Self-study: differential geometry for manifolds

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Contents

1 Definitions

1.1 General and differential manifold

- A *topological manifold*[Munkres, 2000]

- A *differential manifold* [Guillemin and Pollack, 2010]

- is referred as the *parameterization* [Guillemin and Pollack, 2010]

- is referred as a *coordinate system* [Guillemin and Pollack, 2010]

- The *differential* of a map f, df

1.2 Stiefel and Grassmann manifold

- A *Stiefel manifold* $V(r,n) \subset \mathbb{R}^{n\times r}$ is space of all n-by-r matrices whose rank is r (full column rank). It is a submanifold embedded in a $rn-$dimensional Euclidean space of all n-by-r matrices. $\boldsymbol{V} \in V(r,n)$ then $\boldsymbol{V} = \boldsymbol{T}_{n\times n} \begin{bmatrix} \boldsymbol{V}_r \\ \boldsymbol{0} \end{bmatrix}$. The other representation of $V(r,n)$ arises from quotient group. Consider two orthogonal matrices in $O(n)$ are equivalent if their first r-columns are identical, then $U_1 = U_1 \begin{pmatrix} \boldsymbol{I} & 0 \\ 0 & \boldsymbol{Q} \end{pmatrix}$, where $\boldsymbol{Q} \in O(n-r)$. Then $V(r,n) = O(n)/O(n-r)$ as we can add arbitrary $n-r$ more orthogonal basis vectors and make them all orthogonal.

- A *Grassmann manifold* $G(r,n)$ is the space of all $r-$dimensional linear subspace in $\mathbb{R}^n$ and $G(r,n)$ is the quotient space of $V(r,n)$ by identifying the p-dim orthogonal group, i.e. $G(r,n) = V(r,n)/O(p)$. A point in $G(r,n)$ is an equivalence class with respect to the relationship $\boldsymbol{W} \sim_R \boldsymbol{V} \Rightarrow \boldsymbol{W} = \boldsymbol{V}\boldsymbol{U}$, where $\boldsymbol{U} \in O(p)$, and two matrices are equivalent if their column span is the same subspace.

2 Theorems and Properties for topological and differential manifold

3 Theorems and Properties for Stiefel and Grassmann manifold

References

Victor Guillemin and Alan Pollack. *Differential topology*, volume 370. American Mathematical Soc., 2010.

James R Munkres. *Topology, 2nd*. Prentice Hall, 2000.