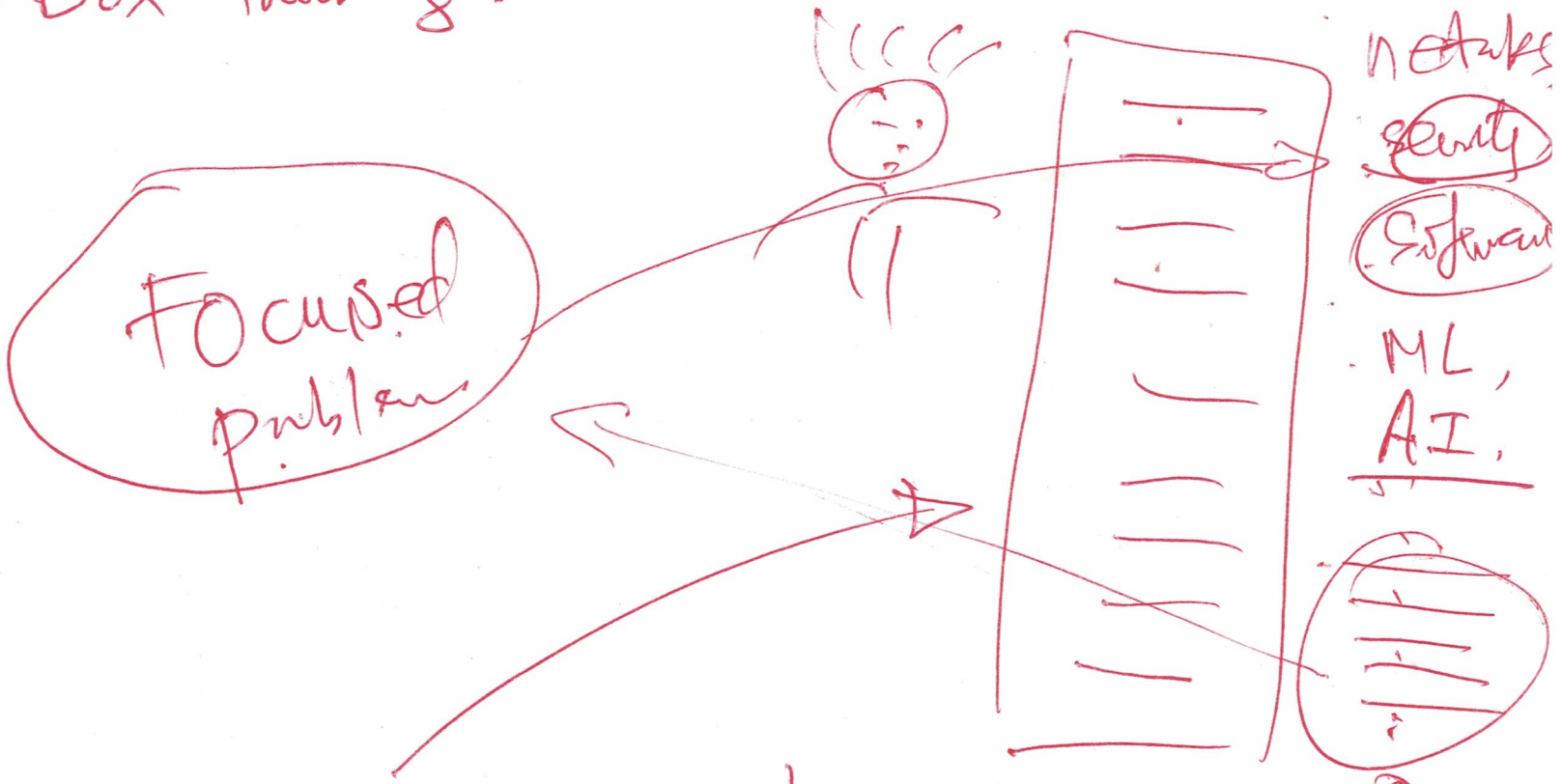
Cpts 515. 10/2/2020.

Out-of-Box thinking:



Key: you must have things to think about!

Solving linear constraints ($\equiv$ Finding a feasible pt in LP instance)

$\Rightarrow$ a new LP problem instance.

Example. I want to find a feasible pt / solution to

$$\begin{cases} x_1 - 2x_2 \leq -3 \\ 2x_1 - x_2 \leq 0 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{cases}$$

Note: $\langle x_1 = 0, x_2 = 0 \rangle$ doesn't satisfy the constraints.

Step 1. Translation. (change "$\leq$" $\Rightarrow$ "$=$" by adding slack vars)

$$x_1 - 2x_2 + y_1 = -3$$
$$2x_1 - x_2 + y_2 = 0$$
$$x_1 \geq 0, \quad y_1 \geq 0$$
$$x_2 \geq 0, \quad y_2 \geq 0.$$

step 2. Change RHS to nonnegative:

$$-x_1 + 2x_2 - y_1 = 3$$
$$2x_1 - x_2 - y_2 = 0$$
$$x_1 \geq 0, \quad y_1 \geq 0$$
$$x_2 \geq 0, \quad y_2 \geq 0.$$

Step3. We add "artificial variables" & then we get an LP instance:

$$\min \quad z_1 + z_2$$

$$\text{Subj. to:} \quad -x_1 + 2x_2 - y_1 + z_1 = 3$$

$$2x_1 - x_2 - y_2 + z_2 = 2$$

$$x_1 \geq 0, \quad y_1 \geq 0, \quad z_1 \geq 0$$

$$x_2 \geq 0, \quad y_2 \geq 0, \quad z_2 \geq 0.$$

feasible pt now is:
$z_1 = 3$, $z_2 = 0$, $x_1 = x_2 = y_1 = y_2 = 0$.

Step4. Observe that ① if the LP in step3 has the $\min z_1 + z_2$ achieved as zero, then the orgnd constraint has solution.

② if the LP in step3 has $\min z_1 + z_2 > 0$, then the orgnd constraint has no solution!

Remark. ILP is not LP.

↑ integer linear programing.

LP can be solved efficiently;

ILP is believed to solve in Exp-time.

(NP-complete).

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① Out-of-Box thinking:

①. approximation. In CS, to find problems that are formulated in ILP and can actually be approx. by solving an LP instance.

(ILP appnx. by. LP).

② Think about ILP.

↳ We have tools to solve it.

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linear relationship between integer variables.

A. integer variables in CS.

examples: program's local variable,

discrete time, counts of events, ...

Example: $\square\Diamond(\#_{proc1} = \#_{proc2})$ ← fairness

(there are infinitely many times in the future that process 1's times take CPU = ...2's

B.

EE	CS
LP is basic tool, linearity is essential.	① integer variables + linear relationship. ② ordering of events. ②': Where is linearity in ordering of events?

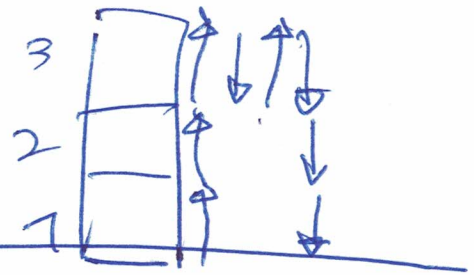
the only thing that mathematicians are good at, and can use in practice.

Why "ordering of events" is important?

Example. events = {1, 2, 3}

good evaluator event ordering:

1, 2, 3, 2, 3, 2, 1.



bad one:

1, 3, 1. (without passing 2).

Cpts 516.

Parikh map & semilinearity.

Exaple. $(abcc)^* a$.

$\Rightarrow$ $\#_a \geq 0$, $\#_c = 2 \cdot \#_b$.

$\#_b \geq 0$, $\#_a = \#_b + 1$.

$\#_c \geq 0$,

Exaple stg:

$abcc\,abcc\,a$.

$\#_a = 3$, $\#_b = 2$,

$\#_c = 4$.