Cpts 515. 9/9/2020

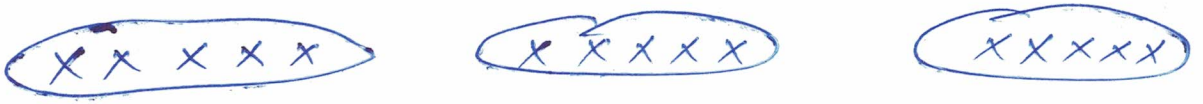
Linear time selection alg.

To select i-th smallest element from an array of n numbers.

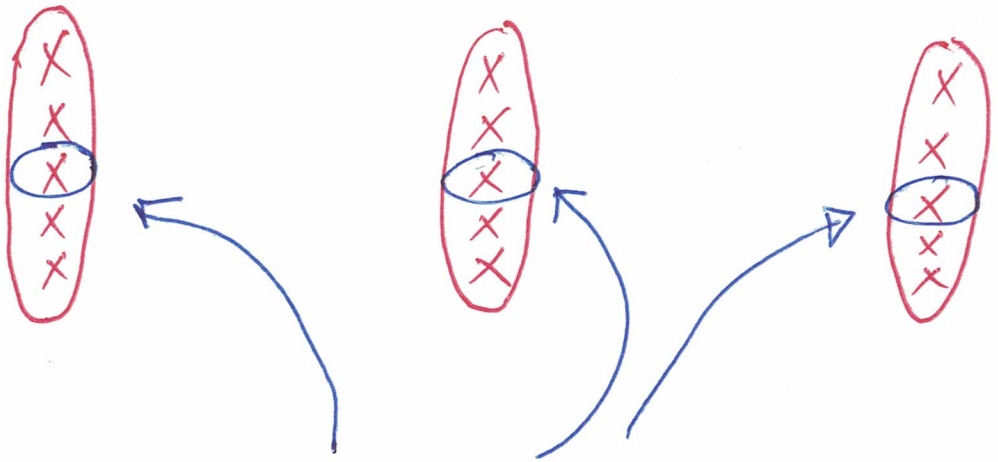
You have a hw to do.

Input array of $n$ numbers

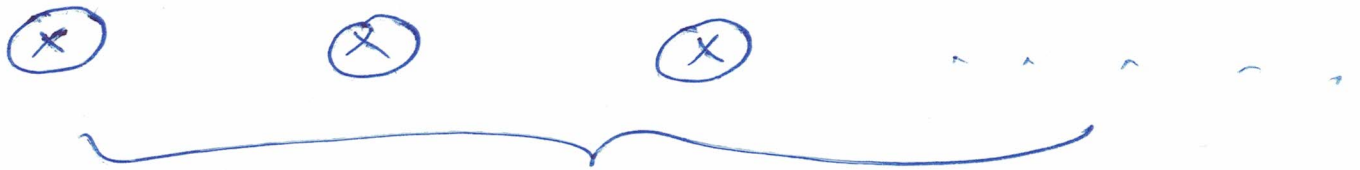
(1). Cut the array into $n/5$ groups where each group has 5 numbers.



(2). Sort each group. ⟵ take $O(n)$ time.



(3). Each group, after sorted, has a median.
Totally, we have $n/5$ medians.



$n/5$ medians.

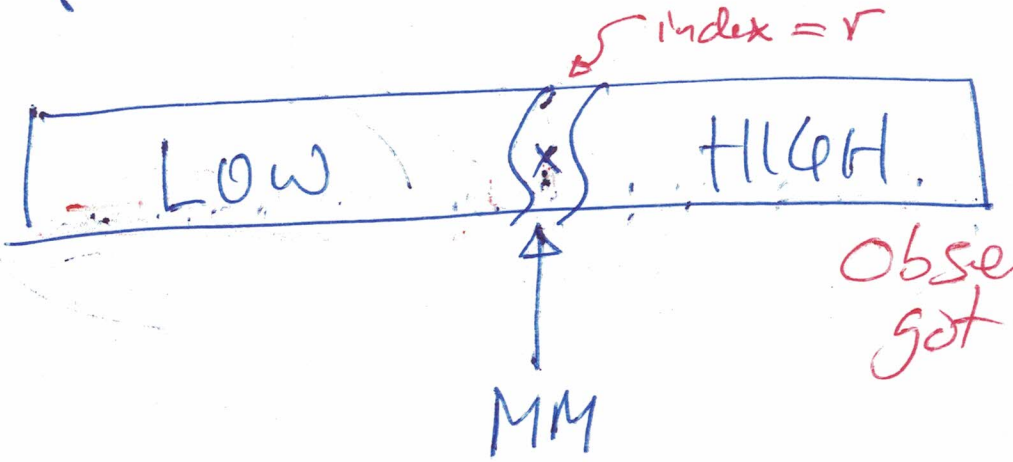
(4). Recursively, we use linear time selection to select $n/10$-th smallest median from the $n/5$ medians.

called MM.

(5). Swap MM with the first element in the original array of $n$ numbers.

if $i > r$, recursively run linear-time selection to select the $(i-r)$-th smallest from the right.

(6). So, now, the original array is updated after the swap. I run Partition on this array:




Observation: this MM got to be the r-th smallest

(7). if $i == r$, return this MM as the $i$-th smallest;

if $i < r$, (so, the $i$-th smallest is one of the numbers in LOW), recursively run linear time selection to select the $i$-th smallest from the LOW;

How many medians? $n/5$ medians.

How many medians that are $\leq MM$?

$n/10$.

Each group has 5 numbers. How many numbers in the group $\leq$ the median of the group? 3

$\Rightarrow$ We have at least $3 \cdot \frac{n}{10}$ numbers in the original array that are $\leq MM$.

$$|LOW| \geq 3 \cdot \frac{n}{10},$$

$$|HIGH| \geq 3 \cdot \frac{n}{10}.$$

$$|LOW| + |HIGH| = n.$$

$$\Rightarrow |LOW|, |HIGH| \leq \frac{7n}{10}.$$

$$\Rightarrow \max \text{ of } T_w(|LOW|), T_w(|HIGH|) \leq T_w\left(\frac{7n}{10}\right).$$

Show it runs in Wust-case linear time.

$$T_w(n) = \underbrace{??}_{\text{step(7)}} + \underbrace{T_w\left(\frac{n}{5}\right)}_{\text{step(4)}} + O(n)$$

↳ the wast-case time for selecting $i$-th smallest from an array of $n$ numbers.

?? = max of $T_w(|LOW|), T_w(|HIGH|)$.

Finally, we have,

$$T_W(n) = T_W\left(\frac{7n}{10}\right) + T_W\left(\frac{n}{5}\right) + \underbrace{a \cdot n}_{\text{~~O(n)~~}}$$

Guess $T_W(n) = O(n) \leq c \cdot n$ for some $c$.

I.H. $\forall j < n$, $T_W(j) \leq c \cdot j$.

Check:

$$T_W(n) = T_W\left(\frac{7n}{10}\right) + T_W\left(\frac{n}{5}\right) + a \cdot n$$

$$\leq c \cdot \frac{7n}{10} + c \cdot \frac{n}{5} + a \cdot n$$

$$= \underbrace{\frac{9}{10}}_{<1} \cdot c \cdot n + a \cdot n$$

$$\leq c \cdot n \quad \text{when } c \gg a.$$

# Tarjan's SCC Algorithm.

deeply

① All my students know this alg

② . this alg is not even well-known.

③ . this alg is one of the most widely used alg in the world!

why? only a few people know how to use it.

↳ barrier is too high!

Today's challenge:

How to know that a given software/hardware program is going to run forever?

You are going to ~~test~~ it?!!!!