Australian National University
Research School of Computer Science

COMP3600/COMP6466 in 2016 – Quiz four

Due: 23:55pm Friday, October 14

Submit your work electronically through Wattle. The total mark of this quiz worths 20 points which is worth 4 of the final mark.

Question 1 (10 points).

(a) Apply the forest consisting of (inverted) directed tree implementation of disjoint-set data structure, using both heuristics (path compression and union by rank), to find the connected components in a graph with 10 vertices and edges provided in this order: 1-3, 2-5, 3-8, 4-7, 6-7, 8-9, 1-10. (3 points)

(b) Consider a red-black tree formed by inserting n nodes with RB-INSERT. Argue if $n > 1$, the tree has at least one red node. (2 points)

(c) Given an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should be run in $O(|V|)$ time, independent of $|E|$. (2 points)

(d) Given a connected undirected graph $G = (V, E)$ where V is the set of nodes and E is the set of edges, devise an $O(|V| + |E|)$ algorithm to verify whether G contains any odd cycles. An odd cycle in a graph is a simple cycle that has odd numbers of edges. (3 points).

Question 2 (10 points).

(a) In which case should the Bellman-Ford algorithm instead of Dijkstra's algorithm be applied to solve the single-source shortest paths problem? (1 point)

(b) Given a connected undirected graph $G = (V, E)$, assume that each edge $e \in E$ has a non-negative weight, let e_{min} be an edge with the minimum weight in a cycle of G, prove or disprove that e_{min} will be contained by any minimum spanning tree in G. **Hints:** you may just give a counter-example if you disprove the claim. (3 points)

(c) You are given a set of cities, along with the pattern of highways between them, in the form of an undirected graph $G = (V, E)$. Each stretch of highway $e \in E$ connects two of the cities, and you know its length in kilometers $l(e)$. You want to get from city $s \in V$ to city $t \in V$. There is one problem: your car can only hold enough petrol to cover L kilometers. There are petrol stations in each city, but not between cities. Therefore, you can only take a route if every one of its edges has length no greater than L.

- [i] Given the limitation on your car's fuel tank capacity, show how to determine in linear time whether there is a feasible route from s to t. (3 points)
- [ii] You are now planning to buy a new car, and you want to know the minimum fuel tank capacity that is needed to travel from s to t. Give an $O((|V|+|E|) \log |V|)$ algorithm to determine this. (3 points)