CptS 515, 11-6-2020

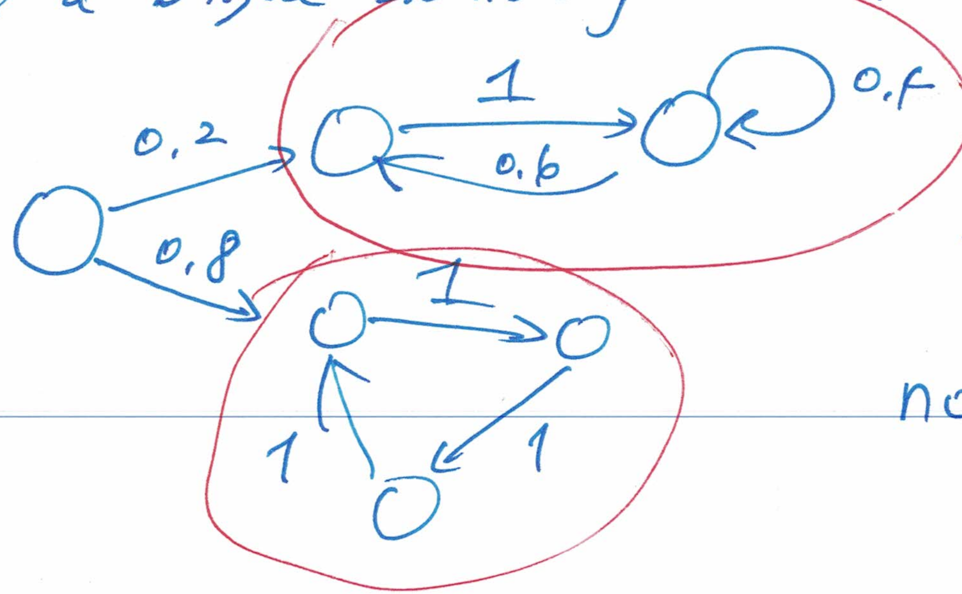
Last time: Markov Chain with transition probability matrix P.

The graph deduced from the chain is:

whenever $P_{ij} > 0$, we put edge $(i) \longrightarrow (j)$ into the graph.

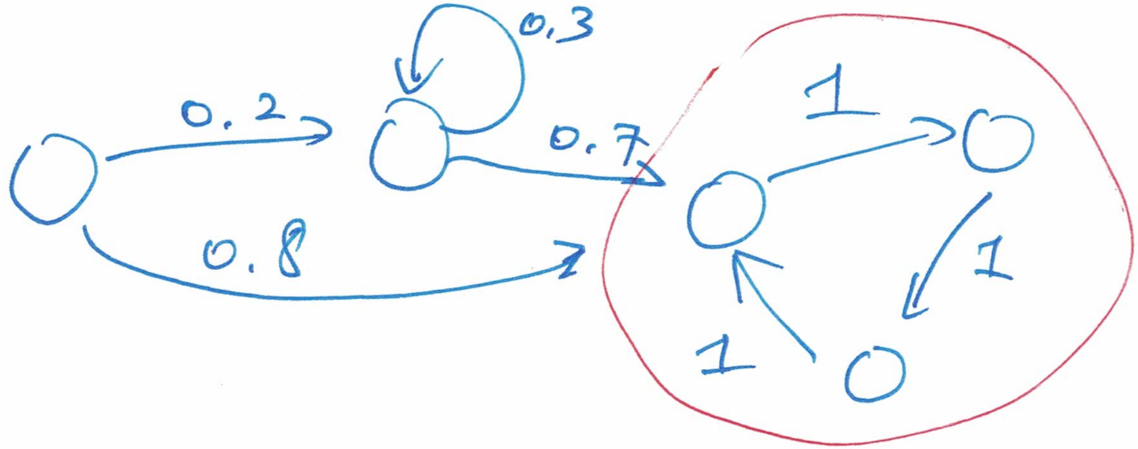
When the graph has only one leaf-SCC, the Markov Chain has a unique stationary distribution.

Example.



two leaf-SCC's.

no-unique stationary distribution.



one leaf-scc.

has a unique stationary distribution.

Special case: The graph is strongly connected (the graph has one scc). The markov chain is called irreducible. Of course, it has unique stationary distribution.

Existence of the limit

$$\lim_{n \to \infty} P^n = P^*.$$

Trillion dollar alg —— Google page rank.

early search engine:

Goofer —— key word match.

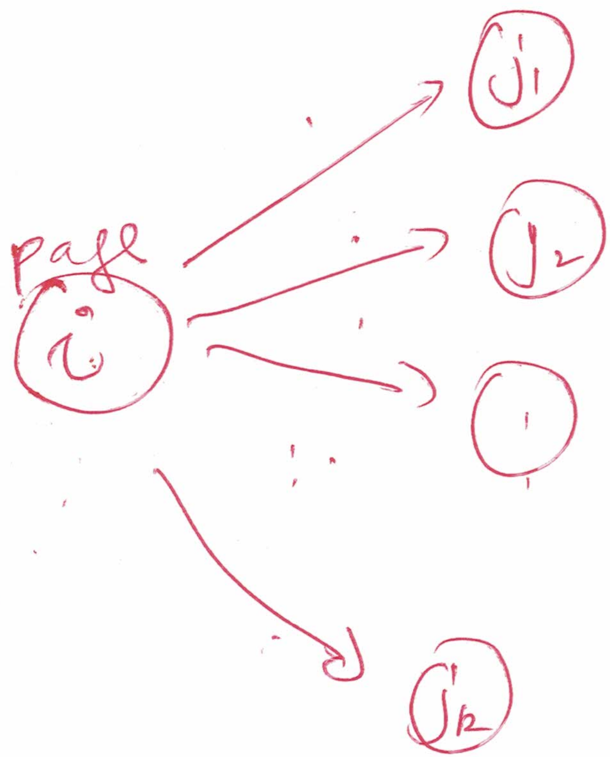
yahoo —— key word match.

google —— use page rank.

Each page is a node.

All the pages form a graph. $\longrightarrow$ Markov chain

Assume that we have N pages.



Small Example. $N = 4$.

$$
\begin{tikzcd}
1 \arrow[r, "1/3"] \arrow[d, "1/3"'] \arrow[dr, "1/3"] & 2 \arrow[d, "1"] \\
3 \arrow[r, "1"'] & 4
\end{tikzcd}
$$

defines a transition probability matrix T.

Define $$R_\alpha = (1-\alpha) \cdot T + \frac{\alpha}{N} \cdot \mathbb{I}$$

where T defined earlier, $\alpha = 0.15$ (adapting factor), $\mathbb{I}$ is all-1-matrix.

T:

$$\begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}$$

node 4 is dangling node.

R_α is

$$\begin{bmatrix} 0.0375 & \frac{0.85}{3}+0.0375 & \frac{0.85}{3}+0.0375 & \frac{0.85}{3}+0.0375 \\ 0.0375 & 0.0375 & 0.0375 & 0.85+0.0375 \\ - & - & - & - \\ - & - & - & \end{bmatrix}$$

Sat. every entry is positive.

R_α has a unique stationary distribution π!

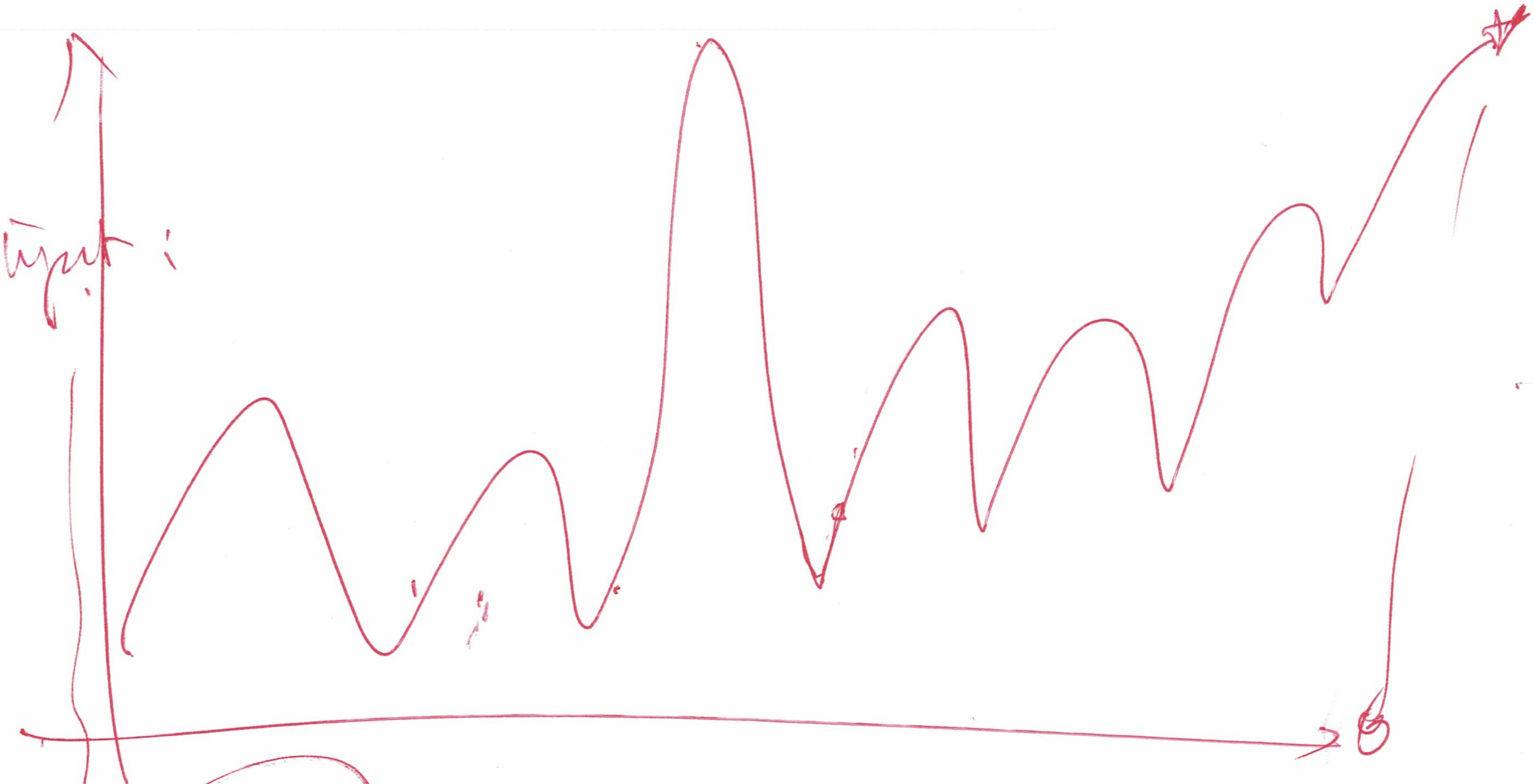
(Since $R_\alpha > 0$).

$$\pi = \pi \cdot R_\alpha .$$

$\Rightarrow$ What is π? π gives a small number (in $[0,1]$) to <u>each</u> page on the internet!

No "key word" matching!

input:



Alg:

output: