



Department of Electrical and Electronic Engineering

MEC104 Experimental, Computer Skills and Sustainability

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**MATLAB Assignment**

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Submission date:05/10/2021

# Catalogue

<b>Problem 1 Matrix Operation</b>	4
<b>1.Equation derivations</b>	4
<b>2.Main programme</b>	4
2.1 For P1-1	4
2.2 For P1-2	4
2.3 For P1-3	5
2.4 For P1-4	5
<b>3.Functions</b>	6
<b>4.Results</b>	6
4.1 For P1-1	6
4.2 For P1-2	7
4.3 For P1-3	9
4.4 For P1-4	10
<b>Problem 2</b>	11
<b>1.Equation derivations</b>	11
1.1 For P5-1	12
1.2 For P5-2	12
<b>2.Initial conditions</b>	12
2.1 For P5-1	12
2.2 For P5-2	13
<b>3.Main programme</b>	13
3.1 For P5-1	13
3.2 For P5-2	13
<b>4.Functions</b>	15
<b>5.Results</b>	16
5.1 For P5-1	16
5.2 For P5-2	18
<b>6.Analysis of results and code</b>	19
6.1 For P5-1	19
6.2 For P5-2	20
<b>Problem 3</b>	20
<b>1.Equation derivation</b>	20
1.1 For Question a	21
1.2 For Question b	21
<b>2.Initial conditions</b>	21
<b>3.Main programme</b>	21
3.1 Question a	22
3.2 Question b	22
<b>4.Functions</b>	23
<b>5.Results</b>	24
5.1 Question a	24

5.2 Question b.....	25
<b>6.Result analysis.....</b>	<b>26</b>
<b>Problem 4.....</b>	<b>27</b>
<b>1.Equation derivation.....</b>	<b>27</b>
<b>2.Initial conditions.....</b>	<b>28</b>
<b>3.Main programme and Simulink model.....</b>	<b>28</b>
<b>4.Functions.....</b>	<b>29</b>
<b>5.Results.....</b>	<b>30</b>
<b>6.Result analysis.....</b>	<b>31</b>
<b>7.Flow chart.....</b>	<b>32</b>
7.1 The flow chart for the Simulink model.....	32
7.2 The flow chart for the whole system.....	33
<b>Problem 5.....</b>	<b>34</b>
<b>1.Equation derivation.....</b>	<b>34</b>
<b>2.Initial conditions and boundary conditions.....</b>	<b>34</b>
<b>3.Main programme.....</b>	<b>36</b>
<b>4.Functions.....</b>	<b>37</b>
<b>5.Results.....</b>	<b>38</b>
<b>6.Result analysis.....</b>	<b>41</b>
<b>7.Flow chart.....</b>	<b>42</b>
7.1 Iterative method function flow chart .....	42
7.2 Input function flow chart.....	43
<b>Reference list.....</b>	<b>44</b>

# Problem 1 Matrix Operation

## 1. Equation derivations

In the first problem, we have four problems, and for each problem, we will provide the corresponding solution including the code, result and result analysis.

For P1-1, we need to change the matrix through the code to create satisfied vectors and arrays.

For P1-2, we first need to transform the matrix A to get the matrix B. Next, we will multiply, divide the elements and then get the max value in matrix A and B.

For P1-3, we need to measure the pressure in every cables by matlab calculation. If we assume the weight of the object is mg, the final expressions should be these values multiplied by mg.

For P1-4, we need to calculate every temperature and the heat loss rate q. In addition, if we assume the area is  $10 \text{ m}^2$ , we also need to calculate the heat loss rate in this condition.

## 2. Main programme

### 2.1 For P1-1:

```
A=[3 7 -4 12;-5 9 10 2;6 13 8 11;15 5 4 1]
v=A(:, 2)
w=A(2, :)
B=A(:, 2:4)
C=A(2:4, :)
D=A([1 2], 2:4)
```

Figure 1: The code required for P1-1

### 2.2 For P1-2:

```

A=[1 4 2;2 4 100;7 9 7;3 pi 42]
B=log(A)
The_second_row_of_B = B(2,:)
The_sum_of_the_second_row_of_B = sum(B(2,:))
The_product_of_B_and_A = B(:,2).*A(:,1)
Maximum_value = max(B(:,2).*A(:,1))
result1=A(1,:)
result2=B(1:3,3)'
result=A(1,:)./B(1:3,3)'
The_sum_of_the_elements = sum(result)

```

Figure 2: The code required for P1-2

## 2.3 For P1-3:

```

mg=1
A=[1/sqrt(35) -3/sqrt(34) 1/sqrt(42);
   3/sqrt(35) 0 -4/sqrt(42);
   5/sqrt(35) 5/sqrt(34) 5/sqrt(42)]
b=[0 0 mg]
T=A\b'
fprintf(' T1=%f\n', T(1,1))
fprintf(' T2=%f\n', T(2,1))
fprintf(' T3=%f\n', T(3,1))
fprintf(' T1=%fmg\n', T(1,1))
fprintf(' T2=%fmg\n', T(2,1))
fprintf(' T3=%fmg\n', T(3,1))

```

Figure 3: The code required for P1-3 when mg=1

## 2.4 For P1-4:

```

%Initial conditions
T1=20;
T0=-10;
R1=0.036;
R2=4.01;
R3=0.408;
R4=0.038;
%Calculate the temperature and q
A=[R1+R2 -R1 0;R3 -R2-R3 R2;0 R4 -R3-R4];
B=[R2*T1;0;-R3*T0]
x=A\b
fprintf(' T1=%f°C\n', x(1,1))
fprintf(' T2=%f°C\n', x(2,1))
fprintf(' T3=%f°C\n', x(3,1))
q=(x(1,1)-x(2,1))/R2

```

Figure 4: Display three temperature and heat loss rate q

```
%Initial conditions
T1=20;
T0=-10;
R1=0.0036;
R2=0.401;
R3=0.0408;
R4=0.0038;
%Calculate the temperature and q
A=[R1+R2 -R1 0;R3 -R2-R3 R2;0 R4 -R3-R4];
B=[R2*T1;0;-R3*T0]
x=A\B
fprintf(' T1=%f°C\n', x(1,1))
fprintf(' T2=%f°C\n', x(2,1))
fprintf(' T3=%f°C\n', x(3,1))
q=(x(1,1)-x(2,1))/R2
```

Figure 5: Display the heat loss rate when the wall's area is 10 m<sup>2</sup>

### 3. Functions

For the first question, we mainly used the example that the teacher taught in class to extract the elements of the matrix in the question.

For the second problem, we calculated it on the basis of extracting elements, and calculated the maximum value with some functions, such as `max()`.

For problem three, we convert the actual example into a matrix, and we compute the matrix.

For problem four, we computed two matrices on the basis of the initial conditions.

### 4. Results

#### 4.1 For P1-1:

```

A =
    3     7    -4    12
   -5     9    10     2
    6    13     8    11
   15     5     4     1

v =
    7
    9
   13
    5

w =
   -5     9    10     2

B =
    7    -4    12
    9    10     2
   13     8    11
    5     4     1

C =
   -5     9    10     2
    6    13     8    11
   15     5     4     1

D =
    7    -4    12
    9    10     2

```

Figure 6 : The result of each small question

Result analysis: we understand the operation of the matrix, successfully realized the requirements of the topic.

## 4.2 For P1-2:

A =

1.0000	4.0000	2.0000
2.0000	4.0000	100.0000
7.0000	9.0000	7.0000
3.0000	3.1416	42.0000

B =

0	1.3863	0.6931
0.6931	1.3863	4.6052
1.9459	2.1972	1.9459
1.0986	1.1447	3.7377

The\_second\_row\_of\_B =

0.6931	1.3863	4.6052
--------	--------	--------

The\_sum\_of\_the\_second\_row\_of\_B =

6.6846

The\_product\_of\_B\_and\_A =

1.3863
2.7726
15.3806
3.4342

Maximum\_value =

15.3806

result1 =

1	4	2
---	---	---

result2 =

0.6931	4.6052	1.9459
--------	--------	--------

result =

1.4427	0.8686	1.0278
--------	--------	--------



```
The_sum_of_the_elements =  
  
3.3391
```

Figure 7: The results for P1-2

Result analysis: we first get the B matrix, and then according to the B matrix, we show the second row of its data. Then we implemented the sum using the sum () function. The max () function finds the maximum value and displays it.

### 4.3 For P1-3:

```
A =  
  
0.1690 -0.5145 0.1543  
0.5071 0 -0.6172  
0.8452 0.8575 0.7715  
  
b =  
  
0 0 1  
  
T =  
  
0.5071  
0.2915  
0.4166  
  
T1=0.507093  
T2=0.291548  
T3=0.416619  
T1=0.507093mg  
T2=0.291548mg  
T3=0.416619mg
```

Figure 8: The results of P1-3, including values when mg =1 and results multiplied by mg

Analysis of the results: we first assumed that  $mg = 1$  and calculated the tension of each rope. And then finally, according to the problem requirement, we put every result in units  $mg$ .

#### 4.4 For P1-4

```
命令行窗口
B =
    80.2000
         0
    4.0800

x =
    19.7596
   -7.0214
   -9.7462

T1=19.759573℃
T2=-7.021371℃
T3=-9.746215℃

q =
    6.6785
```

Figure 9: The result for three temperature and heat loss rate  $q$

```

B =

    8.0200
         0
    0.4080

x =

    19.7596
   -7.0214
   -9.7462

T1=19.759573°C
T2=-7.021371°C
T3=-9.746215°C

q =

    66.7854

```

Figure 10: The result for  $10 \text{ m}^2$  area

Through analysis, we found that when the area was  $10\text{m}^2$ , the numerical value of T1, T2 and T3 remained unchanged. However, the magnitude of q is ten times comparing with the area of  $1\text{m}^2$ . Therefore, we can conclude that as the area of the wall increases, the magnitude of q will also increases.

## Problem 2

### 1.Equation derivations

#### 1.1 For P5-1

We need to solve the equation of motion for a pendulum. Because the equation is given a different condition in every question, we need to solve The equation from the given conditions.

The equation is as follows:

$$L\ddot{\theta} + g \sin \theta = a(t) \cos \theta \quad (1)$$

For the first question:  $a = 5m / s^2$  and  $\theta(0) = 0.5rad$

For the second question:  $a = 5m / s^2$  and  $\theta(0) = 3rad$

For the third question:  $a = 0.5t m / s^2$  and  $\theta(0) = 0.5rad$

## 1.2 For P5-2

We also need to solve an equation for the motion of a body attached to a spring.

The equation is as follows:

$$3\ddot{y} + 75y = f(t) \quad (2)$$

We know  $f(t) = 10 \sin(\omega t)$ .

We're going to be based on  $\omega = 1 rad, 5 rad, 10 rad$ , Plot  $y(t)$  for  $0 \leq t \leq 20 s$  and compare.

## 2. Initial conditions

### 2.1 For P5-1

$$g = 9.81m / s^2$$

$$L = 1m$$

$$\dot{\theta}(0)=0$$

## 2.2 For P5-2

$$y(0) = \dot{y}(0) = 0$$

## 3.Main programme

### 3.1 For P5-1

#### Question a

---

```
%Simplify the equation
f=@(t,theta,a) [theta(2);a(t)*cos(theta(1))-9.81*sin(theta(1))];
a=@(t) 5;
[t1,y1]=ode45(@(t,theta) f(t,theta,a),[0 10],[0.5;0]);
plot(t1,y1(:,1));
xlabel('Time(t)');
ylabel('Theta(t)');
```

Figure 11: The code for the first question:  $a = 5m/s^2$  and  $\theta(0) = 0.5rad$

#### Question b

```
%Simplify the equation
f=@(t,theta,a) [theta(2);a(t)*cos(theta(1))-9.81*sin(theta(1))];
a=@(t) 5;
[t2,y2]=ode45(@(t,theta) f(t,theta,a),[0 10],[3;0]);
plot(t2,y2(:,1));
xlabel('Time(t)');
ylabel('Theta(t)');
```

Figure 12: The code for the first question:  $a = 5m/s^2$  and  $\theta(0) = 3rad$

## Question c

```
% Simplify the equation
f=@(t, theta, a) [theta(2); a(t)*cos(theta(1))-9.81*sin(theta(1))];
a=@(t) 0.5*t;
[t3, y3]=ode45(@(t, theta) f(t, theta, a), [0 10], [3;0]);
plot(t3, y3(:, 1));
xlabel('Time(t)');
ylabel('Theta(t)');
```

Figure 13: The code for the third question:  $a = 0.5t \text{ m/s}^2$  and  $\theta(0) = 0.5\text{rad}$

## 3.2 For P5-2

## Question a

```
syms y(t)
Dy = diff(y);
w=1;
equation = diff(y, t, 2) == (10*sin(w*t)-75*y)/3;
IC1 = y(0) == 0;%initialCondition
IC2 = Dy(0) == 0;
initialConditions = [IC1 IC2];
Sol(t) = dsolve(equation, initialConditions);
t=0:.01:20;
plot(t, Sol(t))
xlabel('Time(t)');
ylabel('y(t)');
```

Figure 14: The code for  $\omega=1$

## Question b

```

syms y(t)
Dy = diff(y);
w=5;
equation = diff(y,t,2) == (10*sin(w*t)-75*y)/3;
IC1 = y(0) == 0;%initialCondition
IC2 = Dy(0) == 0;
initialConditions = [IC1 IC2];
Sol(t) = dsolve(equation, initialConditions);
t=0:.01:20;
plot(t, Sol(t))
xlabel('Time(t)');
ylabel('y(t)');

```

Figure 15: The code for  $\omega=5$

### Question c

```

syms y(t)
Dy = diff(y);
w=10;
equation = diff(y,t,2) == (10*sin(w*t)-75*y)/3;
IC1 = y(0) == 0;%initialCondition
IC2 = Dy(0) == 0;
initialConditions = [IC1 IC2];
Sol(t) = dsolve(equation, initialConditions);
t=0:.01:20;
plot(t, Sol(t))
xlabel('Time(t)');
ylabel('y(t)');

```

Figure 16: The code for  $\omega=10$

## 4.Functions

I use ode45 function which is to solve nonrigid differential equations in P5-1. We solve this problem by constructing the two functions. Moreover, I use the diff() function to solve the question in P5-2.  $Y = \text{diff}(X,n,\text{dim})$  is the nth difference calculated along the dimension specified by dim. The dim input is a positive integer scalar.

## 5.Results

### 5.1 For P5-1

#### Question a

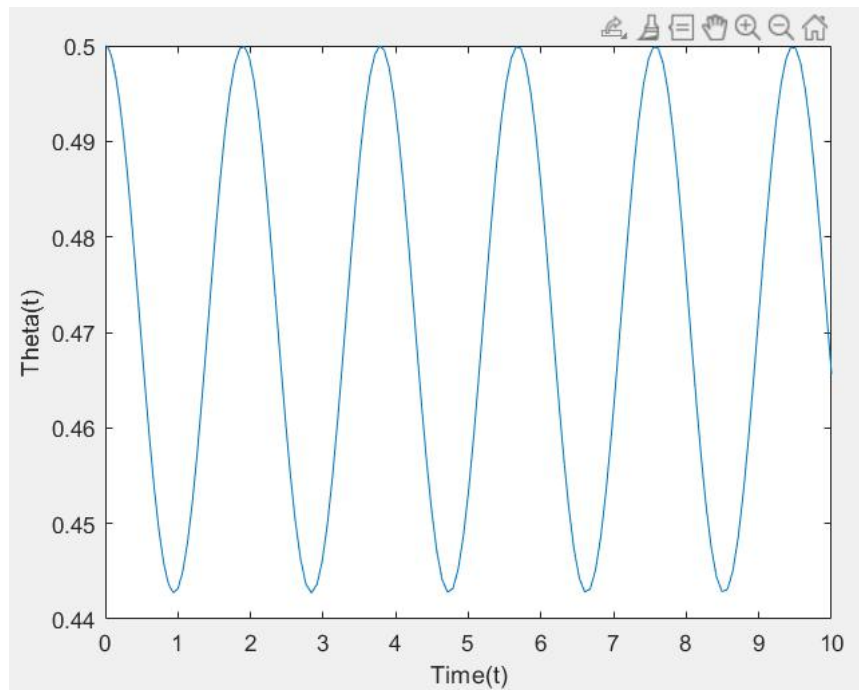


Figure 17: The graph for  $a = 5m/s^2$  and  $\theta(0) = 0.5rad$

#### Question b



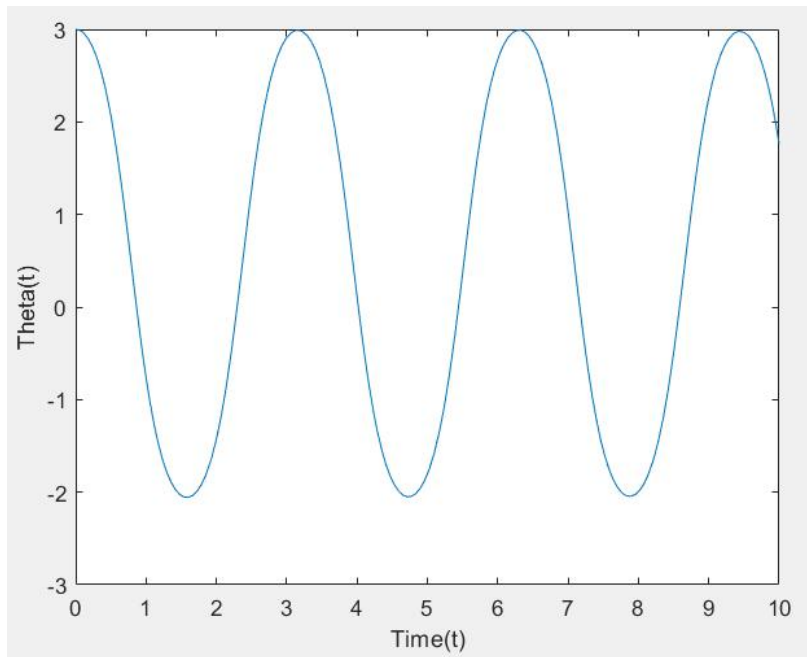


Figure 18: The graph for  $a = 5 \text{ m/s}^2$  and  $\theta(0) = 3 \text{ rad}$

### Question c

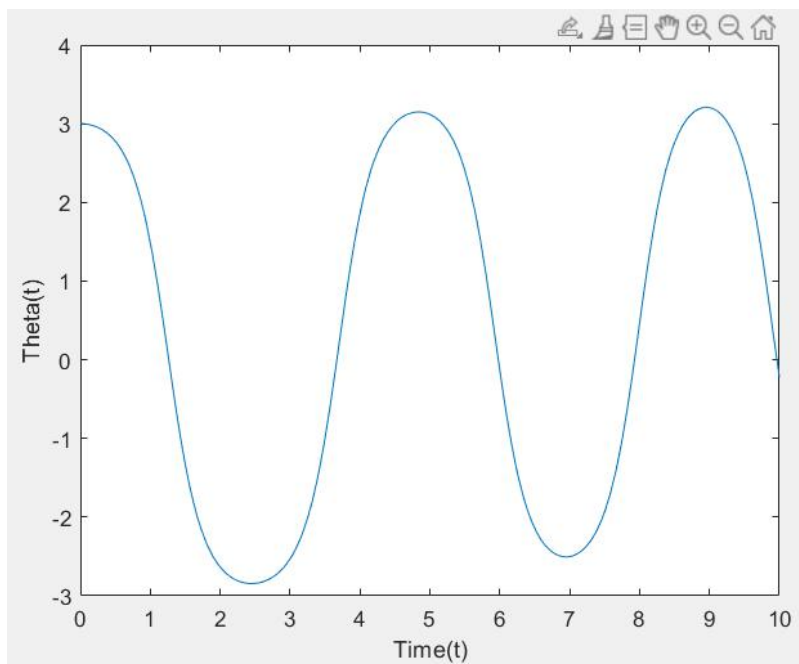


Figure 19: The graph for  $a = 0.5 \text{ m/s}^2$  and  $\theta(0) = 0.5 \text{ rad}$

## 5.2 For P5-2

### Question a

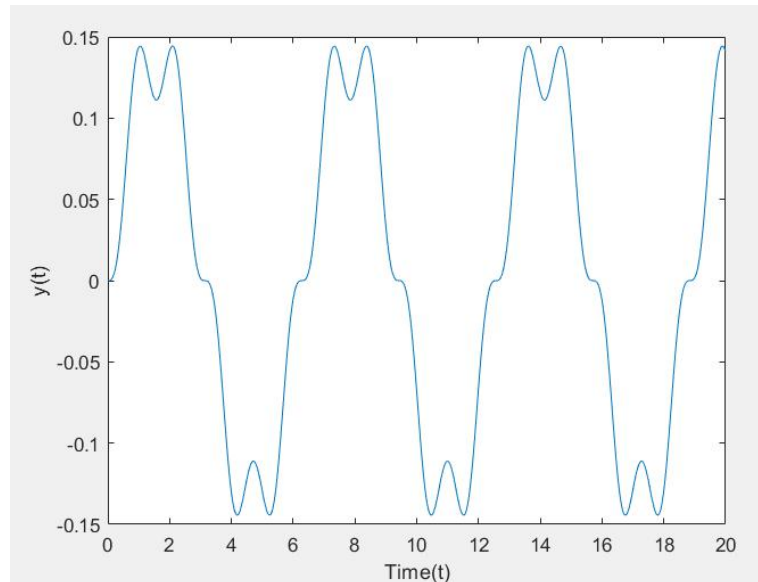


Figure 20: The graph for  $\omega = 1 \text{ rad/s}$

### Question b

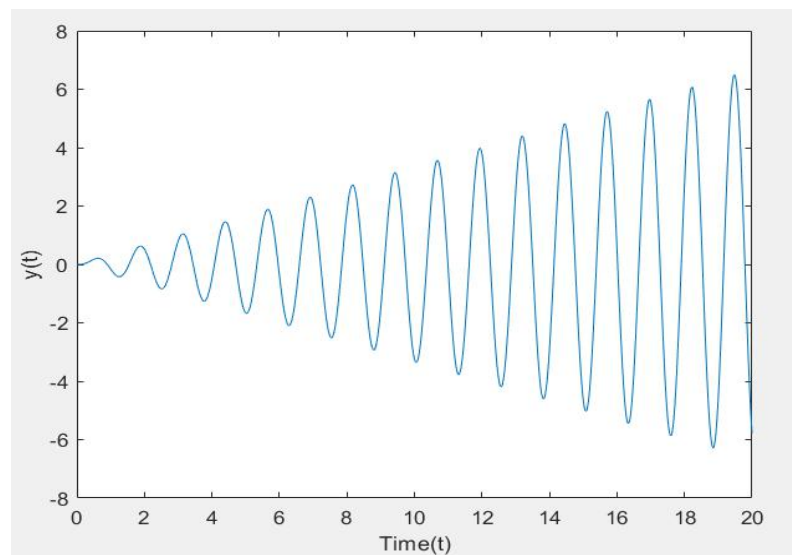


Figure 21: The graph for  $\omega = 5 \text{ rad/s}$

### Question c

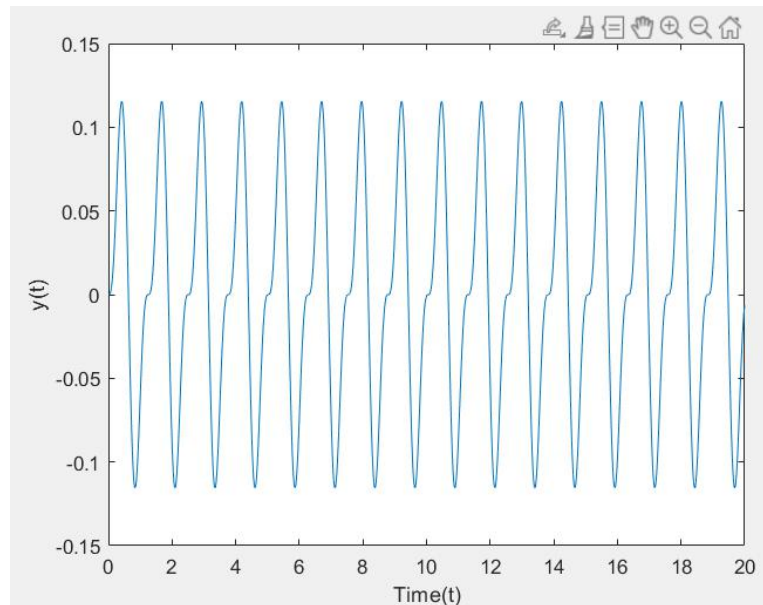


Figure 22: The graph for  $\omega=10 \text{ rad/s}$

## 6. Analysis of results and code

### 6.1 For P5-1

#### Specific steps:

- 1) Because of the condition of the third question, we first take  $\mathbf{a}$  as a function of  $\mathbf{t}$ , let  $\mathbf{a}$  be a constant function  $\mathbf{a}=5$  in the first two questions, and let  $\mathbf{a}=0.5\mathbf{t}$  in the third question.
- 2) And then we simplify the equation so that  $\ddot{\theta}$  is on the left side of the equation, and others will be placed on the right side.
- 3) Next, we use the function: `ode45` to solve the problem
- 4) Finally, we plot the final result: The graph of  $\theta$  with respect to time  $\mathbf{t}$

## 6.2 For P5-2

We first defined the initial condition  $y(0) = \dot{y}(0) = 0$  of the function, and after specifying the range of time  $t$  [0-20s], MATLAB simulated the function image of  $y(t)$  versus time  $t$ .

### Specific steps:

1) Firstly, we put the derivative of  $y$  on the left side of the equation, and put the rest on the right side of the equation

2) We use the `diff()` function to find the solution of the equation

3) Next, we define the initial conditions of the equation

4) We set a time range

5) Draw graphs

**We carefully compared images of  $y(t)$  versus time  $t$  with different values of  $\omega$ . Different  $\omega$  corresponds to different curve shapes. As long as the value of  $\omega$  is changed, it will have a great impact on the whole curve.**

## Problem 3

### 1. Equation derivations

We need to plot the motor's speed and current versus time graph based on the equations for an armature-controlled dc motor.

The equations are as follows:

$$L \frac{di}{dt} = -Ri - K_e \omega + v(t) \quad (1)$$

$$J \frac{d\omega}{dt} = K_T i - c \omega \quad (2)$$

### 1.1 For Question a:

We first assume the applied voltage is 20V, and the time will be in the range of 0-0.5s

### 1.2 For Question b:

We assume the applied voltage is trapezoidal. The equations are as follows:

$$v(t) = \begin{cases} 20 & 0.05 \leq t \leq 0.2 \\ 400t & 0 \leq t < 0.05 \\ -400(t-0.2) + 20 & 0.2 < t \leq 0.25 \\ 0 & t > 0.25 \end{cases} \quad (3)$$

We first need to plot the motor's speed versus time. The time range is [0-0.3s]. At the same time, we will also draw the image of applied voltage versus time.

### 2. Initial conditions:

$R=0.8\Omega$ ,  $L=0.003H$ ,  $K_T=0.05N \cdot m / A$ ,  $K_e=0.05V \cdot s / rad$ ,  $c=0$  and

$J=8 \times 10^{-5} kg \cdot m^2$   $i(0)=0$ ,  $\omega(0)=0$

### 3. Main programme

### 3.1 Question a:

```
syms i(t) speed(t)
eqn1=0.003*diff(i,t,1)==-0.8*i-0.05*speed+20;
eqn2=0.00008*diff(speed,t)==.05*i;
Initial_condition1=i(0)==0;
Initial_condition2=speed(0)==0;
sol=dsolve([eqn1;eqn2],[Initial_condition1;Initial_condition2]);
fplot(sol.i,[0,0.3])
xlabel('time t')
ylabel('i(t)')
figure
fplot(sol.speed,[0,0.3])
xlabel('time t')
ylabel('Speed(t)')
```

Figure 23: The code for the first question(Current and speed versus time)

### 3.2 Qusetion b

```

R=0.8;L=0.003;c=0;
K_T=0.05;K_e=0.05;J=8e-5;
A=[-R/L, -K_e/L;K_T/J, -c/J];
B=[1/L;0];
C1=[0 1];
C2=[1 0];
D=[0];
sys1=ss(A, B, C1, D);
sys2=ss(A, B, C2, D);
time=0:0.0001:0.3;
k=0;
for t=0:0.0001:0.3
    k=k+1
    if t<0.05
        v(k)=400*t;
    elseif t<=0.2
        v(k)=20;
    elseif t<=0.25
        v(k)=-400*(t-0.2)+20;
    else
        v(k)=0;
    end
end
plot(time, v);
xlabel('t(s)');
ylabel('Applied voltage(V)');
axis([0, 0.3, 0, 24]);
figure
[y, t]=lsim(sys2, v, time);
plot(time, y);

xlabel('t(s)');
ylabel('Current(A)');
axis([0, 0.3, -50, 50]);
figure
[y, t]=lsim(sys1, v, time);
plot(time, y);
xlabel('t(s)');
ylabel('Speed(rad/s)');
axis([0, 0.3, 0, 500]);

```

Figure 24: The code for the the motor' s applied voltage、 speed and current versus time

## 4.Functions

For question a, we use `dsolve()` function to draw the image, through the construction of two equations and initial conditions, we can use MATLAB to solve the problem.

For question b, we use `[y,t]=lsim(sys,v,time)` and `plot()` functions to plot. Compared with the method of P5-1, `lsim()` function can set the time, which is more convenient, but both methods can achieve the purpose.

## 5.Results

### 5.1 Question a

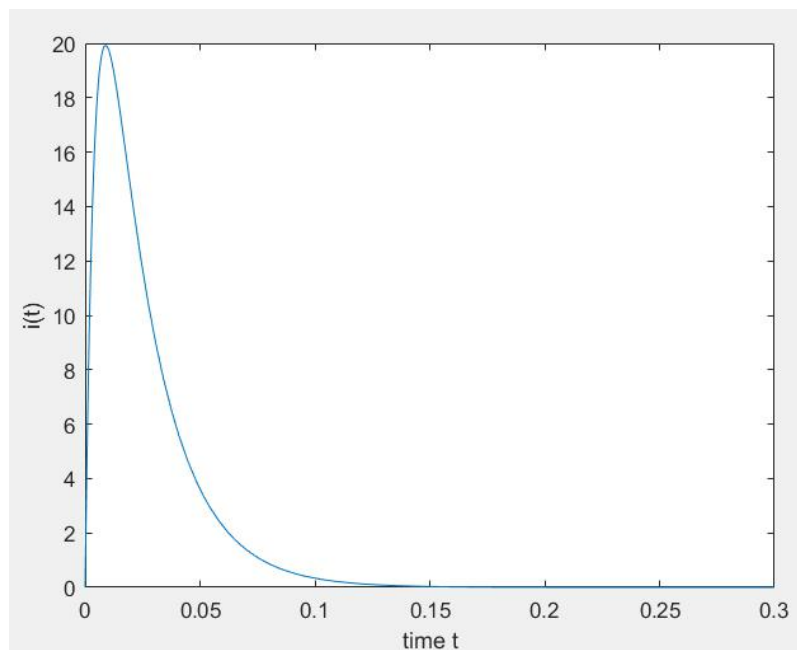


Figure 25: The result of Current versus Time(t)



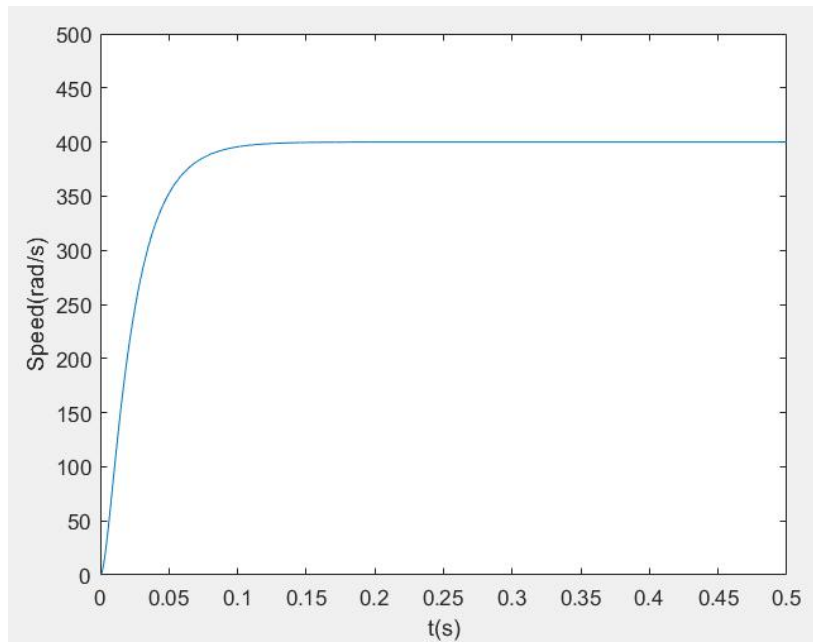


Figure 26: The result of Speed versus Time(t)

## Question b

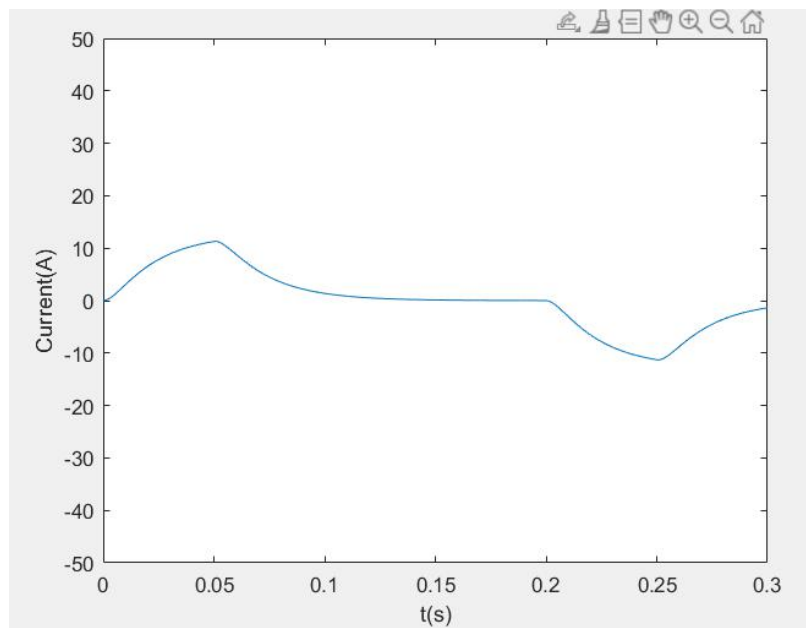


Figure 27: The result of the current versus time

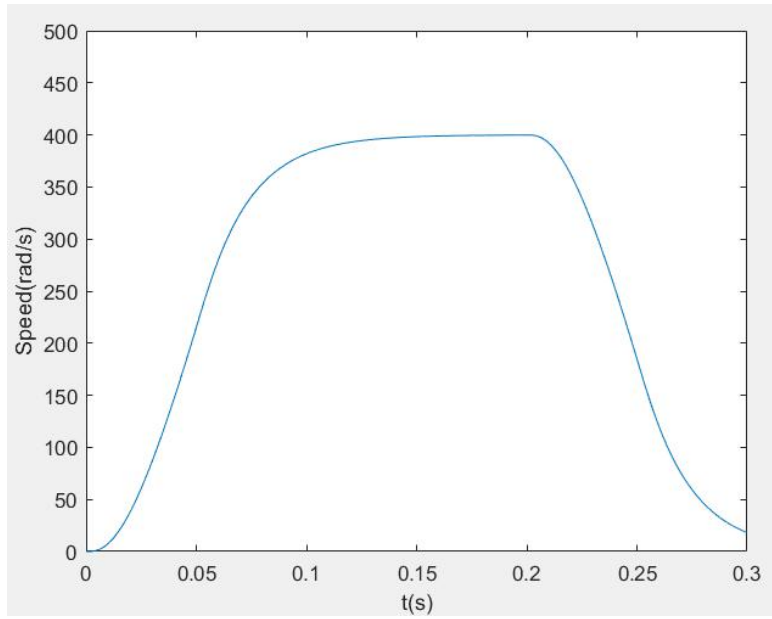


Figure 28: The result of the motor's speed versus time

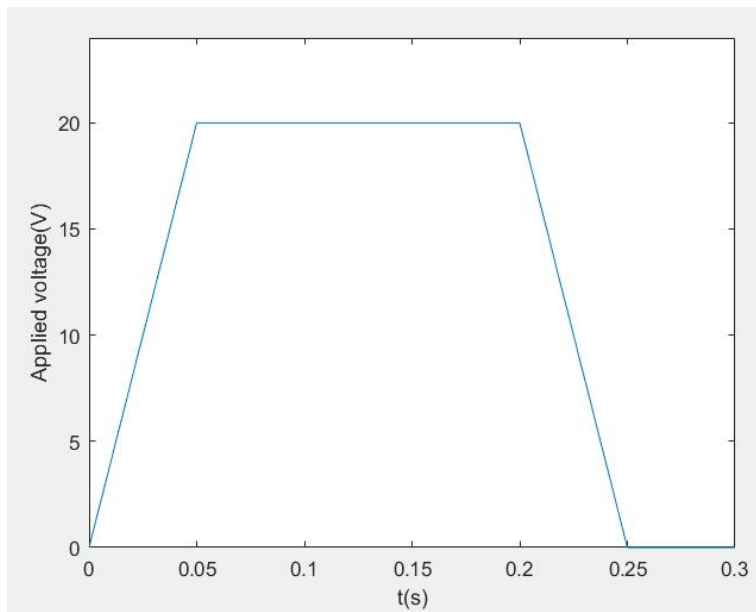


Figure 29: The result of Applied voltage versus time

## 6.Result analysis

We first defined two functions,  $i(t)$  and  $w(t)$ , and then we rewrote the equation into script and defined the initial conditions.

$$1) i(0) = 0$$

$$2) w(0) = 0$$

To get the graph for Current and Speed versus time, we choose the dsolve function. This function can plot the graph for current and speed versus time.

In order to distinguish the first question, when obtaining the image of speed versus Time, we choose the lsim() function, which can plot the image of speed changes over time.

For question b, because outputs for a linear combination of inputs are the same as a linear combination of individual responses to those inputs, we consider the system is Linear time-invariant system. We create 4 matrix(A、 B、 C、 D).

In the second question, we first draw the graph for the speed versus time of the motor. Next, we use the four matrices created in the first question according to the different speeds of the four segments of the motor:

$$v(t) = \begin{cases} 20 & 0.05 \leq t \leq 0.2 \\ 400t & 0 \leq t < 0.05 \\ -400(t - 0.2) + 20 & 0.2 < t \leq 0.25 \\ 0 & t > 0.25 \end{cases} \quad (1)$$

Meanwhile, we also plot the graph of the applied voltage versus time t.

## Problem 4

### 1. Equation derivations:

In this question, we need to plot the graph of the height of two hydraulic system(h1 and h2) versus time t.

The schematic diagram is as follows:

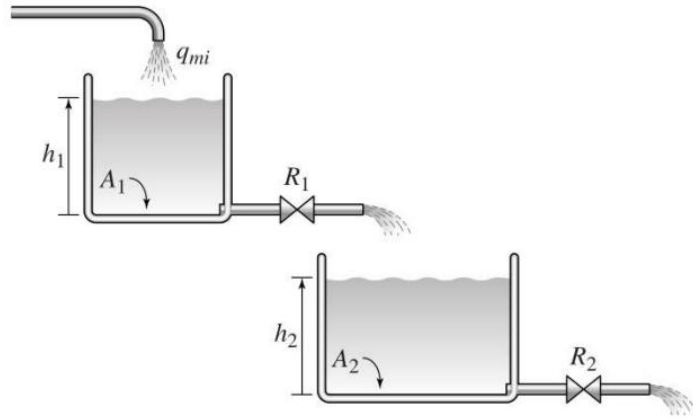


Figure 30: The hydraulic flow system [1]

We know:

$$\rho A \frac{dh}{dt} = q + \frac{1}{R_l} SSR(\rho_l - \rho) - \frac{1}{R_r} SSR(\rho - \rho_r) \quad (1)$$

The equation of SSR is as follows:

$$SSR(\Delta\rho) = \begin{cases} \sqrt{\Delta\rho} & \Delta\rho > 0 \\ -\sqrt{\Delta\rho} & \Delta\rho < 0 \end{cases} \quad (2)$$

## 2.Initial conditions:

$A1=3 \text{ ft}^2$	$R1=30 \text{ ft}^{-1}$	$\rho=1.94 \text{ slug / ft}^3$	$g=32.2 \text{ ft / sec}^2$	$h1=2\text{m}$
$A2=5 \text{ ft}^2$	$R2=40 \text{ ft}^{-1}$	$q_{mi}=0.5 \text{ slug / sec}$		$h2=5\text{m}$

## 3.Main programme and Simulink model

```

A1=3;
A2=5;
R1=30;
R2=40;
q=0.5;
rho=1.94;
h1=2;
h2=5;
g=32.2;

```

Figure 31: The code for the hydraulic system(Initial conditions)

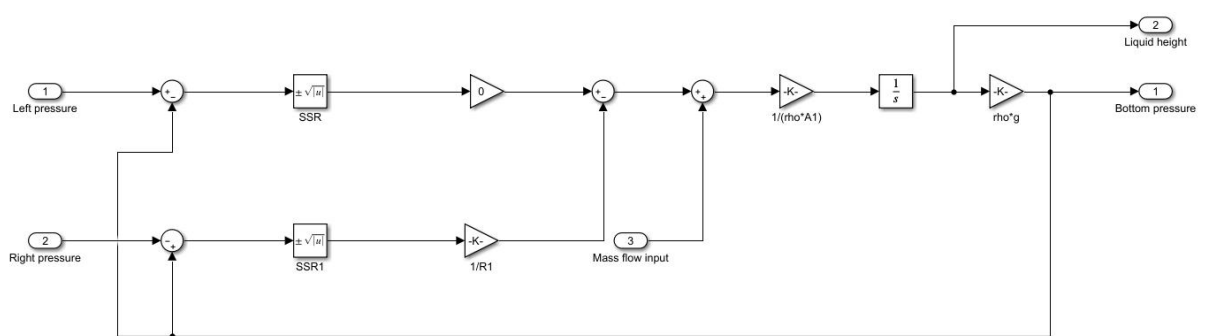


Figure 32: The Simulink model for this hydraulic system

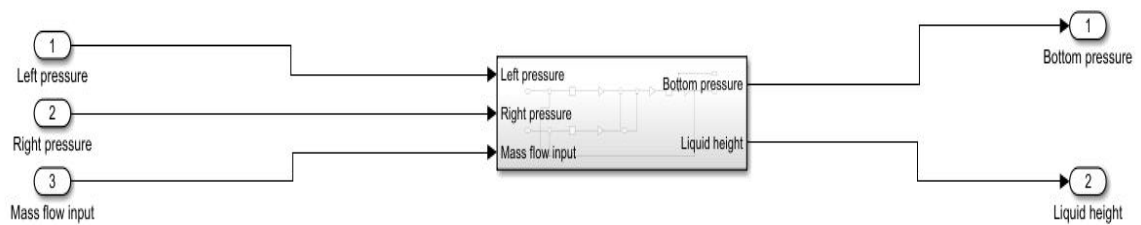


Figure 33: The subsystem from previous opponent

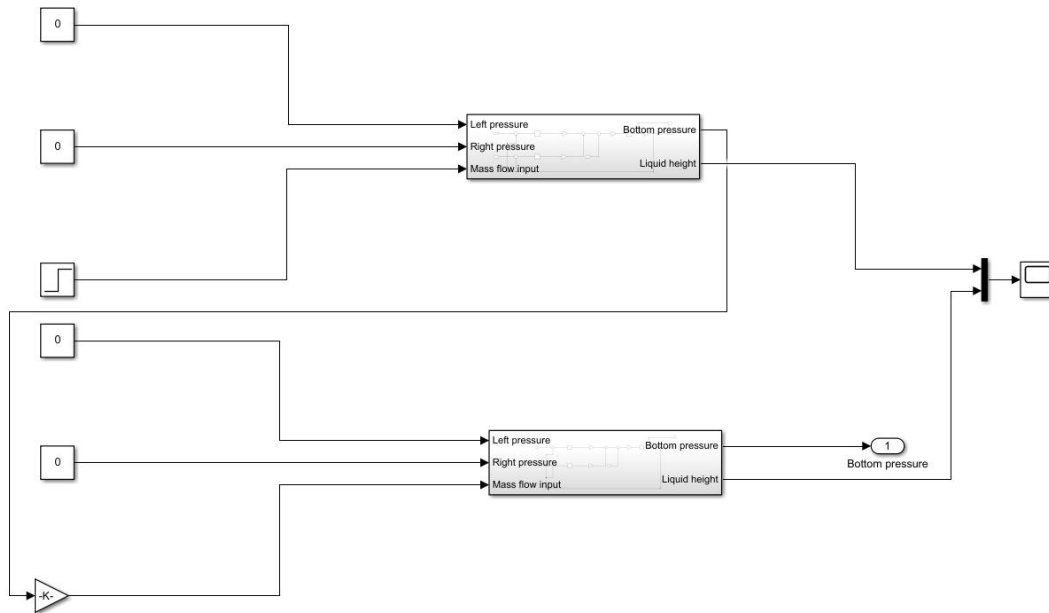


Figure 34: Two subsystems created for the whole system

## 4.Functions

Simulink is a simulation and model-based design environment for dynamic and embedded systems, integrated with MATLAB. We can use the opponents in simulink to simulate the true world. In this problem, i first create a subsystem to be on behalf of one hydraulic. Next, i use simulink to simulate the hydraulic system, which is based on two subsystems. The first bottom pressure provide the second mass flow input.

## 5.Results

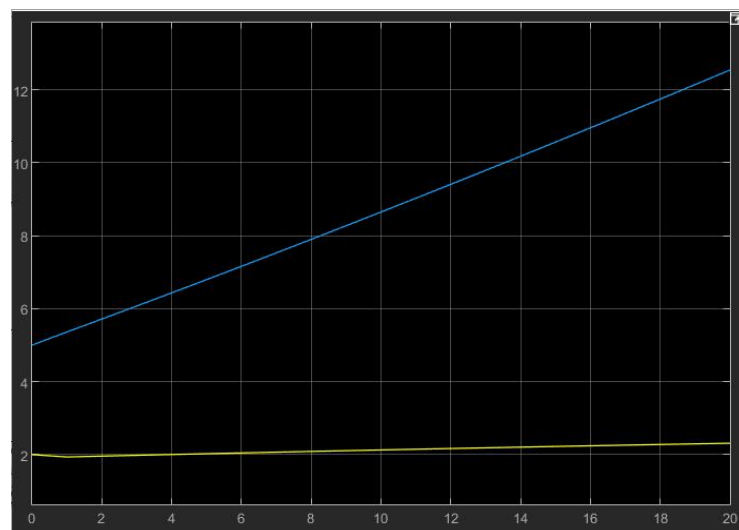


Figure 35: The graph of  $h_1(t)$  and  $h_2(t)$

## 6.Result analysis

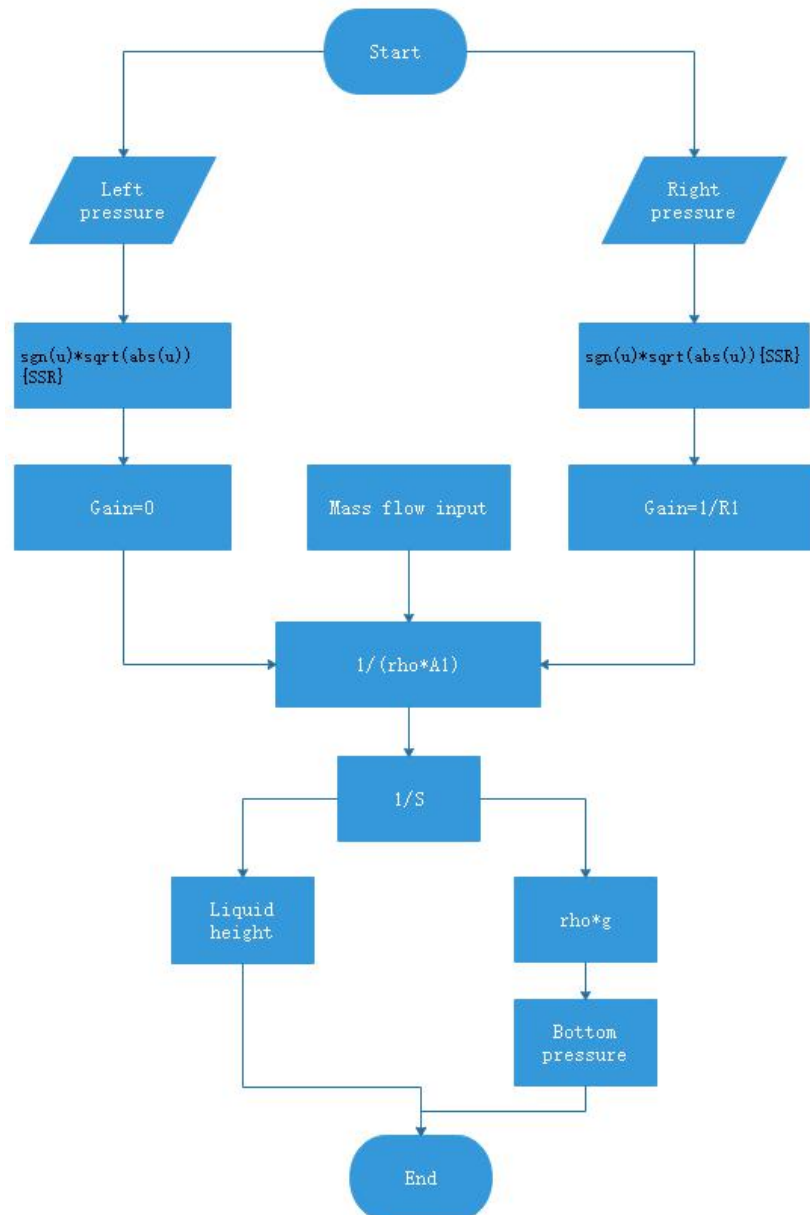
### operating steps:

- 1) We first write the initial conditions provided in the question into script
- 2) According to the example in Tutorial 6 [2], we created the Simulink model for this hydraulic system on the Simulink interface. Also, we filled the corresponding parameters into the corresponding block.
- 3) We create the subsystem based on the Simulink models we have just created
- 4) Since we need two Subsystems, we copy the first subsystem and connect the bottom system of the first to the Mass flow input of the second. We also added gain:  $1/R1$  to the link to make it satisfied the requirements of the question.
- 5) We add initial conditions to each of the two subsystems we just created. Here, we use step block for the first subsystem Mass flow input
- 6) Finally, we connect the two liquid heights to Scope, set the time at 20s, and start running.
- 7) Obtain the graph

In this problem, we use the example in class, learning to use the MATLAB Simulink function.

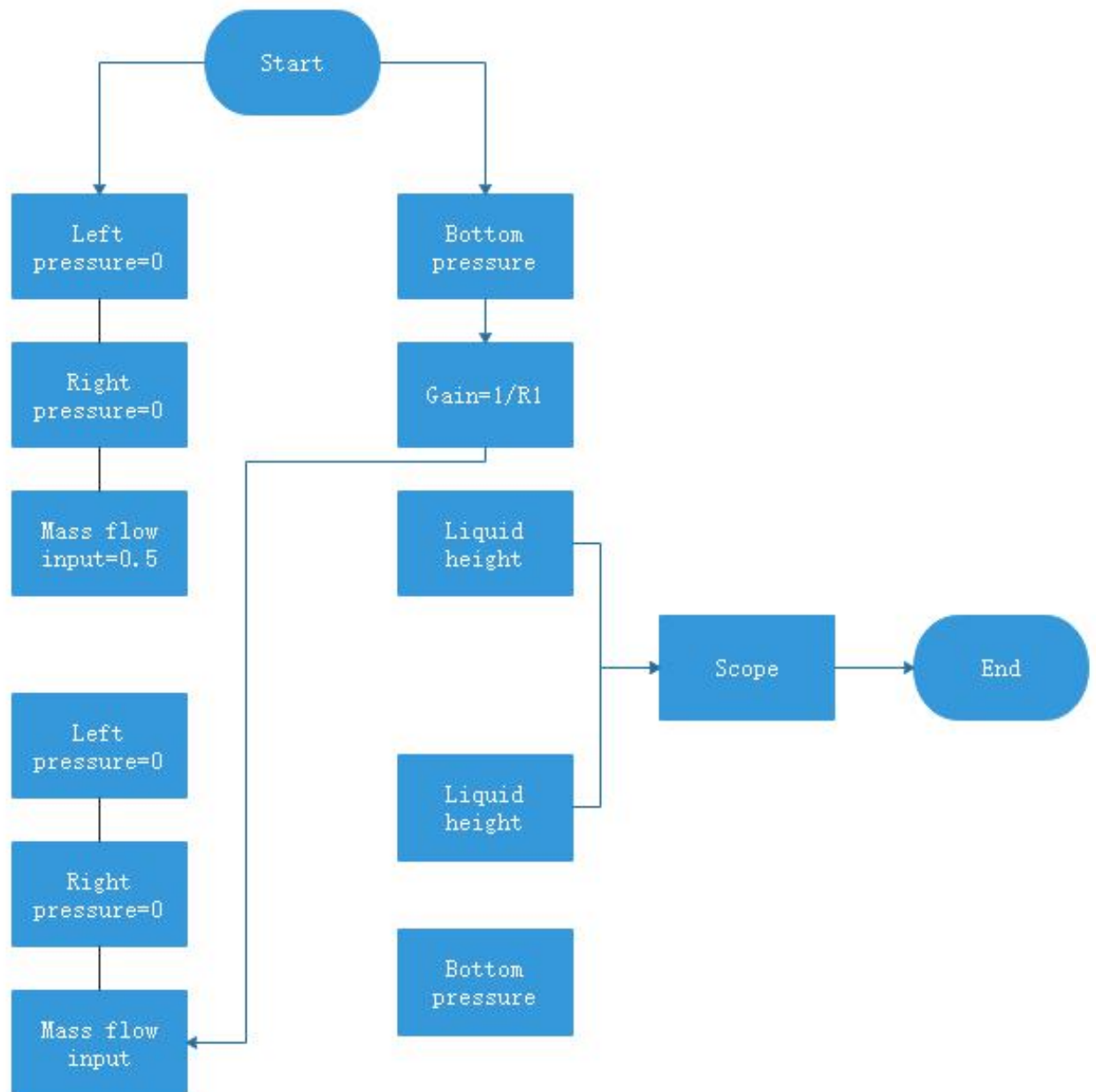
## 7.Flow chart

### 7.1 The flow chart for the Simulink model





## 7.2 The flow chart for the whole system



## Problem 5

### 1. Equation derivations

In a 2-dimensional rectangular area, if we do not consider the internal electrical changes, we believe that the distribution of electric potential satisfies the Laplace equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (1)$$

Since we cannot directly solve this equation, we use Finite Different Method (FDM) to solve this equation using an iterative method.

The equation based on is as follows:

$$\phi_{i,j}^{n+1} = \phi_{i,j}^n + \frac{\omega}{4}(\phi_{i,j+1}^n + \phi_{i,j-1}^{n+1} + \phi_{i-1,j}^{n+1} + \phi_{i+1,j}^n - 4\phi_{i,j}^n) \quad (2)$$

Where, n is iterative, i and j are the node number in x and y directions. We solve the problem by dividing the large region into small regions (meshing). The size of each mesh is h. Therefore, the area will be divided into  $m_1(a/h)$  in x direction and  $m_2(b/h)$  in y direction.

Where, we can get:

$$h_x = m_1 + 1 \quad (3)$$

$$h_y = m_2 + 1 \quad (4)$$

### 2. Initial conditions and boundary conditions

The initial conditions and boundary conditions have been told to us in the question.

If we are given four boundaries and their conditions:

$$1) \phi|_{x=0} = \phi_1$$

$$2) \phi|_{x=a} = \phi_2$$

$$3) \varphi|_{y=0} = \varphi_3$$

$$4) \varphi|_{y=b} = \varphi_4$$

The conditions for each problem are as follows:

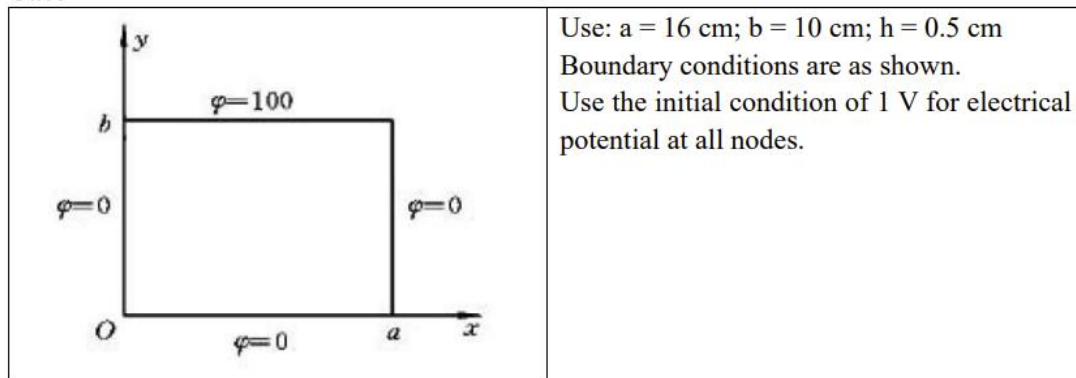


Figure 36: The boundary conditions for case1

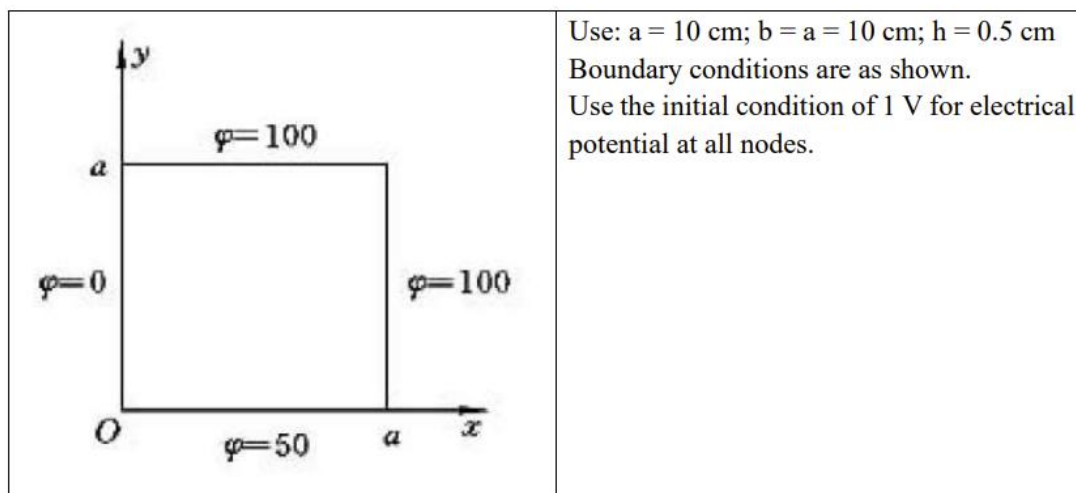


Figure 37: The boundary conditions for case2

### 3. Main programme

```
function [y]=P5_function(number, z, hx, hy)
    omega=2/(1+sqrt(1-((cos(pi/number(1,1))+cos(pi/number(1,2)))/2)^2));
    y=z;
    Tol=100;
    tol=0;
    counter=0;

    while(Tol>0.00001)
        counter=counter+1;
        Tol=0;
        for j=2:hy-1
            for k=2:hx-1
                y(j,k)=z(j,k)+omega*(z(j,k+1)+z(j+1,k)+y(j-1,k)+y(j,k-1)-4*z(j,k))/4;
                tol=abs(y(j,k)-z(j,k));
                if(tol>Tol)
                    Tol=tol;
                end
            end
        end
        z=y;
    end

    disp('The number of cycles of iterative method is: ')
    disp(counter)
```

Figure 38: The code for the iterative function

```

a=input(' Input the x distance (cm):');
b=input(' Input the y distance (cm):');
h=input(' Input the mesh size of h for each small area (cm):');
Ct=input(' Input the top boundary(V): ');
Cb=input(' Input the bottom boundary (V):');
Cl=input(' Input the left boundary (V):');
Cr=input(' Input the right boundary (V):');
%variables
m1=a/h;
m2=b/h;
number=[m1, m2];
hx=m1+1;
hy=m2+1;
z=zeros(hy, hx);
z(hy, :)=ones(1, hx)*Ct;
z(1, :)=ones(1, hx)*Cb;
z(:, 1)=ones(hy, 1)*Cl;
z(:, hx)=ones(hy, 1)*Cr;
[y]=P5_function(number, z, hx, hy);
figure(1)
mesh(linspace(0, a, hx), linspace(0, b, hy), y)
axis([0, a+1, 0, b+1, 0, 100])
xlabel(' x (cm)')
ylabel(' y (cm)')
zlabel(' Electrical potentia (V)')
figure(2)
contour(y, 50)

```

Figure 39: The code for Input function

## 4.Functions

The code of Iterative Method is shown in the figure above. We first loop on the x axis, then make cyclic judgments on the y axis, and finally show the number of cycles of Iterative method.

We also solved the w equation here:

$$\omega = \frac{2}{1 + \sqrt{1 - \left[ \frac{\cos\left(\frac{\pi}{m1}\right) + \cos\left(\frac{\pi}{m2}\right)}{2} \right]^2}}$$

Figure 40: The equation of  $w$  [1]

In the main program, we first write the **Input function** to let the user Input the corresponding data. After inputting the corresponding boundary, the program can calculate and draw according to the boundary conditions input by users.

## 5.Results

### Case 1:

```
Input the x dimension (cm):16
Input the y dimension (cm):10
Input the mesh size (cm):0.5
Input the boundary potential on top boundary(V): 100
Input the boundary potential on bottom boundary (V):0
Input the boundary potential on left boundary (V):0
Input the boundary potential on right boundary (V):0
the number of times of iterative method is:
77
```

Figure 41: The command window for case 1

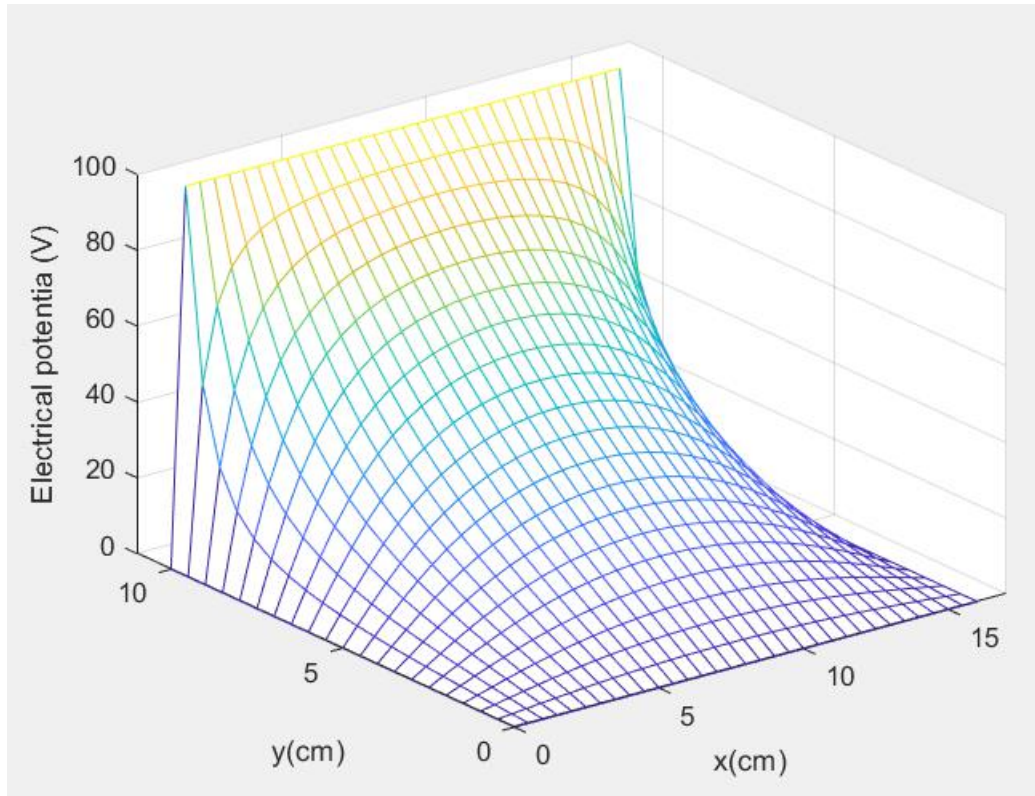


Figure 42: The distribution of the potential for case 1

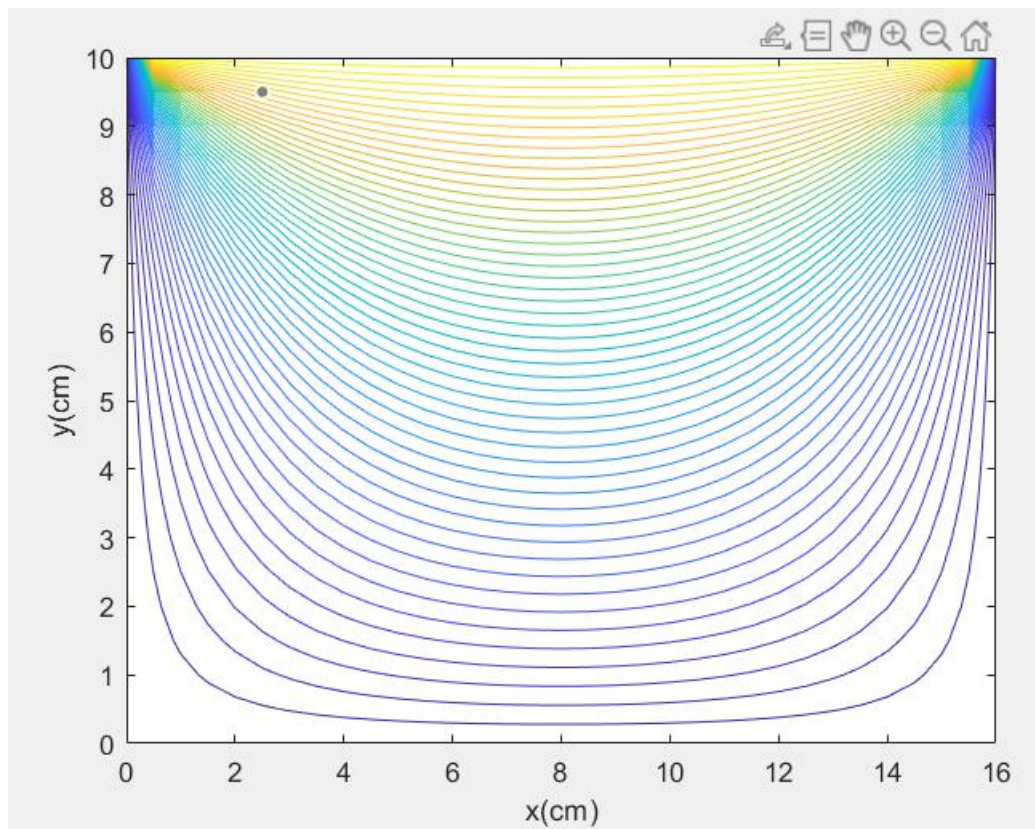


Figure 43: The graph of contour lines



## Case 2:

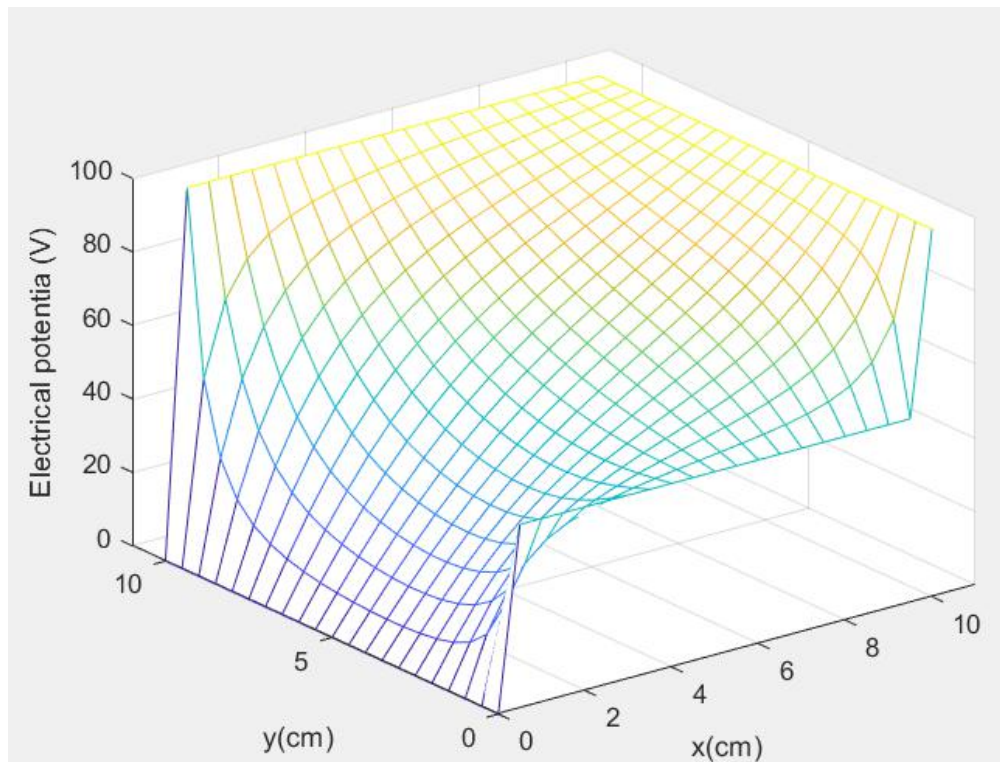


Figure 44: The distribution of the potential for case 1

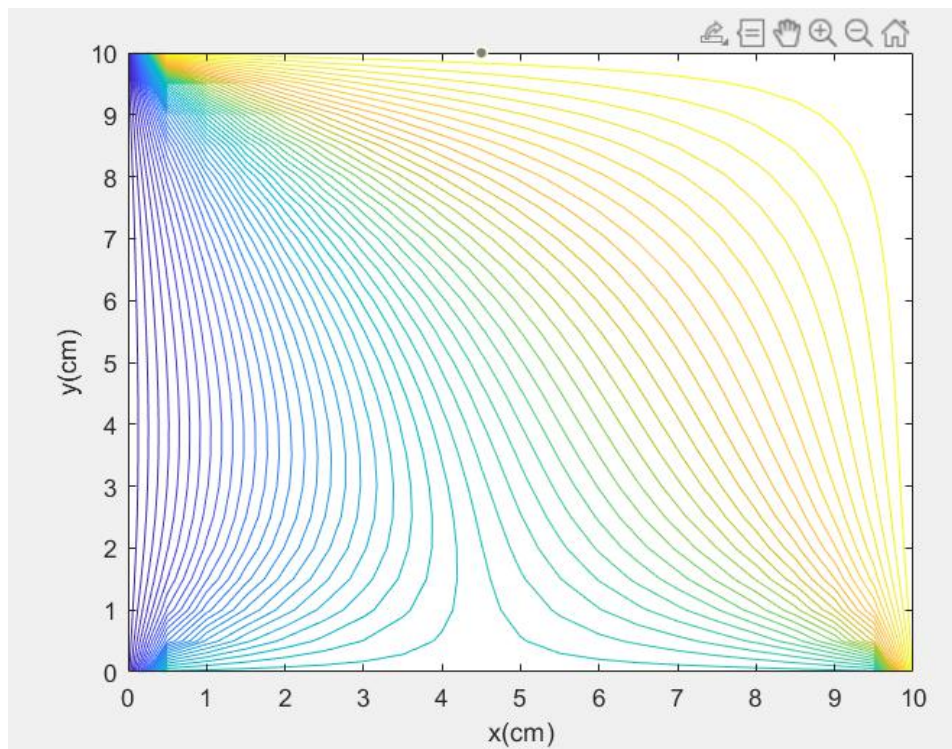


Figure 45: The graph of contour lines



## 6.Results analysis

Through matlab, we firstly calculated the times of the Iterative methods under two different cases with formulas and display the result in the command window.

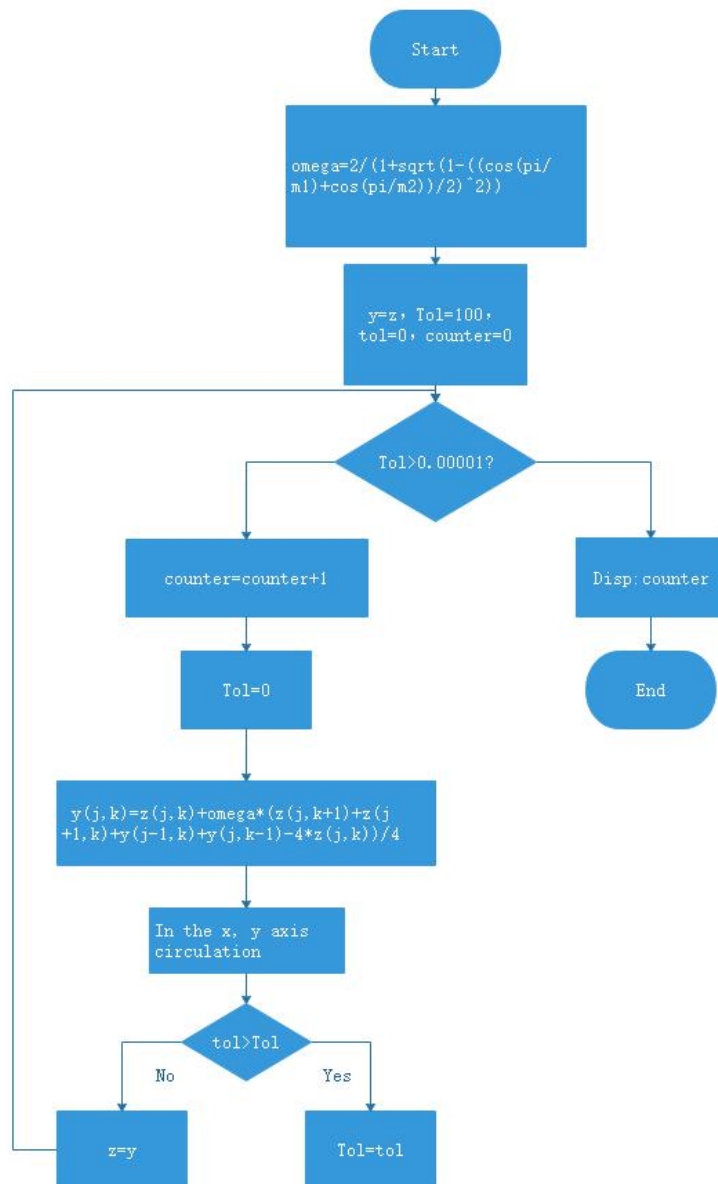
In the main function, we use mesh() and contour() functions to plot the two graphs.

For the first case, the potential is increasing with y increasing. In the x-direction, the values on both sides are higher.

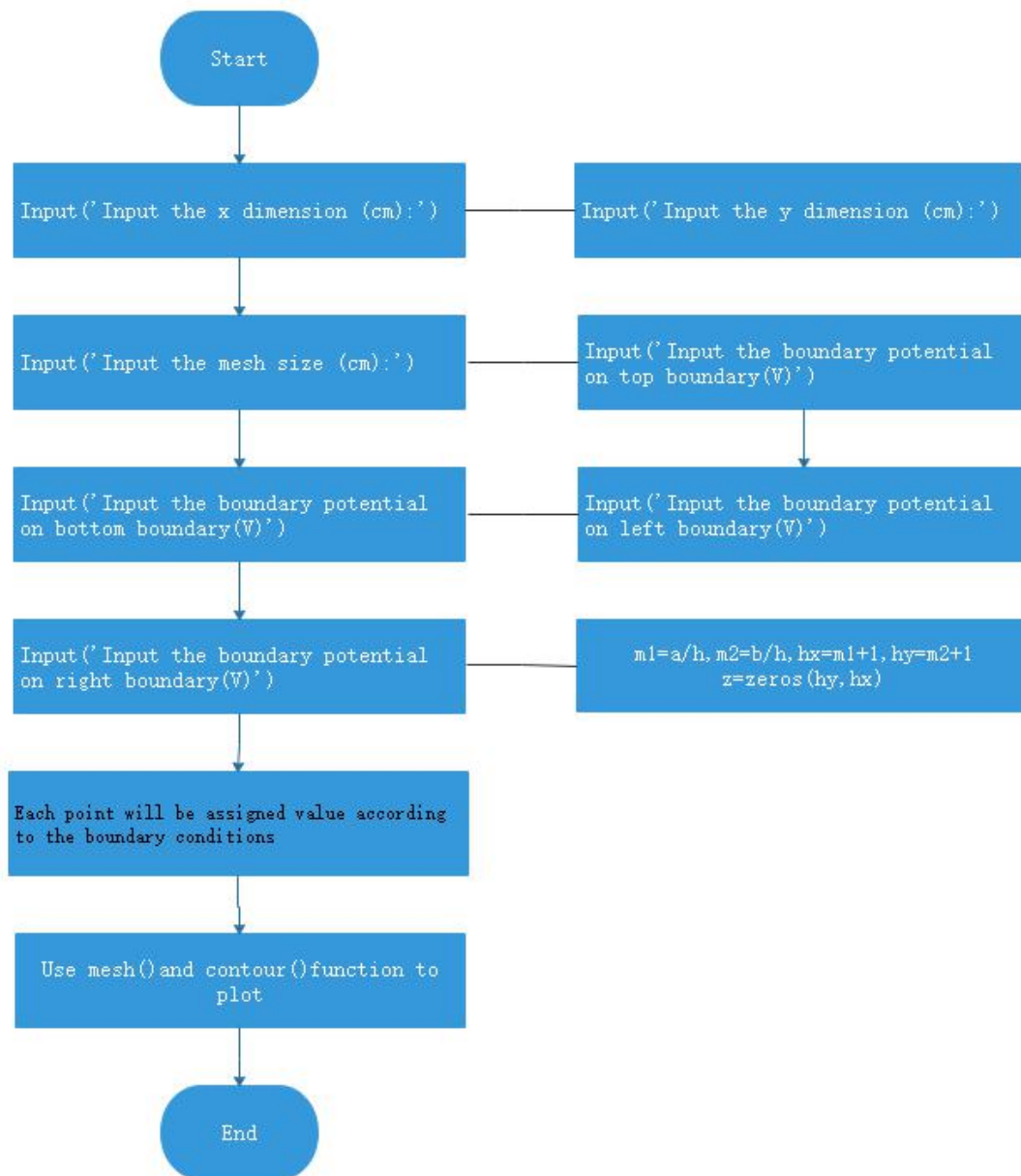
For the second case, the potential is larger at the position (10,10) and is concave in the middle.

## 7.Flow chart

### 7.1 Iterative method function flow chart



## 7.2 Input function flow chart



## Reference list:

- [1] Department of Electrical and Electronic Engineering, "MEC104 MATLAB Assignment 2020-21",  
[Online].Available:<https://learningmall.xjtlu.edu.cn/mod/resource/view.php?id=89259>
- [2] Department of Electrical and Electronic Engineering, "MATLAB lecture notes",  
[Online].Available:<https://learningmall.xjtlu.edu.cn/mod/folder/view.php?id=84509>