Australian National University
Research School of Computer Science

COMP3600/COMP6466 in 2016 – Quiz Three

Due: 23:55pm Friday, September 23

Submit your work electronically through Wattle. The total mark of this quiz worths 20 points, which is worth of 4.5 points of the final mark.

Question 1 (4 points).
Given a positive integer sequence $a_1, a_2, \ldots, a_n$, A maximum weighted decreasing subsequence is a subsequence $a_{i_1}, a_{i_2}, \ldots, a_{i_k}$ such that the weighted sum $\sum_{j=1}^{k} a_{i_j}$ of the sequence is the maximum one and $a_{i_j} \geq a_{i_{j'}}$ if $j < j'$, where $1 \leq i_1 < i_2 < \ldots < i_k \leq n$.

Following the four steps in the design of DP algorithms, devise a DP algorithm that takes a sequence $a[1 \ldots n]$ and returns a maximum weighted decreasing subsequence. Analyse the running time of your proposed algorithm.

Question 2 (5 points).
Given n items with item i having weight $w_i > 0$ and a profit $p_i > 0$ for all i, $1 \leq i \leq n$, assume that each item can be cut into an arbitrary fraction if needed. The fractional knapsack problem is to pack as many items as possible to a knapsack with capacity W such that the total profit of items (or a fraction of an item) in the knapsack is maximized. Show how to solve this fractional knapsack problem in $O(n)$ time. Notice that you here are asked to devise a linear running time algorithm for the problem, while an $O(n \log n)$ time algorithm can easily be devised, assuming that no sorting will be applied to the items. (*Hint: adopt the greedy strategy and the linear selection algorithm*).

Question 3 (2 points).
Show that if we order the characters in an alphabet so that their frequencies are monotonically decreasing, then, there exists an optimal code whose codeword lengths are monotonically increasing. (*Hint: adopt the greedy strategy*)

Question 4 (6 points).
(a) Assuming an initial max-heap is empty, insert the keys $9, 2, 12, 8, 8, 14, 5, 9, 11, 10$ into the max-heap one by one (once a time) until all elements are inserted, then remove the key in the root repeatedly until the heap is empty again. (3 points)

1. Use a diagram to illustrate each step of the insertion and deletion procedure.

2. What is the time complexity of sorting in this fashion if there are n keys to be inserted to and then removed from the max-heap?

(b) In the open addressing schema, three probing techniques: `linear probing`, `quadratic probing`, and `double hashing` have been introduced. (1) How many different probing sequences can be generated for each of the schemes? justify your answer. (2) What are the advantages and disadvantages of each these mentioned techniques? (3 points)

Question 5 (3 points).
Given an element sequence 10, 6, 15, 9, 12, 17, 1, 11, 34, 8, 7, 15, 17.

- Illustrate the final binary search tree by inserting the elements in the sequence one by one
- Assume that a new element 12 will be inserted to the tree, show the resulting tree after the insertion.