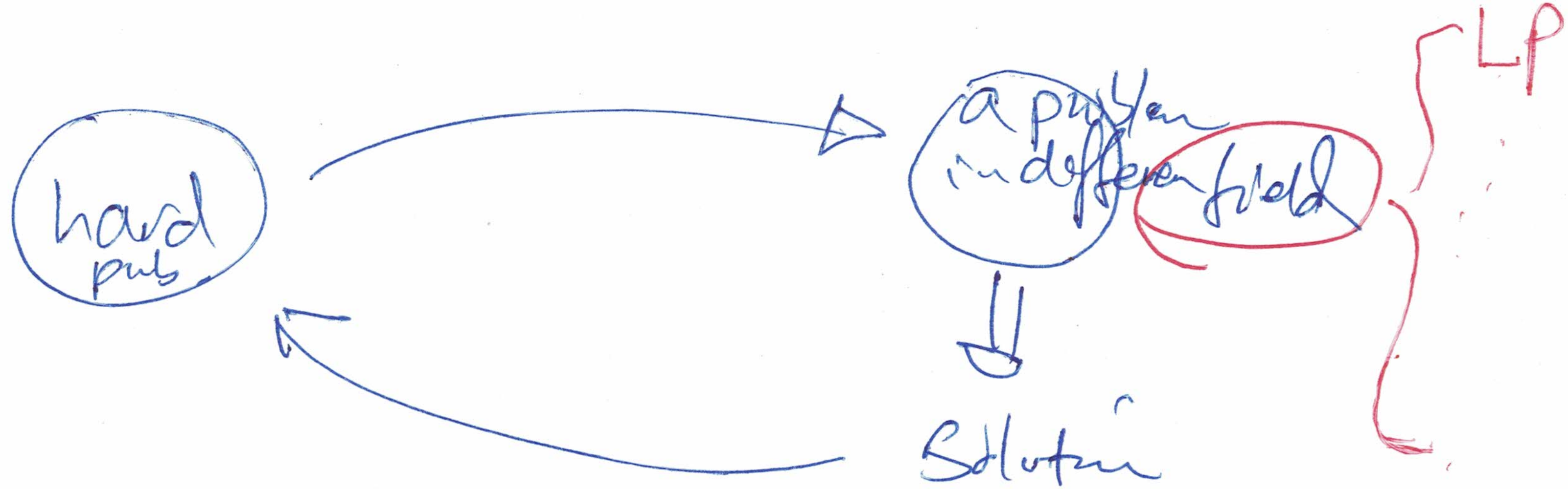
Cpts 515. 9/28/2020.

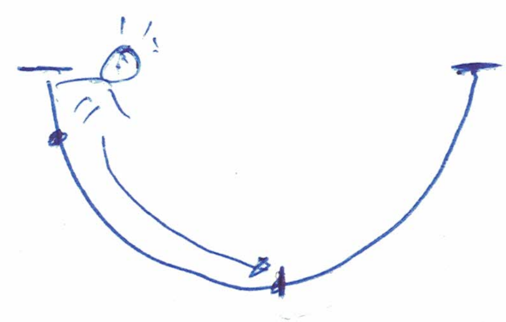
Linear Programming.

// Design alg indirectly.
// to solve problems



— linear optimization, part of convex optimization

= Solvable.



— General Form: all variables take values in $\mathbb{R}$ (reals). // when variables are of
// integer values, LP becomes
// ILP (Integer Linear Program).

$$\max \; c^T x$$

subj to: $\left.\begin{array}{l} Ax \le b \\ x \ge 0 \end{array}\right\}$ linear constraint system

~~$x \in$~~ $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

c: constant ~~matri~~ vector in $\mathbb{R}^n$.

where A is $m \times n$ matrix.

b is a constant vector.

How about $\min \ 2x - 3y$?

$\Rightarrow \quad \max \ -(2x-3y)$

How about $2x - 4y \geq 7$

$\Rightarrow \quad -2x + 4y \leq -7.$

How about $2x - 4y = 7$

$\Rightarrow \quad 2x - 4y \leq 7$

$2x - 4y \geq 7.$

Familiar form:

$$\max \quad c_1 x_1 + \dots + c_n x_n \quad \Longleftarrow \text{ objective function.}$$

Subject to:

$$\left.\begin{aligned} a_{11} x_1 + \dots + a_{1n} x_n &\le b_1 \\ &\vdots \\ a_{m1} x_1 + \dots + a_{mn} x_n &\le b_m \\ x_1, \dots, x_n &\ge 0. \end{aligned}\right| \text{ Constraints.}$$

Each c, a, b are constants in $\mathbb{Z}$ (integers).

Concept: $2x - 4y \leq 7$

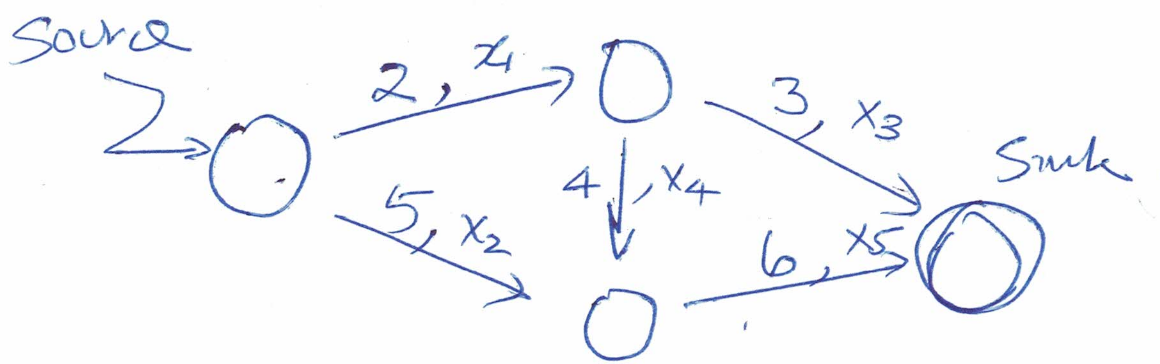
$\Rightarrow$ $2x - 4y + w = 7$

$w \geq 0$

slack variable.

Applications.

1. Network flow is a LP-problem instance.



max $\quad x_1 + x_2$

subject to:
$$x_1 = x_3 + x_4$$
$$x_2 + x_4 = x_5$$
$$x_1 \leq 2, \quad x_3 \leq 3,$$
$$x_2 \leq 5, \quad x_4 \leq 4, \quad x_5 \leq 6$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Applications,

2. Zero-Sum Game.

two players: Alice, Bob.

		Bob Red	Bob Green
Alice		3	-2
Alice		-1	9

} pay-off matrix.

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meaning: if Alice shows  &
Bob shows Red,
then Alice wins 3 \$'s.
(At the same time, Bob loses 3 \$'s. $\Leftarrow$ "zero-sum")

Other entries are similarly defined.

What's The best strategy for Alice?

Suppose That Alice shows  with ~~prob~~ probability p_1

and shows  with probability p_2.

// $p_1 + p_2 = 1$.

Now, we call $\langle p_1, p_2 \rangle$ to be Alice's strategy.

Similarly, we can define $\langle q_1, q_2 \rangle$ to be Bob's ~~stra~~ strategy

with $p_1 + p_2 = 1$.

for Red

for Green.

From Alice view, her strategy will make the following amount of money:

$$(3p_1 + (-1)p_2)\cdot q_1 + ((-2)p_1 + 9p_2)\cdot q_2.$$

This amount is lower & tight boundaly by

$$\min\{3p_1 - p_2,\ -2p_1 + 9p_2\}.$$

(This is because $q_1 + q_2$).

Remark: This is the money that Alice's strategy will win assuming that Bob always takes the best strategy when Bob knows Alice's strategy.

Remark. What's Alice's "best" strategy? She will make the best strategy of Bob's to be least effective! That is,

$$\max_{\langle p_1, p_2 \rangle} \quad \min \{ 3p_1 - p_2 , -2p_1 + 9p_2 \}$$

$$\text{with} \quad p_1 + p_2 = 1 , \quad p_1 , p_2 \geq 0.$$

This is LP problem!

Observations:

(1). It doesn't matter ~~that~~ whether Alice let Bob know her strategy or not. The optimal solution $\langle p_1, p_2 \rangle$ is still the same;

(2). Alice doesn't even need knew Bob's strategy! $\Rightarrow$ A general's victory can't bet on enemy's stupidity.

We can similarly formulate the money that Bob will lose: