

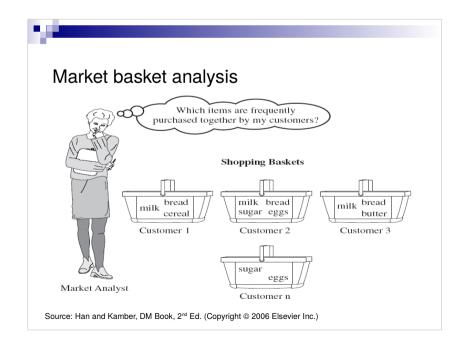
What is association mining?

- Association mining is the task of finding frequent rules / associations / patterns / correlations / causal structures within (large) sets of items in transactional (relational) databases
- *Unsupervised* learning techniques (*descriptive* data mining, not *predictive* data mining)
- The main applications are
 - Market basket analysis (customers who buys X also buys Y)
 - Web log analysis (click-stream)
 - Cross-marketing
 - Sale campaign analysis
 - DNS sequence analysis



Lecture outline

- What is association mining?
- Market basket analysis and association rules examples
- Basic concepts and formalism
- Basic rule measurements
- The Apriori algorithm
- Performance bottlenecks in Apriori
- Improve Apriori's efficiency





Association rules examples

- Rules form: body ⇒ head [support, confidence]
- Market basket:

$$buys(X, `beer') \Rightarrow buys(X, `snacks') [s=1\%, c=60\%]$$

- If a customer X purchased `beer', in 60% she or he also purchased `snacks'
- 1% of all transactions contain the items 'beer' and 'snacks'
- Student grades:

$$major(X, `BIT')$$
 and $takes(X, `COMP3420') \Rightarrow grade(X, `D') [s=3%, c=70%]$

- If a student X, who's major is `BIT', took the course `COMP3420' she or he in 70% achieved a grade `D'
- The combination `BIT', `COMP3420' and `D' appears in 3% of all transactions (records) in the database
- * Disclaimer: This is only an example, it does not mean that 70% of COMP3420 students in the past achieved a 'D' grade.



Formalism

- Set of items $X = \{x_1, x_2, ..., x_k\}$
- Database D containing transactions
- Each transaction T is a set of items, such that T is a subset of X
- Each transaction is associated with a unique identifier, called *TID* (for example, a unique number)
- Let A be a set of items (a subset of X)
- An association rule is an implication of the form $A \Rightarrow B$, where A is a subset of X and B is a subset of X, and the intersection of A and B is empty
 - No item in A can be in B, and vice versa
 - No rule of the form: {`beer', `chips'} ⇒ {`chips', `peanuts'}



Basic concepts

- Given:
 - · A (large) database of transactions
 - Each transaction contains a list of one or more items (e.g. purchased by a customer in a visit)
- Find the rules that correlate the presence of one set of items with that of another set of items
- Normally one is only interested in rules that are frequent
 - For example, 70% of customers who buy tires and car accessories also get their car service done

Question: How can this be improved to 80%? Possibly offer special deals like a 15% reduction of tire costs when the service is done



Basic rule measurements

 A rule A ⇒ B holds in a database D with support s, with s being the percentage of transactions in D that contain A and B

$$support(A \Rightarrow B) = P(A \cup B)$$

 The rule A ⇒ B has a confidence c in a database D if c is the percentage of transactions in D containing A that also contain B

confidence(
$$A \Rightarrow B$$
) = P($B|A$) = P($A \cup B$) / P(A)
confidence($A \Rightarrow B$) = support($A \Rightarrow B$) / support(A)



Rule measurements example

| Transaction ID | Items Bought |
|----------------|--------------|
| 2000 | a, b, c |
| 1000 | a, c |
| 4000 | , - |
| 5000 | b, e, f |

| Itemset | Support |
|---------|---------|
| a | 75.00% |
| b | 50.00% |
| С | 50.00% |
| a, c | 50.00% |

- Minimum support = 50% and confidence = 50%
- Rule $a \Rightarrow c$
 - support (a \Rightarrow c): 50%
 - confidence (a \Rightarrow c) = support(a \Rightarrow c) / support(a) = 50% / 75% = 66.67%



The Apriori algorithm (Agrawal & Srikant, VLDB'94)

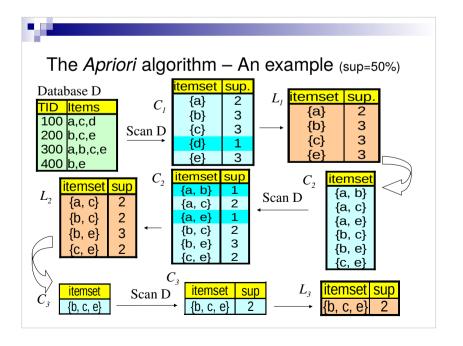
- C_k: Candidate item set of size k
 L_k: Frequent item set of size k
- Pseudo-code:

```
\begin{split} L_{_{1}} &= \{\text{frequent items}\}; \\ &\text{for } (k=1; L_{_{k}} != \{\,\}; \, k++) \text{ do begin} \\ &C_{_{k+1}} = \text{candidates generated from } L_{_{k}}; \\ &\text{for each transaction } t \text{ in database do} \\ &\text{increment the count of all candidates in } C_{_{k+1}} \\ &\text{that are contained in } t \\ &L_{_{k+1}} = \text{candidates in } C_{_{k+1}} \text{ with min\_support} \\ &\text{end do} \\ &\text{return } L = \mathsf{U}_{_{k}} L_{_{k}}; \end{split}
```



Mining frequent item sets

- Key step: Find the *frequent sets of items* that have *minimum support* (appear in at least xx% of all transactions in a database)
- Basic principle (*Apriori* principle): A sub-set of a frequent item set must also be a frequent item set
 - For example, if {a,b} is frequent, both {a} and {b} have to be frequent (if `beer' and 'chips' are purchased frequently together, then `beer' is purchased frequently and `chips' are also purchased frequently)
- Basic approach: Iteratively find frequent item sets with cardinality from 1 to k (k-item sets), k > 1
- Use the frequent item sets to generate association rules
 - For example, frequent 3-item set $\{a,b,c\}$ contains rules: $a \Rightarrow c, b \Rightarrow c, a \Rightarrow b, \{a,b\} \Rightarrow c, \{a,c\} \Rightarrow b, \{b,c\} \Rightarrow a, etc.$
- We are normally only interested in longer rules





The Apriori algorithm - An example (2)

Database D





- Minimum support = 50% and minimum confidence = 50%
- Rules:
 - b \Rightarrow c [s=50%, c=66.67%]
 - b \Rightarrow e [s=75%, c=100%]
 - $c \Rightarrow e [s=50\%, c=66.67\%]$
- $\{b, c\} \Rightarrow e [s=50\%, c=100\%]$
- $\{b, e\} \Rightarrow c [s=50\%, c=66.67\%]$
- {c, e} \Rightarrow b [s=50%, c=100%]



How to generate candidate item-sets?

- Suppose the items in $L_{k,j}$ are listed in an order (e.g. a < b)
- Step 1: Self-joining L_{k-1}

```
insert into C<sub>k</sub>
select p.item<sub>1</sub>, p.item<sub>2</sub>, ..., p.item<sub>k-1</sub>, q.item<sub>k-1</sub>
from L_{k-1} p, L_{k-1} q
where p.item<sub>1</sub> = q.item<sub>1</sub>, ..., p.item<sub>k-2</sub> = q.item<sub>k-2</sub>, p.item<sub>k-1</sub> < q.item<sub>k-1</sub>
```

•Step 2: Pruning

forall item sets c in C, do forall (k-1)-sub-sets s of c do if (s is not in L_{k-1}) then delete c from C_k



Important details of the *Apriori* algorithm

- How to generate candidate sets?
 - Step 1: Self-joining L_{k-1} (C_k is generated by joining L_{k-1} with itself)
 - Step 2: Pruning (any (k-1)-item set that is not frequent cannot be a subset of a frequent *k*-item set)
- Example of candidate generation:
 - L_a = {{a,b,c}, {a,b,d}, {a,c,d}, {a,c,e}, {b,c,d}}
 - Self-joining: $L_a * L_a (\{a,b, \boldsymbol{c},\boldsymbol{d}\} \text{ from } \{a,b,\boldsymbol{c}\} \text{ and } \{a,b,\boldsymbol{d}\}, \text{ and } \{a,c,\boldsymbol{d},\boldsymbol{e}\} \text{ from } \{a,b,\boldsymbol{d}\}, \text{ and } \{a,c,d,e\} \text{ from } \{a,b,d\}, \text{ and } \{a,c,d,e\}, \text{ from } \{a,b,d\}, \text{ and } \{a,c,d,e\}, \text{ from } \{a,b,d\}, \text{ and } \{a,c,d,e\}, \text{ from } \{a,b,d\}, \text{ from$ {a,c,**d**} and {a,c,**e**})
 - Pruning: {a,c,d,e} is removed because {a,d,e} is not in L
 - $C_A = \{\{a,b,c,d\}\}$
- How to count supports for candidates?



Apriori performance bottlenecks

- The core of the Apriori algorithm is to
 - Use frequent (k-1) item sets to generate candidate frequent k item sets
- Use database scan and pattern matching to collect counts for candidate item sets
- Candidate generation is the main bottleneck
 - 10⁴ frequent 1-item sets (sets of length 1) will generate 10⁷ candidate
 - To discover a frequent pattern of size 100 (for example {a,, a,, ..., a,,,}) one needs to generate 2¹⁰⁰ = 10³⁰ candidates
 - Multiple scans of the database are needed (n+1 scans if the longest pattern is *n* items long)



Methods to improve *Apriori's* efficiency

- Reduce the number of scans of the database
 - Any item set that is potentially frequent in the database must be frequent in at least one of the partitions of the database
 - Scan 1: Partition database and find local frequent patterns
 - Scan 2: Consolidate global frequent patterns

Shrink number of candidates

- Select a sample of the database, mine frequent patterns within sample using *Apriori*
- Scan database once to verify frequent item sets found in sample
- Scan database again to find missed frequent patterns

• Facilitate support of counting candidates

 For example, use special data structures like Frequent-Pattern tree (FP-tree)