CptS 515. 11/4/2020

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HW hint.

Problem 1.

pump instruction: $P_{\{T_1, T_2\}}, 1.5, 2.4$

$$T_1, T_2, \ldots \underbrace{\quad\ldots\quad T_k}_{\text{unchanged}}.$$

$$\uparrow \quad \uparrow$$

$$1.5 \le x \le 2.4$$

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Wait: a given graph $G$.

So, for each $\#_p(\alpha)$ on the $\alpha$, we obtain a formula

$$W_p(x_1, \ldots, x_k, \#_p(\alpha)).$$

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After reading the entire $\alpha$, the water levels (from 0's) sat.

// I use $x_1, \ldots, x_k$ for
// the water levels

$\exists x_{1,p}, \dots, x_{k,p}, \quad p \in$ pump inst. on $G$,

$$x_1 = \sum_p x_{1,p} \quad \wedge$$

$$x_2 = \sum_p x_{2,p} \quad \wedge$$

$$\vdots$$

$$x_k = \sum_p x_{k,p} \quad \wedge$$

$$\bigwedge_p W_p(x_{1,p}, \dots, x_{k,p}, \#_p(\alpha))$$

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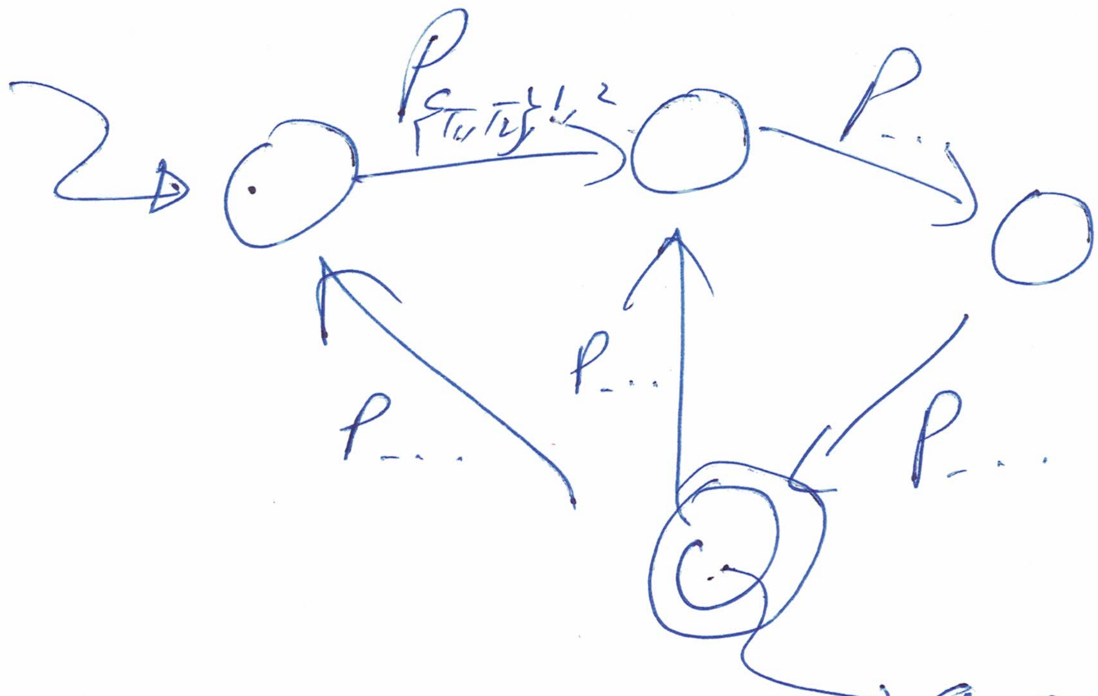
I denote this formula as

$$W_\alpha(x_1, \dots, x_k).$$

Need prove: (by exaple).

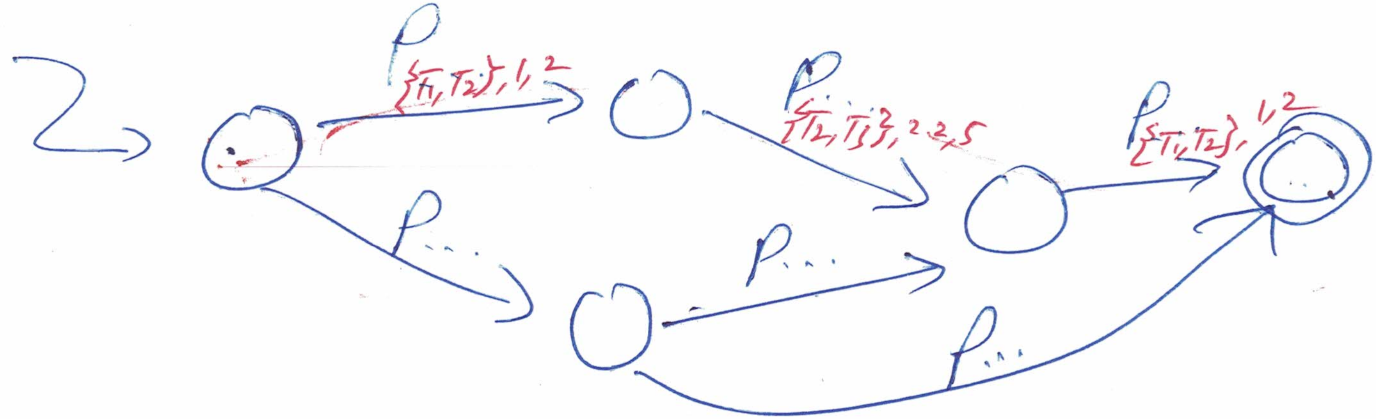



$\to$ a condition $\underline{Bad(x_1, \dots, x_k)}$.

whether there is a walk from init to final s.t. at final node, the water levels sat. Bad.

~~Hint~~: to use LP.

Simple case. No cycles/loops in $G$.




Let's assume that, on a walk $\alpha$ from init to final, we have

$\#_p(\alpha)$ to denote the # of pump instructions $p$ on the $\alpha$.

(On a graph, there are only finitely many pump instructions.)

count.

$\Longrightarrow W(x_1, x_2, \ldots, x_k, 2)$

subscripted with $W_\beta$, where $\beta = P_{\{T_1, T_2\}, 1, 2}$

$\Longrightarrow W_\beta(x_1, \ldots, x_k, 1)$, where $\beta = P_{\{T_2, T_5\}, 2, 2, 5}$.

$$2 \cdot 1''' \leq x_1 = x_2 \leq 2 \cdot 2''' \wedge$$

$$x_3 = 0 \wedge \cdots \wedge x_k = 0 .$$

is to spec. the water levels for all tanks after 2 pumping instructions $P_{\{T_1, T_2\}}, 1''', 2'''$.

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$$1 \cdot 2.2''' \leq x_2 = x_5 \leq 1 \cdot 5''' \wedge$$

$$x_1 = 0 \wedge x_3 = 0 \wedge x_4 = 0 \wedge x_6 = 0 \cdots \wedge x_k = 0,$$

is to spec the water levels for all tanks after 1 pump instruction $P_{\{T_2, T_5\}}, 2.2, 5$.

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Both of above are from water levels $= 0$.

Suppose that

$$\alpha = P_{\{T_1,T_2\},1,2}\ ;\ P_{\{T_2,T_5\},2,2,5}\ ;\ P_{\{T_1,T_2\},1,2}.$$

$$\#_{P_{\{T_1,T_2\},\text{"}1\text{"},\text{"}2\text{"}}}(\alpha) = 2$$

$$\#_{P_{\{T_2,T_5\},\text{"}2.2\text{"},\text{"}5\text{"}}}(\alpha) = 1$$

two counts.

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Can you write a formula for water levels at the end of $\alpha$, in terms of these counts $\#_{\ldots}(\alpha)$.

For each fixed $\alpha$, $W_\alpha$ is a Boolean combination $(\wedge, \vee, \neg)$ of linear constraints over real variables.

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Given: $G$ (without loop).

init, final and pump inst's on edges.

Quest: $\exists \alpha$ from init. to final s.t.

$W_\alpha(x_1, \ldots, x_k) \wedge Bad(x_1 \ldots x_k)$ is sat.?

Case 2. We do have loops on the $G$.

Difficulty: you have inf. many $\alpha$ from init to final. Can't check one by one.

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Let $\Gamma$ be the set of all $\alpha$ from init to final. For each $\alpha$, we have a formula for water levels:

$$W_\alpha(x_1, \dots, x_k).$$

I define a vector

$$\#_p(\alpha), \quad \text{for all } p \text{ on the } G.$$

(for instance, if we only have 5 pump instances on $G$, then the vector is of 5-arity).

Such a vector is denoted by $\#(\alpha)$.

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For all $\alpha \in \Gamma$, we obtain a set

only dep. on $G$ !

of vectors $V = \{ \#(\alpha) : \alpha \in P \}$.

Theorem. There is a integer linear constraint system $\cancel{Q(V}$ $Q(v)$ s.t. $\forall v$,

$v \in V$ iff $Q(v)$ holds.

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(linear constraints over integers also with Boolean combinations and with quantifiers).

Summary:

For each $\alpha \in \Gamma$, we have

$$W_\alpha(x_1, \ldots, x_k).$$

For all $\alpha$, we have a

$$V = \{\#(\alpha) ; \alpha \in P\}$$

that is characterized by $Q$.

$\Rightarrow$ linear constraint system on reals & integers?