Australian National University

Computer Science COMP3600/COMP6466 in 2016 – Tutorial One

Question 1. (i) Study the definitions of the basic notations such as Θ, O, Ω, o, ω
(ii) Rank the following functions in the order of growth:
$n^2 \lg n$, $n^{3/2} \lg^4 n$, $3^{0.5n}$, $\lg^* n \lg^2 n$, $n^{1.44}$, $\sqrt{n^{1/3} \lg n}$, $n^{\lg \lg n}$, $\sqrt{n}$, n^5.

Question 2. (i) Study the introduced bounding techniques for summations
(ii) Estimate the following functions, using $\Theta(\,)$ notation:

$$(a) \sum_{k=1}^{n} \frac{k^2}{7^k}, \qquad (b) \sum_{k=2}^{n} \frac{k^{1/2}}{\lg k}, \qquad (c) \sum_{k=1}^{n} k^{7/2}$$

Question 3. Reread the two approaches for solving recurrences, and apply them to solve the following recurrences.

(a) $T(n) = T(n/3) + n^{5/4}$
(b) $T(n) = 3T(n/2) + 4n$
(c) $T(n) = T(\lfloor \sqrt{n} \rfloor) + n \log n$ (optional)

Question 4. Today is your birthday, you are 37 years old! You are a great cybersecurity expert and your company has just been breached by a hacker who turned the power system off. Now, we need the backup generator which however is protected by a 3 digit code. Unfortunately, you cannot remember that code, but the following clues will help you recover the code.

(i) It is a number greater then triple your age today, square the number, you get 11664.

(ii) The note is two months old.

Assume that you can use a manual algorithm - the old Babylonian method to crack the code, which is described as follows.

The Babylonian Algorithm

INPUT: n and $i_{\max}$
OUTPUT: approximation of $\sqrt{n}$

1. Start with an initial value x_0;
2. for $1 \leq i \leq i_{\max}$ do
3. $x_i \leftarrow \frac{x_{i-1}+\frac{n}{x_{i-1}}}{2}$;
4. return $x_{i_{max}}$

One important property of the Babylonian algorithm is that *for any positive integer i_{max}, its approximate solution is always greater or equal to its square root.*

1. Show that $\frac{x_{i-1}+\frac{n}{x_{i-1}}}{2} \geq \sqrt{n},\ \forall x_{i-1} > 0$, using the fact that $(x_{i-1} - \sqrt{n})^2 \geq 0$.
2. Using the Babylonian algorithm, starting with $x_0 = 100$, find the three digit code for your backup generator.

Great job! The power is on again. You computer is now able to communicate with all the other servers in your company. Your initial goal is to check whether the main server is being accessed by the hacker. You can access a list of the servers that are currently being hacked. The servers are sorted in increasing order of their id numbers. You have 10 seconds to check if the main server is being hacked. Assume that there are 240,00 servers interconnected in your company, and each comparison on server id takes one millisecond.

1. Write an algorithm that is able to achieve your initial goal.
2. Specify the input(s) and output(s) of the algorithm.
3. Show that your algorithm will always return a solution within the time limit.