COMP3420: Advanced Databases and Data Mining

Classification and prediction: Bayesian classification and evaluating classifier accuracy

Lecture outline

- Bayesian classification
- Bayesian theorem
- Naïve Bayesian classifier
- Naïve Bayesian classifier example
- Zero-probability problem
- Comments on naïve Bayesian classifier
- Bayesian belief networks
- Measuring classifier accuracy
- Evaluating the accuracy of a classifier

Bayesian classification: Why?

- A statistical classifier: performs *probabilistic prediction, i.e.,* predicts class membership probabilities
- Foundation: Based on *Bayes' Theorem*
- Performance: A simple Bayesian classifier, called *naïve Bayesian classifier*, has comparable performance with decision tree and selected neural network classifiers
- Works incremental: Each training example (training record) can incrementally increase/decrease the probability that a hypothesis is correct — prior knowledge can be combined with observed data
- Standard: even when Bayesian methods are computationally intractable, they can provide a standard of optimal decision making against which other methods can be measured

Bayesian theorem: Basics

- Let **X** be a data sample ("*evidence*"): class label is unknown
- Let H be a *hypothesis* that **X** belongs to class **C**
- Classification is to determine *P(H|**X**)*, the probability that the hypothesis holds given the observed data sample **X**
- *P(H)* (*prior,* or *a priori probability*): The initial probability
  - For example, **X** will buy computer, regardless of age, income, …
- *P(**X**)*: Probability that sample data is observed
- *P(**X**|H)*: The probability of observing the sample **X**, given that the hypothesis holds
  - For example, given that **X** will buy computer, the probability that **X** is of age 31..40, has medium income, etc.

Bayesian theorem

- Given training data **X**, the *posteriori probability of a hypothesis* H, *P(H|**X**)*, follows the Bayes Theorem

$$P(H|X)=\frac{P(X|H)P(H)}{P(X)}$$

- Predicts data sample **X** belongs to class C_j if probability *P(C_j|**X**)* is the highest among all the *P(C_k|**X**)* for all the classes *k*
- Practical difficulty: requires initial knowledge of many probabilities, and significant computational costs

Towards naïve Bayesian classifier

- Let **D** be a training set of tuples and their associated class labels, and each tuple is represented by an *n*-dimensional attribute vector **X** = (x_1, x_2, …, x_n)
- Suppose there are *m* classes: C_1, C_2, …, C_m (often, m=2)
- Classification is to derive the maximum posteriori, i.e., the maximal *P(C_i|**X**)*
- This can be derived from Bayes' theorem

$$P(C_i|X)=\frac{P(X|C_i)P(C_i)}{P(X)}$$

- Since *P(X)* is constant for all classes, only needs to be maximised $P(C_i|X)=P(X|C_i)P(C_i)$

Derivation of naïve Bayesian classifier

- A simplified assumption: attributes are conditionally independent (i.e., no dependence relation between attributes): $P(X|C_i)=\prod_{k=1}^{n}P(x_k|C_i)=P(x_1|C_i)\times P(x_2|C_i)\times\ldots\times P(x_n|C_i)$
- This greatly reduces the computation cost: Only counts the class distribution
- If attribute A_k is categorical, *P(x_k|C_i)* is the number of tuples in C_i that have value x_k for A_k divided by $|C_{i, D}|$ (number of tuples of C_i in D)
- If A_k is a continuous-valued attribute, *P(x_k|C_i)* is usually computed based on Gaussian (normal) distribution
  - Formulas 8.13 and 8.14 in text book, page 352

Naïve Bayesian classifier: Training data set

Classes:

C1: buys_computer = "yes"

C2: buys_computer = "no"

Data sample (test record):

X = *(age <="30", income = "medium", student = "yes", credit_rating = "fair")*

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Naïve Bayesian classifier: An example

- $P(C_i)$: P(buys_computer = "yes") = 9/14 = 0.643
  P(buys_computer = "no") = 5/14= 0.357
- Compute $P(\mathbf{X}|C_i)$ for each class:
  P(age = "<=30" | buys_computer = "yes") = 2/9 = 0.222
  P(age = "<= 30" | buys_computer = "no") = 3/5 = 0.6
  P(income = "medium" | buys_computer = "yes") = 4/9 = 0.444
  P(income = "medium" | buys_computer = "no") = 2/5 = 0.4
  P(student = "yes" | buys_computer = "yes) = 6/9 = 0.667
  P(student = "yes" | buys_computer = "no") = 1/5 = 0.2
  P(credit_rating = "fair" | buys_computer = "yes") = 6/9 = 0.667
  P(credit_rating = "fair" | buys_computer = "no") = 2/5 = 0.4
- **X** = (age <= "30", income = "medium", student = "yes", credit_rating = "fair")

$P(X|C_i)$**:** P(X|buys_computer = "yes") = 0.222 x 0.444 x 0.667 x 0.667 = 0.044
P(X|buys_computer = "no") = 0.6 x 0.4 x 0.2 x 0.4 = 0.019

$P(X|C_i)*P(C_i)$: P(X|buys_computer = "yes") * P(buys_computer = "yes") = 0.028
P(X|buys_computer = "no") * P(buys_computer = "no") = 0.007

Therefore, the naïve Bayesian classifier predicts **X** belongs to class: "buys_computer = yes"

Avoiding the Zero-probability problem

- Naïve Bayesian prediction requires each conditional probability to be non-zero, as otherwise the predicted probability will be zero:

$$P(X|C_i)=\prod_{k=1}^{n} P(x_k|C_i)$$

- For example, suppose a data set with 1000 tuples
  - income= "low" (0), income= "medium" (990), income = "high" (10)
  - Use *Laplacian* correction (or *Laplacian* estimator)
  - Adding 1 to each case

    Prob(income = "low") = 1/1003

    Prob(income = "medium") = 991/1003

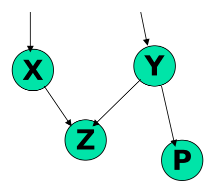
    Prob(income = "high") = 11/1003
- The "corrected" probability estimates are close to their "uncorrected" counterparts

Naïve Bayesian classifier: Some comments

- Advantages
  - Easy to implement
  - Good results obtained in most of the cases
- Disadvantages
  - Assumption: class conditional independence, therefore loss of accuracy
  - Practically, dependencies exist among variables
  - For example:
    - Hospital patients with attributes *age*, *family history*, etc.
    - Symptoms: *fever*, *cough* etc.
    - Diseases: *lung cancer*, *diabetes*, etc.
  - Dependencies among these cannot be modeled by naïve Bayesian classifier
- How to deal with these dependencies?
  - Bayesian Belief Networks

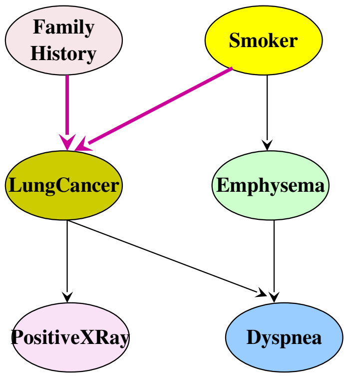
Bayesian belief networks

- Bayesian belief network allows a *subset* of the variables conditionally independent
- A graphical model of causal relationships
  - Represents dependency among the variables
  - Gives a specification of joint probability distribution



- Nodes: random variables (attributes)
- Links: dependency
- *X* and *Y* are the parents of *Z*, and *Y* is the parent of *P* (*P* is dependent on *Y)*
- No dependency between *Z* and *P*
- Has no loops or cycles

Bayesian belief network: An example



The *conditional probability table* (CPT) for variable (attribute) LungCancer:

~ = not)

	(FH, S)	(FH, ~S)	(~FH, S)	(~FH, ~S)
LC	0.8	0.5	0.7	0.1
~LC	0.2	0.5	0.3	0.9

CPT shows the conditional probability for each possible combination of its parents

Derivation of the probability of a particular combination of values of **X**, from CPT:

$$P(x_1,\ldots,x_n)=\prod_{i=1}^{n} P(x_i \mid Parents(Y_i))$$

P(LC = "yes" | FH = "yes", S = "yes") = 0.8
P(LC = "no" | FH = "no", S = "no") = 0.9

Source: Han and Kamber, DM Book, 2nd Ed. (Copyright © 2006 Elsevier Inc.)

Training Bayesian networks

- Several scenarios:
  - Given both the network structure and all variables observable: *learn only the CPTs*
  - If network structure known, some hidden variables: *gradient descent* (greedy hill-climbing) method, analogous to neural network learning
  - If network structure unknown, all variables observable: search through the model space to *reconstruct network topology*
  - If unknown structure, all hidden variables: No good algorithms known for this purpose

Measuring classifier accuracy

- Classification of new tuples/records results in a *confusion matrix* with positive and negative examples

Predicted class

		C_1	C_2
True class	C_1	True positive	False negative
	C_2	False positive	True negative

- Given *m* classes, $C_{i,j}$ (an entry in a confusion matrix) indicates the number of tuples in class *i* that are labeled by the classifier as class *j*
- *Accuracy* of a classifier *M*, *acc(M)* is the percentage of test set tuples that are correctly classified by the model *M*
  - *accuracy = (true_pos + true_neg) / (all_class_pos + all_class_neg)*
- *Error rate* (misclassification rate) of *M = 1 – acc(M)*

Classifier accuracy measures

classes	buy_computer = yes	buy_computer = no	total	recognition(%)
buy_computer = yes	6954	46	7000	99.34
buy_computer = no	412	2588	3000	86.27
total	7366	2634	10000	95.42

- Accuracy in example here is: (6954+2588) / 10000 = 95.42%
- Ideally, non-diagonal entires should be zero or close to it
- Alternative accuracy measures:
  - *sensitivity = true_pos / all_true_pos* (true positive recognition rate) (also called *recall)*, with *all_true_pos = true_pos+false_neg*
  - *specificity = true_neg / all_true_neg* (true negative recognition rate)
  - *precision = true_pos / (true_pos + false_pos)* (also called *positive predictor value*)
- This model can also be used for cost-benefit analysis

Evaluating the accuracy of a classifier (1)

- *Holdout* method
  - Given data is randomly partitioned into two independent sets
    - Training set (for example, 2/3 of all records in data) for model construction
    - Test set (for example, 1/3 of all records in data) for accuracy estimation
  - Random sampling: a variation of holdout
    - Repeat holdout k times, accuracy = average of the accuracies obtained
- Cross-validation (k-fold, where k=10 is most popular)
  - Randomly partition the data into k mutually exclusive subsets, each approximately equal size
  - At i-th iteration, use D_i as test set and others as training set
  - *Leave-one-out*: k folds where k = number of tuples, for small sized data
  - *Stratified cross-validation*: folds are stratified so that class distribution in each fold is approximately the same as that in the initial data
- 'Partition' box in Rattle: training/testing/validation

Evaluating the accuracy of a classifier (2)

- *Bootstrap* method
  - Works well with small data sets
  - Samples the given training tuples (records) uniformly *with replacement*
  - Each time a tuple is selected, it is equally likely to be selected again and re-added to the training set
- Several bootstrap methods, and a common one is *.632 bootstrap*
  - Suppose we are given a data set of d tuples (records), which is sampled d times (with replacement) resulting in a training set of d samples
  - Records that did not make it into the training set end up forming the test set. About 63.2% of the original data will end up in the *bootstrap* (training set), and the remaining 36.8% will form the test set (since $(1 - 1/d)^d \approx e^{-1} = 0.368$)
  - Repeat the sampling procedure k times, overall accuracy of the model:

$$acc(M)=\sum_{i=1}^{k}(0.632\times acc(M_i)_{testset}+0.368\times acc(M_i)_{trainset})$$

What now... things to do

- No lab next week (last lab - 6 - in two weeks)
- Start reading chapter 8 in text book (Sections 8.1 to 8.3, and 8.5), as well as Section 9.1
- Continue working on assignment 2!
  Due Thursday 19 May 5 pm