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We can relate the body and inertial frame by a rotation matrix R which goes from the body frame to the inertial frame. This matrix is derived by using the ZYZ Euler angle conventions and successively “undoing” the yaw, pitch, and roll.

$$R = \begin{bmatrix} c_\phi c_\psi - c_\theta s_\phi s_\psi & -c_\psi s_\phi - c_\phi c_\theta s_\psi & s_\theta s_\psi \\ c_\theta c_\psi s_\phi + c_\phi s_\psi & c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\psi s_\theta \\ s_\phi s_\theta & c_\phi s_\theta & c_\theta \end{bmatrix}$$

For a given vector \vec{v} in the body frame, the corresponding vector is given by $R\vec{v}$ in the inertial frame.

Unfortunately there is a mistake here. The rotation matrix as a function of roll, pitch, yaw angles is actually derived from rotating around 3 different axes e. g. XYZ (not rotating around 2 axes ZYZ). The final formula, if we make the following association:

Roll – φ – X-axis

Pitch – θ – Y-axis

Yaw – ψ – Z-axis

is given by

$$R = \begin{bmatrix} \cos \psi \cos \theta & \cos \psi \sin \theta \sin \varphi - \cos \varphi \sin \psi & \sin \theta \sin \psi + \cos \varphi \cos \psi \sin \theta \\ \cos \theta \sin \psi & \cos \varphi \cos \psi + \sin \varphi \sin \psi \sin \theta & \cos \varphi \sin \psi \sin \theta - \cos \psi \sin \varphi \\ -\sin \theta & \cos \theta \sin \varphi & \cos \varphi \cos \theta \end{bmatrix}$$

This formula can be derived from elementary rotations around X,Y,Z axes. The derivations can be easily obtained with Matlab’s symbolic toolbox:

```
syms phi theta psi;

Rx = [1      0      0;
      0 cos(phi) -sin(phi); ...
      0 sin(phi)  cos(phi)];

Ry = [ cos(theta) 0 sin(theta);
      0 1 0; ...
      -sin(theta) 0 cos(theta)];

Rz = [cos(psi) -sin(psi) 0; ...
      sin(psi)  cos(psi) 0; ...
      0 0 1];

R = Rz*Ry*Rx
```

Also, see https://en.wikipedia.org/wiki/Rotation_matrix