****Appendix A****

The complete derivation steps for transforming the bi-level problem in (10) into an equivalent MILP problem are as follows.

### *KKT conditions derivation of the lower-level problem*

Here, the complete primal problem of the lower-level problem is expressed as follows:

 (A1)

Then the primal problem with constraints can be converted to the equivalent optimization problem without constraints by Lagrangain relaxation. The Lagrangian function of (A1) is as

 (A2)

*1)* *KKT stationarity conditions*

Base on the Lagrangian function in (A2), the stationarity conditions can be obtained as (A3) by requiring that the gradient of with respect to the decision variables is zero.

* *The gradient for*

 (A3a)

 (A3b)

 (A3c)

where, taking the superscript as an example, it represents the element in the first row and second column of the corresponding matrix.

* *The gradient for*

 (A4a)

 (A4b)

 (A4c)

* *The gradient for*

For the root node of PDN, it has

 (A5a)

 (A5b)

 (A5c)

For the non-root and non-terminal nodes in PDN, it has

 (A6a)

 (A6b)

 (A6c)

For the terminal nodes in PDN, it has

 (A7a)

 (A7b)

 (A7c)

* *The gradient for*

 (A8a)

 (A8b)

 (A8c)

* *The gradient for*

 (A9a)

 (A9b)

 (A9c)

* *The gradient for*

 (A10a)

 (A10b)

 (A10c)

 (A10d)

 (A10e)

 (A10f)

*2)* *Dual feasibility and complementary slackness conditions*

 (A11a)

 (A11b)

 (A11c)

 (A11d)

 (A11e)

 (A11f)

 (A11g)

 (A11h)

 (A11i)

 (A11j)

 (A11k)

 (A11l)

The bilinear terms in (A11) can be relaxed by using Big-M method, presented as

 (A12a)

 (A12b)

 (A12c)

 (A12d)

 (A12e)

 (A12f)

 (A12g)

 (A12h)

 (A12i)

 (A12j)

 (A12k)

 (A12l)

where, M is a large constant and all the *Z*-type variables are 0-1 variables.

*3)* *Primal feasibility conditions*

The primal feasible conditions have been detailed in (A1).

### *Dual objective function reformulation of the lower-level problem*

By substituting the KKT stationarity conditions (A3)-(A10) into the Lagrangian function in (A2), can be reformulated as the objective function of the dual problem in (A1), expressed as (A13).

 (A13)

According to the strong duality theorem, the objective function of the primal problem is equal to that of the dual problem, that is

 (A14)

### *The equivalent MILP problem*

Based on the equation (9) and , we can derive an equivalent form of the objective function in the upper-level problem by transposing the terms in (A14), thereby effectively eliminating the bilinear terms from the original objective function.

 (A15)

Thus, the original objective function in upper-level problem can be replaced by (A16)

 (A16)

The constraints of the upper-level problem include (5i), (6p), (7l), (8k) and (9). The KKT conditions of the lower-level problem consist of the primal feasibility constraints (2) and (4), the KKT stationarity conditions (A3)-(A10), as well as the dual feasibility and relaxed complementary slackness conditions (A12). In summary, the equivalent MILP problem can be formulated as

 (A17)