Tiangong Falling - ES98B Group Project

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Background

- The project dives into the task of designing a software capable predicting the trajectory of a de-orbiting satellite.
- An Extended Kalman Filter (EKF) lies at the heart of the predictor to tackle the inherent non-linearity of dynamics.
- A plethora of features are tackled alongside the baseline model, ranging from radar line of sight detection to non-equatorial orbits, non-equidistant radar locations and an ellipsoidal earth.

What are we Assuming?

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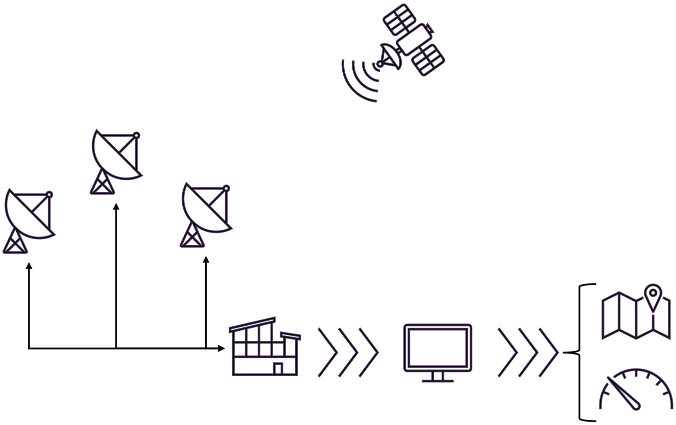
As this project focuses on the real-life scenario of a de-orbiting space station, we have taken the following assumptions into account.

- The satellite does not lose mass or cross-sectional area during the descent.
- There are no perturbations from other celestial bodies.
- We are not taking atmospheric uncertainties like storms into account.
- The orbital inclination of the satellite and the position of its perigee are known.
- Satellite position (with error) is the only information available.

Setup



Governing Dynamics

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- Gravitation acts as the major force between the Earth and satellite, given by $\frac{Gm_S M_E}{r^2}$, where G is the gravitational constant, with M_E and m_s as their respective masses.
- Standard atmospheric drag opposing velocity is assumed, given by $F_D = \frac{1}{2}\rho v^2 C_d A_S$, for air density ρ (an exponentially decreasing function of height h), velocity v, drag coefficient C_d, and effective satellite area A_S[1]

$$\dot{r} = v_r \tag{1}$$

$$\ddot{r} = -\frac{GM_E}{r^2} + r\dot{\phi}^2 \tag{2}$$

$$\dot{\phi} = \frac{v_\phi}{r} \tag{3}$$

$$\ddot{\phi} = -\frac{1}{2}Br\rho(h)\dot{\phi}^2 \tag{4}$$

with ballistic coefficient $B = \frac{A_S C_d}{m_S}$.

[1] Nwankwo, V. and Chakrabarti, S. (2014) [1]

Atmosphere Model

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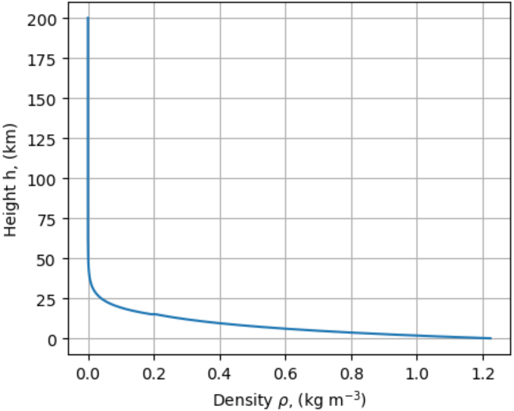
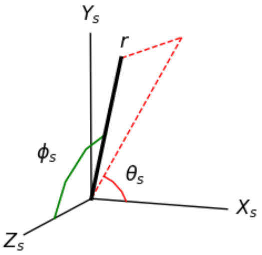
For this project a simple atmosphere model was approximated using the barometric formula resulting in the below density accurate up to 500km. [2]



[2] M. Foilhais. et al. (2023) [2], N. G. D. Centre. (1976) [3]

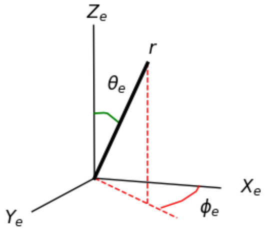
Coordinate Systems

Polar angle ϕ_s
Azimuthal angle θ_s



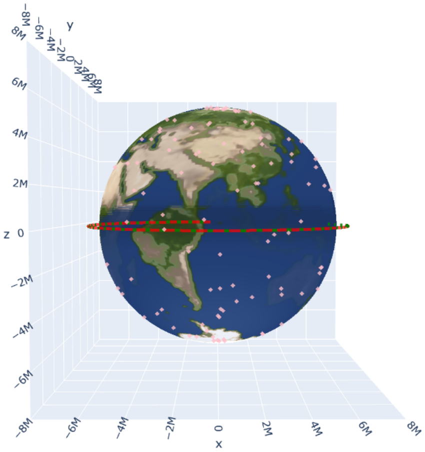
(a) Satellite Coordinates axes

Polar angle θ_e
Azimuthal angle ϕ_e

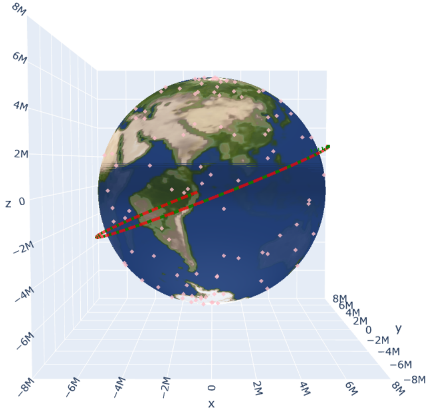


(b) Earth Coordinates axes

Non-Equatorial Orbit



(a) Initial polar angle $\theta_E = \frac{\pi}{2}$

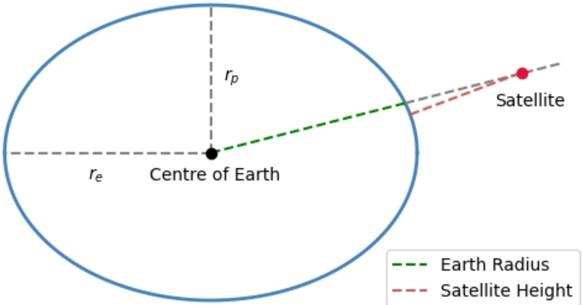


(b) Initial polar angle $\theta_E = 1.2$

Ellipsoidal Earth

The equation for our ellipsoidal earth model in the spherical coordinates is given by:

$$\frac{r^2 \sin^2 \theta_E \cos^2 \phi_E}{r_e^2} + \frac{r^2 \sin^2 \theta_E \sin^2 \phi_E}{r_e^2} + \frac{r^2 \cos^2 \theta_E}{r_p^2} = 1 \tag{5}$$



Height Calculation

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For the calculation of the height of the satellite the third coordinate, the azimuthal angle, can be disregarded as it has no affect on the resultant height. Therefore we only consider, $(r, \theta_E, \cancel{\phi_E}) \rightarrow (x_p, y_p)$.

$$\theta_0 = \arctan 2\,(x_p, y_p) \tag{6}$$

$$S_0 = (r_e \cos(\theta_0), r_p \sin(\theta_0)) \tag{7}$$

$$S_i{}' = (-r_e \sin(\theta_i), r_p \cos(\theta_i)) \tag{8}$$

$$S_i{}'' = (-r_e \cos(\theta_i), -r_p \sin(\theta_i)) \tag{9}$$

Newton routine: [3]

$$\theta_{i+1} = \theta_i - \frac{S_i{}'(\theta_i)}{S_i{}''(\theta_i)} \tag{10}$$

$$\mathrm{err} = \left| \frac{\Delta S_i{}'(\theta_i)}{\Delta S_i{}''(\theta_i)} \right| \tag{11}$$

[3] Pewsey, M. (2022). [4]

Radar System

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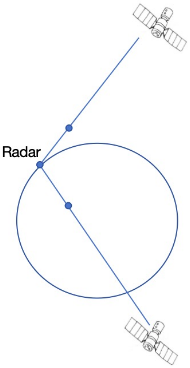


Fig. 3: How radars work

Whether points on line of sight are in earth

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \tag{12}$$

Generate positions on earth randomly

$$e^2 = 1-\left(\frac{r_p}{r_e}\right)^2 \tag{13}$$

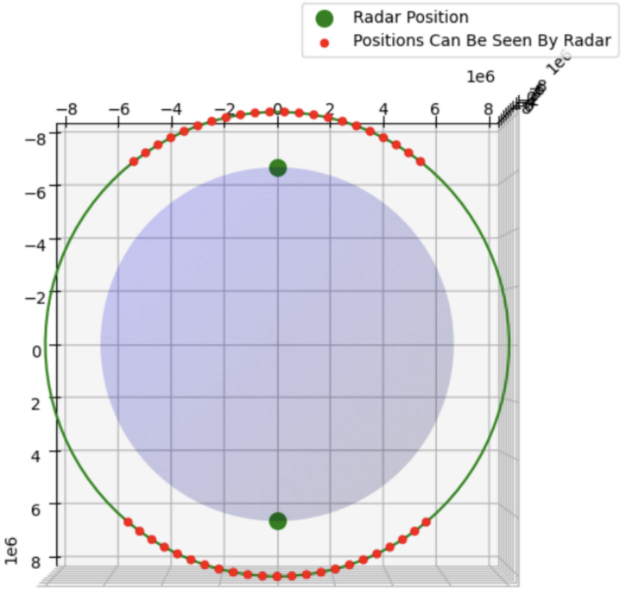
$$N=\frac{r_e}{\sqrt{1-e^2\sin^2(\text{polar})}} \tag{14}$$

$$x = N\cos(\text{polar})\cos(\text{azimuthal}) \tag{15}$$

$$y = N\cos(\text{polar})\sin(\text{azimuthal}) \tag{16}$$

$$z = \left(N\times(1-e^2)\right)\sin(\text{polar}) \tag{17}$$

Radar System Behaviour



Extended Kalman Filter

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- **State Vector Initialisation:** The state vector **x** which are following consists of position, velocity, and acceleration components for both radial and angular dimensions within the orbital plane, note that v_ϕ is tangential:

$$\mathbf{x}_0 = \begin{bmatrix} r & v_r & \dot{v_r} & \phi & v_\phi & \dot{v_\phi} \end{bmatrix}^T$$

- **Equations:** For the given model, the radial and angular update calculations are based on current state and physical dynamics:

$$r^{i+1} = r^i + \Delta t\, v_r^i \tag{18}$$

$$v_r^{i+1} = v_r^i + \Delta t\, \dot{v}_r^i \tag{19}$$

$$\dot{v}_r^{i+1} = -\frac{GM_E}{(r^i)^2} + \frac{(v_\phi^i)^2}{r^i} \tag{20}$$

$$\phi^{i+1} = \phi^i + \frac{\Delta t v_\phi^i}{r^i} \tag{21}$$

$$v_\phi^{i+1} = v_\phi^i + \Delta t \dot{v}_\phi^i \tag{22}$$

$$\dot{v}_\phi^{i+1} = -\frac{1}{2}\rho^i (v_\phi^i)^2 B + \frac{v_\phi^i\, v_r^i}{r^i} \tag{23}$$

Extended Kalman Filter

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- **Atmosphere Density:** given it's approximate distance from the planet's surface.

$$\rho^i = f(\hat{r}^i, \hat{\phi}^i) \tag{24}$$

- **State Transition Matrix F:** The calculation of the Jacobian **F** with respect to the state variables, crucial for the prediction step in the EKF is given by:

$$\mathbf{F}_i = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 1 & \Delta t & 0 & 0 & 0 \\ \frac{2GM_E}{r^3} - (\frac{v_\phi}{r})^2 & 0 & 0 & 0 & \frac{2v_\phi}{r} & 0 \\ \frac{-\Delta t\, v_\phi}{r^2} & 0 & 0 & 1 & \frac{\Delta t}{r} & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta t \\ \frac{-v_\phi\, v_r}{r^2} & \frac{v_\phi}{r} & 0 & 0 & -\rho\, v_\phi\, B + \frac{v_r}{r} & 0 \end{bmatrix}$$

Extended Kalman Filter

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- **Observation Matrix H:** How many of the Kalman states are being observed.

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

- **Observation Noise Covariance Matrix R:** How much noise is in the observed data.

$$\mathbf{R} = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix}$$

- **Process Noise Covariance Matrix Q:** The deviation, or uncertainty, of the true dynamics of the object from the chosen model described by the transition matrix **F**. In this case, we found that despite the atmosphere having minor deviation from the truth, setting this matrix to zero gave the most stable results.

Code Structure and Flow

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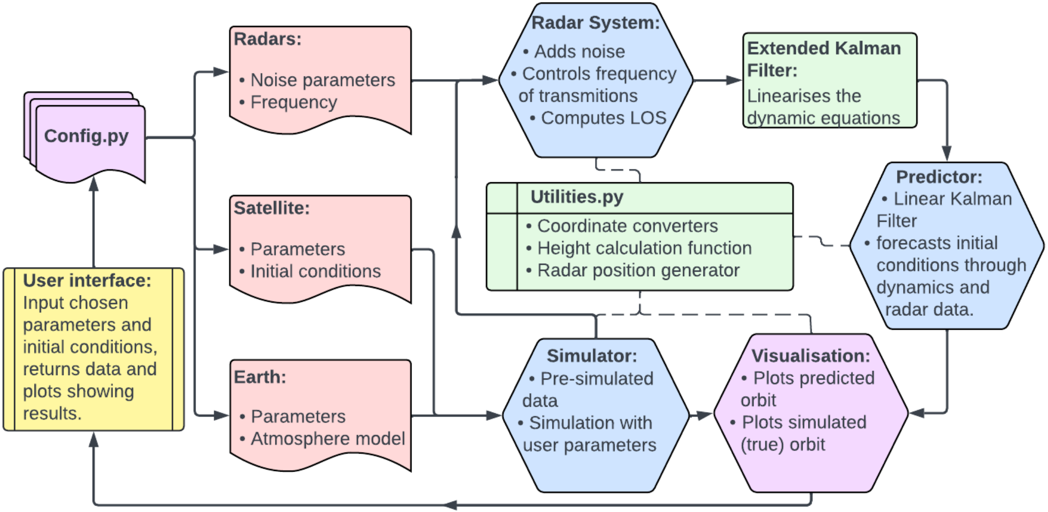
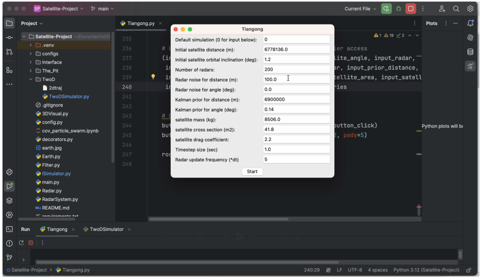
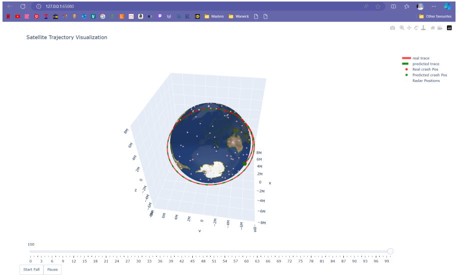


User Interface, Visuals

- After launching ‘Tiangong.py‘, a dashboard will pop up, with some default parameters.
- The user will be able to modify the inputs.
- Once the simulation starts, the user will be able to generate 2D and 3D plots.

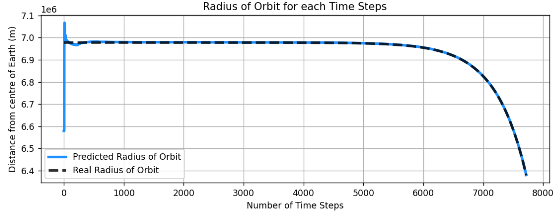


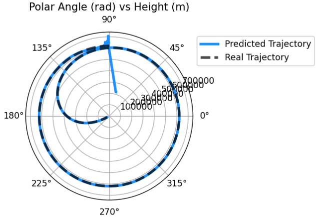
(a) Initial view in a PyCharm editor

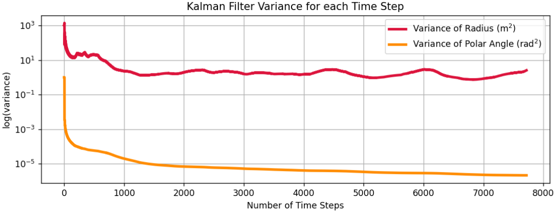


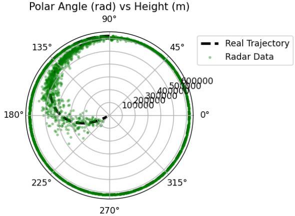
(b) Second view

Results and Discussion









Satellite Distance (m)	Satellite Angle (rad)	Radar Counts	Distance Noise	Polar Noise	Prior Distance (m)	Prior Angle (rad)	Prior v_r	Prior v_ϕ	Satellite Mass (kg)	Satellite Area(m^2)	Satellite Drag	Time Interval	Radar Frequency
6978136.0	1.2	100	100.0	0.2	6579136.0	1.3	0.0	4000.0	8506.0	41.8	2.2	1.0	5

Results and Discussion cont.

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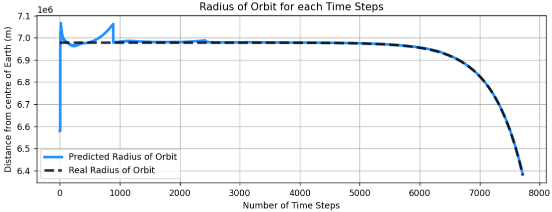

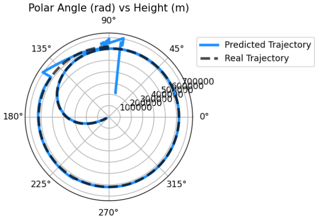

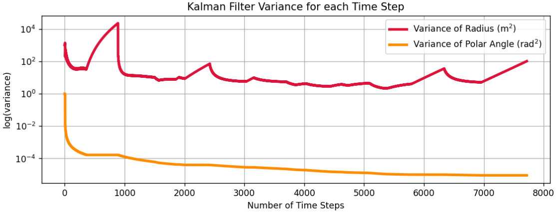

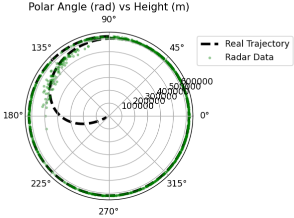

Satellite Distance (m)	Satellite Angle (rad)	Radar Counts	Distance Noise	Polar Noise	Prior Distance (m)	Prior Angle (rad)	Prior v_r	Prior v_ϕ	Satellite Mass (kg)	Satellite Area(m^2)	Satellite Drag	Time Interval	Radar Frequency
6978136.0	1.2	50	100.0	0.2	6579136.0	1.3	0.0	4000.0	8506.0	41.8	2.2	1.0	5

References I

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[1] V. Nwankwo and S. Chakrabarti, "Theoretical model of drag force impact on a model international space station satellite due to solar activity," *TRANSACTIONS OF THE JAPAN SOCIETY FOR AERONAUTICAL AND SPACE SCIENCES SPACE TECHNOLOGY JAPAN*, vol. 12, pp. 47–53, Jul. 2014. DOI: 10.2322/tastj.12.47.

[2] M. Fiolhais, L. Gonzalez-Urbina, T. Milewski, C. Chaparro, and A. Ferroglia, "Orbital decay in the classroom," *The Physics Teacher*, vol. 61, no. 3, pp. 182–185, Mar. 2023, ISSN: 1943-4928. DOI: 10.1119/5.0063725. [Online]. Available: http://dx.doi.org/10.1119/5.0063725.

[3] N. G. D. Center, "U.S. standard atmosphere (1976),", vol. 40, no. 4, pp. 553–554, Apr. 1992. DOI: 10.1016/0032-0633(92)90203-Z.

[4] M. Pewsey, "Minimum distance between ellipse and point," (2022), [Online]. Available: https://mpewsey.github.io/2021/11/07/minimum-distance-between-ellipse-and-point.html.