CptS 515. 10/16/2020.

Midterm: Monday, take home,

due back Wed. midnight.

Content: write a paper.

---

will upload a video for the midterm

<u>after</u> I send out the exam pdf.

Don't give me a laundry list!

1.3. unfolding a program/design

```
1   x := y + z;
2   while (x > y) {
3       x++;
4       y := z + 15;
5   }
```

lines.

I run the program for 10 steps.

(x_0, y_0, z_0) ← vals at the beginning.

⋮

(x_{10}, y_{10}, z_{10}) ← vals after 10 steps.

The relationship on $(x_0, y_0, z_0;\ x_{10}, y_{10}, z_{10})$ is simply (almost) a linear constraint system. Then,

Suppose $P(x_0, y_0, z_0;\ x_{10}, y_{10}, z_{10})$ is a property for the program. Then,

$$\exists\ x_0, y_0, z_0;\ x_{10}, y_{10}, z_{10}.\quad \text{sat.}$$

ℓ

$$\neg P(x_0, y_0, z_0;\ x_{10}, y_{10}, z_{10}) \wedge$$

$$\ell(x_0, y_0, z_0;\ x_{10}, y_{10}, z_{10}).$$

When P is linear, the above can be solved in ILP.

2. Randomness.

2.1. R.V.

Fair-coin is a r.v. s.t.

$$\mathrm{Prob}(X = \text{head}) = 1/2$$

$$\mathrm{Prob}(X = \text{tail}) = 1/2.$$

(there is no randomness in def. of r.v.)

What's probability?

pizza.

each dot is labeled with either head or tail.

$\mathrm{Prob}(X = \text{head}) = 1/2 \iff$

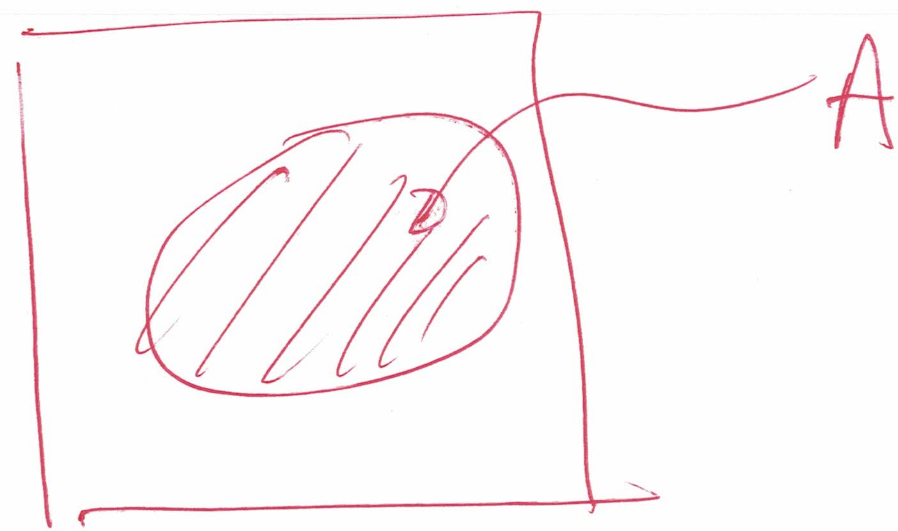
$\Rightarrow$ next page

the total area of the dots labeled with "head"

= 50% of the whole pizza.

what's area?

measure theory.



measure.

probality of event A $\equiv \mu(A)$

which sat.

$$\sum_i \mu(A_i) = \mu\left(\sum A_i\right).$$

when A_i's are disjoint.

Probabity is a pre-defined measure.

Let X take values $a_1, \dots, a_k$ with prob.

$p_1, \dots, p_k$ s.t. $\sum p_i = 1$.

Then:
$$E(X) = \sum_{i=1}^{k} p_i a_i$$

$$Var(X) = \sum_{i=1}^{k} p_i (a_i - E(X))^2$$

$$\sigma^2 = Var(X).$$

A r.v. can take inf. values

Chebyshev Ineq: for all $d > 0$,

$$\text{Prob}\left(|X - \mu| \geq d \cdot \sigma \right) \leq \frac{1}{d^2}$$

where μ is $E(X) < \infty$.

Consider a seq of i.i.d seq of r.v.'s

$$X_1, \ldots, X_n, \ldots$$

Then, as $n \to \infty$, $\frac{1}{n}(X_1 + \cdots + X_n) \longrightarrow \mu$ with probability 1.

// Central Limit
// Theorem.

Law of large numbers:

When $\sigma(X_i) = \sigma$ is finite, then

$$\frac{\frac{1}{n}(X_1 + \cdots + X_n) - \mu}{\sqrt{n}} \sim N(0, \sigma^2).$$

Normal dist.