Australian National University

COMP3600/6466 Algorithms

Review and Applications 1

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Exercise 1

$$\text{Show that } 10^5 \cdot 2^n = o(3^n)$$

Exercise 2

$$\text{Show that } 2^n = \omega(n^3)$$

Exercise 3

Let $p(n) = \sum_{i=0}^{d} a_i n^i$ where $a^d > 0$. $p(n)$ is a degree-d polynomial in n. Given a constant k, prove the following properties:

1. If $k \geq d$, then $p(n) = O(n^k)$.
2. If $k \leq d$, then $p(n) = \Omega(n^k)$.
3. If $k = d$, then $p(n) = \Theta(n^k)$.
4. If $k > d$, then $p(n) = o(n^k)$.
5. If $k < d$, then $p(n) = \omega(n^k)$.

Exercise 4

Explain why the following statement is meaningless:

"The running time of algorithm A is at least $O(n^2)$"

Exercise 5

$$\text{Show that } 2^{n+1} = O(2^n), \text{ but } 2^{2n} \neq O(2^n)$$

Exercise 6

Show that $\lceil \lg n \rceil!$ is not polynomially bounded, but $\lceil \lg \lg n \rceil!$ is.

Exercise 7

```
COUNTING-SORT(A, B, k)
let C[0..k] be a new array
for i = 0 to k
    C[i] = 0
for j = 1 to A.length
    C[A[j]] = C[A[j]] + 1
// C[i] now contains the number of elements equal to i.
for i = 1 to k
    C[i] = C[i] + C[i - 1]
// C[i] now contains the number of elements less than or equal to i.
for j = A.length downto 1
    B[C[A[j]]] = A[j]
    C[A[j]] = C[A[j]] - 1
```

How much time does counting sort require in Θ notation?
(Analyse each "for" loop. Don't bother proving it formally.)

Exercise 8

Give an asymptotic upper bound for $\sum_{k=1}^{n} k^{11/4}$

Exercise 9

Give an asymptotic upper bound for

1. $T(n) = 8T(n/2) + n^3$
2. $T(n) = T(\sqrt{n}) + 1$