Computer Vision
and Geometry Lab

Computer Vision

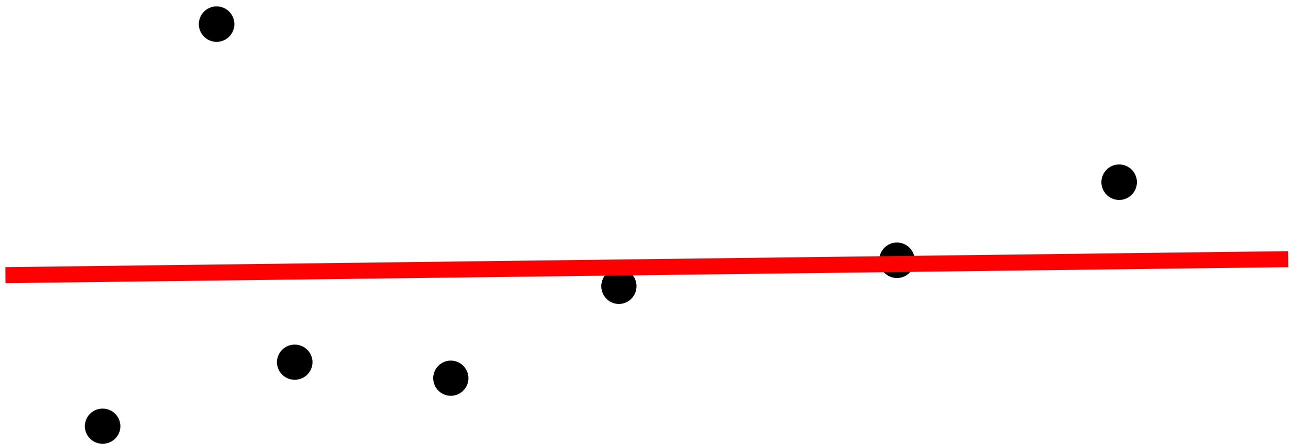
Exercise Session 4

Assignment 4

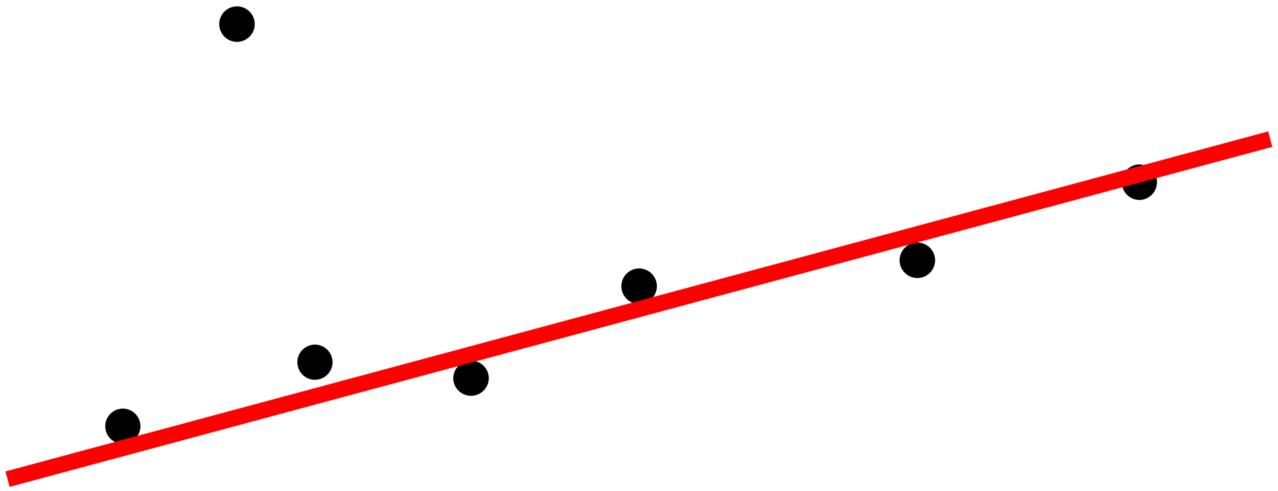
- 3 Tasks:
  - 2D line fitting with RANSAC.
  - 8-Point algorithm (fundamental/essential matrix)
  - Fundamental matrix fitting with RANSAC

RANSAC

- Least squares solution is dramatically effected by outliers:



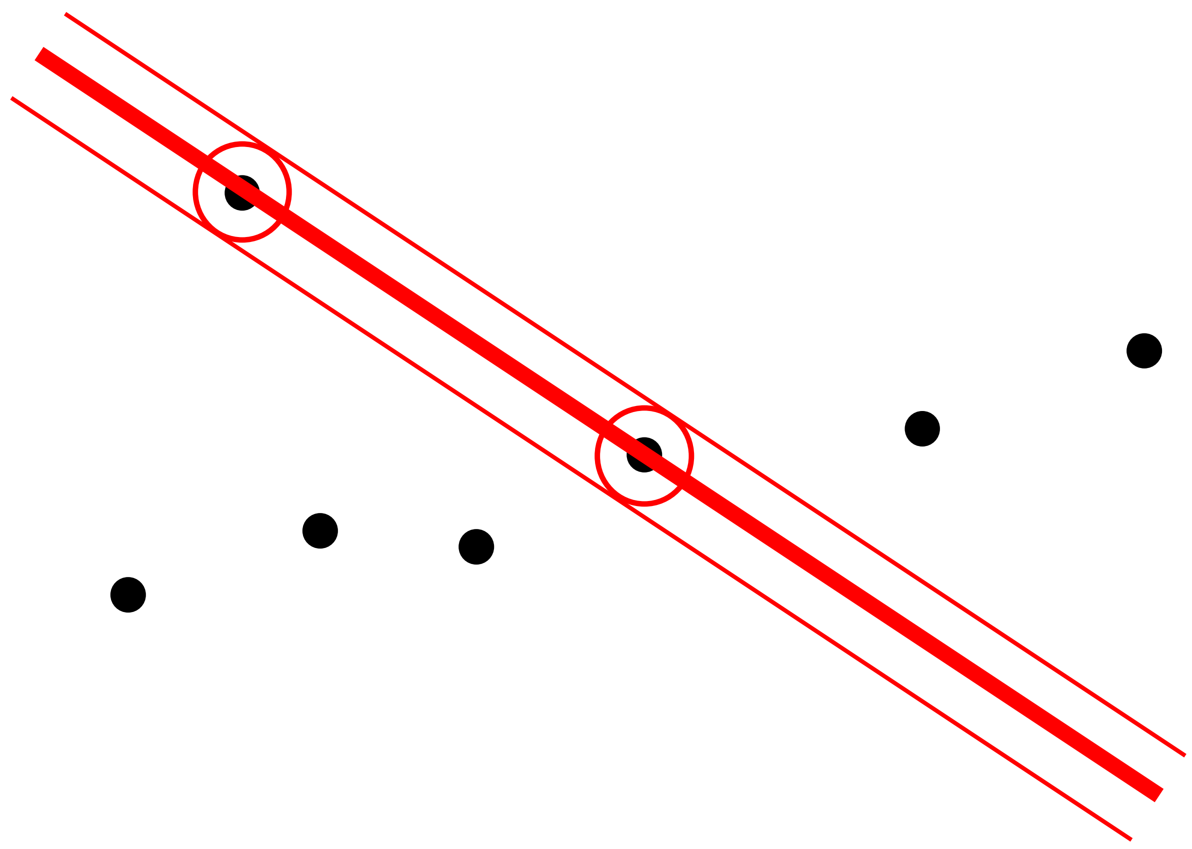
What we want to have:



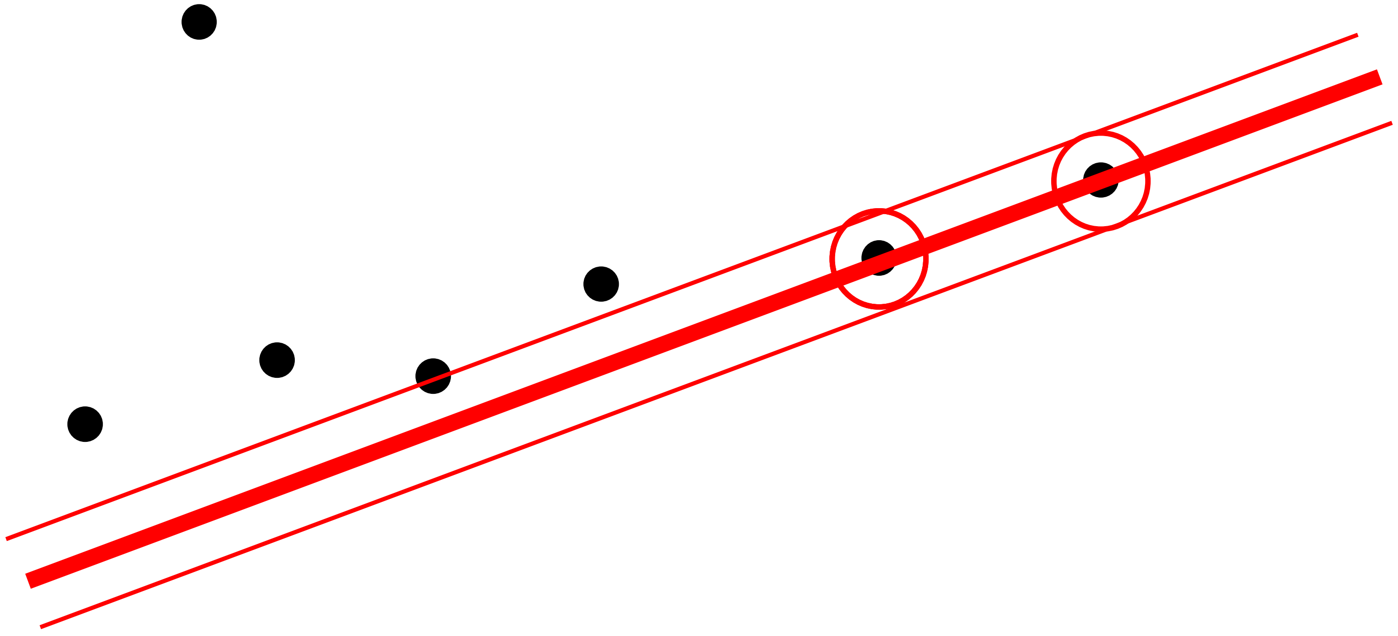
RANSAC

- Algorithm
  1. Guess N points that you hope are inliers.
  2. Compute the solution.
  3. Check how many other points fit within some threshold, i.e. are inliers.
  4. Repeat 1-3 until you’re sure the solution has been found.
  5. (Optional) Take the solution that has the most inliers, and compute least-squares solution from inliers.

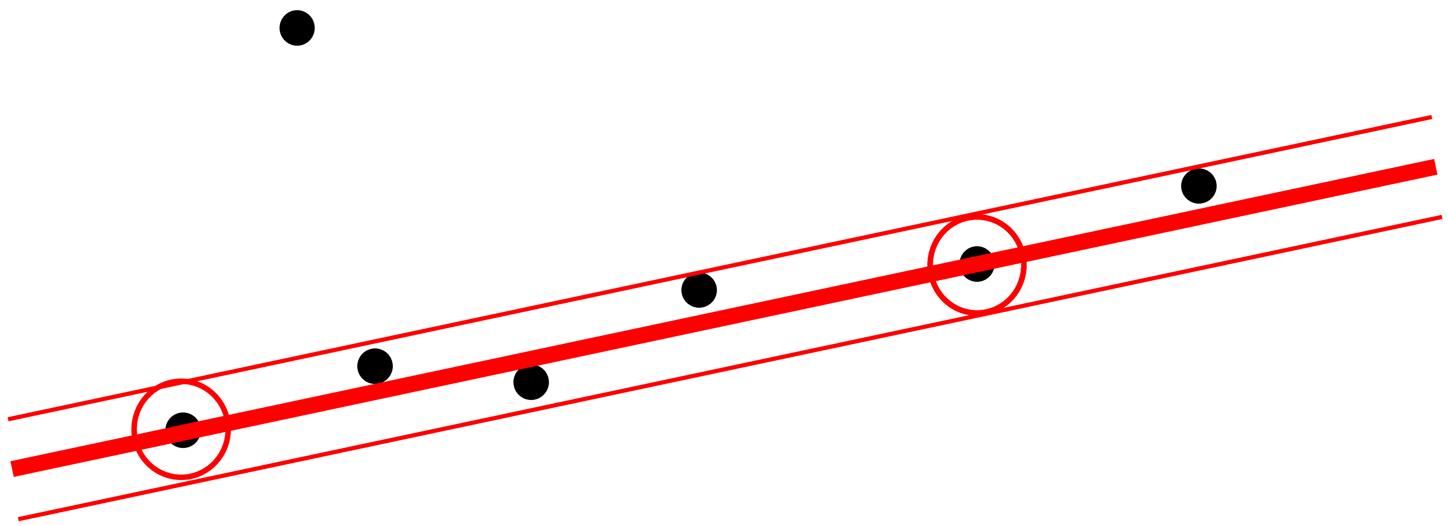
RANSAC



RANSAC

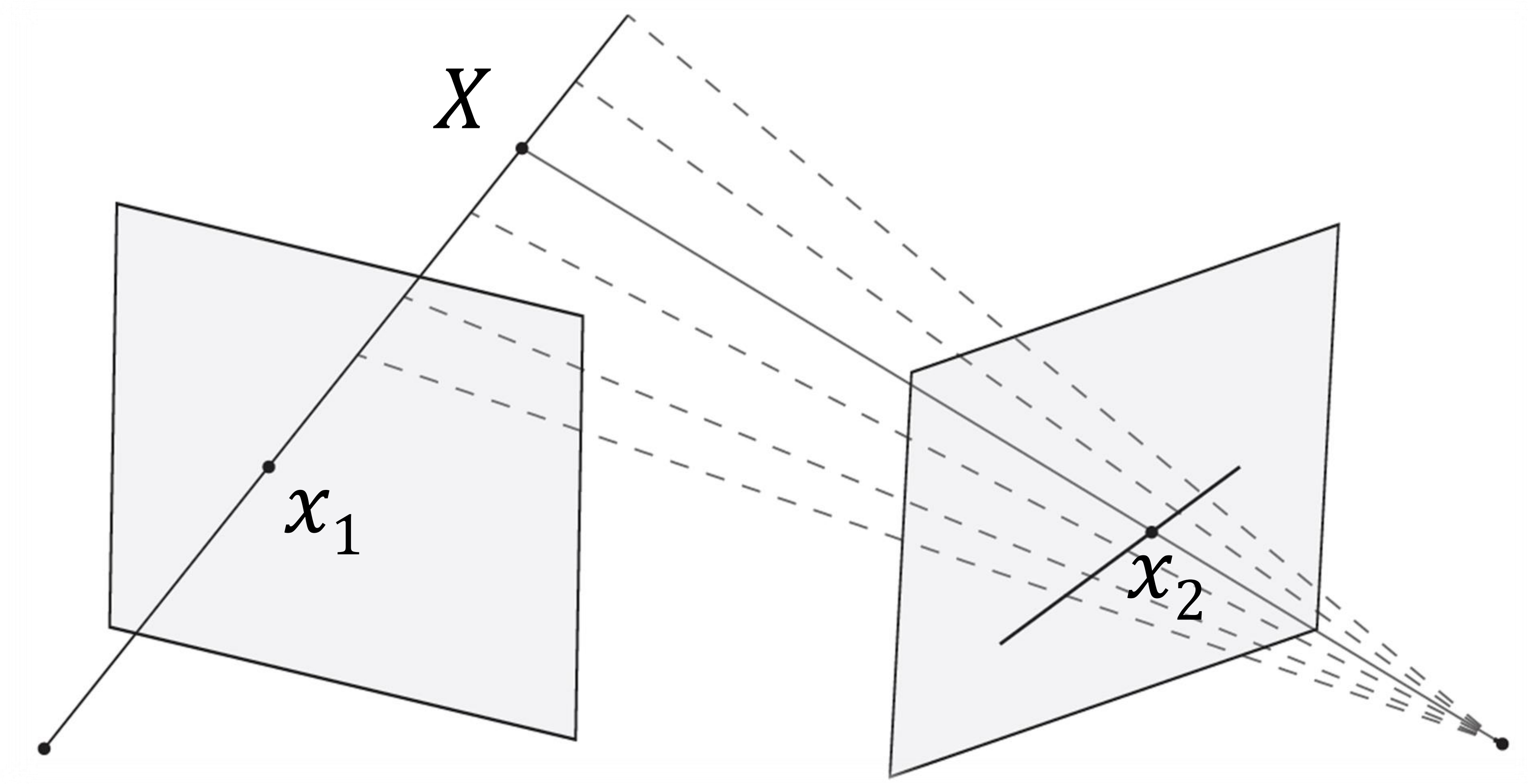


RANSAC



Fundamental matrix

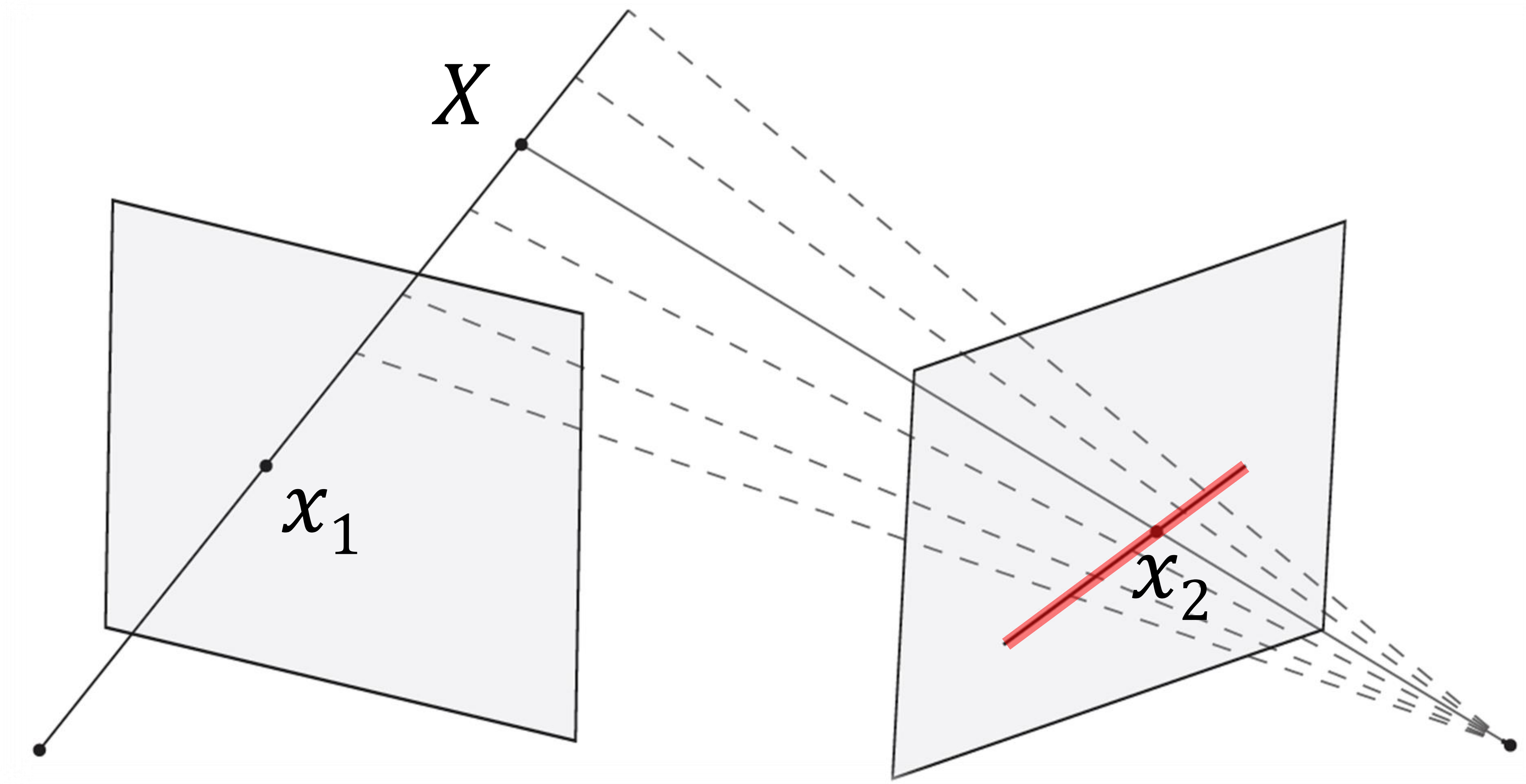
- 3x3 matrix
- $\det(F) = 0$
- $x_2^T F x_1 = 0$



Fundamental matrix

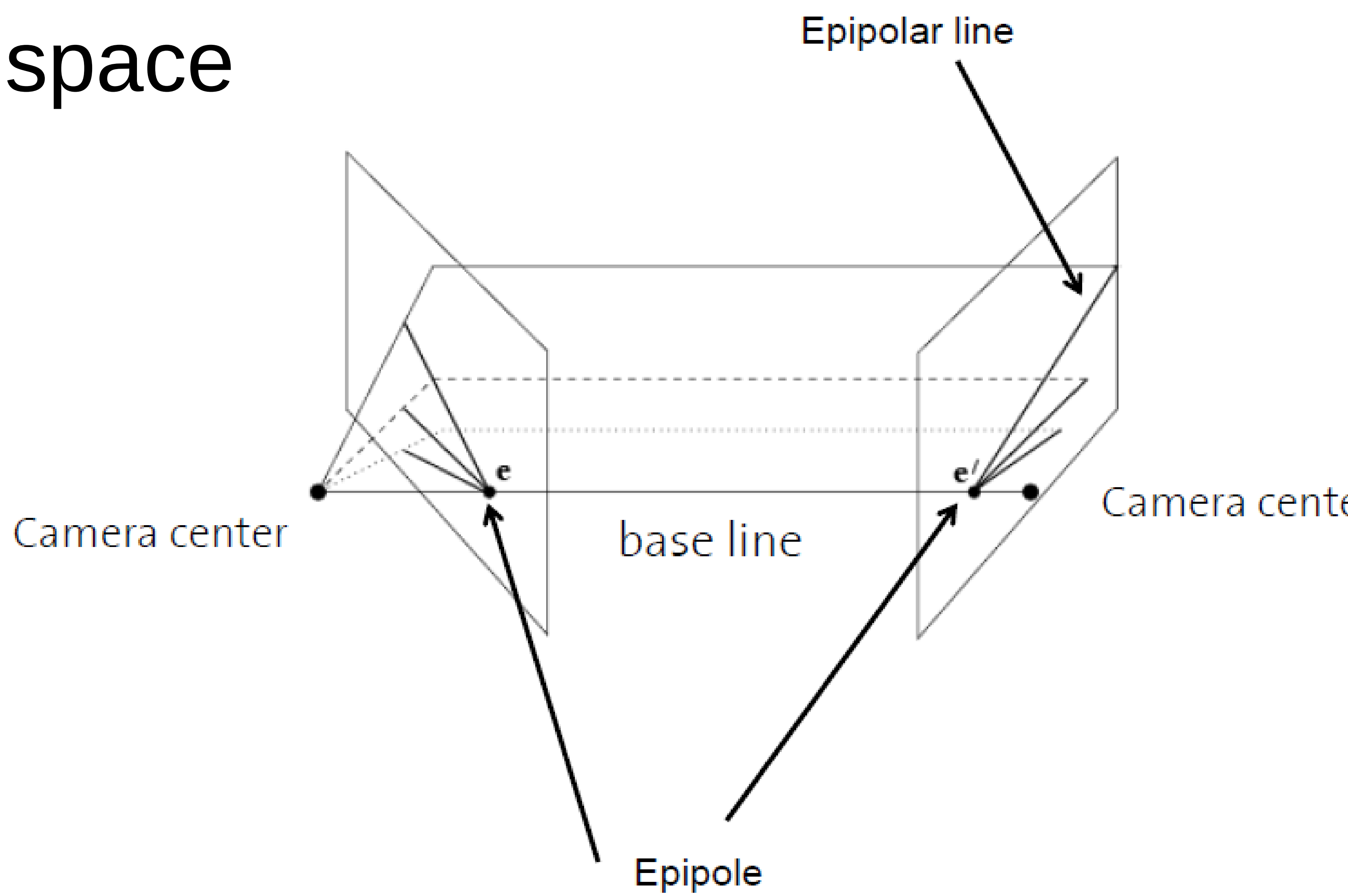
- 3x3 matrix
- $\det(F) = 0$
- $x_2^T F x_1 = 0$

$$l = Fx_1$$



Fundamental matrix

- Epipoles = Projections of the camera centers
- $Fe = F^T e' = 0$
- Left and right nullspace



Eight point algorithm

- Linear constraint

$$x'^T F x = 0$$

- $(xx', xy', x, yx', yy', y, x', y', 1) * vec(F) = 0$

Eight point algorithm

- Linear constraint

$$x'^T F x = 0$$

- $(xx', xy', x, yx', yy', y, x', y', 1) * vec(F) = 0$

$$\begin{pmatrix} x_1x_1', x_1y_1', x_1, y_1x_1', y_1y_1', y_1, x_1', y_1', 1 \\ x_2x_2', x_2y_2', x_2, y_2x_2', y_2y_2', y_2, x_2', y_2', 1 \\ x_3x_3', x_3y_3', x_3, y_3x_3', y_3y_3', y_3, x_3', y_3', 1 \\ \vdots \end{pmatrix} \mathbf{f} = 0$$

- Homogeneous least squares (solve using SVD)

$$\min_{\boldsymbol{f}} |A\boldsymbol{f}|^2 \; s.t. \; |\boldsymbol{f}|^2 = 1$$

Eight point algorithm

- Points need to be normalized (center and rescale)
  - Don't forget to remove normalization afterwards!
  - Why normalize?

$$\left(\underbrace{xx', xy'}_{\sim 10^6}, x, yx', yy', \underbrace{y, x', y'}_{\sim 10^3}, \underbrace{1}_{1}\right) * vec(F) = 0$$

- Enforce singularity constraint $\det(F) = 0$
  - Project to closest singular matrix using SVD (set last singular value to zero)

Essential matrix (calibrated cameras)

- Essential matrix is Fundamental matrix for calibrated cameras

$$\hat{x}_2^T E \hat{x}_1 = 0$$

where $\hat{x}_1$ and $\hat{x}_2$ are calibrated image coordinates

$$\hat{x}_1 = K_1^{-1} x_1 \text{ and } \hat{x}_2 = K_2^{-1} x_2$$

- K is the camera calibration matrix
- First two singular values equal, third zero

$$E = [t]_\times R \qquad [\mathrm{t}]_\times = \begin{bmatrix} 0 & t_z & -\mathrm{t}_y \\ -\mathrm{t}_z & 0 & t_x \\ t_y & -\mathrm{t}_x & 0 \end{bmatrix}$$

Essential matrix (calibrated cameras)

- Linear solution using 8 points
- Enforce constraints using SVD

$$E = U \begin{bmatrix} s_1 & & \\ & s_2 & \\ & & s_3 \end{bmatrix} V^T$$

Set first two singular values equal and last to zero

$$\hat{E} = U \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^T$$

Assignment 4.2 and 4.3

- Manually click points in images
- Estimate Fundamental/Essential matrix using normalized 8 point algorithm (see [1])
- Draw epipolar lines + epipoles
- Tip: Use more than 8 points

[1] Hartley, In defence of the 8-point algorithm, ICCV'95

Assignment 4.4

- Decompose essential matrix, i.e. find R, t such that $E = [t]_x R$

- $E = USV^T$ $\quad\quad$ $W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_1 = UWV^T$ and $R_2 = UW^TV^T$

$t_1 = U_3$ and $t_2 = -U_3$ (where U_3 is third col. of U_3)

- Cameras are $P_1 = [I \; 0]$ and P_2 is one of

$$[R_1 \; t_1], \quad [R_1 \; t_2], \quad [R_2 \; t_1], \quad [R_2 \; t_2]$$

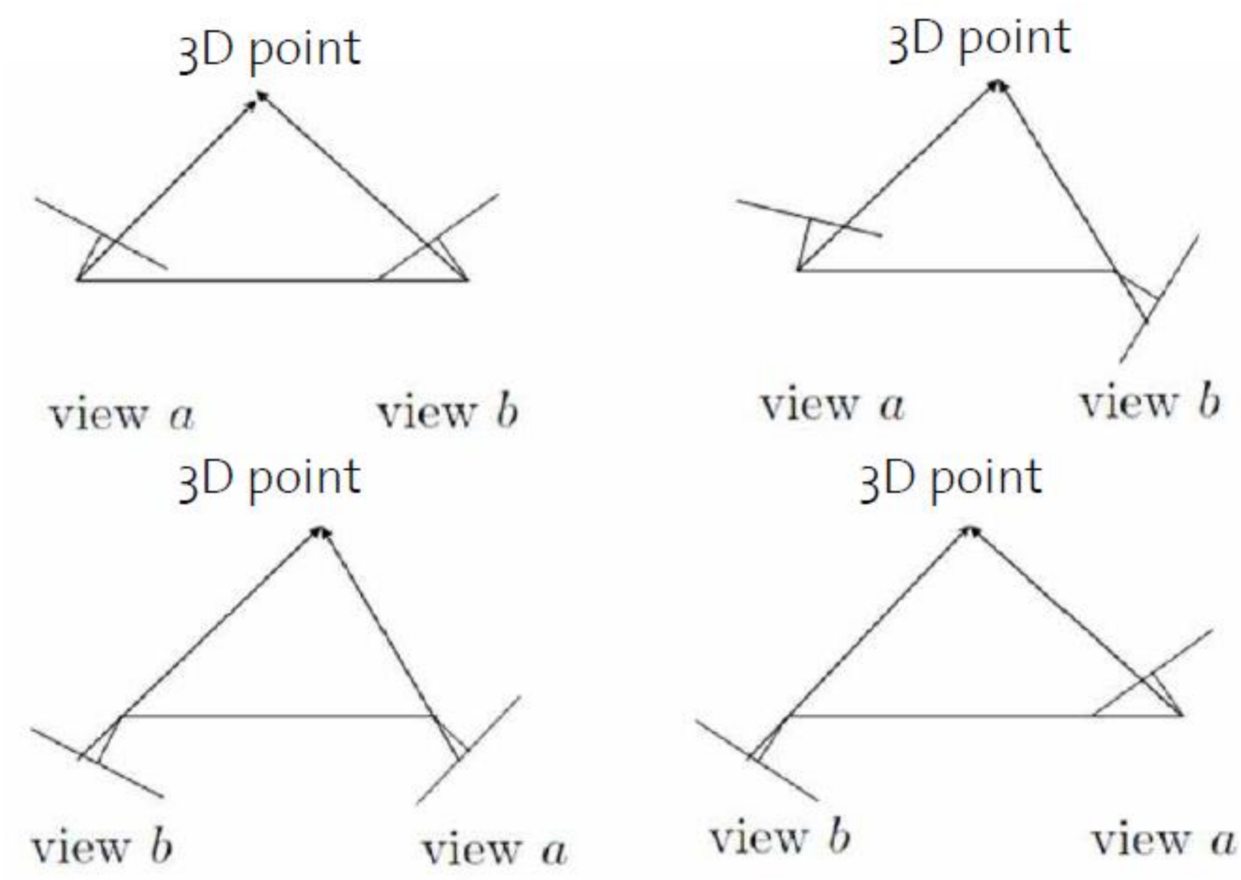
- For RHS coordinate system $\det(R) = 1$

Choosing correct cameras from E

- Cameras are $P_1 = [I\ 0]$ and P_2 is one of

$$[R_1\ t_1], \qquad [R_1\ t_2], \qquad [R_2\ t_1], \qquad [R_2\ t_2]$$

- Correct solution -> 3D points in front of cameras



Choosing correct cameras from E

- Cameras are $P_1 = [I\ 0]$ and P_2 is one of

$$[R_1\ t_1], \qquad [R_1\ t_2], \qquad [R_2\ t_1], \qquad [R_2\ t_2]$$

- Correct solution -> 3D points in front of cameras
- Easy check

  In front of P1 X(3,:) > 0

  In front of P2 P2(3,:)*X > 0

- Notice that PX is the coordinate of X in camera P

Assignment 4.5

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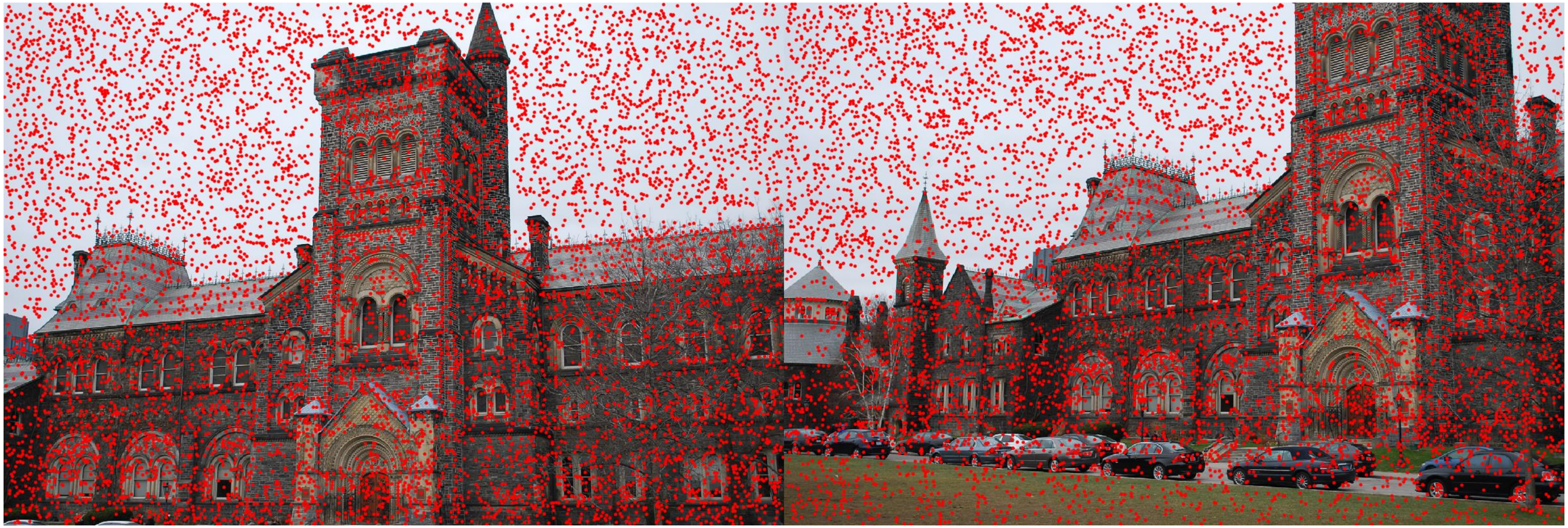
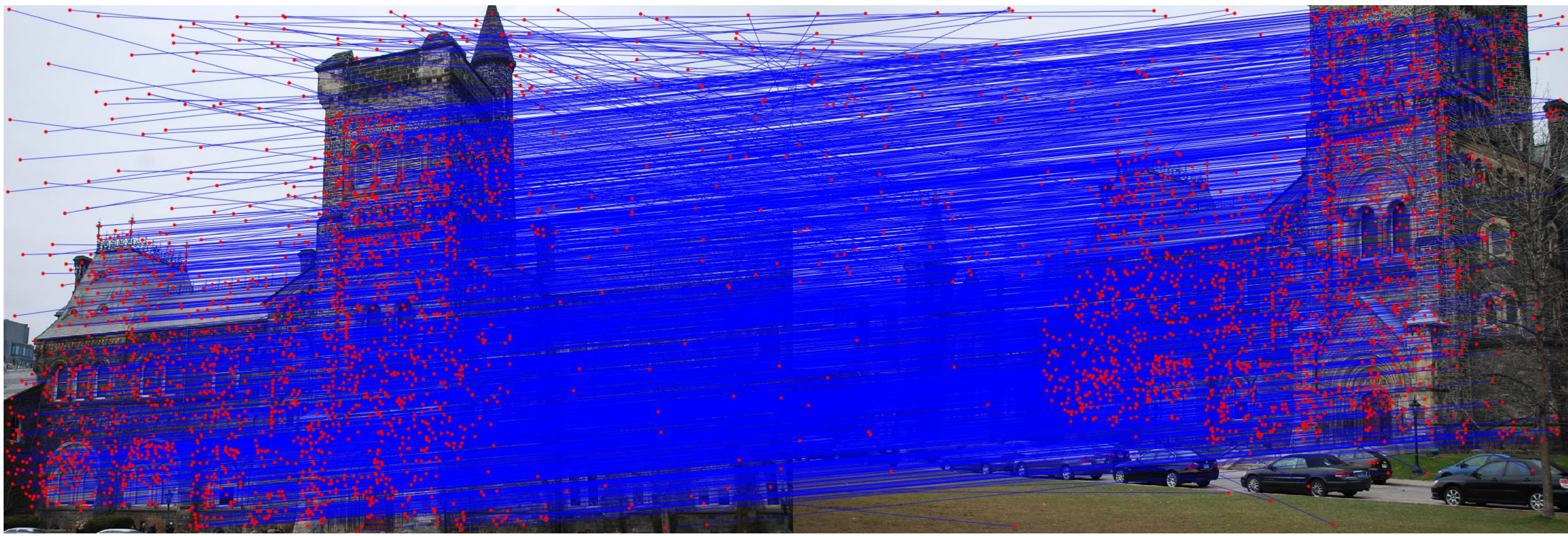
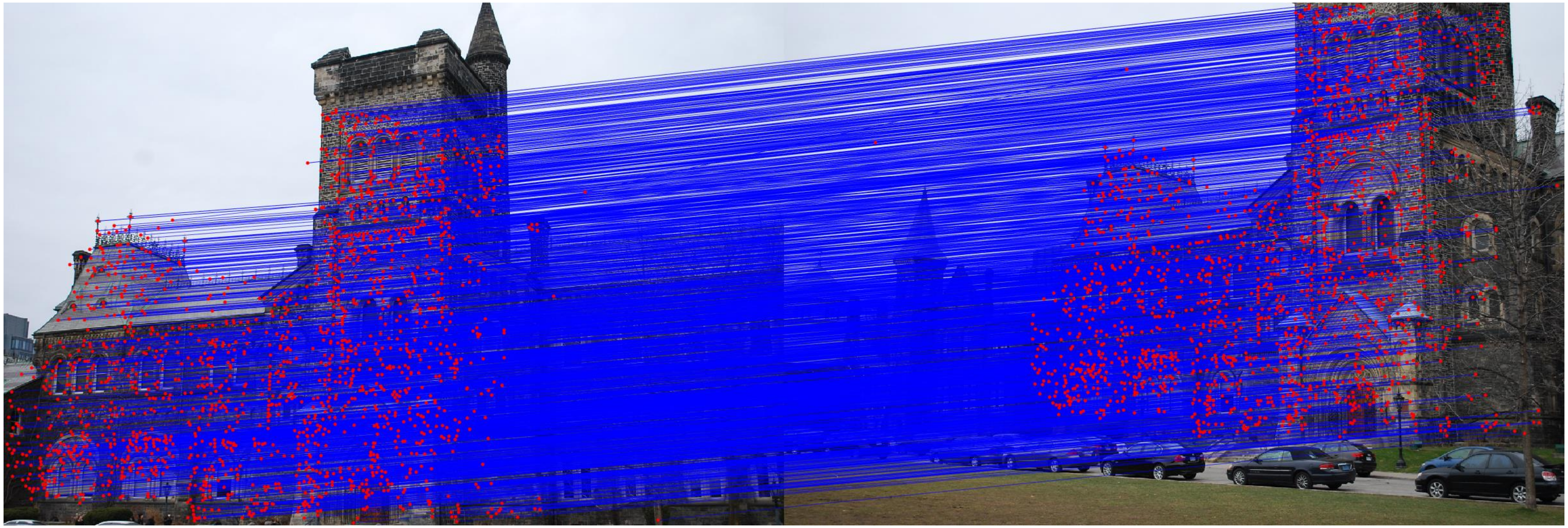


Assignment 4.5



Assignment 4.6 (RANSAC 8 point alg.)



RANSAC

- Probability of having found solution:

$$p = 1-(1-r^N)^M$$

  - r is inlier ratio
  - N is number of samples drawn (i.e. 8 for fundamental matrix)
  - M is number of iterations
- Adaptive RANSAC:
  - Use the largest number of inliers found so far as a lower bound on p
  - Stop iterating once the solution probability lower bound is above 0.99

Hand-in

- Assignment 4 should be submitted latest by
  - 15:00, 25th Oct 2018

Tip: Kronecker product

- Useful identity

$$vec(AXB) = (B^T \otimes A) * vec(X)$$

- Fundamental matrix

$$x_2^T F x_1 = (x_1^T \otimes x_2^T) * vec(F)$$

- In MATLAB

```
x2'*F*x1  ==  kron(x1',x2')*F(:)
```