Assignment#

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Question 1

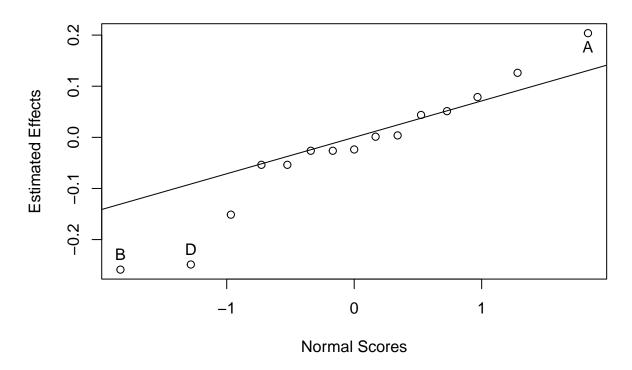
```
# Clear the environment
rm(list = ls())
require("conf.design")
## Loading required package: conf.design
# Define the factors and response variable
A = rep(c(-1, 1), 8)
B = rep(c(rep(-1, 2), rep(1, 2)), 4)
C = rep(c(rep(-1, 4), rep(1, 4)), 2)
D = c(rep(-1, 8), rep(1, 8))
y = c(2.45, 3.36, 2.16, 2.29, 2.49, 3.39, 2.32, 2.44,
      1.84, 2.24, 1.69, 1.87, 2.29, 2.92, 2.04, 2.03)
# Create a data frame
credit_card_data <- data.frame(y, A, B, C, D)</pre>
# Fit a linear model
res.lm <- lm(y ~ A * B * C * D, data = credit_card_data)
# Summary of the linear model to check p-values
summary_res_lm <- summary(res.lm)</pre>
# Print the summary
print(summary_res_lm)
## lm(formula = y ~ A * B * C * D, data = credit_card_data)
##
## ALL 16 residuals are 0: no residual degrees of freedom!
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.36375
                                NaN
                                        NaN
                                                  NaN
## A
                0.20375
                                NaN
                                        NaN
                                                  NaN
## B
               -0.25875
                                \mathtt{NaN}
                                        NaN
                                                  NaN
## C
               0.12625
                                NaN
                                        NaN
                                                  NaN
## D
                                NaN
                                        NaN
                                                  NaN
               -0.24875
```

```
## A:B
                -0.15125
                                 {\tt NaN}
                                          {\tt NaN}
                                                    NaN
## A:C
                0.00125
                                 {\tt NaN}
                                          {\tt NaN}
                                                    NaN
## B:C
                -0.02375
                                 {\tt NaN}
                                          {\tt NaN}
                                                    \mathtt{NaN}
## A:D
                                          NaN
                                                    NaN
                -0.05375
                                 {\tt NaN}
## B:D
                 0.05125
                                 {\tt NaN}
                                          {\tt NaN}
                                                    NaN
## C:D
                 0.07875
                                 {\tt NaN}
                                          {\tt NaN}
                                                    NaN
## A:B:C
                -0.02625
                                 \mathtt{NaN}
                                          NaN
                                                    NaN
## A:B:D
                                                    NaN
                0.04375
                                 \mathtt{NaN}
                                          {\tt NaN}
## A:C:D
                0.00375
                                 NaN
                                          NaN
                                                    NaN
## B:C:D
                                 NaN
                                          {\tt NaN}
                                                    NaN
                -0.05375
## A:B:C:D
                -0.02625
                                 NaN
                                          NaN
                                                    NaN
## Residual standard error: NaN on O degrees of freedom

    Adjusted R-squared:

## Multiple R-squared:
## F-statistic: NaN on 15 and 0 DF, p-value: NA
# Alternatively, you can use ANOVA
res.aov <- aov(y ~ A * B * C * D, data = credit_card_data)
summary_res_aov <- summary(res.aov)</pre>
# Print the ANOVA summary
print(summary_res_aov)
##
                Df Sum Sq Mean Sq
## A
                1 0.6642 0.6642
## B
                 1 1.0712 1.0712
## C
                 1 0.2550 0.2550
                1 0.9900 0.9900
## D
## A:B
                1 0.3660 0.3660
## A:C
                1 0.0000 0.0000
## B:C
                 1 0.0090 0.0090
## A:D
                1 0.0462 0.0462
## B:D
                1 0.0420 0.0420
## C:D
                 1 0.0992 0.0992
## A:B:C
                 1 0.0110 0.0110
                 1 0.0306 0.0306
## A:B:D
                 1 0.0002 0.0002
## A:C:D
                 1 0.0462 0.0462
## B:C:D
                 1 0.0110 0.0110
## A:B:C:D
library(daewr)
## Warning: package 'daewr' was built under R version 4.2.3
fullnormal(coef(res.lm)[-1],alpha=.05)
```

Normal Q-Q Plot



Part a

From the plot, we can conclude that factor C(Initial interest rate) are not significant.

Part b

```
credit_card_data_Reduced <- within(credit_card_data, rm(C))</pre>
# Fit an ANOVA model to the reduced dataset
res.aov_reduced <- aov(y~A*B*D, data=credit_card_data_Reduced)
summary(res.aov_reduced)
##
              Df Sum Sq Mean Sq F value Pr(>F)
## A
               1 0.6642 0.6642 12.306 0.00798 **
               1 1.0712 1.0712 19.847 0.00213 **
## B
                         0.9900 18.342 0.00268 **
## D
                1 0.9900
## A:B
               1 0.3660 0.3660
                                  6.781 0.03142 *
## A:D
                1 0.0462 0.0462
                                  0.856 0.38181
               1 0.0420
                         0.0420
                                  0.779 0.40330
## B:D
## A:B:D
               1 0.0306
                         0.0306
                                  0.567 0.47288
## Residuals
               8 0.4318
                         0.0540
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
# Fit a linear model to the reduced dataset
res.lm_reduced <- lm(y~A*B*D, data=credit_card_data_Reduced)
```

```
summary(res.lm_reduced)
```

```
##
## Call:
## lm(formula = y ~ A * B * D, data = credit_card_data_Reduced)
##
## Residuals:
##
     Min
              1Q Median
                            3Q
                                  Max
   -0.34 -0.08
                   0.00
                          0.08
                                 0.34
##
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 2.36375
                           0.05808
                                    40.697 1.46e-10 ***
## A
                0.20375
                           0.05808
                                     3.508
                                           0.00798 **
## B
               -0.25875
                           0.05808
                                    -4.455
                                            0.00213 **
## D
               -0.24875
                           0.05808
                                    -4.283
                                            0.00268 **
               -0.15125
                           0.05808
                                    -2.604
                                            0.03142 *
## A:B
## A:D
               -0.05375
                           0.05808
                                    -0.925
                                            0.38181
## B:D
                0.05125
                           0.05808
                                     0.882 0.40330
## A:B:D
                0.04375
                           0.05808
                                     0.753 0.47288
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2323 on 8 degrees of freedom
## Multiple R-squared: 0.8814, Adjusted R-squared:
## F-statistic: 8.497 on 7 and 8 DF, p-value: 0.003616
```

Part c

The coefficient for B in the model output is -0.25875. This is the estimate of the main effect of the account-opening fee on the response rate. It means that, all else being equal, introducing an account-opening fee is associated with a decrease in the response rate by 0.25875 units on the scale of the response variable being measured.

part D

According to the ANOVA summary and the regression output, the account-opening fee (factor B) is significant. The p-value for factor B is 0.00213, which is less than 0.05, which means there is a statistically significant association between the account-opening fee and the response rate. The "Estimate" value of -0.25875 for B and its corresponding low p-value suggest that the account-opening fee has a significant negative effect on the response rate.

part e

```
## Block 1 Block 2
## 2.4075 2.3200
```

Question 2

Part a

```
treatments <- c("1", "a", "b", "ab", "c", "ac", "bc", "abc", "d", "ad", "bd", "abd", "cd", "acd", "bcd" replicate1 <- c(90, 74, 81, 83, 77, 81, 88, 73, 98, 72, 87, 85, 99, 79, 87, 80)
```

```
replicate2 <- c(93, 78, 85, 80, 78, 80, 82, 70, 95, 76, 83, 86, 90, 75, 84, 80)
# Calculating average yields
average_yields <- (replicate1 + replicate2) / 2</pre>
n <- length(treatments)</pre>
# Main effects
effect_A \leftarrow (sum(average\_yields[c(2,4,6,8,10,12,14,16)]) - sum(average\_yields[c(1,3,5,7,9,11,13,15)]))
effect_B <- (sum(average_yields[c(3,4,7,8,11,12,15,16)]) - sum(average_yields[c(1,2,5,6,9,10,13,14)]))
effect_D \leftarrow (sum(average_yields[c(9,10,11,12,13,14,15,16)]) - sum(average_yields[c(1,2,3,4,5,6,7,8)]))
# Output main effects
list(A = effect_A, B = effect_B, C = effect_C, D = effect_D)
## $A
## [1] -9.0625
##
## $B
## [1] -1.3125
##
## $C
## [1] -2.6875
##
## $D
## [1] 3.9375
part b
# Define the treatment combinations
coded\_combinations \leftarrow expand.grid(A = c(-1, 1), B = c(-1, 1), C = c(-1, 1), D = c(-1, 1))
coded_combinations$Yield <- average_yields</pre>
# Check the data frame
print(coded_combinations)
      A B C D Yield
##
## 1 -1 -1 -1 91.5
     1 -1 -1 -1 76.0
## 3 -1 1 -1 -1 83.0
## 4
     1 1 -1 -1 81.5
## 5 -1 -1 1 -1 77.5
## 6
     1 -1 1 -1 80.5
     -1 1 1 -1 85.0
## 7
## 8
      1 1 1 -1 71.5
## 9 -1 -1 -1 1 96.5
## 10 1 -1 -1 1 74.0
## 11 -1 1 -1 1 85.0
## 12 1 1 -1 1 85.5
## 13 -1 -1 1 1 94.5
## 14 1 -1 1 1 77.0
## 15 -1
        1
           1 1
## 16 1 1 1 1 80.0
```

```
# Create a linear model with all interactions
model <- lm(Yield ~ A*B*C*D, data = coded_combinations)</pre>
# Summary of the model to see coefficients and statistics
summary(model)
##
## Call:
## lm(formula = Yield ~ A * B * C * D, data = coded_combinations)
## Residuals:
## ALL 16 residuals are 0: no residual degrees of freedom!
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 82.78125
                                         NaN
                                 NaN
                                         NaN
                                                   NaN
               -4.53125
                                 NaN
## B
               -0.65625
                                 NaN
                                         NaN
                                                   NaN
## C
                -1.34375
                                 {\tt NaN}
                                         NaN
                                                   NaN
## D
                1.96875
                                 NaN
                                         NaN
                                                   NaN
## A:B
                                 {\tt NaN}
                                         {\tt NaN}
                                                   NaN
                2.03125
## A:C
                0.34375
                                 {\tt NaN}
                                         {\tt NaN}
                                                   NaN
## B:C
                                 NaN
                -0.28125
                                         NaN
                                                   NaN
## A:D
               -1.09375
                                 NaN
                                         NaN
                                                   NaN
## B:D
               -0.09375
                                 {\tt NaN}
                                         {\tt NaN}
                                                   NaN
## C:D
                                 {\tt NaN}
                                         NaN
                                                   NaN
                0.84375
## A:B:C
                -2.59375
                                 NaN
                                         NaN
                                                   NaN
## A:B:D
                                         NaN
                                                   NaN
                2.34375
                                 NaN
## A:C:D
                -0.46875
                                 NaN
                                         NaN
                                                   NaN
## B:C:D
                -0.46875
                                 NaN
                                         NaN
                                                   NaN
## A:B:C:D
                 1.21875
                                 NaN
                                         NaN
                                                   NaN
## Residual standard error: NaN on O degrees of freedom
                              1, Adjusted R-squared:
## Multiple R-squared:
## F-statistic: NaN on 15 and 0 DF, p-value: NA
# Perform ANOVA analysis
anova_result <- anova(model)</pre>
## Warning in anova.lm(model): ANOVA F-tests on an essentially perfect fit are
## unreliable
# Print the ANOVA table
print(anova_result)
## Analysis of Variance Table
##
## Response: Yield
             Df Sum Sq Mean Sq F value Pr(>F)
## A
              1 328.52 328.52
                                     NaN
                                            NaN
## B
                   6.89
                           6.89
                                     NaN
                                            NaN
## C
              1 28.89
                          28.89
                                     NaN
                                            NaN
## D
              1 62.02
                          62.02
                                     {\tt NaN}
                                            NaN
## A:B
              1 66.02
                         66.02
                                     {\tt NaN}
                                            NaN
## A:C
                  1.89
                          1.89
                                     {\tt NaN}
                                            NaN
## B:C
                   1.27
                                            NaN
              1
                          1.27
                                     NaN
```

```
## A:D
              1 19.14
                          19.14
                                    NaN
                                            NaN
## B:D
                  0.14
                           0.14
                                    NaN
                                            NaN
              1
## C:D
                11.39
                          11.39
                                    NaN
                                            NaN
                                            NaN
## A:B:C
              1 107.64
                        107.64
                                    NaN
## A:B:D
                 87.89
                          87.89
                                    NaN
                                            NaN
## A:C:D
                  3.52
                           3.52
                                    NaN
                                            NaN
              1
## B:C:D
                  3.52
                                            NaN
              1
                           3.52
                                    NaN
                          23.77
## A:B:C:D
              1
                 23.77
                                    NaN
                                            NaN
## Residuals 0
                  0.00
                            NaN
```

Part C

 $\label{eq:Yield=82.78125-4.53125A-0.65625B-1.34375C+1.96875D+2.03125(AB)+0.34375(AC)-0.28125(BC)-1.09375(AD)-0.09375(BD)+0.84375(CD)-2.59375(ABC)+2.34375(ABD)-0.46875(ACD)-0.46875(BCD)+1.21875(ABCD)-1.09375(ABC)$

Part D

NO, indicated by zero residuals for all observations, suggests an overfitting scenario rather than a meaningful assessment of model adequacy.

Question3

Part a

B(-8.2045), C(-6.5304), D(-6.0275) The larger estimates would be more significant.

part b

##

This would mean that the combined effect of factors A and B is highly significant and should be a primary focus in understanding the system's behavior or in optimizing the response variable.

Question 4

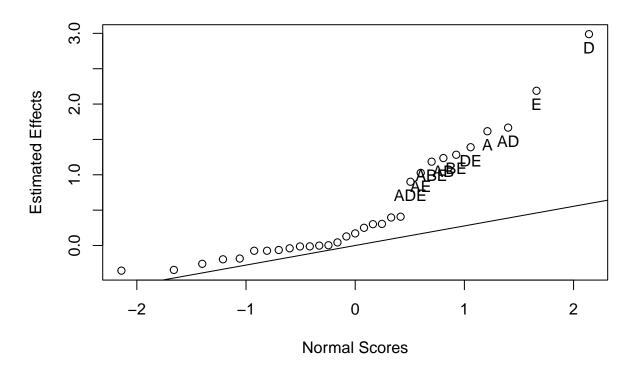
```
A = rep(c(-1, 1), times = 16)
B = rep(c(rep(-1, 2), rep(1, 2)), times = 8)
C = rep(c(rep(-1, 4), rep(1, 4)), times = 4)
D = rep(c(rep(-1, 8), rep(1, 8)), times = 2)
E = rep(c(-1, 1), each = 16)
y = c(8.11, 5.56, 5.77, 5.82, 9.17, 7.8, 3.23, 5.69, 8.82, 14.23, 9.2, 8.94,
      8.68, 11.49, 6.25, 9.12, 7.93, 5, 7.47, 12, 9.86, 3.65, 6.4, 11.61,
      12.43, 17.55, 8.87, 25.38, 13.06, 18.85, 11.78, 26.05)
# Create the data frame
experiment_data <- data.frame(A, B, C, D, E, y)
full_model <- lm(y ~ A*B*C*D*E, data = experiment_data)</pre>
# Summary of the full model to see coefficients
summary(full_model)
##
## Call:
## lm(formula = y ~ A * B * C * D * E, data = experiment_data)
```

```
## Residuals:
## ALL 32 residuals are 0: no residual degrees of freedom!
## Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
## (Intercept) 10.180312
                                    {\tt NaN}
                                             \mathtt{NaN}
                                                       NaN
                                             NaN
                                                       NaN
                 1.615938
                                    NaN
## B
                  0.043438
                                    {\tt NaN}
                                             {\tt NaN}
                                                       NaN
## C
                 -0.012187
                                    NaN
                                             NaN
                                                       NaN
## D
                                             NaN
                                                       NaN
                 2.988437
                                    NaN
## E
                  2.187813
                                    {\tt NaN}
                                             NaN
                                                       NaN
## A:B
                                             NaN
                                                       NaN
                 1.236562
                                    {\tt NaN}
## A:C
                -0.001563
                                    {\tt NaN}
                                             NaN
                                                       NaN
## B:C
                                             NaN
                -0.195313
                                    NaN
                                                       NaN
## A:D
                                    NaN
                                             NaN
                                                       NaN
                 1.666563
## B:D
                 -0.013438
                                    NaN
                                             {\tt NaN}
                                                       NaN
## C:D
                                             {\tt NaN}
                                                       NaN
                 0.003437
                                    NaN
## A:E
                 1.027188
                                    {\tt NaN}
                                             NaN
                                                       NaN
## B:E
                                             NaN
                                                       NaN
                  1.283437
                                    NaN
## C:E
                 0.301563
                                    {\tt NaN}
                                             NaN
                                                       NaN
## D:E
                  1.389687
                                    NaN
                                             NaN
                                                       NaN
## A:B:C
                 0.250313
                                    NaN
                                             NaN
                                                       NaN
## A:B:D
                -0.345312
                                             {\tt NaN}
                                                       NaN
                                    {\tt NaN}
## A:C:D
                -0.063437
                                    NaN
                                             NaN
                                                       NaN
## B:C:D
                                                       NaN
                 0.305312
                                    {\tt NaN}
                                             NaN
## A:B:E
                 1.185313
                                    NaN
                                             NaN
                                                       NaN
## A:C:E
                 -0.259062
                                    NaN
                                             NaN
                                                       NaN
## B:C:E
                 0.170938
                                    {\tt NaN}
                                             NaN
                                                       NaN
## A:D:E
                                    NaN
                                             NaN
                                                       NaN
                 0.901563
## B:D:E
                -0.039687
                                    {\tt NaN}
                                             NaN
                                                       NaN
## C:D:E
                 0.395938
                                    {\tt NaN}
                                             NaN
                                                       NaN
## A:B:C:D
                -0.074063
                                    {\tt NaN}
                                             NaN
                                                       NaN
## A:B:C:E
                -0.184688
                                    NaN
                                             NaN
                                                       NaN
## A:B:D:E
                                                       NaN
                 0.407187
                                    NaN
                                             NaN
## A:C:D:E
                  0.127812
                                    {\tt NaN}
                                             NaN
                                                       NaN
## B:C:D:E
                -0.074688
                                    NaN
                                             NaN
                                                       NaN
## A:B:C:D:E
                -0.355312
                                    {\tt NaN}
                                             NaN
                                                       NaN
## Residual standard error: NaN on O degrees of freedom
## Multiple R-squared:
                               1, Adjusted R-squared:
## F-statistic:
                   NaN on 31 and 0 DF, p-value: NA
# Perform ANOVA analysis
anova_full_model <- anova(full_model)</pre>
## Warning in anova.lm(full model): ANOVA F-tests on an essentially perfect fit
## are unreliable
# Print the ANOVA table
print(anova_full_model)
## Analysis of Variance Table
##
## Response: y
                  Sum Sq Mean Sq F value Pr(>F)
## A
               1 83.560 83.560
                                        {\tt NaN}
                                                NaN
```

##	В	1	0.060	0.060	NaN	NaN
##	C	1	0.005	0.005	NaN	NaN
##	D	1	285.784	285.784	NaN	NaN
##	E	1	153.169	153.169	NaN	NaN
##	A:B	1	48.931	48.931	NaN	NaN
##	A:C	1	0.000	0.000	NaN	NaN
##	B:C	1	1.221	1.221	NaN	NaN
##	A:D	1	88.878	88.878	NaN	NaN
##	B:D	1	0.006	0.006	NaN	NaN
##	C:D	1	0.000	0.000	NaN	NaN
##	A:E	1	33.764	33.764	NaN	NaN
##	B:E	1	52.711	52.711	NaN	NaN
##	C:E	1	2.910	2.910	NaN	NaN
##	D:E	1	61.799	61.799	NaN	NaN
##	A:B:C	1	2.005	2.005	NaN	NaN
##	A:B:D	1	3.816	3.816	NaN	NaN
##	A:C:D	1	0.129	0.129	NaN	NaN
##	B:C:D	1	2.983	2.983	NaN	NaN
##	A:B:E	1	44.959	44.959	NaN	NaN
##	A:C:E	1	2.148	2.148	NaN	NaN
##	B:C:E	1	0.935	0.935	NaN	NaN
##	A:D:E	1	26.010	26.010	NaN	NaN
##	B:D:E	1	0.050	0.050	NaN	NaN
##	C:D:E	1	5.017	5.017	NaN	NaN
##	A:B:C:D	1	0.176	0.176	NaN	NaN
##	A:B:C:E	1	1.092	1.092	NaN	NaN
##	A:B:D:E	1	5.306	5.306	NaN	NaN
##	A:C:D:E	1	0.523	0.523	NaN	NaN
##	B:C:D:E	1	0.179	0.179	NaN	NaN
##	A:B:C:D:E	1	4.040	4.040	NaN	NaN
##	Residuals	0	0.000	NaN		

fullnormal(coef(full_model)[-1],alpha=.025)

Normal Q-Q Plot



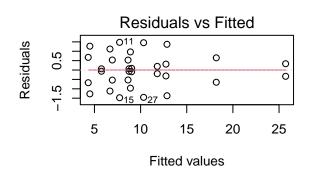
Through the plot, the factor C seems to be non-significant terms, so we remove C.

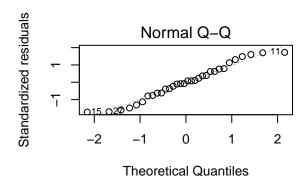
Part b

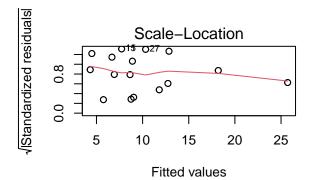
```
partB_model <- lm(y ~ A*B*D*E, data = experiment_data)</pre>
summary(partB_model)
##
## Call:
## lm(formula = y ~ A * B * D * E, data = experiment_data)
##
## Residuals:
##
       Min
                 1Q Median
                                 3Q
                                         Max
##
   -1.4750 -0.5637
                    0.0000
                            0.5637
                                      1.4750
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 10.18031
                            0.21360
                                      47.661 < 2e-16 ***
## A
                 1.61594
                            0.21360
                                       7.565 1.14e-06 ***
## B
                 0.04344
                            0.21360
                                       0.203 0.841418
## D
                                      13.991 2.16e-10 ***
                2.98844
                            0.21360
## E
                 2.18781
                            0.21360
                                      10.243 1.97e-08 ***
                                       5.789 2.77e-05 ***
## A:B
                 1.23656
                            0.21360
## A:D
                 1.66656
                            0.21360
                                       7.802 7.66e-07 ***
                                      -0.063 0.950618
## B:D
                -0.01344
                            0.21360
## A:E
                1.02719
                            0.21360
                                       4.809 0.000193 ***
                                       6.009 1.82e-05 ***
                1.28344
                            0.21360
## B:E
```

```
## D:E
               1.38969
                          0.21360
                                    6.506 7.24e-06 ***
## A:B:D
              -0.34531
                          0.21360 -1.617 0.125501
## A:B:E
               1.18531
                          0.21360
                                    5.549 4.40e-05 ***
## A:D:E
                          0.21360
                                    4.221 0.000650 ***
               0.90156
## B:D:E
               -0.03969
                          0.21360
                                   -0.186 0.854935
## A:B:D:E
               0.40719
                          0.21360
                                    1.906 0.074735 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.208 on 16 degrees of freedom
## Multiple R-squared: 0.9744, Adjusted R-squared: 0.9504
## F-statistic: 40.58 on 15 and 16 DF, p-value: 7.07e-10
anova partB model <- anova(partB model)</pre>
print(anova_partB_model)
## Analysis of Variance Table
## Response: y
##
            Df Sum Sq Mean Sq F value
                                           Pr(>F)
## A
             1 83.560 83.560 57.2328 1.136e-06 ***
## B
             1
                 0.060
                         0.060
                                0.0414 0.8414184
## D
             1 285.784 285.784 195.7422 2.161e-10 ***
## E
             1 153.169 153.169 104.9099 1.966e-08 ***
## A:B
             1 48.931 48.931 33.5142 2.767e-05 ***
             1 88.878 88.878 60.8751 7.661e-07 ***
## A:D
## B:D
                 0.006
                        0.006
                                0.0040 0.9506177
             1
## A:E
             1 33.764 33.764 23.1257 0.0001928 ***
## B:E
             1 52.711 52.711
                                36.1032 1.822e-05 ***
## D:E
             1 61.799 61.799 42.3283 7.243e-06 ***
                        3.816
## A:B:D
                 3.816
                                2.6135 0.1255014
             1
## A:B:E
             1 44.959 44.959
                                30.7937 4.402e-05 ***
## A:D:E
             1 26.010 26.010 17.8151 0.0006496 ***
## B:D:E
             1
                 0.050
                         0.050
                                 0.0345 0.8549347
## A:B:D:E
             1
                 5.306
                         5.306
                                 3.6340 0.0747350 .
## Residuals 16 23.360
                         1.460
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#model adequacy checking
# Plot to check for normality of residuals
par(mfrow=c(2,2))
plot(partB_model)
## hat values (leverages) are all = 0.5
```

and there are no factor predictors; no plot no. 5







Part C

Factor A: Positive coefficient - High level (+1) Factor B: Not significant, but its interactions are significant Factor D: Positive coefficient - High level (+1) Factor E: Positive coefficient - High level (+1)

Question 5

```
# Determine the block assignment based on the ABCDE interaction
experiment_data$Block <- with(experiment_data, A * B * C * D * E)</pre>
experiment_data$Block <- ifelse(experiment_data$Block == 1, "Block1", "Block2")</pre>
blocked_model <- lm(y ~ A*B*C*D*E + Block, data = experiment_data)</pre>
# Summary of the blocked model to see coefficients
summary(blocked_model)
##
## Call:
## lm(formula = y ~ A * B * C * D * E + Block, data = experiment_data)
##
## Residuals:
## ALL 32 residuals are 0: no residual degrees of freedom!
## Coefficients: (1 not defined because of singularities)
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 9.825000
                                 NaN
                                          NaN
                                                   NaN
## A
                1.615938
                                 NaN
                                         NaN
                                                   NaN
```

```
## B
                 0.043438
                                   NaN
                                            NaN
                                                      NaN
## C
                                   NaN
                                            NaN
                                                     NaN
                -0.012187
## D
                 2.988437
                                   NaN
                                            NaN
                                                     NaN
## E
                                           NaN
                                                     NaN
                 2.187813
                                   NaN
## BlockBlock2 0.710625
                                   NaN
                                            NaN
                                                      NaN
                                           NaN
                                                     NaN
## A:B
                 1.236562
                                   NaN
                -0.001563
## A:C
                                   NaN
                                           NaN
                                                     NaN
## B:C
                -0.195313
                                   NaN
                                            NaN
                                                     NaN
## A:D
                 1.666563
                                   NaN
                                            NaN
                                                     NaN
## B:D
                -0.013438
                                   NaN
                                            NaN
                                                      NaN
## C:D
                 0.003437
                                   NaN
                                            NaN
                                                      NaN
## A:E
                                            NaN
                                                      NaN
                 1.027187
                                   NaN
                 1.283437
## B:E
                                   NaN
                                            NaN
                                                      NaN
## C:E
                 0.301562
                                   NaN
                                            NaN
                                                      NaN
## D:E
                                            NaN
                 1.389687
                                   NaN
                                                      NaN
## A:B:C
                 0.250313
                                   NaN
                                            NaN
                                                      NaN
## A:B:D
                -0.345312
                                   NaN
                                            NaN
                                                     NaN
## A:C:D
                -0.063437
                                   NaN
                                            NaN
                                                      NaN
## B:C:D
                                                     NaN
                 0.305313
                                   NaN
                                           NaN
## A:B:E
                 1.185313
                                   NaN
                                            NaN
                                                      NaN
## A:C:E
                -0.259062
                                   NaN
                                           NaN
                                                     NaN
## B:C:E
                 0.170938
                                   NaN
                                            NaN
                                                      NaN
## A:D:E
                 0.901563
                                   {\tt NaN}
                                           {\tt NaN}
                                                     NaN
## B:D:E
                -0.039687
                                           NaN
                                                     NaN
                                   NaN
## C:D:E
                 0.395938
                                   NaN
                                           NaN
                                                     NaN
## A:B:C:D
                -0.074063
                                   NaN
                                           NaN
                                                     NaN
## A:B:C:E
                -0.184688
                                            NaN
                                                      NaN
                                   NaN
## A:B:D:E
                 0.407187
                                   NaN
                                            NaN
                                                      NaN
## A:C:D:E
                 0.127812
                                            NaN
                                                      NaN
                                   NaN
## B:C:D:E
                -0.074688
                                   NaN
                                            NaN
                                                      NaN
## A:B:C:D:E
                        NA
                                    NA
                                             NA
                                                      NA
##
## Residual standard error: NaN on O degrees of freedom
                              1, Adjusted R-squared:
## Multiple R-squared:
                                                            NaN
## F-statistic:
                   NaN on 31 and 0 DF, p-value: NA
anova_blocked_model <- anova(blocked_model)</pre>
## Warning in anova.lm(blocked_model): ANOVA F-tests on an essentially perfect fit
## are unreliable
print(anova_blocked_model)
## Analysis of Variance Table
##
## Response: y
##
                  Sum Sq Mean Sq F value Pr(>F)
              Df
## A
               1
                  83.560 83.560
                                       NaN
                                               NaN
## B
               1
                   0.060
                            0.060
                                       NaN
                                               NaN
## C
                   0.005
                            0.005
                                       NaN
                                               NaN
## D
               1 285.784 285.784
                                       NaN
                                               NaN
## E
               1 153.169 153.169
                                       NaN
                                               NaN
## Block
                   4.040
               1
                            4.040
                                       {\tt NaN}
                                               NaN
## A:B
                  48.931
                           48.931
                                               NaN
                                       {\tt NaN}
## A:C
                   0.000
                            0.000
                                       NaN
                                               NaN
```

##	B:C	1	1.221	1.221	NaN	${\tt NaN}$
##	A:D	1	88.878	88.878	NaN	${\tt NaN}$
##	B:D	1	0.006	0.006	NaN	${\tt NaN}$
##	C:D	1	0.000	0.000	NaN	${\tt NaN}$
##	A:E	1	33.764	33.764	NaN	${\tt NaN}$
##	B:E	1	52.711	52.711	NaN	${\tt NaN}$
##	C:E	1	2.910	2.910	NaN	${\tt NaN}$
##	D:E	1	61.799	61.799	NaN	${\tt NaN}$
##	A:B:C	1	2.005	2.005	NaN	${\tt NaN}$
##	A:B:D	1	3.816	3.816	NaN	${\tt NaN}$
##	A:C:D	1	0.129	0.129	NaN	${\tt NaN}$
##	B:C:D	1	2.983	2.983	NaN	${\tt NaN}$
##	A:B:E	1	44.959	44.959	NaN	${\tt NaN}$
##	A:C:E	1	2.148	2.148	NaN	${\tt NaN}$
##	B:C:E	1	0.935	0.935	NaN	${\tt NaN}$
##	A:D:E	1	26.010	26.010	NaN	${\tt NaN}$
##	B:D:E	1	0.050	0.050	NaN	${\tt NaN}$
##	C:D:E	1	5.017	5.017	NaN	${\tt NaN}$
##	A:B:C:D	1	0.176	0.176	NaN	NaN
##	A:B:C:E	1	1.092	1.092	NaN	NaN
##	A:B:D:E	1	5.306	5.306	NaN	NaN
##	A:C:D:E	1	0.523	0.523	NaN	${\tt NaN}$
##	B:C:D:E	1	0.179	0.179	NaN	${\tt NaN}$
##	Residuals	0	0.000	NaN		

All 32 residuals are 0, which means the model has a perfect fit to the data with no residual variation. This shows that the model is over-parameterized, there are as many parameters being estimated as there are observations. The significance test are not valid in this test.