Electromagnetism and Electromechanics

MEC102

Lab 1 Electrostatic Field Plotting

Objective:

- To illustrate the pattern of electric field lines associated with systems of conductors.
- To recognize:
  - i) electric charges as the sources and sinks of field lines;
  - ii) the electric field as the (negative) gradient of potential.

1. Introduction

Electrostatics as implied by the component parts of the word relates to the influence and interaction of electric charges at rest. This influence is described in terms of a field of force in the vicinity of electric charges which constitutes the Electric Field (**E**). Formally, the strength of the electric field **E** at a given point in space is defined as the force on unit positive test charge placed at that point.

Pre-lab Q1: Can you think of other fields of force that you have encountered in your science studies?

It is important for electrical engineers to know the distribution of the electric fields and potentials in many practical situations, For example, in the vicinity of high voltage insulators and conductors used in the electricity supply industry, where excessive field stress may lead to flashover phenomena. In the course of the present experiment we set out to identify the distribution of potentials in the region of systems of conductors comprising of i) two parallel plates, and ii) two concentric cylinders. The basis of the experimental method is a conducting-paper plotting technique, which allows the voltage distribution to be assessed by analogy between the flow of steady current in a resistive medium and the electrostatic field in a charge-free space.

The voltage (V) at a point in an electrostatic field, more correctly identified as the electrostatic potential at the point, is defined as the amount of work (J) performed in bringing a unit positive charge from infinity to that point. The electric field and potential are closely related; formally we express the electric field as the (negative) gradient of potential. The term gradient has a strict mathematical meaning when relating a vector quantity such as the **E** field, to a scalar quantity (the potential). By way of introduction to this experiment, however, it is useful to visualize the two parameters by analogy with the presentation of contour lines and associated slopes on an ordinance survey map. The slope is perpendicular to the contour lines and represents the steepest descent at a given point. In the same way the **E** field is directed perpendicular to the lines of constant or equi-potential, and presents the maximum "slope" of potential at a point. In the case of a field directed uniquely along a direction *x*, the expression linking the magnitude of the field and potential simplifies to:

$$E = -\frac{dV}{dx}$$

Our immediate task in this experiment is the determination of lines of constant potential, the so called equipotentials. The **E** field can be deduced by constructing lines at right angles to the equipotentials.

Further to an experimental investigation, the opportunity may also be available to investigate the fields by numeric, computer solution of the equation (Laplace's) governing the behavior of the electrostatic field.

2. Gauss's Law

Positive and negative electric charges are the sources and sinks of electric fields and this underlines the depiction in Figures 1 and 2 (in Appendix) of the fields and charges with the two systems of conductors used in the experiment. Note the field lines originating at the surface of one (positively) charged conductor and terminating with the negative charge at the companion electrode. The correspondence between field and charge is contained in Gauss's Law which states that the integral of the electric field (strictly the normal component) over a closed surface (S) is equal to the total charge Q contained by the closed surface, divided by the permittivity ε, i.e.

$$\int \mathbf{E} \bullet d\mathbf{S} = \frac{Q}{\varepsilon_0 \varepsilon_r}$$

Here the permittivity is expressed in terms of the permittivity of free-space ε_0 by writing $\varepsilon = \varepsilon_0\varepsilon_r$, where ε_r is the relative permittivity of the medium in which the electrodes are situated. By forming the product of the **E** field and the permittivity the equation is rewritten as:

$$\int \mathbf{D} \bullet d\mathbf{S} = Q$$

which when integrated over the closed surface is now seen to be directly linked to the total charge. **D** is customarily known as the Electric Flux Density and is a very convenient parameter, particularly when examining electric phenomena in mixed dielectric media.

a) Parallel plate conductors

In the case of the parallel plate conductors, we can draw a series of closed surfaces similar to S_1 as shown, but set at different distances between the parallel plates - in each case the density of field lines between the plates is constant.

b) Concentric or coaxial cylinders

The density of field lines passing through a closed surface such as S_2 in Figure 2 behaves differently. The pattern is symmetric about the centre for any closed surface formed with radius $a < r < b$, there is a constant number of field lines, but the surface area increases with increasing radius.

***Pre-lab Q*2**: In terms of the information already cited in the introduction, why is it the negative and not the positive of the gradient of potential that is involved here?

***Pre-lab Q*3**: What are the units of electric flux density and electric field?
What is the relationship between them?

3. Experimental procedure

The complete electrical circuit for this experiment is shown in Figure 3 (Appendix).

Start by placing the electrodes in the experiment set as shown in Figure 1. Make connections to the electrodes. Pour clean water into the experiment set. The connecting wires are led to the power supply and the graphite probe connected to the high-impedance voltmeter. The negative terminal of the voltmeter is connected to the corresponding negative terminal of the voltage supply. Set the frequency to be around 200Hz, and set the voltage to be +8V to the positive electrode.

The position of points at a common, fixed potential can now be traced by moving the probe tip across the water. When the desired voltage is located, read the location of the point and write it in your laboratory log book. By joining together the points we form the equipotential line at the set potential.

With the parallel plate geometry, probe the equipotentials (e.g. at 1, 2, 4 and 5 volt), ensuring that:

i) your plots extend a few centimeters beyond the inter-electrode region,
ii) the equipotential pattern for the 1 and 5 V contour is probed even in the region to the rear of the electrodes, and
iii) at least six to eight points should be measured for each equipotential line.

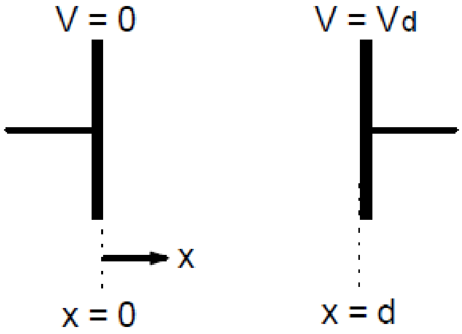
For the concentric circle geometry proceed along similar lines with 8 V at the centre electrode, but as noted earlier confine your plotting to the hemi-cylindrical region opposite the slot bearing the connecting wire. Investigate equipotentials (e.g. at 5, 4, 2.5 and 1 V).

Make a tracing of your equipotential distribution and electrode geometry and construct the field distribution for the both parallel and cylindrical geometries. (Remember what was stated in the introduction concerning the relative directions of **E** and lines of constant voltages).

4. Predictions of voltage and field distributions

A gradient of potential unique to one direction leads to the simplest form of the gradient operator as discussed on page 1. For the two sets of electrodes used here this simplest form is applicable, since the field is predicted to be directed perpendicular to the plates for the parallel plate conductors, and directed along the radial direction in the case of the concentric cylinders.

a) Parallel plate geometry



For parallel plate geometry shown above, $E = -\frac{dV}{dx} = \text{constant}$. Integration leads to $V = -Ex + B$, where B is a constant.

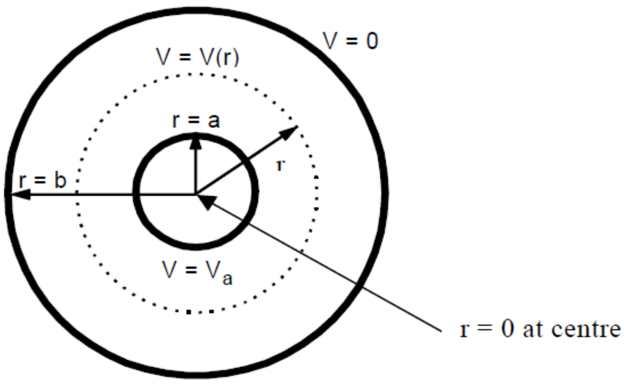
Consider the boundary conditions:

i) $V = 0$ for $x = 0$, it therefore follows that $B = 0$, and

ii) $V = V_d$ for $x = d$, thus $|E| = V_d / d$

By substitution, we have: $V(x) = \frac{V_d}{d} x$

b) Concentric cylinders



In this case, $E(r) = -\frac{dV}{dr} = \frac{Q}{2\pi r\varepsilon}$. Follows from Gauss's Law applied at a closed surface drawn at an arbitrary radius $a < r < b$. (recall, Q is the total charge at the enclosed). Rearranging the relation between E and V to integral form leads to:

$$V_{rb} = -\int_b^r e(r)dr$$

where the subscripts rb indicate the potential at radius r taken with the outer cylinder as reference. Substituting for E gives:

$$V_{rb} = -\frac{Q}{2\pi\varepsilon}\int_b^r \frac{dr}{r} = \frac{Q}{2\pi\varepsilon}\ln\left(\frac{b}{r}\right)$$

The boundary conditions are:
i) $V = 0$ for $r = b$, which implies that V_{rb} express the absolute potential V_r,
ii) $V = V_a$ at the centre electrode situated at $r = a$.

Making use of condition ii) we can formulate:

$$V_a = V_{ab} = \frac{Q}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right), \quad \text{or} \quad Q = \frac{2\pi\varepsilon V_a}{\ln(b/a)}$$

Substituting for Q in the expression for V_{rb} above leads to:

$$V_{rb} = V_a \frac{\ln(b/r)}{\ln(b/a)}$$

$E(r)$ can be found in terms V_a, b and a by using $E = -dV/dr$, or by substituting for Q in terms of V_a, a and b in the initial expression for E. Check both procedures and display the calculated results for E in your earlier graph.

5. Laboratory report

Your report must have the following:

a) Cover page

Including the experiment title, your name, ID number, the name who are in your group, the lab's date.

b) Abstract (10%)

A short section of between 50 and 300 words which must be capable of being read and understood independently of the rest of the report. This section should briefly summarise

- The purpose and scope of the experiment,
- The experimental procedures that were carried out,
- The main conclusions.

c) Introduction (10%)

This section describes, in general terms, the scope of the experiment and its relevance to the field of study you are engaged in. A statement of objectives should be given along with general comments about how the experiment will be carried out.

d) Main body (45%)

***Task* 1:** Comment the main feature of the **E** field in the both parallel and cylindrical geometries.

***Task* 2**: Using Gauss' Law to express the electric field in terms of the radial position r and the total charge Q at the center cylindrical electrode. (Starting by writing the result for the curved surface area of a cylinder of unit length drawn at any arbitrary radius $a<r<b$).

***Task* 3**: Make a tracing of your equipotential distribution and electrode geometry and construct the field distribution for the cylindrical geometries. (Remember what was stated in the introduction concerning the relative directions of **E** and lines of constant voltages) Draw graphs showing the electrostatic potential as a function of the distance from the center ($r = 0$) for $a < r < b$ to the position of the outer electrode for the cylindrical electrodes respectively.

***Task* 4:** Draw graphs showing the electrostatic potential as a function of the distance from the negative electrode for the parallel electrodes. Make a tracing of your equipotential distribution and electrode geometry and construct the field distribution for the parallel geometries. (Remember what was stated in the introduction concerning the relative directions of E and lines of constant voltages).

From the distribution of potential deduce and plot a graph of the field strength as a function of the distance. In this task, follow the equipotentials along the central axis for the parallel conductors, and select about four points at which to draw a tangent to you graph for the cylindrical electrodes.

***Task* 5:** Using the results developed in the section of predictions of voltage and field distributions you should calculate the theoretical voltages and fields for the two sets of electrodes, and you should plot these on your graphs for comparison between theory and experiment.

Comment on the gradient of your graphs.

e) Error analysis and Discussion (5%)

Including: individual equipment and process/method used in experiment, the equivalent circuits and others.

f) Conclusion (5%)

Briefly describe the experiment as you undertook it, the major findings, any errors or difficulties that you found, and comment on the relevance to your study.

g) References (5%)

You may refer to textbooks, research papers, magazine articles, data sheets and lecture notes as references. References should be listed in a separate section at the end of the report, in the proper form.

h) Appendix (20%)

Please include all answers of the pre-lab questions in the appendix section of your report.

6. Assessment

You are required to submit a formal report individually for this experiment. The deadline for your group can be found from Learning Mall Online.

This report contributes 10% to module MEC102.

Appendix

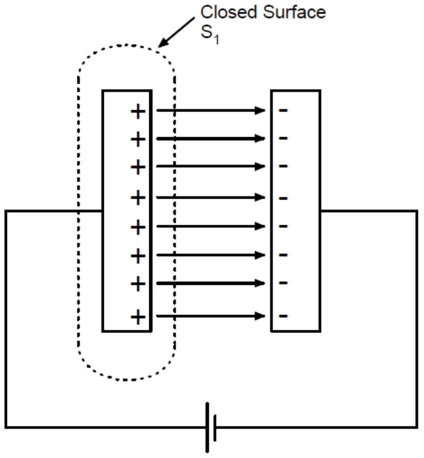


Figure 1 Charges and **E** Field with Parallel Plates

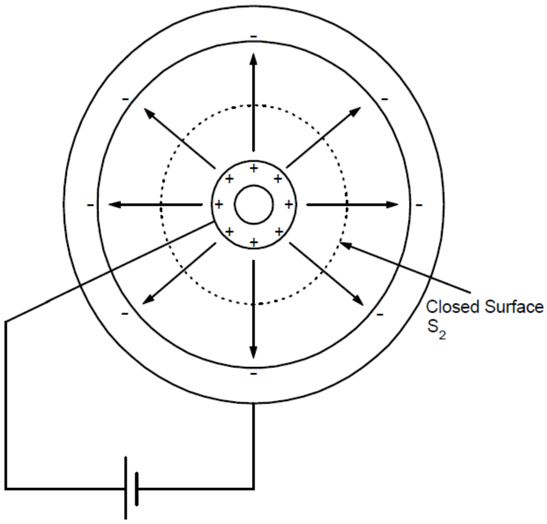


Figure 2 Charges and **E** Field with Concentric Cylinders

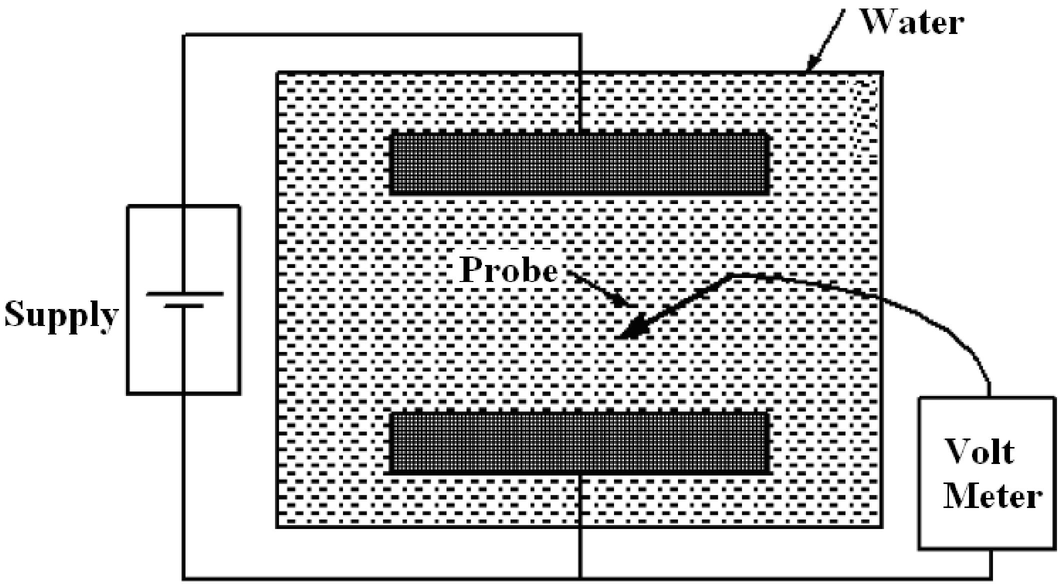


Figure 3 The Electrical Circuit for Plotting Equipotential Lines