

Lecture 5: Concentration of Measure and Isoperimetry

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Contents

1	Concentration of Measures	2
1.1	Blow-Up of Sets	2
1.2	Levy's Inequalities	2
1.3	Concentration on the Hypercube	2
1.4	Gaussian Concentration: The Tsirelson-Ibragimov-Sudakov Inequality	2
1.5	Gaussian Random Projections	2
1.6	Convex Distance Inequality	2
1.7	Concentration of Convex Lipschitz Function	2
2	The Classic Isoperimetry Theorems	2
2.1	Brunn-Minkowski Inequality	2
2.2	Isoperimetric Inequalities on the Unit Sphere	2
2.3	Gaussian Isoperimetry Inequalities	2
2.4	Concentration of Lipschitz Function for the Sphere	2
2.5	Edge Isoperimetric Inequality on the Binary Hypercube	2
2.6	Vertex Isoperimetric Inequality on the Hypercube	2

1 Concentration of Measures

1.1 Blow-Up of Sets

1.2 Levy's Inequalities

1.3 Concentration on the Hypercube

1.4 Gaussian Concentration: The Tsirelson-Ibragimov-Sudakov Inequality

1.5 Lipschitz Function of Gaussian Random Variables

1.6 Gaussian Random Projections

1.7 Convex Distance Inequality

1.8 Concentration of Convex Lipschitz Function

2 The Classic Isoperimetry Inequalities

2.1 Brunn-Minkowski Inequality

2.2 Gaussian Isoperimetric Inequalities

2.3 Concentration of Lipschitz Function for the Sphere

2.4 Isoperimetric Inequalities on the Unit Sphere

2.5 Edge Isoperimetric Inequality on the Binary Hypercube

2.6 Vertex Isoperimetric Inequality on the Binary Hypercube