Cpts 515. 8/31/2020

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Last week: concepts & def's on ALG & Complexities.

From Today: Content of cpts 350.

Spend two wks:

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Complexity analysis through recurrence solving.

$$= c(n-1)^2 + a \cdot n$$

$$= c(n^2 - 2n + 1) + a \cdot n$$

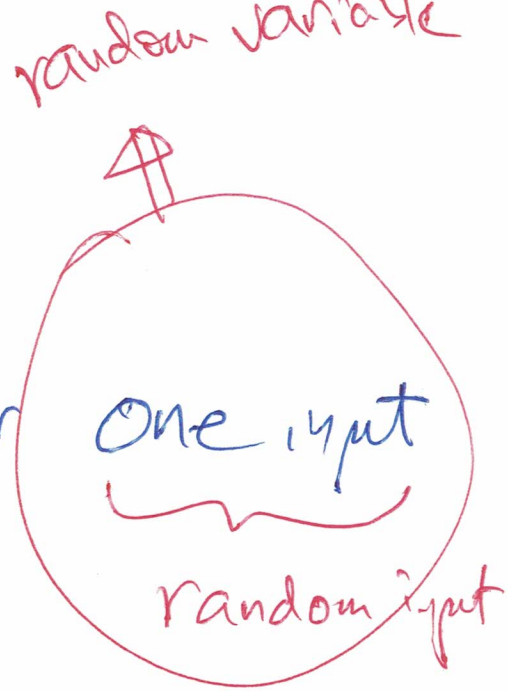
$$= cn^2 - 2cn + a \cdot n + c$$

$$\leq cn^2 \quad \text{when } c \gg a.$$

Logic:

avg-case time complexity

= the running of the alg under one input

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avg-score of the class

= the score of one student.

average student.
random student.

avg-case time complexity of QS.

1. Write a formula

$$T_{AVG}(n) = \frac{1}{n}\sum_{r=1}^{n}\left\{T_{AVG}(r-1) + T_{AVG}(n-r) + O(n)\right\}$$

2. Guess $T_{AVG}(n) = O(n\log n)$. Want $C$ s.t.

$$T_{AVG}(n) \le Cn\log n.$$

3. Check

$$T_{AVG}(n) = \frac{1}{n}\sum_{r=1}^{n}\left\{T_{AVG}(r-1) + T_{AVG}(n-r) + a\cdot n\right\}$$

given

I.H.

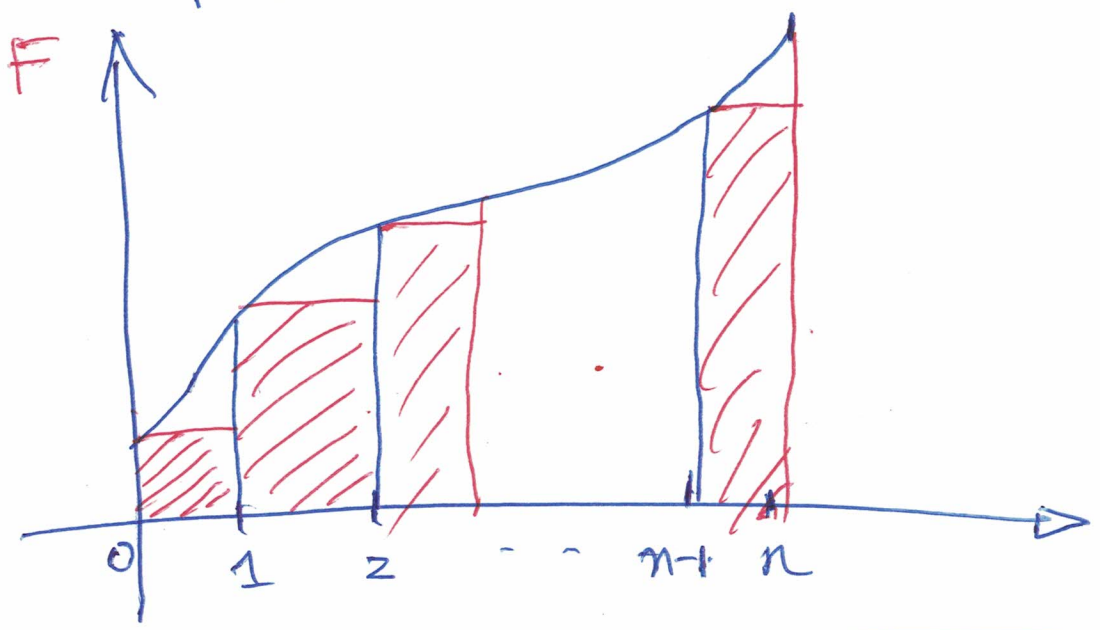
$\forall i < n$,

$T_{AVG}(i) \le c\cdot i\cdot \log i$.

"$\int$" $\equiv$ SUM

I now want to

$$\sum_{r=1}^{n} F(r-1) = F(0)+F(1)+\ldots+F(n-1)$$




Conclusion:

$$\sum_{r=1}^{n} F(r-1) \leq \int_{0}^{n} F(r)\,dr$$

Google:

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$\log x = \frac{\ln x}{\ln 2}$$

$$\int x \log x \, dx = \frac{1}{\ln 2} \int x \ln x$$

$$= \frac{x^2}{2} \log x - \frac{x^2}{4 \ln 2} + C.$$

$$\leq \frac{1}{n} \sum_{r=1}^{n} \left\{ c(r-1)\log(r-1) + c\cdot(n-r)\log(n-r) + a\cdot n \right\}$$

$$= \frac{c}{n} \sum_{r=1}^{n} (r-1)\log(r-1) + \frac{c}{n} \sum_{r=1}^{n} (n-r)\log(n-r) + a\cdot n$$

$0\log 0 + 1\log 1 + \ldots + (n-1)\log(n-1)$ $\qquad$ $(n-1)\log(n-1) + \ldots + 0\log 0$

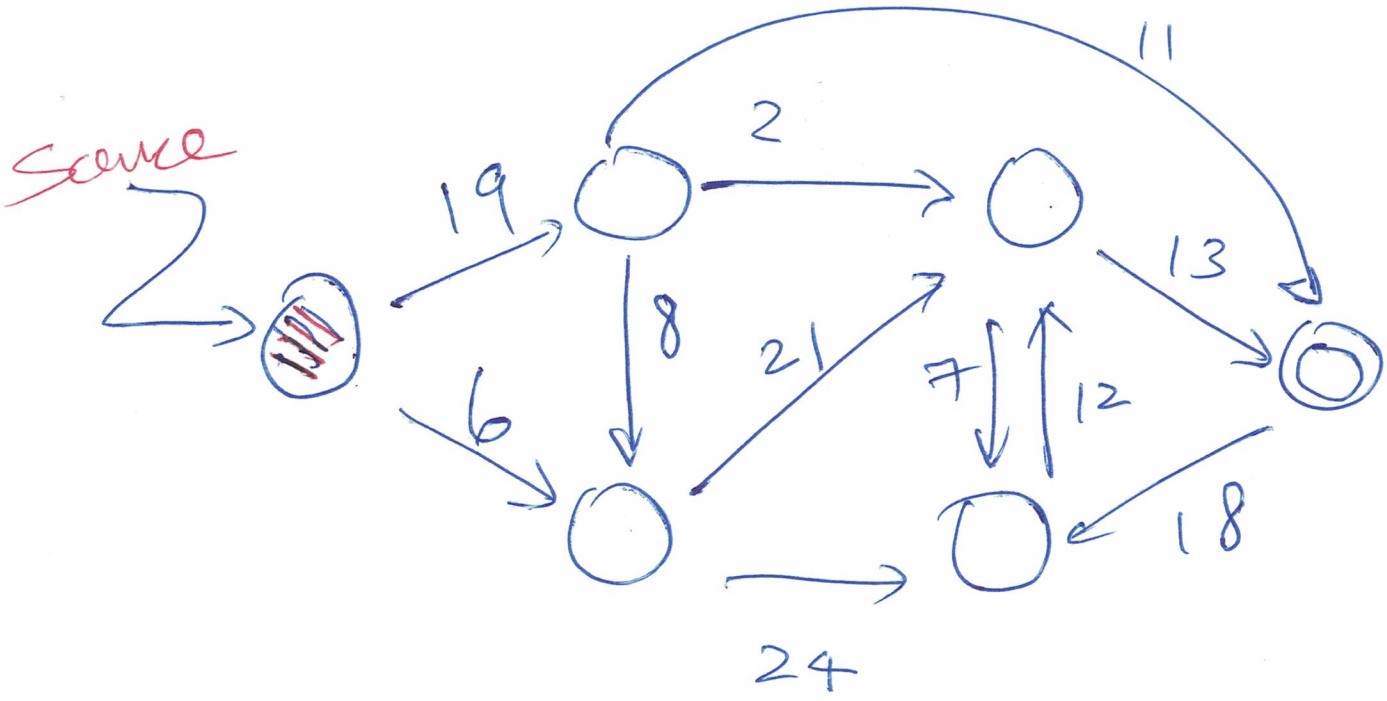
$$= \frac{2c}{n} \sum_{r=1}^{n} \underbrace{(r-1)\log(r-1)}_{F(r-1)} + a\cdot n$$

$$\leq \frac{2c}{n} \int_{0}^{n} x\log x \, dx + a\cdot n$$

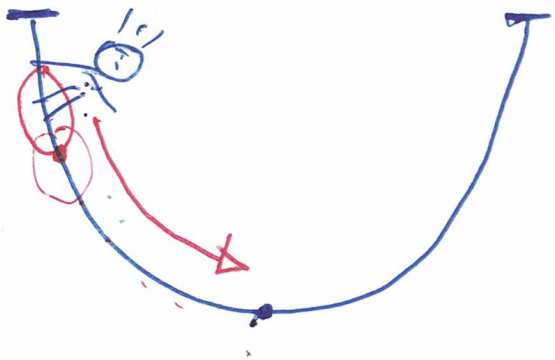
$$= \frac{2c}{n} \left( \frac{n^2}{2}\log n - \frac{n^2}{4\ln 2} \right) + a\cdot n$$

$$\leq c\, n\log n \quad \text{when } c \gg a$$

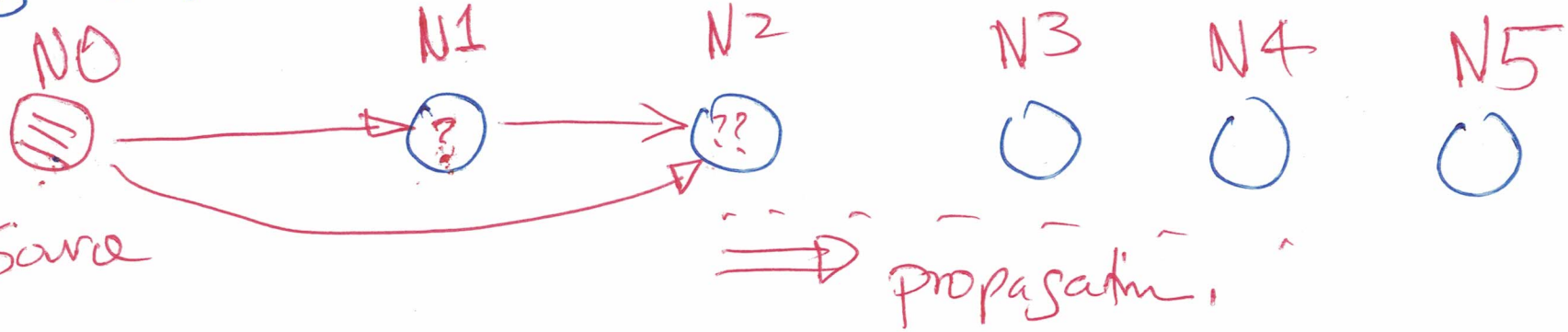
# Shortest Path. Alg. Dijstra. Alg.




Inside any greedy alg, there is something called "monotonic".



imagine :




nodes are sorted according to the shortest distance from the source.

↑ the total weight of the shortest path.