CptS 515. 9/30/2020

Zero-Sum Game.

From Bob's view, the money that he loses

$$p_1(3q_1 - 2q_2) + p_2(-q_1 + 9q_2)$$

$$\leq \max\{3q_1 - 2q_2, -q_1 + 9q_2\}.$$

Then, Bob will have his strategy $\langle q_1, q_2 \rangle$ to min the loss:

$$\min_{\langle q_1, q_2 \rangle} \max\{3q_1 - 2q_2, -q_1 + 9q_2\}$$

$$q_1 + q_2 = 1, \quad q_1, q_2 \geq 0$$

(3). Von Neuman's Min-Max Theorem.

Under Alice's best strategy, the money she made $= U_A$

Under Bob's best strategy, the money he lost $= U_B$

we have, $U_A = U_B$.

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(4). The minmax can be formulated as LP instance.

max z

subject to: $z \leq 3p_1 - p_2$

$z \leq -2p_1 + 9p_2$

$p_1 + p_2 = 1$

$p_1, p_2 \geq 0$.

Remark: if you are interested in Computer security, Then you shall spend 7 nights in studying Game Theory.

Security.

hard problem.

math problem

area1

area2

Zero-Knowledge proof,

Dantzig's Alg. (1947), through an Example.

$$\max \quad 2x_1 + 9x_2$$

subject to:

$$5x_1 + 6x_2 \le 8$$

$$2x_1 - 3x_2 \le 3$$

$$-x_1 + 4x_2 \le 15$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

Starting feasible point (feasible $\equiv$ sat. all constraints)

$$x_1 = 0, \quad x_2 = 0,$$

Current obj. function value $= 2 \cdot 0 + 9 \cdot 0 = 0.$

$$
\begin{aligned}
\max \quad & 2x_1 + 9x_2 \\
\text{subj to:} \quad & 5x_1 + 6x_2 \le 8 \\
& 2x_1 - 3x_2 \le 3 \\
& -x_1 + 4x_2 \le 15 \\
& \left.\begin{aligned} x_1 &\geq 0 \\ x_2 &\geq 0 . \end{aligned}\right\} \text{ are tight for } x_1, x_2 = 0
\end{aligned}
$$

Current $x_1 = 0$, $x_2 = 0$. Obj val $= 0$.

I want: increase objval when

I pick x_2 to increase.

// Note: the coefficien "9" of x_2 is ≥ 0.

Max $2x_1 + 9x_2$

subj to: $5x_1 + 6x_2 \le 8$

$2x_1 - 3x_2 \le 3$

$-x_1 + 4x_2 \le 15$

$x_1 \ge 0$

$x_2 \ge 0$.

Curret: $x_1 = 0$, $x_2 = 0$ curret: objval $= 0$

pick x_2 to increase. How much?

I can max. increase x_2 by (so max. increasing objval).

$= \min\{8/6, \infty, \frac{15}{4}\}$

$= 8/6 = 4/3$.

max $\quad 2x_1 + 9x_2$

subj to

$5x_1 + 6x_2 \le 8$ ← is tight for current $x_1 = 0$, $x_2 = 4/3$

$2x_1 - 3x_2 \le 3$

$-x_1 + 4x_2 \le 15$

$x_1 \ge 0$ ← is tight for current $x_1 = 0$.

$x_2 \ge 0$

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current: $x_1 = 0$, $x_2 = 4/3$, $val = 2 \cdot 0 + 9 \cdot \frac{4}{3}$

$= 12$.

most important step: def. a new co-ordinate system $\langle y_1, y_2 \rangle$ s.t. the curr $\langle x_1, x_2 \rangle$ will be the new origin.

$y_1 = x_1$ ← from

$y_2 = 8 - 5x_1 - 6x_2$. ← from

So, the new system is with:

$$x_1 = y_1$$

$$x_2 = -\frac{5}{6} y_1 - \frac{1}{6} y_2 + 8/6$$

$$y_1 \geq 0, \quad y_2 \geq 0.$$

current val = 12.

So, the original LP instance is transformed into one in $\langle y_1, y_2 \rangle$:

max $12 - 5.5y_1 - 1.5y_2$

sbj to: $y_2 \geq 0$

$4.5y_1 + 0.5y_2 \leq 7$

$-\frac{13}{3} \cdot y_1 - \frac{2}{3}y_2 \leq \frac{29}{3}$

$y_1 \geq 0$

$\frac{5}{6}y_1 + \frac{1}{6}y_2 \leq \frac{4}{3}$.

No way! Since the coefficients -5.5 and -1.5 are < 0.

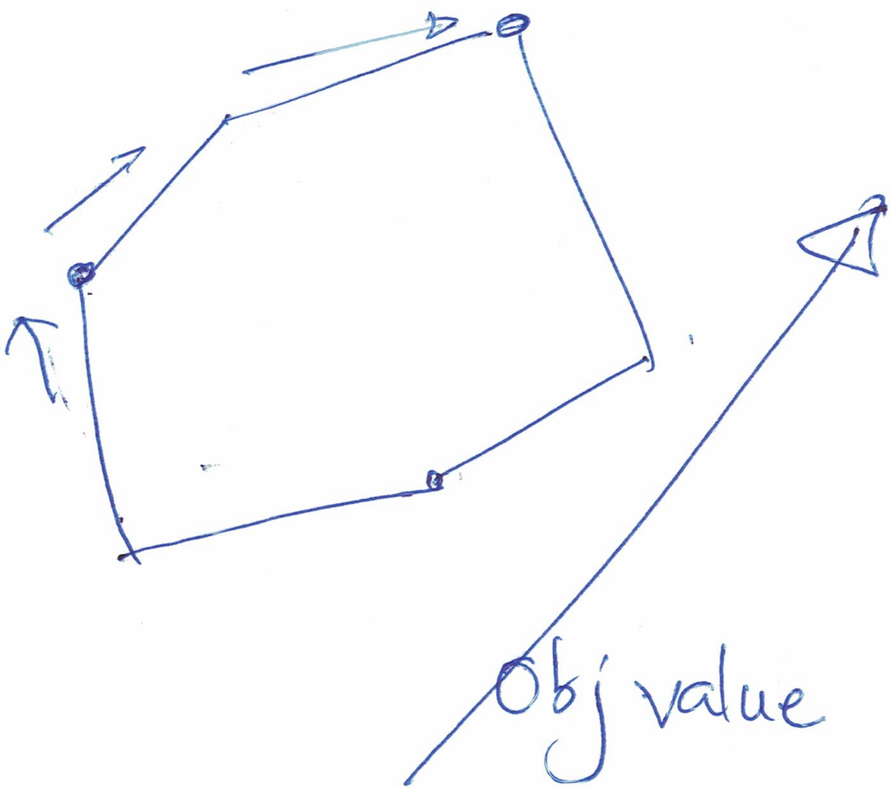
We start with $\langle y_1 = 0, y_2 = 0 \rangle$ ← feasible point.

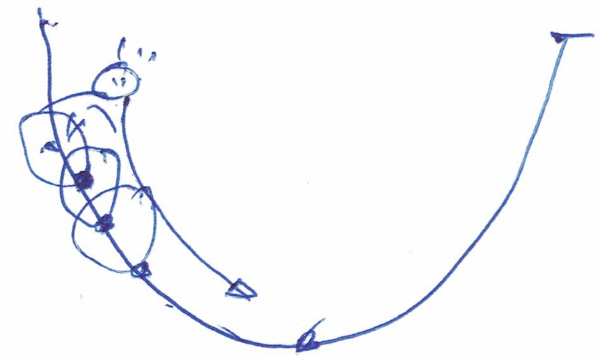
Pick one of y_1, y_2 to increase so that current val $= 12 - 5.5 \cdot 0 - 1.5 \cdot 0$ will also increase.

Done!

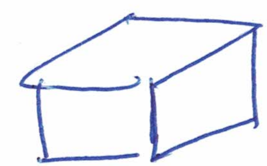
Intuition of the alg. ———— efficient alg.

(1). Greedy Alg. Local opt $\xrightarrow{\text{move to}}$ Global opt.





Dimension Curse:



a cube in k-dimension space has 2^k corners.

So, Don't try all corners!

(2). to "move" to a next neighboring point, we simply change the co-ordinate system. (So that we can repeat the whole process - -)

(3). What if the origin is <u>not</u> feasible?

Trick we use:

Translate "finding a feasible point" to a new LP instance!

(This example shows that how to reduce solving linear constraints into solving LP.)

Remark.

For LP problems, the variables are taking real-values.

If var's take only integer values, then we call them ILP problems

LP has efficient algs.

ILP is NP-complete. (we don't know any efficient solutions existed).

⇒ discrete val problems are harder
(CS)

than continuous val problems.
(EE)