Plan of Talk

- **Zero Knowledge proofs**
- **Ali baba's cave story.**
- **Zero knowledge proof based on discrete logarithms.**
- **DSA-Digital Signature Algorithm of NIST**

Advanced Cryptographic Concepts

There are many esoteric protocols in cryptography which facilitate wide varieties of modern day network security applications.

Some of the important concepts:

- Zero Knowledge Protocols
- Threshold Cryptography
- Oblivious Transfer
- Anonymous Protocols

Here we discuss briefly Zero-Knowledge protocols.

Zero Knowledge proofs
Interactive proof (IP) Systems

- Is an interactive proof by a party (Prover) to a an another party (Verifier) that a mathematical statement is true, without revealing anything more than what is conveyed in the interaction.
- Usually the Prover holds some secret protected by a hard problem (a problem in NP) and describe a mathematical statement involving the secret which otherwise could not have been made without the secret.
- Protocols which use such proofs are known as Zero Knowledge protocols.

Zero Knowledge Protocols

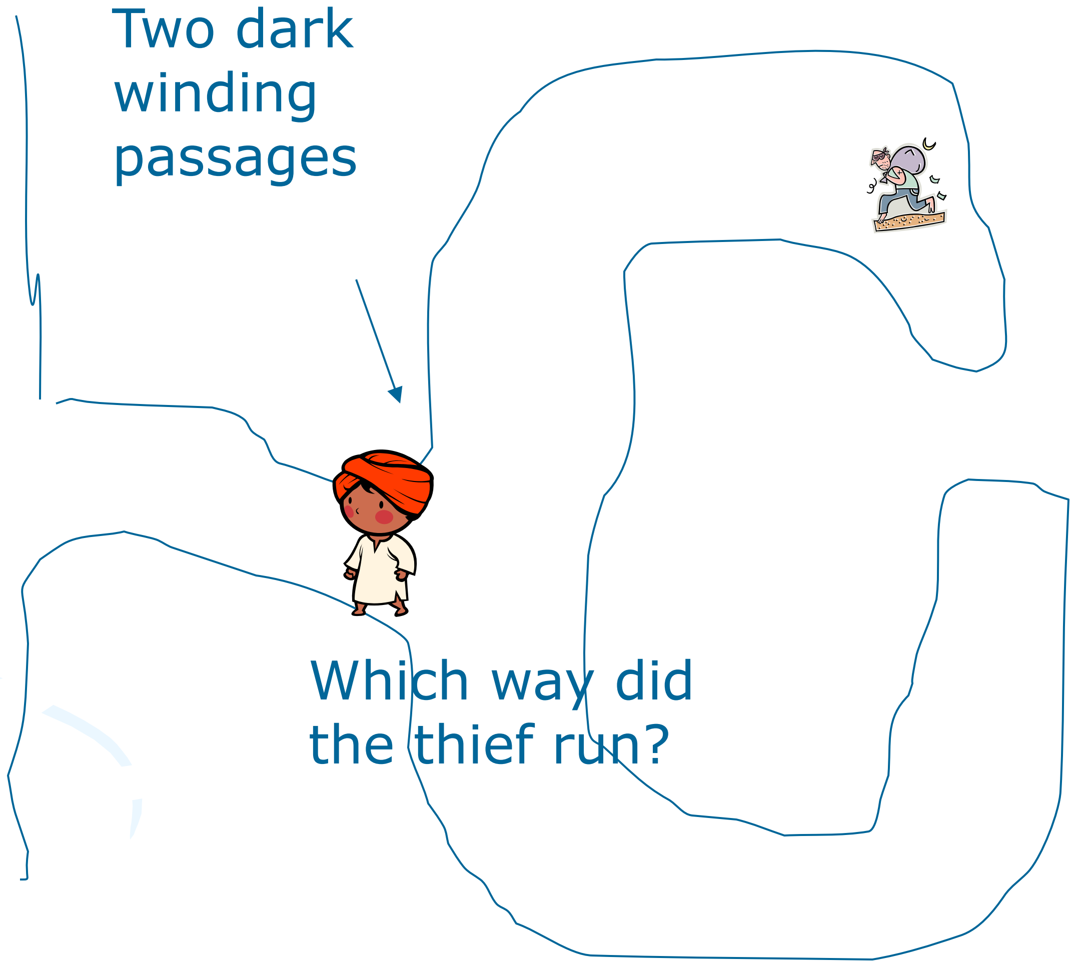
- Two party interactive game where Alice (called the prover) proves to Bob (called the verifier) that a predicate of statement holds true without letting Bob learn the method of Alice's proof.
- The game uses **Interactive proof (IP) System**.
- Sometimes it is called as "**proof in the dark**".
- Verifier after been convinced the validity of the what is being proved cannot have learned the knowledge possessed by the prover.
- Any third party watching the game learns nothing.

Zero Knowledge Protocols

General Ideas:

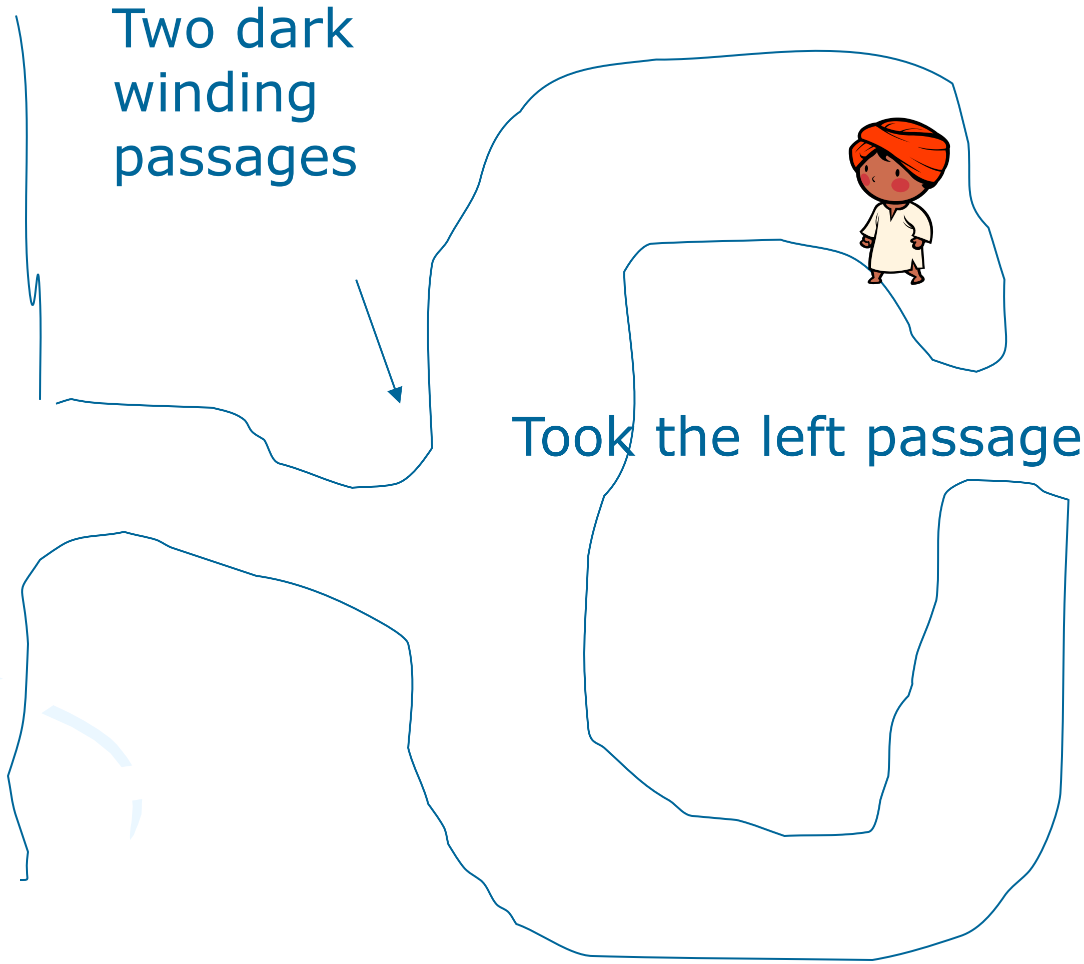
- The prover has certain knowledge (protected by some one-way function and an hard problem).
- The verifier is ignorant of this knowledge.
- The prover uses his knowledge to convince the verifier that he holds that knowledge.
- The information leaked while proving is **zero**.

Ali baba's cave story



Read the article on this topic by Quisquarter et.al on the internet

First day he misses him



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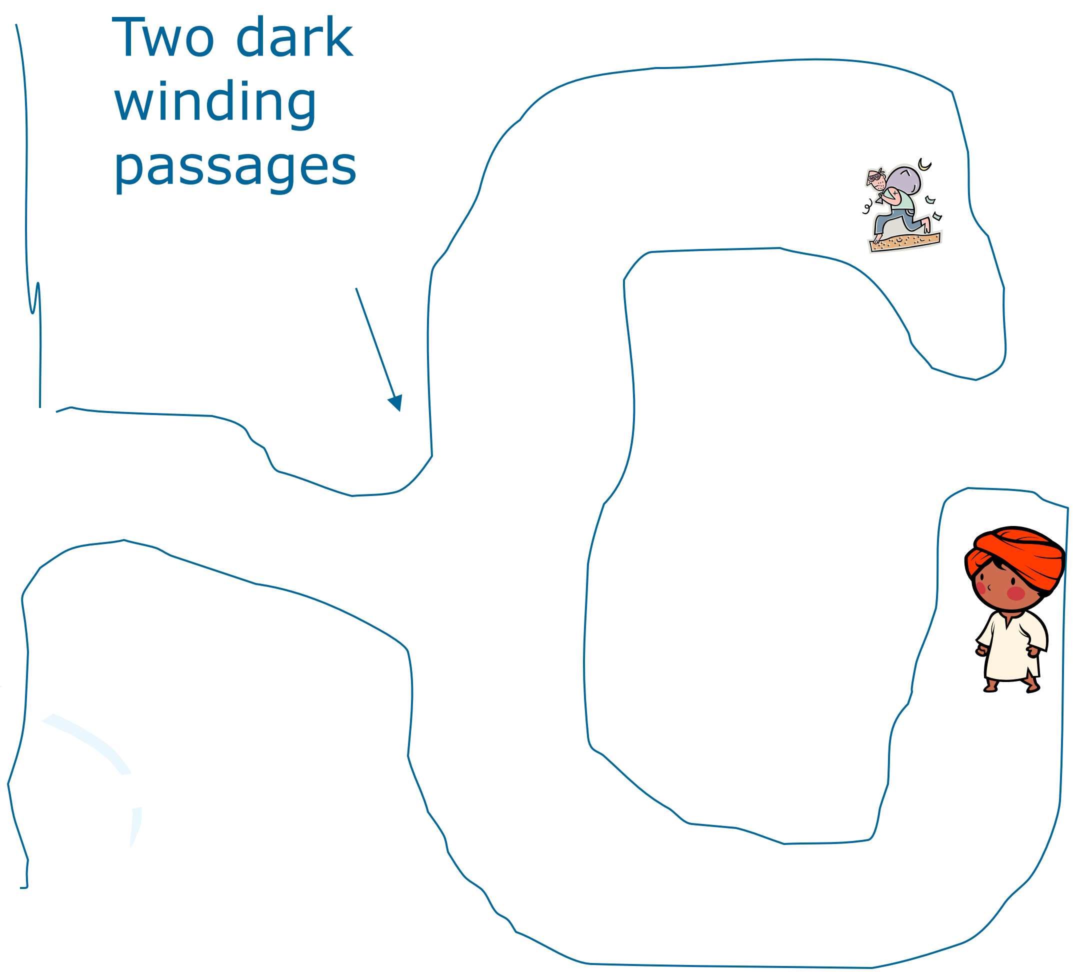
Next day he misses him too



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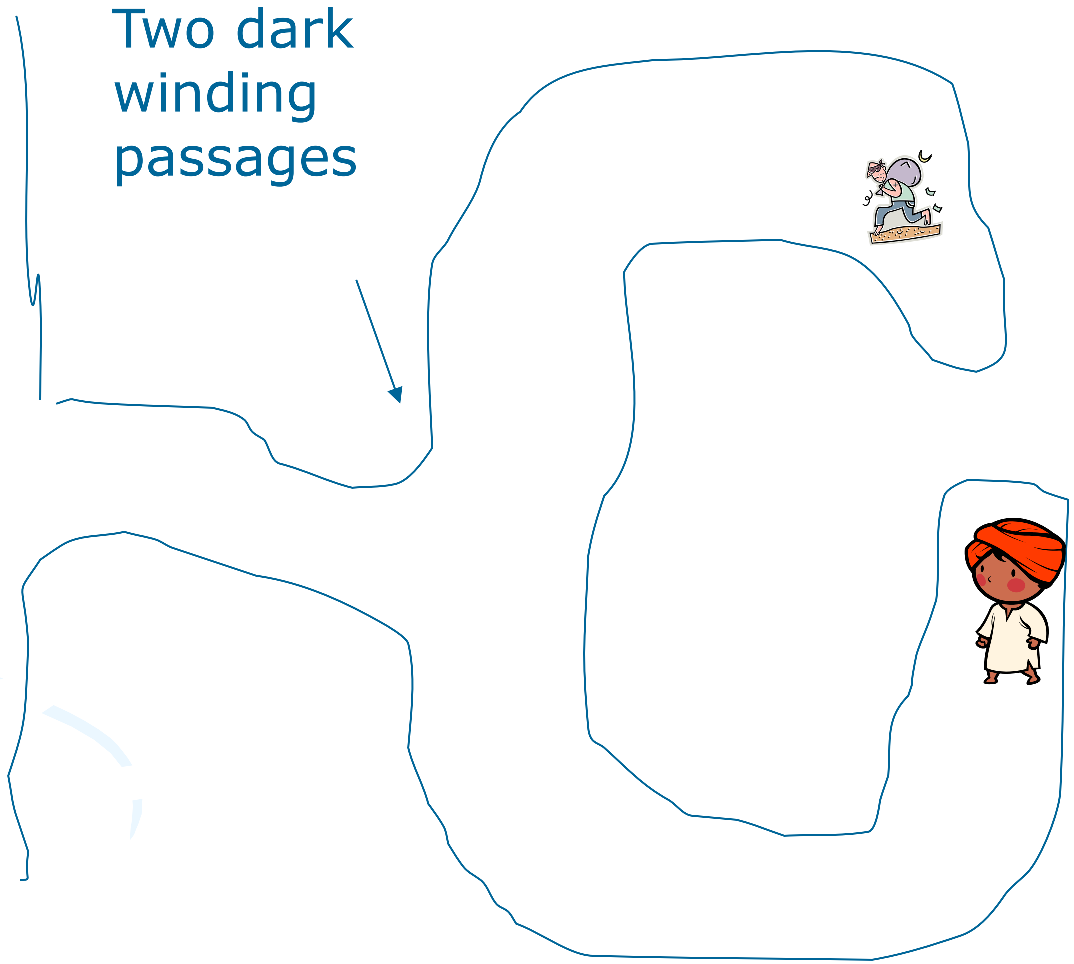
Was thief lucky for 40 times



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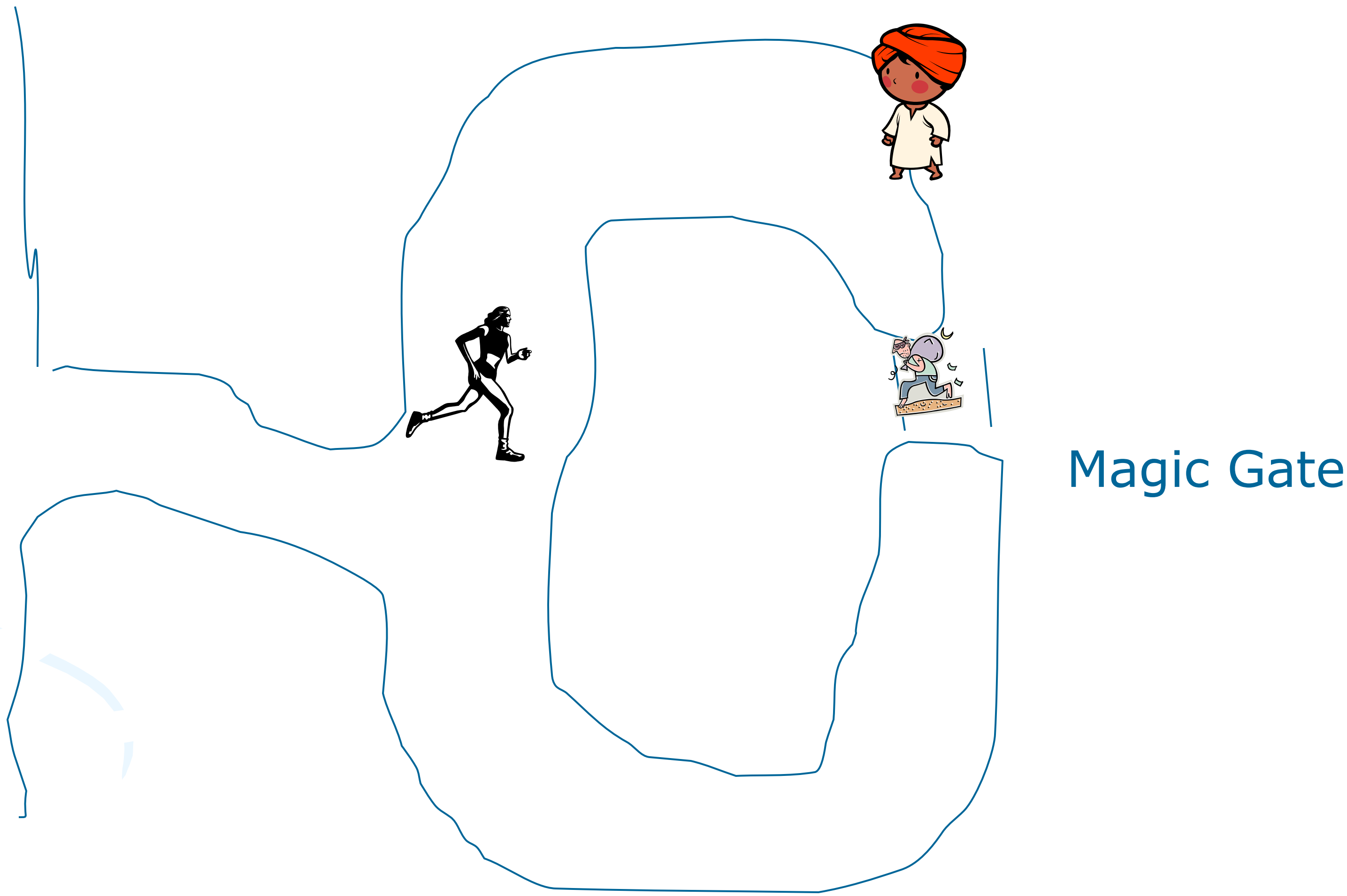
Ali Baba learns the secret



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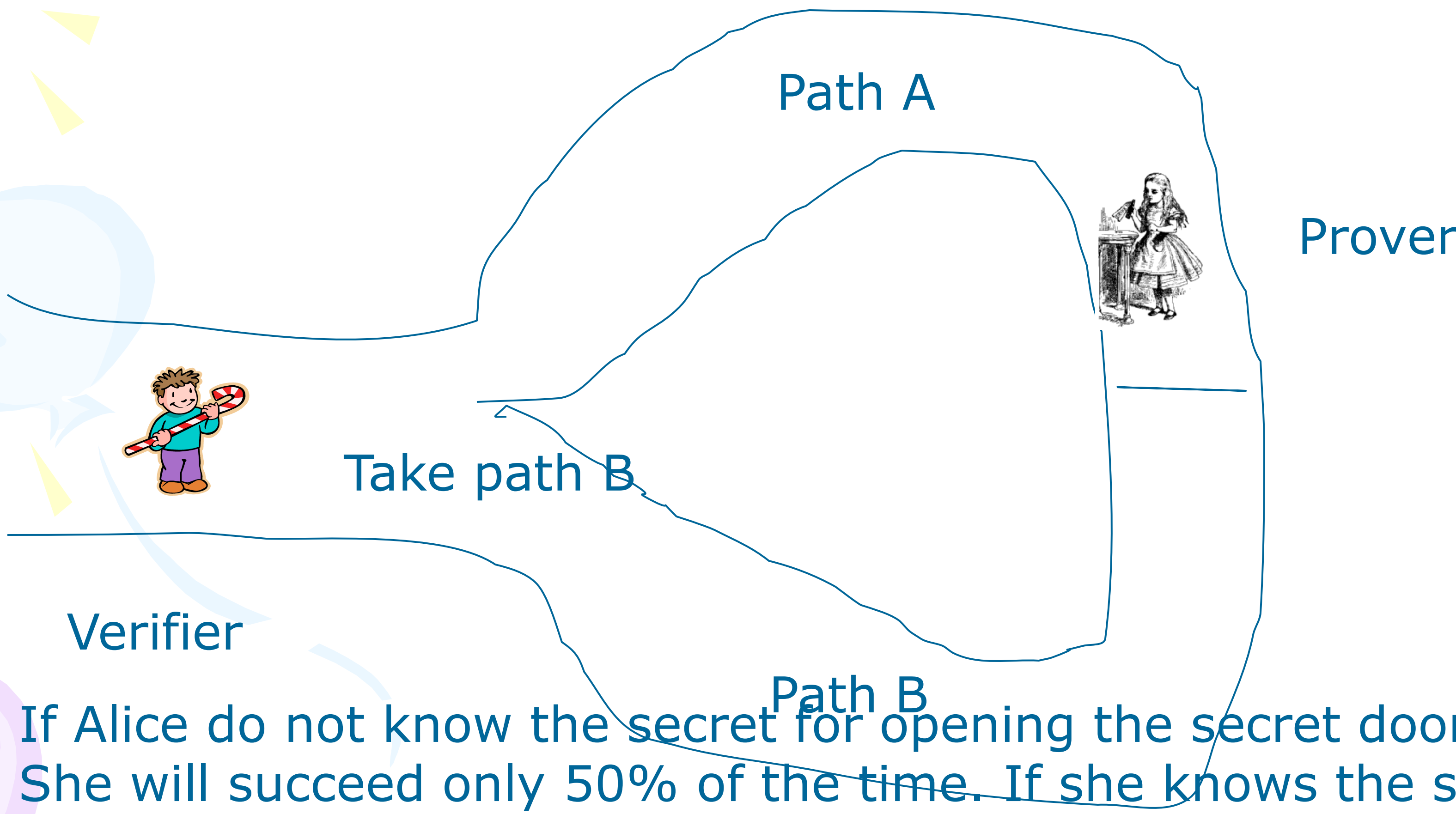
Modern version

- Manuscript to modern times-one of descendent of Ali broke the code.
- Sold the story to a tv network –
- Jealous reporter
- Court judgement- a proof obtained by one person cannot be transferred to others.

Cont.

- Tests in parallel-
- Jealous reporter’s tale is a case of a prior agreement.
- A single test with million passages.

A protocol



If Alice do not know the secret for opening the secret door, She will succeed only 50% of the time. If she knows the secret She will succeed always. Repeating the above steps increases the confidence of the verifier

Nature of Zero Knowledge protocols

- Alice holds some secret and a corresponding public protected by some hard problem.
- At the end of the protocol Bob is convinced that Alice holds the secret.
- Alice(Prover) Bob (Verifier)
- Repeat the following m times
- {
- Computes **Commit** ---------------→
- <----------------- **Challenge**
- **Response** -------→ Bob verifies **Response**
- reject if verification fails
- }
- Bob accepts

Simple Scheme

- Let p be a prime and g be a generator of Z_p
- Let A's Secret be a, $1 \le a < p$ and its public parameter is $y = g^a$.
- A will prove this to B that A knows this secret `a' by announcing a witness g^r, where r is a random number.
- B's Challenge is a random c, $1 \le c < p$
- A's response is $u = r + c\,a$.
- B can verify that $g^u = g^r y^c$

$y = g^a$: public information of A

Choose random r

Witness g^r

g^r →

Challenge c

←

Challenge
c = random

$U = r + ac$ →

U

Response

Mod (p-1)

Verification
Accept if
$g^{\mathbf{u}} = g^r y^{\mathbf{c}}$ **Mod p**

Operations on Integer Ring

Operations in Finite Field

Schnorr's Identification Protocol

- COMMON PARAMETERS
- P, q: two primes, q divides (p-1) ($q \mid (p-1)$;
- Size of p = 1024 bits, size of q = 160 bits;
- g: An Element of order q, i.e $g^q = 1 \bmod p$;
- y: $y = g^{-a} \pmod{p}$;
- Alice's Public Key Material (p,q,g,y), certified by Certificate Authority
- Alice's Private Key information: $a < q$;
- After the protocol Bob is certain that Alice knows some a in Z_q
- with a property $y = g^{-a} \pmod{p}$;

Schnorr's Identification Protocol. Contd.

- Repeat the following steps m times
- Alice picks k in Z^q computes **Commit** ← g^k (mod p)
- Alice Sends **Commit** to Bob
- Bob picks **Challenge** and Sends it to Alice
- Alice Computes **Response** ← k + a * **challenge** (mod q) and Sends **Response** to Bob
- Bob checks if Commit = $g^{\textbf{Response}} y^{\textbf{Challenge}}$ (mod p);
- he rejects and abort if the checking shows error

- Bob accepts the fact that Alice knows a such that $y = g^{-a}$ (mod p);
-

Verification Equation

- The verification equation:
- Commit = $g^{\textbf{Response}}\ y^{\textbf{Challenge}}$ (mod p);

- RHS:=$g^{(k + a * \textbf{challenge})} * g^{\textbf{(-a*Challenge)}}$
- = g^{k}
- = Commit = LHS

Summary

- Zero Knowledge proofs
- Schnor’s ZKN protocol