Lecture 4: The Tychonoff Theorem

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Contents

1 The Tychonoff Theorem

- **Lemma 1.1** ***(Existance of Maximal Collection with Finite Intersection Property)*** *[Munkres, 2000]*
  Let X be a set; let $\mathscr{A}$ be a collection of subsets of X having the ***finite intersection property****. Then there is a collection $\mathscr{D}$ of subsets of X such that $\mathscr{D}$* ***contains*** *$\mathscr{A}$, and $\mathscr{D}$ has the finite intersection property, and no collection of subsets of X that properly contains $\mathscr{D}$ has this property.*

  [Hint: apply Zorn's Lemma to *the collection of collections of subsets* with finite intersection property]

- **Definition** We often say that a collection $\mathscr{D}$ satisfying the conclusion of this theorem is ***maximal*** *with respect to* ***the finite intersection property****.*

- **Lemma 1.2** ***(Elements of Maximal Collection with Finite Intersection Property)*** *[Munkres, 2000]*
  Let X be a set; let $\mathscr{D}$ be a collection of subsets of X that is ***maximal*** *with respect to* ***the finite intersection property****. Then:*

  1. *Any* ***finite intersection of elements*** *of $\mathscr{D}$ is an* ***element*** *of $\mathscr{D}$.*
  2. *If A is a subset of X that* ***intersects every element*** *of $\mathscr{D}$, then A is an element of $\mathscr{D}$.*

- **Theorem 1.3** ***(Tychonoff Theorem)****. [Munkres, 2000]*
  An ***arbitrary product*** *of* ***compact*** *spaces is* ***compact*** *in the* ***product topology****.*

2 The Stone-Ĉech Compactification

References

James R Munkres. *Topology, 2nd*. Prentice Hall, 2000.