COMP3600/6466 Algorithms
Review and Applications 3

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Exercise 1

What are the minimum and maximum numbers of elements in a heap of height h?

Exercise 2

Show that an n-element heap has height $\lfloor \lg n \rfloor$.

Exercise 3

Show that, with the array representation for storing an n-element heap, the leaves are the nodes indexed by $\lfloor n/2 \rfloor + 1, \lfloor n/2 \rfloor + 2, \ldots, n$.
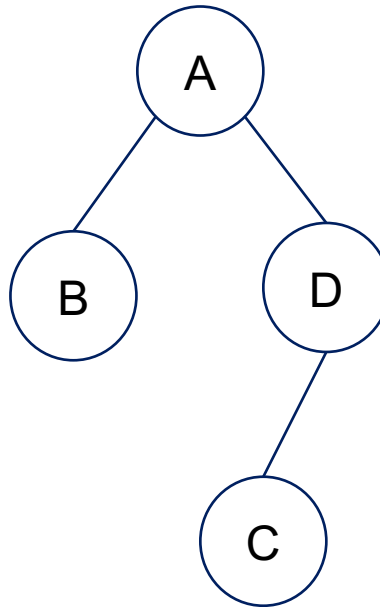
Exercise 4

Suppose we use a hash function h to hash n distinct keys into an array T of length m. Assuming simple uniform hashing, what is the expected number of collisions? More precisely, what is the expected cardinality of $\{\{k, l\} : k \neq l$ and $h(k) = h(l)\}$?

Exercise 5

Is the operation of deletion "commutative" in the sense that deleting x and then y from a binary search tree leaves the same tree as deleting y and then x? Argue why it is or give a counterexample.

Delete A then B

Delete B then A

Exercise 6

In a red-black tree, show that the longest simple path from a node x to a leaf has length at most twice that of the shortest simple path from x to a leaf.

Exercise 7

Give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in $O(V)$ time, independent of $|E|$.