

Cpts 515. 8/26/2020

I will stay 10 more minutes after each
Zoom lecture. You may ask
questions then,

Last time: Graduate alg design — Solving problems
indirectly.

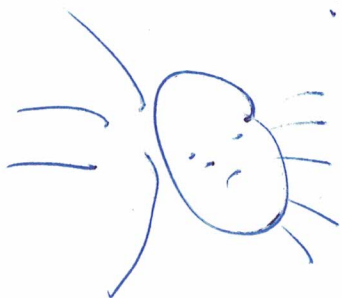
Today: Mathematical def of algs.

Ideas:

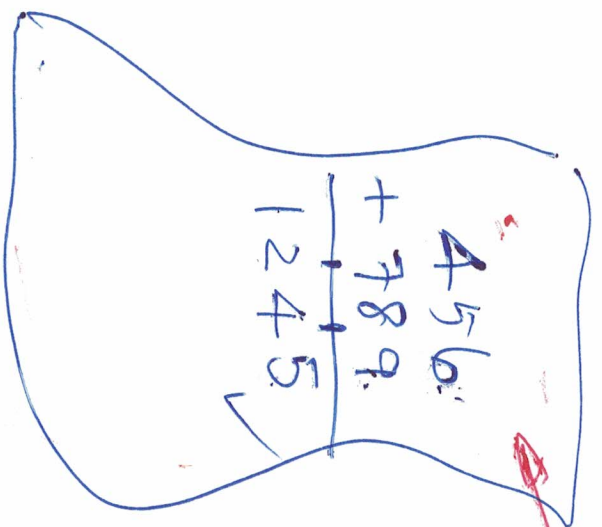
- (A). Define a universal computer.
 - (B). Define/study algs on the computer.
-

The universal computer = Turing machine
(1936).

Intuition.



works on a piece of paper.



a piece of paper

local

Read/write on a piece of paper.

Finite number of rules
(can hold to read/write/move).

piece of paper

infinite memory

This is Turing Machine.

Turing-Church Thesis:

All Computers (Past, current, future) Can't be more powerful than Turing Machines.

⇒ TMs are the most powerful machine humans can create.

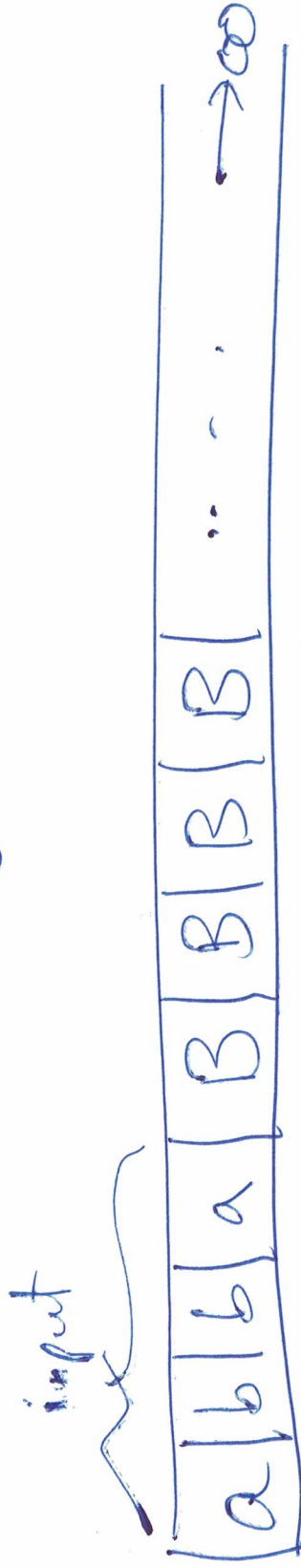
Thinking: humans are TM?

Cats
dogs,
tractors,

mathematical model,

(1). make the piece of paper linear;

(2). Better illustration of the controller.



program stored here

Instructions:

tape symbols

direction of move

$$\delta(p, a) = (q, b, R)$$

$$\delta(p, a) = (q, b, L)$$

Remark: "state" ^{control} q, f

in C, PC values,
(line numbers).



When a $\overline{\text{TM}}$ M runs on input w , ^{exactly} one

of the following will happen:

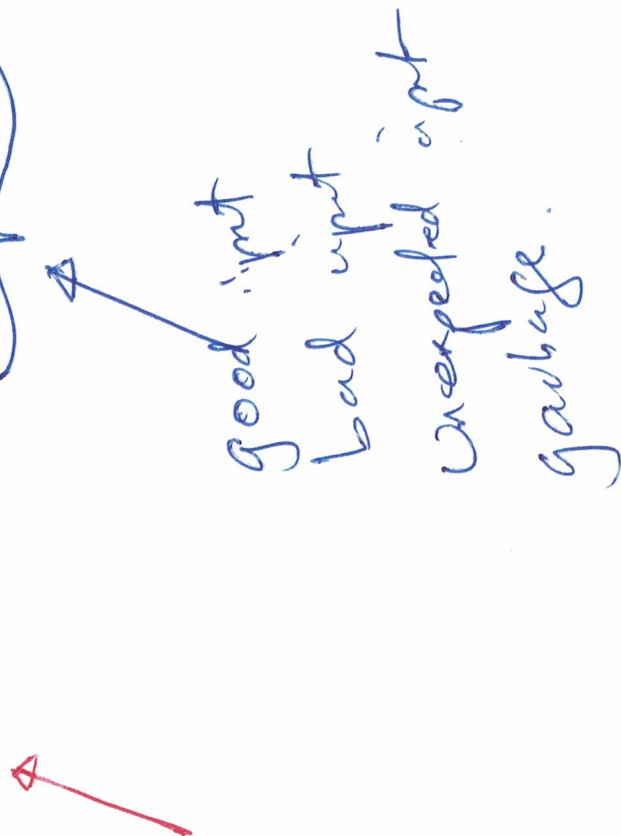
(1). M enters an accepting state. ("saying yes to w ")

↑
designated

(2). M crashes ("saying No to w ")

(3). M runs forever without saying yes !

Alg = a TM that waits on all input.



good input
bad input
unexpected input
garbage.

Alg = a C-program that hatts on
all input;

Java ...

Examples.

"EVEN" { Given: a number n .
Ques: Is n an even number?

Alg?

input n
return $(n \% 2 == 0)$;

yes.

Alg?

input n
for each $k \leq n$
if $2 \cdot k == n$ return yes
return NO.

yes.

Stupid Alg.

Alg? NO.

input n
for each k
if $2 \cdot k == n$, ret yes
ret NO.

~~Integers are of 32-bits~~

~~So, the for loop will stop~~
~~So it can't run forever in C.~~

Integers in Alg desgn are NOT
Integers in C, Java, ...
are unbounded!

Job interview:

Engineers are NOT mathematicians.

They don't care.

"

sort

x

numbers

stored in an

array"

Ask: They are unbounded
of 32-bits/64-bit.



if

asks you:

"What's the running time of your alg?"

you can't answer:

3 seconds!

X

you shall answer with
time complexity.

Time Complexity

Space Complexity



① functions of input size

② Don't have a unit.

(you can't say 1 year,
days, seconds.)

Input Size is the size of the input stored
in memory!

For ex-ple, $n = 52356$. Size of n ?

Size of $n = \log_2 n$.

Let A be the input to some alg.

A is an array of k integers.

Size of $A = k$. // This is rough but
// everybody does it.

Remark on space complexity.

① "space" means "additional memory" you need.

When the input is read-only, the input doesn't even take memory.

String - Comparison: n

abbcda * abbcda

↑
p

↑
q

while

*p++ == *q++

...

Space complexity =

Size of p + size of q

$$= O(\log_2 n)$$

Mathematical def. of time & space complexities.
(Next lecture.)