi_3}} = \pi_{i,t}^{p,\phi_3} - \pi_{j,t}^{p,\phi_3} - 2\left(\tilde{\boldsymbol{r}}_{ij}^{\mathrm{A}_{13}}\theta_{j,t}^{v,\phi_1} + \tilde{\boldsymbol{r}}_{ij}^{\mathrm{A}_{23}}\theta_{j,t}^{v,\phi_2} + \tilde{\boldsymbol{r}}_{ij}^{\mathrm{A}_{33}}\theta_{j,t}^{v,\phi_3}\right) = 0, \forall(i,j)\in\Omega_{\mathrm{L}}, \forall t\in\Omega_{\mathrm{T}} \tag{A3c}$$

where, taking the superscript A_{12} as an example, it represents the element in the first row and second column of the corresponding matrix.

- *The gradient for $Q_{ij,t}^{\phi}$*

$$\frac{\partial L_{\mathrm{DSO}}}{\partial Q_{ij,t}^{\phi_1}} = \pi_{i,t}^{q,\phi_1} - \pi_{j,t}^{q,\phi_1} - 2\left(\tilde{\boldsymbol{x}}_{ij}^{\mathrm{A}_{11}}\theta_{j,t}^{v,\phi_1} + \tilde{\boldsymbol{x}}_{ij}^{\mathrm{A}_{21}}\theta_{j,t}^{v,\phi_2} + \tilde{\boldsymbol{x}}_{ij}^{\mathrm{A}_{31}}\theta_{j,t}^{v,\phi_3}\right) = 0, \forall(i,j)\in\Omega_{\mathrm{L}}, \forall t\in\Omega_{\mathrm{T}} \tag{A4a}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial Q_{ij,t}^{\phi_2}}=\pi_{i,t}^{q,\phi_2}-\pi_{j,t}^{q,\phi_2}-2\left(\tilde{\boldsymbol{x}}_{ij}^{\mathrm{A}_{12}}\theta_{j,t}^{v,\phi_1}+\tilde{\boldsymbol{x}}_{ij}^{\mathrm{A}_{22}}\theta_{j,t}^{v,\phi_2}+\tilde{\boldsymbol{x}}_{ij}^{\mathrm{A}_{32}}\theta_{j,t}^{v,\phi_3}\right)=0,\forall(i,j)\in\Omega_{\mathrm{L}},\forall t\in\Omega_{\mathrm{T}} \tag{A4b}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial Q_{ij,t}^{\phi_3}}=\pi_{i,t}^{q,\phi_3}-\pi_{j,t}^{q,\phi_3}-2\left(\tilde{\boldsymbol{x}}_{ij}^{\mathrm{A}_{13}}\theta_{j,t}^{v,\phi_1}+\tilde{\boldsymbol{x}}_{ij}^{\mathrm{A}_{23}}\theta_{j,t}^{v,\phi_2}+\tilde{\boldsymbol{x}}_{ij}^{\mathrm{A}_{33}}\theta_{j,t}^{v,\phi_3}\right)=0,\forall(i,j)\in\Omega_{\mathrm{L}},\forall t\in\Omega_{\mathrm{T}} \tag{A4c}$$

- *The gradient for* $\upsilon_{j,t}^{\phi}$

For the root node of PDN, it has

$$\frac{\partial L_{\mathrm{DSO}}}{\partial v_{1,t}^{\phi_1}}=\theta_{2,t}^{v,\phi_1}-\theta_{1,t}^{v,\phi_1}=0,\forall t\in\Omega_{\mathrm{T}} \tag{A5a}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial v_{1,t}^{\phi_2}}=\theta_{2,t}^{v,\phi_2}-\theta_{1,t}^{v,\phi_2}=0,\forall t\in\Omega_{\mathrm{T}} \tag{A5b}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial v_{1,t}^{\phi_3}}=\theta_{2,t}^{v,\phi_3}-\theta_{1,t}^{v,\phi_3}=0,\forall t\in\Omega_{\mathrm{T}} \tag{A5c}$$

For the non-root and non-terminal nodes in PDN, it has

$$\frac{\partial L_{\mathrm{DSO}}}{\partial v_{j,t}^{\phi_1}}=-\theta_{j,t}^{v,\phi_1}+\sum_{k\in C_j}\theta_{k,t}^{v,\phi_1}-\gamma_{j,t}^{v,\phi_1,\min}+\gamma_{j,t}^{v,\phi_1,\max}-\gamma_{j,t}^{\phi_1\phi_2,\min}+\gamma_{j,t}^{\phi_1\phi_2,\max}+\gamma_{j,t}^{\phi_3\phi_1,\min}-\gamma_{j,t}^{\phi_3\phi_1,\max}=0, \\ \forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}} \tag{A6a}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial v_{j,t}^{\phi_2}}=-\theta_{j,t}^{v,\phi_2}+\sum_{k\in C_j}\theta_{k,t}^{v,\phi_2}-\gamma_{j,t}^{v,\phi_2,\min}+\gamma_{j,t}^{v,\phi_2,\max}+\gamma_{j,t}^{\phi_1\phi_2,\min}-\gamma_{j,t}^{\phi_1\phi_2,\max}-\gamma_{j,t}^{\phi_2\phi_3,\min}+\gamma_{j,t}^{\phi_2\phi_3,\max}=0, \\ \forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}} \tag{A6b}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial v_{j,t}^{\phi_3}}=-\theta_{j,t}^{v,\phi_3}+\sum_{k\in C_j}\theta_{k,t}^{v,\phi_3}-\gamma_{j,t}^{v,\phi_3,\min}+\gamma_{j,t}^{v,\phi_3,\max}+\gamma_{j,t}^{\phi_2\phi_3,\min}-\gamma_{j,t}^{\phi_2\phi_3,\max}-\gamma_{j,t}^{\phi_3\phi_1,\min}+\gamma_{j,t}^{\phi_3\phi_1,\max}=0, \\ \forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}} \tag{A6c}$$

For the terminal nodes in PDN, it has

$$\frac{\partial L_{\mathrm{DSO}}}{\partial v_{j,t}^{\phi_1}}=-\theta_{j,t}^{v,\phi_1}-\gamma_{j,t}^{v,\phi_1,\min}+\gamma_{j,t}^{v,\phi_1,\max}-\gamma_{j,t}^{\phi_1\phi_2,\min}+\gamma_{j,t}^{\phi_1\phi_2,\max}+\gamma_{j,t}^{\phi_3\phi_1,\min}-\gamma_{j,t}^{\phi_3\phi_1,\max}=0, \\ \forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}} \tag{A7a}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial v_{j,t}^{\phi_2}}=-\theta_{j,t}^{v,\phi_2}-\gamma_{j,t}^{v,\phi_2,\min}+\gamma_{j,t}^{v,\phi_2,\max}+\gamma_{j,t}^{\phi_1\phi_2,\min}-\gamma_{j,t}^{\phi_1\phi_2,\max}-\gamma_{j,t}^{\phi_2\phi_3,\min}+\gamma_{j,t}^{\phi_2\phi_3,\max}=0, \\ \forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}} \tag{A7b}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial v_{j,t}^{\phi_3}}=-\theta_{j,t}^{v,\phi_3}-\gamma_{j,t}^{v,\phi_3,\min}+\gamma_{j,t}^{v,\phi_3,\max}+\gamma_{j,t}^{\phi_2\phi_3,\min}-\gamma_{j,t}^{\phi_2\phi_3,\max}-\gamma_{j,t}^{\phi_3\phi_1,\min}+\gamma_{j,t}^{\phi_3\phi_1,\max}=0, \\ \forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}} \tag{A7c}$$

- *The gradient for* $P_{\mathrm{sub},t}^{\phi}$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial P_{\mathrm{sub},t}^{\phi_1}}=\lambda_t^{\phi_1}-\pi_{1,t}^{p,\phi_1}-\gamma_{1,t}^{p,\phi_1,\min}+\gamma_{1,t}^{p,\phi_1,\max}=0,\forall t\in\Omega_{\mathrm{T}} \tag{A8a}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial P_{\mathrm{sub},t}^{\phi_2}}=\lambda_t^{\phi_2}-\pi_{1,t}^{p,\phi_2}-\gamma_{1,t}^{p,\phi_2,\min}+\gamma_{1,t}^{p,\phi_2,\max}=0,\forall t\in\Omega_{\mathrm{T}} \tag{A8b}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial P_{\mathrm{sub},t}^{\phi_3}}=\lambda_t^{\phi_3}-\pi_{1,t}^{p,\phi_3}-\gamma_{1,t}^{p,\phi_3,\min}+\gamma_{1,t}^{p,\phi_3,\max}=0,\forall t\in\Omega_{\mathrm{T}} \tag{A8c}$$

- *The gradient for* $Q_{\mathrm{sub},t}^{\phi}$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial Q_{\mathrm{sub},t}^{\phi_1}} = -\pi_{1,t}^{q,\phi_1} - \gamma_{1,t}^{q,\phi_1,\min} + \gamma_{1,t}^{q,\phi_1,\max} = 0, \forall t \in \Omega_{\mathrm{T}} \tag{A9a}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial Q_{\mathrm{sub},t}^{\phi_2}} = -\pi_{1,t}^{q,\phi_2} - \gamma_{1,t}^{q,\phi_2,\min} + \gamma_{1,t}^{q,\phi_2,\max} = 0, \forall t \in \Omega_{\mathrm{T}} \tag{A9b}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial Q_{\mathrm{sub},t}^{\phi_3}} = -\pi_{1,t}^{q,\phi_3} - \gamma_{1,t}^{q,\phi_3,\min} + \gamma_{1,t}^{q,\phi_3,\max} = 0, \forall t \in \Omega_{\mathrm{T}} \tag{A9c}$$

- *The gradient for $P_{j,t}^{\mathrm{MT},\phi}$ and $Q_{j,t}^{\mathrm{MT},\phi}$*

$$\frac{\partial L_{\mathrm{DSO}}}{\partial P_{j,t}^{\mathrm{MT},\phi_1}} = c_j^{\mathrm{MT}} - \pi_{j,t}^{p,\phi_1} - \gamma_{j,t}^{\mathrm{MT},p,\phi_1,\min} + \gamma_{j,t}^{\mathrm{MT},p,\phi_1,\max} - \gamma_{j,t}^{\mathrm{MT},q,\phi_1,\max} \tan(\arccos(\varphi^{\mathrm{pf}})) = 0, \forall j \in \Omega_{\mathrm{MT}}, \forall t \in \Omega_{\mathrm{T}} \tag{A10a}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial P_{j,t}^{\mathrm{MT},\phi_2}} = c_j^{\mathrm{MT}} - \pi_{j,t}^{p,\phi_2} - \gamma_{j,t}^{\mathrm{MT},p,\phi_2,\min} + \gamma_{j,t}^{\mathrm{MT},p,\phi_2,\max} - \gamma_{j,t}^{\mathrm{MT},q,\phi_2,\max} \tan(\arccos(\varphi^{\mathrm{pf}})) = 0, \forall j \in \Omega_{\mathrm{MT}}, \forall t \in \Omega_{\mathrm{T}} \tag{A10b}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial P_{j,t}^{\mathrm{MT},\phi_3}} = c_j^{\mathrm{MT}} - \pi_{j,t}^{p,\phi_3} - \gamma_{j,t}^{\mathrm{MT},p,\phi_3,\min} + \gamma_{j,t}^{\mathrm{MT},p,\phi_3,\max} - \gamma_{j,t}^{\mathrm{MT},q,\phi_3,\max} \tan(\arccos(\varphi^{\mathrm{pf}})) = 0, \forall j \in \Omega_{\mathrm{MT}}, \forall t \in \Omega_{\mathrm{T}} \tag{A10c}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial Q_{j,t}^{\mathrm{MT},\phi_1}} = -\pi_{j,t}^{q,\phi_1} - \gamma_{j,t}^{\mathrm{MT},q,\phi_1,\min} + \gamma_{j,t}^{\mathrm{MT},q,\phi_1,\max} = 0, \forall j \in \Omega_{\mathrm{MT}}, \forall t \in \Omega_{\mathrm{T}} \tag{A10d}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial Q_{j,t}^{\mathrm{MT},\phi_2}} = -\pi_{j,t}^{q,\phi_2} - \gamma_{j,t}^{\mathrm{MT},q,\phi_2,\min} + \gamma_{j,t}^{\mathrm{MT},q,\phi_2,\max} = 0, \forall j \in \Omega_{\mathrm{MT}}, \forall t \in \Omega_{\mathrm{T}} \tag{A10e}$$

$$\frac{\partial L_{\mathrm{DSO}}}{\partial Q_{j,t}^{\mathrm{MT},\phi_3}} = -\pi_{j,t}^{q,\phi_3} - \gamma_{j,t}^{\mathrm{MT},q,\phi_3,\min} + \gamma_{j,t}^{\mathrm{MT},q,\phi_3,\max} = 0, \forall j \in \Omega_{\mathrm{MT}}, \forall t \in \Omega_{\mathrm{T}} \tag{A10f}$$

2) Dual feasibility and complementary slackness conditions

$$0 \le \gamma_{j,t}^{v,\phi,\min} \perp v_{j,t}^{\phi} - \left(V_j^{\min,\phi}\right)^2 \ge 0, \forall j \in \Omega_{\mathrm{N}}, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A11a}$$

$$0 \le \gamma_{j,t}^{v,\phi,\max} \perp \left(V_j^{\max,\phi}\right)^2 - v_{j,t}^{\phi} \ge 0, \forall j \in \Omega_{\mathrm{N}}, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A11b}$$

$$0 \le \gamma_{1,t}^{p,\phi,\min} \perp P_{\mathrm{sub},t}^{\phi} + P_{\max} \ge 0, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A11c}$$

$$0 \le \gamma_{1,t}^{p,\phi,\max} \perp P_{\max} - P_{\mathrm{sub},t}^{\phi} \ge 0, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A11d}$$

$$0 \le \gamma_{1,t}^{q,\phi,\min} \perp Q_{\mathrm{sub},t}^{\phi} + Q_{\max} \ge 0, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A11e}$$

$$0 \le \gamma_{1,t}^{q,\phi,\max} \perp Q_{\max} - Q_{\mathrm{sub},t}^{\phi} \ge 0, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A11f}$$

$$0 \le \gamma_{j,t}^{\mathrm{MT},p,\phi,\min} \perp P_{j,t}^{\mathrm{MT},\phi} \ge 0, \forall j \in \Omega_{\mathrm{MT}}, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A11g}$$

$$0 \le \gamma_{j,t}^{\mathrm{MT},p,\phi\max} \perp P_j^{\mathrm{MT},\phi,\max} - P_{j,t}^{\mathrm{MT},\phi} \ge 0, \forall j \in \Omega_{\mathrm{MT}}, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A11h}$$

$$0 \le \gamma_{j,t}^{\mathrm{MT},q,\phi,\min} \perp Q_{j,t}^{\mathrm{MT},\phi} \ge 0, \forall j \in \Omega_{\mathrm{MT}}, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A11i}$$

$$0 \le \gamma_{j,t}^{\mathrm{MT},q,\phi\max} \perp P_{j,t}^{\mathrm{MT},\phi} \tan(\arccos(\varphi^{\mathrm{pf}})) - Q_{j,t}^{\mathrm{MT},\phi} \ge 0, \forall j \in \Omega_{\mathrm{MT}}, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A11j}$$

$$0 \le \gamma_{j,t}^{\phi\varphi,\min} \perp v_{j,t}^{\phi} - v_{j,t}^{\varphi} + K_v \ge 0, \forall j \in \Omega_{\mathrm{N}}, \forall t \in \Omega_{\mathrm{T}}, \forall \phi, \varphi \in \Phi, \phi \ne \varphi \tag{A11k}$$

$$0 \le \gamma_{j,t}^{\phi\varphi,\max} \perp K_v - v_{j,t}^{\phi} + v_{j,t}^{\varphi} \ge 0, \forall j \in \Omega_{\mathrm{N}}, \forall t \in \Omega_{\mathrm{T}}, \forall \phi, \varphi \in \Phi, \phi \ne \varphi \tag{A11l}$$

The bilinear terms in (A11) can be relaxed by using Big-M method, presented as

$$0 \le \gamma_{j,t}^{v,\phi,\min} \le \mathrm{M} Z_{j,t}^{v,\phi,\min}, 0 \le v_{j,t}^{\phi} - \left(V_j^{\min,\phi}\right)^2 \le \mathrm{M}\left(1 - Z_{j,t}^{v,\phi,\min}\right), \forall j \in \Omega_{\mathrm{N}}, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A12a}$$

$$0 \le \gamma_{j,t}^{v,\phi,\max} \le \mathrm{M} Z_{j,t}^{v,\phi,\max}, 0 \le \left(V_j^{\max,\phi}\right)^2 - v_{j,t}^{\phi} \le \mathrm{M}\left(1 - Z_{j,t}^{v,\phi,\max}\right), \forall j \in \Omega_{\mathrm{N}}, \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A12b}$$

$$0 \le \gamma_{1,t}^{p,\phi,\min} \le \mathrm{M} Z_{1,t}^{p,\phi,\min}, 0 \le P_{\mathrm{sub},t}^{\phi} + P_{\max} \le \mathrm{M}\left(1 - Z_{1,t}^{p,\phi,\min}\right), \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A12c}$$

$$0 \le \gamma_{1,t}^{p,\phi,\max} \le \mathrm{M} Z_{1,t}^{p,\phi,\max}, 0 \le P_{\max} - P_{\mathrm{sub},t}^{\phi} \le \mathrm{M}\left(1 - Z_{1,t}^{p,\phi,\max}\right), \forall t \in \Omega_{\mathrm{T}}, \forall \phi \in \Phi \tag{A12d}$$

$$0\le\gamma_{1,t}^{q,\phi,\min}\le \mathrm{M}Z_{1,t}^{q,\phi,\min},0\le Q_{\mathrm{sub},t}^{\phi}+Q_{\max}\le \mathrm{M}\left(1-Z_{1,t}^{q,\phi,\min}\right),\forall t\in\Omega_{\mathrm{T}},\forall\phi\in\Phi \tag{A12e}$$

$$0\le\gamma_{1,t}^{q,\phi,\max}\le \mathrm{M}Z_{1,t}^{q,\phi,\max},0\le Q_{\max}-Q_{\mathrm{sub},t}^{\phi}\le \mathrm{M}\left(1-Z_{1,t}^{q,\phi,\max}\right),\forall t\in\Omega_{\mathrm{T}},\forall\phi\in\Phi \tag{A12f}$$

$$0\le\gamma_{j,t}^{\mathrm{MT},p,\phi,\min}\le \mathrm{M}Z_{j,t}^{\mathrm{MT},p,\phi,\min},0\le P_{j,t}^{\mathrm{MT},\phi}\le \mathrm{M}\left(1-Z_{j,t}^{\mathrm{MT},p,\phi,\min}\right),\forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}},\forall\phi\in\Phi \tag{A12g}$$

$$0\le\gamma_{j,t}^{\mathrm{MT},p,\phi,\max}\le \mathrm{M}Z_{j,t}^{\mathrm{MT},p,\phi,\max},\ \ 0\le P_{j}^{\mathrm{MT},\phi,\max}-P_{j,t}^{\mathrm{MT},\phi}\le \mathrm{M}\left(1-Z_{j,t}^{\mathrm{MT},p,\phi,\max}\right),\forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}},\forall\phi\in\Phi \tag{A12h}$$

$$0\le\gamma_{j,t}^{\mathrm{MT},q,\phi,\min}\le \mathrm{M}Z_{j,t}^{\mathrm{MT},q,\phi,\min},0\le Q_{j,t}^{\mathrm{MT},\phi}\le \mathrm{M}\left(1-Z_{j,t}^{\mathrm{MT},q,\phi,\min}\right),\forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}},\forall\phi\in\Phi \tag{A12i}$$

$$0\le\gamma_{j,t}^{\mathrm{MT},q,\phi,\max}\le \mathrm{M}Z_{j,t}^{\mathrm{MT},q,\phi,\max},\ \ 0\le P_{j,t}^{\mathrm{MT},\phi}\tan(\arccos(\varphi^{\mathrm{pf}}))-Q_{j,t}^{\mathrm{MT},\phi}\le \mathrm{M}\left(1-Z_{j,t}^{\mathrm{MT},q,\phi,\max}\right),\forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}},\forall\phi\in\Phi \tag{A12j}$$

$$0\le\gamma_{j,t}^{\phi\varphi,\min}\le \mathrm{M}Z_{j,t}^{\phi\varphi,\min},0\le v_{j,t}^{\phi}-v_{j,t}^{\varphi}+K_v\le \mathrm{M}\left(1-Z_{j,t}^{\phi\varphi,\min}\right),\forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}},\forall\phi,\varphi\in\Phi,\phi\ne\varphi \tag{A12k}$$

$$0\le\gamma_{j,t}^{\phi\varphi,\max}\le \mathrm{M}Z_{j,t}^{\phi\varphi,\max},0\le K_v-v_{j,t}^{\phi}+v_{j,t}^{\varphi}\le \mathrm{M}\left(1-Z_{j,t}^{\phi\varphi,\max}\right),\forall j\in\Omega_{\mathrm{N}},\forall t\in\Omega_{\mathrm{T}},\forall\phi,\varphi\in\Phi,\phi\ne\varphi \tag{A12l}$$

where, M is a large constant and all the *Z*-type variables are 0-1 variables.

3) Primal feasibility conditions

The primal feasible conditions have been detailed in (A1).

Dual objective function reformulation of the lower-level problem

By substituting the KKT stationarity conditions (A3)-(A10) into the Lagrangian function L_{DSO} in (A2), L_{DSO} can be reformulated as the objective function of the dual problem in (A1), expressed as (A13).

$$\begin{aligned}
&L_{\mathrm{DSO}}\left(\begin{array}{l}\pi_{j,t}^{p,\phi},\pi_{j,t}^{q,\phi},\theta_{j,t}^{v,\phi},\gamma_{j,t}^{v,\phi,\min},\gamma_{j,t}^{v,\phi,\max},\gamma_{1,t}^{p,\phi,\min},\gamma_{1,t}^{p,\phi,\max},\gamma_{1,t}^{q,\phi,\min},\gamma_{1,t}^{q,\phi,\max},\gamma_{j,t}^{\mathrm{MT},p,\phi,\min},\gamma_{j,t}^{\mathrm{MT},p,\phi,\max},\\ \gamma_{j,t}^{\mathrm{MT},q,\phi,\min},\gamma_{j,t}^{\mathrm{MT},q,\phi,\max},\gamma_{j,t}^{\phi_1\phi_2,\min},\gamma_{j,t}^{\phi_1\phi_2,\max},\gamma_{j,t}^{\phi_2\phi_3,\min},\gamma_{j,t}^{\phi_2\phi_3,\max},\gamma_{j,t}^{\phi_3\phi_1,\min},\gamma_{j,t}^{\phi_3\phi_1,\max}\end{array}\right)\\
&=\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\pi_{j,t}^{p,\phi}\left(P_{j,t}^{\mathrm{D},\phi}+P_{j,t}^{\mathrm{Hub},\phi}\right)+\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\pi_{j,t}^{q,\phi}Q_{j,t}^{\mathrm{D},\phi}+\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\theta_{1,t}^{v,\phi}\left(V^{ref}\right)^2\\
&+\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\min}\left(V_j^{\min,\phi}\right)^2-\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\max}\left(V_j^{\max,\phi}\right)^2\\
&-\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\min}P_{\max}-\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\max}P_{\max}\\
&-\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\min}Q_{\max}-\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\max}Q_{\max}\\
&-\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_1\phi_2,\min}K_v-\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_1\phi_2,\max}K_v\\
&-\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_2\phi_3,\min}K_v-\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_2\phi_3,\max}K_v\\
&-\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_3\phi_1,\min}K_v-\sum_{j\in\Omega_{\mathrm{N}}}\sum_{t\in\Omega_{\mathrm{T}}}\gamma_{j,t}^{\phi_3\phi_1,\max}K_v-\sum_{j\in\Omega_{\mathrm{MT}}}\sum_{t\in\Omega_{\mathrm{T}}}\sum_{\phi\in\Phi}\gamma_{j,t}^{\mathrm{MT},p,\phi,\max}P_j^{\mathrm{MT},\phi,\max}
\end{aligned}\tag{A13}$$

According to the strong duality theorem, the objective function of the primal problem is equal to that of the dual problem, that is

$$
\begin{aligned}
&\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\lambda_t^{\phi}P_{\mathrm{sub},t}^{\phi}+\sum_{j\in\Omega_\mathrm{MT}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}c_j^{\mathrm{MT}}P_{j,t}^{\mathrm{MT},\phi}\\
&=\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\pi_{j,t}^{p,\phi}\left(P_{j,t}^{\mathrm{D},\phi}+P_{j,t}^{\mathrm{Hub},\phi}\right)+\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\pi_{j,t}^{q,\phi}Q_{j,t}^{\mathrm{D},\phi}+\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\theta_{1,t}^{v,\phi}\left(V^{ref}\right)^2\\
&+\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\min}\left(V_j^{\min,\phi}\right)^2-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\max}\left(V_j^{\max,\phi}\right)^2\\
&-\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\min}P_{\max}-\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\max}P_{\max}\\
&-\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\min}Q_{\max}-\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\max}Q_{\max}\\
&-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_1\phi_2,\min}K_v-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_1\phi_2,\max}K_v\\
&-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_2\phi_3,\min}K_v-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_2\phi_3,\max}K_v\\
&-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_3\phi_1,\min}K_v-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_3\phi_1,\max}K_v-\sum_{j\in\Omega_\mathrm{MT}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{j,t}^{\mathrm{MT},p,\phi,\max}P_j^{\mathrm{MT},\phi,\max}
\end{aligned}
\tag{A14}
$$

The equivalent MILP problem

Based on the equation (9) and $\lambda_{j,t}^{\mathrm{DLMP},\phi}=\pi_{j,t}^{p,\phi}$, we can derive an equivalent form of the objective function in the upper-level problem by transposing the terms in (A14), thereby effectively eliminating the bilinear terms from the original objective function.

$$
\begin{aligned}
&\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\sum_{j\in\Omega_\mathrm{N}}\left\{\lambda_{j,t}^{\mathrm{DLMP},\phi}\left(P_{j,t}^{\mathrm{PV},\phi}-P_{j,t}^{\mathrm{ES},\phi}-P_{j,t}^{\mathrm{EVA,C},\phi}+P_{j,t}^{\mathrm{EVA,D},\phi}-P_{j,t}^{\mathrm{FEV},\phi}\right)+f^{\mathrm{PV}}P_{j,t}^{\mathrm{PV},\phi}\right\}\\
&=\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\sum_{j\in\Omega_\mathrm{N}}\left\{-\pi_{j,t}^{p,\phi}P_{j,t}^{\mathrm{Hub},\phi}+f^{\mathrm{PV}}P_{j,t}^{\mathrm{PV},\phi}\right\}\\
&=-\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\lambda_t^{\phi}P_{\mathrm{sub},t}^{\phi}-\sum_{j\in\Omega_\mathrm{MT}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}c_j^{\mathrm{MT}}P_{j,t}^{\mathrm{MT},\phi}+\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\sum_{j\in\Omega_\mathrm{N}}f^{\mathrm{PV}}P_{j,t}^{\mathrm{PV},\phi}\\
&+\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\pi_{j,t}^{p,\phi}P_{j,t}^{\mathrm{D},\phi}+\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\pi_{j,t}^{q,\phi}Q_{j,t}^{\mathrm{D},\phi}+\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\theta_{1,t}^{v,\phi}\left(V^{ref}\right)^2\\
&+\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\min}\left(V_j^{\min,\phi}\right)^2-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\max}\left(V_j^{\max,\phi}\right)^2\\
&-\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\min}P_{\max}-\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\max}P_{\max}\\
&-\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\min}Q_{\max}-\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\max}Q_{\max}\\
&-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_1\phi_2,\min}K_v-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_1\phi_2,\max}K_v\\
&-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_2\phi_3,\min}K_v-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_2\phi_3,\max}K_v\\
&-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_3\phi_1,\min}K_v-\sum_{j\in\Omega_\mathrm{N}}\sum_{t\in\Omega_\mathrm{T}}\gamma_{j,t}^{\phi_3\phi_1,\max}K_v-\sum_{j\in\Omega_\mathrm{MT}}\sum_{t\in\Omega_\mathrm{T}}\sum_{\phi\in\Phi}\gamma_{j,t}^{\mathrm{MT},p,\phi,\max}P_j^{\mathrm{MT},\phi,\max}
\end{aligned}
\tag{A15}
$$

Thus, the original objective function in upper-level problem can be replaced by (A16)

$$\max \sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\sum_{j\in\Omega_N}\left\{\lambda_{j,t}^{\mathrm{DLMP},\phi}\left(P_{j,t}^{\mathrm{PV},\phi}-P_{j,t}^{\mathrm{ES},\phi}-P_{j,t}^{\mathrm{EVA,C},\phi}+P_{j,t}^{\mathrm{EVA,D},\phi}-P_{j,t}^{\mathrm{FEV},\phi}\right)+f^{\mathrm{PV}}P_{j,t}^{\mathrm{PV},\phi}\right\}$$

$$=\max\left\{\begin{array}{l}
-\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\lambda_t^{\phi}P_{\mathrm{sub},t}^{\phi}-\sum_{j\in\Omega_{MT}}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}c_j^{\mathrm{MT}}P_{j,t}^{\mathrm{MT},\phi}+\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\sum_{j\in\Omega_N}f^{\mathrm{PV}}P_{j,t}^{\mathrm{PV},\phi}\\
+\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\pi_{j,t}^{p,\phi}P_{j,t}^{\mathrm{D},\phi}+\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\pi_{j,t}^{q,\phi}Q_{j,t}^{\mathrm{D},\phi}+\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\theta_{1,t}^{v,\phi}\left(V^{ref}\right)^2\\
+\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\min}\left(V_j^{\min,\phi}\right)^2-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\max}\left(V_j^{\max,\phi}\right)^2\\
-\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\min}P_{\max}-\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\max}P_{\max}\\
-\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\min}Q_{\max}-\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\max}Q_{\max}\\
-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_1\phi_2,\min}K_v-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_1\phi_2,\max}K_v\\
-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_2\phi_3,\min}K_v-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_2\phi_3,\max}K_v\\
-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_3\phi_1,\min}K_v-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_3\phi_1,\max}K_v-\sum_{j\in\Omega_{MT}}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{j,t}^{\mathrm{MT},p,\phi,\max}P_j^{\mathrm{MT},\max}
\end{array}\right\} \quad (A16)$$

The constraints of the upper-level problem include (5i), (6p), (7l), (8k) and (9). The KKT conditions of the lower-level problem consist of the primal feasibility constraints (2) and (4), the KKT stationarity conditions (A3)-(A10), as well as the dual feasibility and relaxed complementary slackness conditions (A12). In summary, the equivalent MILP problem can be formulated as

$$\max\left\{\begin{array}{l}
-\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\lambda_t^{\phi}P_{\mathrm{sub},t}^{\phi}-\sum_{j\in\Omega_{MT}}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}c_j^{\mathrm{MT}}P_{j,t}^{\mathrm{MT},\phi}+\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\sum_{j\in\Omega_N}f^{\mathrm{PV}}P_{j,t}^{\mathrm{PV},\phi}\\
+\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\pi_{j,t}^{p,\phi}P_{j,t}^{\mathrm{D},\phi}+\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\pi_{j,t}^{q,\phi}Q_{j,t}^{\mathrm{D},\phi}+\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\theta_{1,t}^{v,\phi}\left(V^{ref}\right)^2\\
+\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\min}\left(V_j^{\min,\phi}\right)^2-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{j,t}^{v,\phi,\max}\left(V_j^{\max,\phi}\right)^2\\
-\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\min}P_{\max}-\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{1,t}^{p,\phi,\max}P_{\max}\\
-\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\min}Q_{\max}-\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{1,t}^{q,\phi,\max}Q_{\max}\\
-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_1\phi_2,\min}K_v-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_1\phi_2,\max}K_v\\
-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_2\phi_3,\min}K_v-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_2\phi_3,\max}K_v\\
-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_3\phi_1,\min}K_v-\sum_{j\in\Omega_N}\sum_{t\in\Omega_T}\gamma_{j,t}^{\phi_3\phi_1,\max}K_v-\sum_{j\in\Omega_{MT}}\sum_{t\in\Omega_T}\sum_{\phi\in\Phi}\gamma_{j,t}^{\mathrm{MT},p,\phi,\max}P_j^{\mathrm{MT},\max}
\end{array}\right\} \quad (A17)$$

$$s.t.\quad (2),(4),(9),(\mathrm{A3})-(\mathrm{A10}),(\mathrm{A12}),\text{cons-PV},\text{cons-ES},\text{cons-SEV},\text{cons-FEV}$$