	PMF	$E(X)$	$Var(X)$	MGF	Sum	Symbol	transformation
Binomial	$P_X(x;n,p)=\begin{cases}\binom{n}{x}p^x(1-p)^{n-x}, & \text{if } x=0,1,2\cdots n;\\ 0 & \text{otherwise}\end{cases}$	np	$np(1-p)$	$M(t)=[(1-p)+pe^t]^n$	$\sum_{i=1}^{m}X_i\sim b(\sum_{i=1}^{m}n_i,p)$	$X\sim b(n,p)$	$n\to\infty \Rightarrow poi(np)$
hypergeometric	$P_X(x;n,M,N)=\begin{cases}\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}, & \text{if } x=0,1,2\cdots n;\\ 0, & \text{otherwise}\end{cases}$	$n\cdot\frac{M}{N}$	$\left(\frac{N-n}{N-1}\right)\cdot n\cdot\frac{M}{N}\left(1-\frac{M}{N}\right)$			$X\sim h(n,M,N)$	
Negative Binomial	$P_X(x;r,p)=\begin{cases}\binom{x+r-1}{r-1}p^r(1-p)^x, & \text{if } x=0,1,2\cdots\\ 0, & \text{otherwise}\end{cases}$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$M(t)=\frac{p^r}{[1-(1-p)e^t]^r},\ t<-\ln(1-p)$	$\sum_{i=1}^{m}X_i\sim nb(\sum_{i=1}^{m}r_i,p)$	$X\sim nb(r,p)$	$r=1 \Rightarrow Geo$
poisson	$P_X(x;\lambda)=\begin{cases}\frac{e^{-\lambda}\lambda^x}{x!}, & \text{if } x=0,1,2\cdots\\ 0, & \text{otherwise}\end{cases}$	λ	λ	$M(t)=e^{\lambda(e^t-1)}$	$\sum_{i=1}^{m}X_i\sim Poi(\sum_{i=1}^{n}\lambda_i)$	$X\sim Poi(\lambda)$	
Gamma	$f_X(x;\alpha,\beta)=\begin{cases}\frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-\frac{x}{\beta}}, & \text{if } x>0\\ 0, & \text{if } x\le 0\end{cases}$	$\alpha\beta$	$\alpha\beta^2$	$M(t)=\frac{1}{(1-\beta t)^\alpha},\ t<\frac{1}{\beta}$	$\sum_{i=1}^{n}X_i\sim\Gamma(\sum_{i=1}^{n}\alpha_i,\beta)$	$X\sim\Gamma(\alpha,\beta)$	
Exponential	$f_X(x;\lambda)=\begin{cases}\lambda e^{-\lambda x}, & \text{if } 0<x<\infty;\\ 0, & \text{otherwise}\end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$		$\sum_{i=1}^{n}X_i\sim\Gamma(n,\frac{1}{\lambda})$	$X\sim Exp(\lambda)$	$\Gamma(\alpha=1,\beta=\frac{1}{\lambda})\Rightarrow Exp(\lambda)$
Chi-Square	$f_X(x;r)=\begin{cases}\frac{1}{\Gamma(\frac{r}{2})2^{r/2}}x^{\frac{r}{2}-1}e^{-\frac{x}{2}}, & \text{if } 0<x<\infty\\ 0, & \text{otherwise}\end{cases}$	r	$2r$		$\sum_{i=1}^{n}X_i\sim\chi^2(\sum_{i=1}^{n}r_i)$	$X\sim\chi^2(\lambda)$	$\Gamma(\alpha=\frac{r}{2},\beta=2)\Rightarrow\chi^2(r)$
Beta	$f_X(x;\alpha,\beta)=\begin{cases}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, & \text{if } 0<x<1\\ 0, & \text{otherwise}\end{cases}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$			$X\sim Beta(\alpha,\beta)$	$\frac{\Gamma(\alpha_1,\beta)}{\Gamma(\alpha_1,\beta)+\Gamma(\alpha_2,\beta)}\sim Beta(\alpha_1,\alpha_2)$
Normal (Gaussian)	$f_X(x;\mu,\sigma)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}},\ -\infty<x<\infty$	μ	σ^2	$M(t)=e^{\mu t+\frac{\sigma^2t^2}{2}},\ -\infty<t<\infty$	$\sum_{i=1}^{n}a_iX_i\sim N(\sum_{i=1}^{n}a_i\mu_i,\sum_{i=1}^{n}a_i^2\sigma_i^2)$	$X\sim N(\mu,\sigma^2)$	$Z^2\sim\chi^2(1)$
Standard Normal	$f_Z(z;0,1)=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}},\ -\infty<z<\infty$					$Z\sim N(0,1)$	$Z=\frac{X-\mu}{\sigma}\sim N(0,1)$; $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i\sim N(\mu,\frac{\sigma^2}{n})$

Multivariate Normal: $X\sim N(\mu_1,\sigma_1^2),\ Y\sim N(\mu_2,\sigma_2^2)$

Corr $=0$, x,y independent: $f(x,y)=f(x)\cdot f(y)$, Corr $\neq 0$: $f_{X,Y}(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\left\{-\frac{q}{2(1-\rho^2)}\right\}$ where $q=\left(\frac{x-\mu_1}{\sigma_1}\right)^2-2\rho\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)^2$

Corr $=0$, $f_{Y|X}(y|x)=f_Y(y)=\frac{1}{\sqrt{2\pi}\sigma_2}\exp\left\{-\frac{(y-\mu_2)^2}{2\sigma_2^2}\right\}$; Corr $\neq 0$ $f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\frac{1}{\sqrt{2\pi}\sigma_2\sqrt{1-\rho^2}}\exp\left\{-\frac{[y-(\mu_2+\rho\sigma_1^{-1}\sigma_2(x-\mu_1))]^2}{2\sigma_2^2(1-\rho^2)}\right\}$

	PMF	$E(X)$	$Var(X)$	Symbol	transformation
t	$f(x;r)=\frac{\Gamma(\frac{r+1}{2})}{\Gamma(\frac{r}{2})}\cdot\frac{1}{\sqrt{r\pi}}\cdot\frac{1}{(1+x^2/r)^{(r+1)/2}}$, for $-\infty<x<\infty$	0 (if $r>1$)	$\frac{r}{r-2}$ (if $r>2$)	$X\sim t(r)$	$\frac{Z}{\sqrt{\chi^2(r)/r}}\sim t(r)$
F	$f(x;r_1,r_2)=\begin{cases}\frac{\Gamma(\frac{r_1+r_2}{2})}{\Gamma(\frac{r_1}{2})\Gamma(\frac{r_2}{2})}\left(\frac{r_1}{r_2}\right)^{\frac{r_1}{2}}\frac{x^{\frac{r_1}{2}-1}}{[1+(r_1/r_2)x]^{(r_1+r_2)/2}}, & \text{if } 0<x<\infty\\ 0, & \text{otherwise}\end{cases}$	$\frac{r_2}{r_2-2}$ (if $r_2>2$)	$2\left(\frac{r_2}{r_2-2}\right)^2\frac{r_1+r_2-2}{r_1(r_2-4)}$ if $r_2>4$		$\frac{\chi^2(r_1)/r_1}{\chi^2(r_2)/r_2}\sim F(r_1,r_2)$

note: gamma function: $\Gamma(\alpha)=\int_0^\infty x^{\alpha-1}e^{-x}dx$ for $\alpha>0$, $\Gamma(m)=(m-1)!$

Student's Theorem: ① $\bar{X}\sim N(\mu,\frac{\sigma^2}{n})$ ② $\bar{X}$ and S^2 are independent ③ $(n-1)S^2/\sigma^2\sim\chi^2(n-1)$ ④ $\frac{\bar{X}-\mu}{S/\sqrt{n}}\sim t(n-1)$

unbiased estimation = $E(\hat{\theta})=\theta$, A point estimator $\hat{\theta}$ for θ is consistent if $\hat{\theta} \xrightarrow{P} \theta$

$$\boxed{CLT \Rightarrow \bar{X}_n \sim N(\mu, \tfrac{\sigma^2}{n})}$$

$$\begin{cases} E[E(X_2|X_1)] = E(X_2) \\ Var[E(X_2|X_1)] \le Var(X_2) \\ Var(X_2) = E[Var(X_2|X_1)] + Var[E(X_2|X_1)] \end{cases}$$

$Cov(X,Y) = E[(X-\mu_x)(Y-\mu_y)] = E(XY) - \mu_x\mu_Y$, $Cov(X,X) = Var(X)$

$\rho_{X,Y} = Corr(X,Y) = \dfrac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$

$Corr(aX+b, cY+d) = \dfrac{ac\,Cov(X,Y)}{|ac|\sqrt{Var(X)}\sqrt{Var(Y)}} = \begin{cases} Corr(X,Y) & ac>0 \\ -Corr(X,Y) & ac<0 \end{cases}$

X_1 and X_2 are independent iff $f(x_1,x_2) = f(x_1)f(x_2)$

iff there exist non negative functions $g(x_1)$ and $h(x_2)$ such that for every $x_1 \in \mathcal{X}_1^+$, $x_2 \in \mathcal{X}_2^+$ $f(x_1,x_2) = g(x_1)h(x_2)$

$\Rightarrow E(u(X_1)v(X_2)) = E(u(X_1))E(v(X_2))$

$\Rightarrow M_{X_1+X_2}(t) = M_{X_1}(t)\cdot M_{X_2}(t) = E(e^{tX_1})E(e^{tX_2})$

$\Rightarrow Cov(X,Y) = 0$

$E\left(\sum_{i=1}^n a_iX_i\right) = \sum_{i=1}^n a_iE(X_i)$

if $X_1, X_2 \cdots X_n$ are dependent: $Var\left(\sum_{i=1}^n a_iX_i\right) = \sum_{i=1}^n\sum_{j=1}^n a_ia_j Cov(X_i,X_j)$

$= \sum_{i=1}^n a_i^2 Var(X_i) + \underbrace{2\sum_{i<j} a_ia_j Cov(X_i,X_j)}_{=0}$

if independent

Order Statistics:

$f_{X_{(1)}}(x_{(1)}) = \begin{cases} nf_X(x_{(1)})[1-F_X(x_{(1)})]^{n-1}, & \text{if } a<x_{(1)}<b \\ 0, & o/w \end{cases}$

$f_{X_{(n)}}(x_{(n)}) = \begin{cases} nf_X(x_{(n)})\cdot[F_X(x_{(n)})]^{n-1}, & \text{if } a<x_{(n)}<b \\ 0, & o/w \end{cases}$

Joint PDF: $f_{X_{(1)},X_{(n)}}(x_{(1)},x_{(n)}) = n(n-1)f_X(x_{(1)})f_X(x_{(n)})[F_X(x_{(n)})-F_X(x_{(1)})]^{n-2}$, $a<x_{(1)}<x_{(n)}<b$.

Joint PDF of all: $f_{X_{(1)}\cdots X_{(n)}}(x_{(1)}\cdots x_{(n)}) = \begin{cases} n!\prod_{i=1}^n f_X(x_{(i)}), & \text{if } a<x_{(1)}<x_{(2)}<\cdots x_{(n)}<b; \\ 0, & o/w \end{cases}$

Chebyshev's Inequality: $P(|X-\mu| \ge k\sigma) \le \frac{1}{k^2}$ or $P(|X-\mu|<k\sigma) \ge 1-\frac{1}{k^2}$

Markov's Inequality: Suppose $u(X)$ is a nonnegative function of X. If $E[u(X)]<\infty$, then for any positive constant a,

$P(u(X)\ge a) \le \dfrac{E(u(X))}{a}$

Jensen's Inequality: $v(x)$ is a convex function ($\rightarrow v''(x)\ge 0$), then $E[v(X)] \ge v(E(X))$. if concave $\Rightarrow \le$

$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix}$

Δ-method: $\sqrt{n}(X_n-\mu) \xrightarrow{D} N(0,\sigma^2)$

if $g(\cdot)$ is differentiable at μ and $g'(\mu)\neq 0$, then

$\sqrt{n}(g(X_n)-g(\mu)) \xrightarrow{D} N(0, \sigma^2\{g'(\mu)\}^2)$

Converges in Probability: $\lim_{n\to\infty} P(|X_n-X|\ge\varepsilon)=0$

or $\lim_{n\to\infty} P(|X_n-X|<\varepsilon)=1$

① $X_n \xrightarrow{P} X$, $Y_n \xrightarrow{P} Y \Rightarrow X_n+Y_n \xrightarrow{P} X+Y$

② $X_n \xrightarrow{P} X \Rightarrow aX_n \xrightarrow{P} aX$

③ $X_n \xrightarrow{P} X$, $g(\cdot)$ is a continuous function $\Rightarrow g(X_n) \xrightarrow{P} g(X)$

④ $X_n \xrightarrow{P} X$, $Y_n \xrightarrow{P} Y \Rightarrow X_nY_n \xrightarrow{P} XY$

Convergence in Distribution: $X_n \xrightarrow{D} X$

X_n converges in distribution to X if $\lim_{n\to\infty} F_{X_n}(x) = F_X(x)$ CDF

$\lim_{n\to\infty}\{1+\frac{a_n}{n}\}^n = e^a$, $\lim_{n\to\infty}\{1+\frac{a}{n}+\frac{b_n}{n}\}^{cn} = e^{ac}$

$X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{D} X$

$X_n \xrightarrow{P} a$ iff $X_n \xrightarrow{D} a$

$X_n \xrightarrow{D} X$, $g(\cdot)$ is continuous $\Rightarrow g(X_n) \xrightarrow{D} g(X)$

$X_n \xrightarrow{D} X$, $Y_n \xrightarrow{P} a \Rightarrow Y_nX_n \xrightarrow{D} aX$, $X_n+Y_n \xrightarrow{D} X+a$

$\lim_{n\to\infty} M_{X_n}(t) = M_X(t) \Rightarrow X_n \xrightarrow{D} X$