Australian National University
Research School of Computer Science

COMP3600/COMP6466 in 2016 – Quiz One

Due: 5pm Friday, July 29

Submit your work electronically through Wattle. The total mark of this quiz worths 10 points, which is worth of 3 points of the final mark.

Question 1 (2 points).
Given the following sequence, order them into a sorted sequence in the order of **growth** when n approaches infinity.

$\sqrt{n}$, $n^4 \log n$, $n^{3.5} \log n + n^3$, $3^{0.01n}$, n^6, $5^{75} \log^2 n$, $n^3 + n \log \log n$, $n^2 \log^{0.5} n$.

Question 2 (2 points).
Let $f(n) = an^3 + bn^2 + cn + d$ and $g(n) = n^3$ where a, b, c and d are nonnegative constants. Show that

$$f(n) = \Theta(g(n)).$$

Question 3 (2 points).
Provide the simplest expression for $\sum_{k=1}^{n} k^{9/4}$, using the $\Theta(\,)$ notation. Explain your reasoning clearly.

Question 4 (2 points).
Give an asymptotic upper bound on $T(n)$ for the following recurrence, using the $O(\,)$ notation. Justify your answers.

$$T(n) = 3T(2n/5) + n^2.$$

Question 5 (2 points).

(i) Show that $\ln x \leq x, \forall x \in R^+$, and $x \geq 1$, where R^+ is the set of positive real numbers.

(ii) Show that $\log n = O(n^\epsilon)$, $\forall \epsilon > 0$.

Hints: (i) use the fact that $\ln 1 = 0$ and $(\ln x)' = 1/x$. (ii) use a variable substitution such as $n = k^2$, $\ln x^k = k \ln x$, and $\log x = \frac{\ln x}{\ln 2}$.