COMP90043: Cryptography and Security

Week 5: RSA and Diffie-Hellman

Recap:

1. What is public key cryptography?
2. What is the integer factorization problem?
3. RSA Algorithm

   $C = M^e \bmod n$

   $M = C^d \bmod n = (M^e)^d \bmod n = M^{ed} \bmod n$

4. Man in the Middle Attack

Exercises:

1. Given the parameters below, fill in the blanks accordingly for the relevant RSA parameter:

   p =13   q = 7   n = p.q = ________

   a) Using Euler's Totient Function, calculate

      $\phi(n) = \phi(\quad) =$ ____

2. For the RSA algorithm to work, it requires two coefficients – e and d. Where e represents the encryption component (generally the public key) and d represents the decryption component (generally the private key)

   In order to calculate d, we can use Extended Euclidean Algorithm which can be summarized as follows for any a and b such that (a > b).

GCD(a,b) $a = q_1 b + r_1$ $b = q_2 r_1 + r_2$ $r_1 = q_3 r_2 + r_3$ $r_2 = q_4 r_3 + r_4$ … (1) $r_{n-2} = q_n r_{n-1} + r_n$ (2) $r_{n-1} = q_{n+1} r_n + r_{n+1}$, **where $r_{n+1} = 1$ (GCD exists)** (3) $r_n = q_{n+2} r_{n+1} + r_{n+2}$, **where $r_{n+2} = 0$**	Now we can perform a back substitution to get d as follows: From (2) we get $r_{n+1} = 1 = r_{n-1} - q_{n+1} r_n$ We know r_n from (1), so we can substitute $= r_{n-2} - q_{n+1}(r_{n-2} - q_n r_{n-1})$ We continue this for each r while simplifying each step until we can represent the r_{n+1} in terms of b.

a) For the following, for each of the given values of e, calculate the value of d such that

$$d.e = 1 \bmod \phi(n)$$

e = 5	e = 7
$GCD(\phi(n), e) = GCD(72, 5)$ $\phi(n) = 72 = q_1e + r_1 = 14 * 5 + 2$ $e = 5 = q_2r_1 + r_2 = 2 * 2 + 1$ $r_1 = 2 = q_3r_2 + r_3 = 2 * 1 + 0$ Back Substitution we get $1 = [e - q_2r_1]\ \phi(n) = [5 – (2*2)] \bmod \phi(n)$ $1 = [e – q_2 (\phi(n) - q_1e)]\ \phi(n)$ $= [5 – (2*(72-(14*5)))]\ \phi(n)$ $= [5 + (-2*(72-(14*5)))]\ \phi(n)$ $= [5 +(-2*72 + 2*(14*5))]\ \phi(n)$ $= [5 + (-2*72 + 28*5)]\ \phi(n)$ $= [5 + 28*5 – 2*72]\ \phi(n)$ $= [29*5 – 2*72]\ \phi(n)$ From the above if we want to determine $d.e = 1 \bmod \phi(n)$ where e = 5, then **d = 29**	$GCD(\phi(n), e) = GCD(\mathbf{72}, 7)$ $\phi(n) = 72 = q_1e + r_1 =$ 10*7+2 $e = 7 = q_2r_1 + r_2 =$ 3*2+1 $r_1 =$ 2 $= q_3r_2 + r_3 =$ 2*1+0 Back Substitution we get $1 = [e - q_2r_1]\ \phi(n) = [\qquad\qquad] \bmod \phi(n)$ $1 = [e – q_2 (\phi(n) - q_1e)]\ \phi(n)$ = ____________________ = ____________________ = ____________________ = ____________________ = ____________________ = ____________________ From the above if we want to determine $d.e = 1 \bmod \phi(n)$ where e = 7, then **d =** __________

b) For the following, for each of the given values of e, calculate the value of d such that

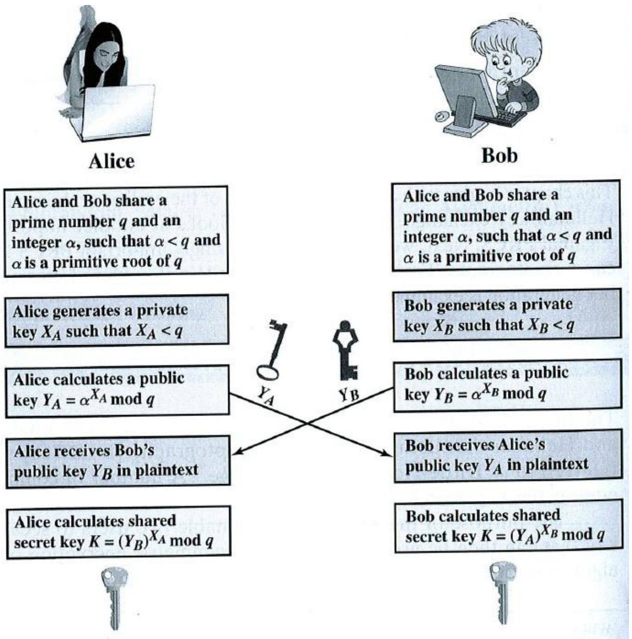
$$d.e = 1 \bmod \phi(n)$$

p =23 q = 37 n = p.q =__________ $\phi(n)$ = _

e = 5	e = 61
$GCD(\phi(n), e) = GCD($____$, 5)$ $\phi(n) = \qquad = q_1e + r_1 =$ ____________________ $e = 5 = q_2r_1 + r_2 =$ ____________________	$GCD(\phi(n), e) = GCD(\qquad, 61)$ $\phi(n) = \qquad = q_1e + r_1 =$ ____________________ $e = 61 = q_2r_1 + r_2 =$ ____________________

r_1 = _____ = $q_3r_2 + r_3$ = ______________________	r_1 = ____ = $q_3r_2 + r_3$ = ______________________
Back Substitution we get	Back Substitution we get
$1 = [e - q_2r_1]\ \phi(n) = [\qquad\qquad]\ \text{mod}\ \phi(n)$	$1 = [e - q_2r_1]\ \phi(n) = [\qquad\qquad]\ \text{mod}\ \phi(n)$
$1 = [e - q_2\ (\phi(n) - q_1e)]\ \phi(n)$	$1 = [e - q_2\ (\phi(n) - q_1e)]\ \phi(n)$
= ____________________________________	= ____________________________________
= ____________________________________	= ____________________________________
= ____________________________________	= ____________________________________
= ____________________________________	= ____________________________________
= ____________________________________	= ____________________________________
= ____________________________________	= ____________________________________
From the above if we want to determine	From the above if we want to determine
$d.e = 1\ \text{mod}\ \phi(n)$	$d.e = 1\ \text{mod}\ \phi(n)$
where e = 5, then **d =** ____________	where e = 7, then **d =** __________

3. The Diffie-Hellman key exchange algorithm can be defined as follows:



(Image borrowed from Cryptography and Network Security, Stallings, 6th Edition)

Using the above algorithm, can you show that Diffie-Hellman can be subject to a man-in-the-middle attack?

4. Given the encryption and decryption formulas for RSA as follow:

   $C = M^e \bmod n$

   $M = C^d \bmod n = (M^e)^d \bmod n = M^{ed} \bmod n$

   Calculate the encryption and decryption for the given values of p, q, e and M

   a) p =3; q = 13; e = 5; M = 10

      n = _ ϕ(n) = _ d = _ _

      $C = M^e \bmod n = 10^5 \bmod$ _ =_ M =

      $C^d \bmod n$ =_mod _ =______

   b) p =5; q = 7; e = 7; M = 12

      n = _ ϕ(n) = _ d = _ _

      $C = M^e \bmod n = 12^7 \bmod$ _ =_ M =

      $C^d \bmod n$ =_mod _ =______

   c) p =11; q = 7; e = 11; M = 7

      n = _ ϕ(n) = _ d = _ _

      $C = M^e \bmod n = 7^{11} \bmod$ _ =_ M =

      $C^d \bmod n$ =_mod _ =______

5. In a public-key system using RSA, you intercepted the cipher text C = 8 sent to a user whose public key is e = 13; n = 33. What is the plaintext M ?

   M =

Homework:

Show that the RSA encryption and decryption functions are inverse operations by trying with some example messages. You can use the package magma online (http://magma.maths.usyd.edu.au/calc/).