COMP90043: Cryptography and Security

Week 5: RSA and Diffie-Hellman

Recap:

1. What is public key cryptography?
2. What is the integer factorization problem?
3. RSA Algorithm

   $C = M^e \bmod n$

   $M = C^d \bmod n = (M^e)^d \bmod n = M^{ed} \bmod n$
4. Man in the Middle Attack

Exercises:

1. Given the parameters below, fill in the blanks accordingly for the relevant RSA parameter:

   p =13 q = 7 n = p.q = 91

   a) Using Euler's Totient Function, calculate

   $\phi(n) = \underline{\phi(91) = \phi(7 \cdot 13) = \phi(7) \cdot \phi(13) = (7-1).(13-1) = 6.12 = 72}$
2. For the RSA algorithm to work, it requires two coefficients – e and d. Where e represents the encryption component (generally the public key) and d represents the decryption component (generally the private key)

   In order to calculate d, we can use Extended Euclidean Algorithm which can be summarized as follows for any a and b such that (a > b).

GCD(a,b) $a = q_1 b + r_1$ $b = q_2 r_1 + r_2$ $r_1 = q_3 r_2 + r_3$ $r_2 = q_4 r_3 + r_4$ … (1) $r_{n-2} = q_n r_{n-1} + r_n$ (2) $r_{n-1} = q_{n+1} r_n + r_{n+1}$, **where $r_{n+1} = 1$ (GCD exists)** (3) $r_n = q_{n+2} r_{n+1} + r_{n+2}$, **where $r_{n+2} = 0$**	Now we can perform a back substitution to get d as follows: From (2) we get $r_{n+1} = 1 = r_{n-1} - q_{n+1} r_n$ We know r_n from (1), so we can substitute $= r_{n-2} - q_{n+1}(r_{n-2} - q_n r_{n-1})$ We continue this for each r while simplifying each step until we can represent the r_{n+1} in terms of b.

a) For the following, for each of the given values of e, calculate the value of d such that

$$d.e = 1 \bmod \phi(n)$$

e = 5	e = 7
GCD(ϕ(n), e) = GCD(72, 5)	GCD(ϕ(n), e) = GCD(72, 7)
ϕ(n) = 72 = q_1e + r_1 = 14 * 5 + 2	ϕ(n) = 72 = q_1e + r_1 = 10 * 7 + 2
e = 5 = q_2r_1 + r_2 = 2 * 2 + 1	e = 7 = q_2r_1 + r_2 = 3 * 2 + 1
r_1 = 2 = q_3r_2 + r_3 = 2 * 1 + 0	r_1 = 2 = q_3r_2 + r_3 = 2 * 1 + 0
Back Substitution we get	Back Substitution we get
1 = [e - q_2r_1] ϕ(n) = [5 – (2*2)] mod ϕ(n)	1 = [e - q_2r_1] ϕ(n) = [7 – (3*2)] mod ϕ(n)
1 = [e – q_2 (ϕ(n)- q_1e)] ϕ(n)	1 = [e – q_2 (ϕ(n)- q_1e)] ϕ(n)
= [5 – (2*(72-(14*5)))] ϕ(n)	= [7 – (3*(72-(10*7)))] ϕ(n)
= [5 + (-2*(72-(14*5)))] ϕ(n)	= [7 + (-3*(72-(10*7)))] ϕ(n)
= [5 +(-2*72 + 2*(14*5))] ϕ(n)	= [7 +(-3*72 + 3*(10*7))] ϕ(n)
= [5 + (-2*72 + 28*5)] ϕ(n)	= [7 + (-3*72 + 30*7)] ϕ(n)
= [5 + 28*5 – 2*72] ϕ(n)	= [7 + 30*7 – 3*72] ϕ(n)
= [29*5 – 2*72] ϕ(n)	= [31*7 – 3*72] ϕ(n)
From the above if we want to determine d.e = 1 mod ϕ(n)	From the above if we want to determine d.e = 1 mod ϕ(n)
where e = 5, then **d = 29**	where e = 7, then **d = 31**

b) For the following, for each of the given values of e, calculate the value of d such that

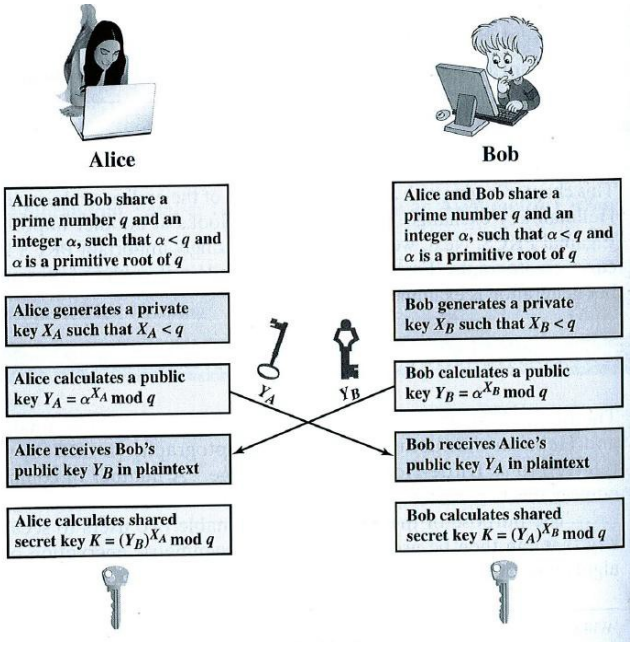
$$d.e = 1 \bmod \phi(n)$$

p =23 q = 37 n = p.q = 851 ϕ(n) = 792

e = 5	e = 61
GCD(ϕ(n), e) = GCD(792, 5)	GCD(ϕ(n), e) = GCD(792, 61)
ϕ(n) = 792 = q_1e + r_1 = 158 * 5 + 2	ϕ(n) = 792 = q_1e + r_1 = 12 * 61 + 60
e = 5 = q_2r_1 + r_2 = 2 * 2 + 1	e = 61 = q_2r_1 + r_2 = 1 * 60 + 1

$r_1 = 2 = q_3 r_2 + r_3 = 2 * 1 + 0$ Back Substitution we get $1 = [e - q_2 r_1]\ \phi(n) = [5 - (2*2)] \bmod \phi(n)$ $1 = [e - q_2 (\phi(n) - q_1 e)]\ \phi(n)$ $= [5 - (2*(792-(158*5)))]\ \phi(n)$ $= [5 + (-2*(792-(158*5)))]\ \phi(n)$ $= [5 + (-2*792 + 2*(158*5))]\ \phi(n)$ $= [5 + (-2*792 + 316*5)]\ \phi(n)$ $= [5 + 316*5 - 2*792]\ \phi(n)$ $= [317*5 - 2*792]\ \phi(n)$ From the above if we want to determine $d.e = 1 \bmod \phi(n)$ where e = 5, then **d = 317**	$r_1 = 60 = q_3 r_2 + r_3 = 60 * 1 + 0$ Back Substitution we get $1 = [e - q_2 r_1]\ \phi(n) = [61 - (1*60)] \bmod \phi(n)$ $1 = [e - q_2 (\phi(n) - q_1 e)]\ \phi(n)$ $= [61 - (1*(792-(12*61)))]\ \phi(n)$ $= [61 + (-1*(792-(12*61)))]\ \phi(n)$ $= [61 + (-1*792 + 1*(12*61))]\ \phi(n)$ $= [61 + (-1*792 + 12*61)]\ \phi(n)$ $= [61 + 12*61 - 1*792]\ \phi(n)$ $= [13*61 - 1*72]\ \phi(n)$ From the above if we want to determine $d.e = 1 \bmod \phi(n)$ where e = 7, then **d = 13**

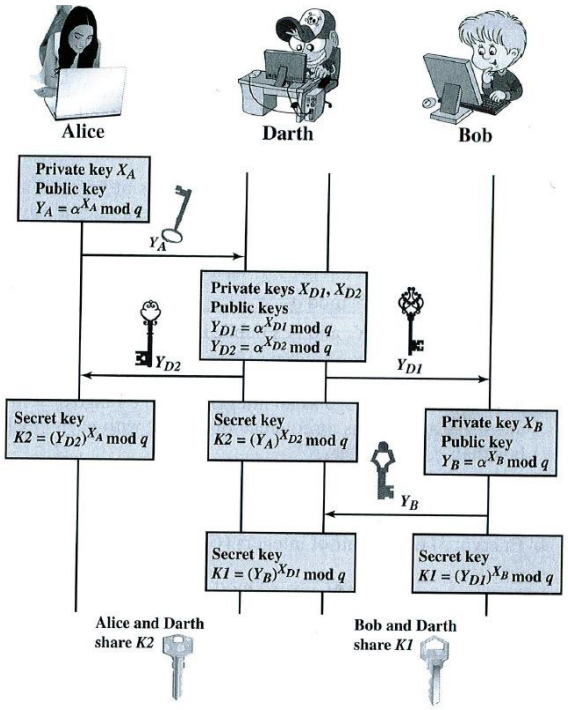
3. The Diffie-Hellman key exchange algorithm can be defined as follows:



(Image borrowed from Cryptography and Network Security, Stallings, 6th Edition)

Using the above algorithm, can you show that Diffie-Hellman can be subject to a man-in-the-middle attack?

A man in the middle attack is possible as shown in the below figure, where an attacker generated two separate keys and then intercepts the communication between Alice and Bob. The communication is compromised as the attacker uses the generated key to convince Alice or Bob that it belong to the other person. When this key is used to establish the connection, what Alice or Bob are actually doing is establishing a connection with the attacker who then is establishing another simultaneous connection to the other person after reading everything sent on the first connection.



((Image borrowed from Cryptography and Network Security, Stallings, 6th Edition)

4. Given the encryption and decryption formulas for RSA as follow:

   $C = M^e \bmod n$

   $M = C^d \bmod n = (M^e)^d \bmod n = M^{ed} \bmod n$

   Calculate the encryption and decryption for the given values of p, q, e and M

   a) p =3; q = 13; e = 5; M = 10

      n = 39 $\phi(n)$ = 24 d = 5

      $C = M^e \bmod n = 10^5 \bmod 39 = 4$

      $M = C^d \bmod n = 4^5 \bmod 39 = 10$

   b) p =5; q = 7; e = 7; M = 12

      n = 35 $\phi(n)$ = 24 d = 7

      $C = M^e \bmod n = 12^7 \bmod 35 = 33$

      $M = C^d \bmod n = 33^7 \bmod 35 = 12$

c) p =11; q = 7; e = 11; M = 7

n = _77_ $\phi(n)$ = _60_ d = _11_

$C = M^e \bmod n = 7^{11} \bmod$ _77_ =7_

$M = C^d \bmod n =$ _7^{11}_ mod _77_ =7_

5. In a public-key system using RSA, you intercepted the cipher text C = 8 sent to a user whose public key is e = 13; n = 33. What is the plaintext M ?

   M = 2.

   To show this, note that we know that n = 33, which has only two prime dividers. Therefore, p = 3 and q = 11. $\phi(n) = 2 \times 10 = 20$. Using the Extended Euclidean Algorithm, d, the multiplicative inverse of e mod $\phi(n)$ = 11 mod 20, is found to be 17. Therefore, we can determine M to be $M = C^d \bmod n = 8^{17} \bmod 33 = 2$.

Homework:

Show that the RSA encryption and decryption functions are inverse operations by trying with some example messages. You can use the package magma online (http://magma.maths.usyd.edu.au/calc/).