MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) If government need money, they can borrow money by issuing bonds but not simply print money, otherwise, it will lead to serious inflation problem.

   (b) For instance, when fed is going to raise interest rates, the economic condition is very uncertain, in this case the yield curve is flat for both long term and short term as investors prefer to buy bonds for different maturities in order to be benefited no matter what will happened.

   (c) Quantitative easing is that government purchases securities from open market which will raise the price of those securities and decreases their yield meanwhile increases money supply in the market to stimulate economic and for U.S, the fed largely purchases U.S bond and mortgaged backed securities to against economic downturn caused by Covid pandemic.

2. In order to construct an efficient yield and spot curve, we need to choose similar bonds but different year to maturities. I will choose the bonds with year to maturities of roughly four months, ten months, a year and four months, a year and ten months and so on. Besides maturities, I wish all those bonds are as much as similar in other conditions like coupon rate, Moody's rating. The reason I choose those is based on definition of yield curve as yield curve is a line that plots yields of similar bonds but differing maturity dates and I wish my lists to be representative. Also, the reason I want my maturities spread evenly within five years is it can make my curve more precise. Also in order to use bootstrapping, we need to find 10 bonds which their number of payments left is one, two, three, four, five, six, seven, eight, nine, ten. The list of bonds that I choose is : year 2022 :CAN 0.75 Apr 30, CAN 0.125 Oct 31, Year 2023: CAN 0.125 APR 30, CAN 0.25 Oct 31, Year 2024: CAN 0.125 Mar 31, CAN 0.375 Sep 30, Year 2025: CAN 0.625 Feb 28, CAN 0.25 Aug 31, Year 2026: CAN 0.125 Feb 28, CAN 0.5 Aug 31.

3. PCA is to reduce the variables of data meanwhile preserve as much information as possible. Eigenvectors is direction. Each eigenvector is a particular direction in your scatterplot of data. And eigenvalue represent the how important there is of data in that direction or how the data spread out along that direction, the higher the eigenvalue, the more important the direction. The principle component is the eigenvector with highest eigenvalue. So basically, we can use eigenvalue and eigenvector to pick the most important variables.

Empirical Questions - 75 points

4.

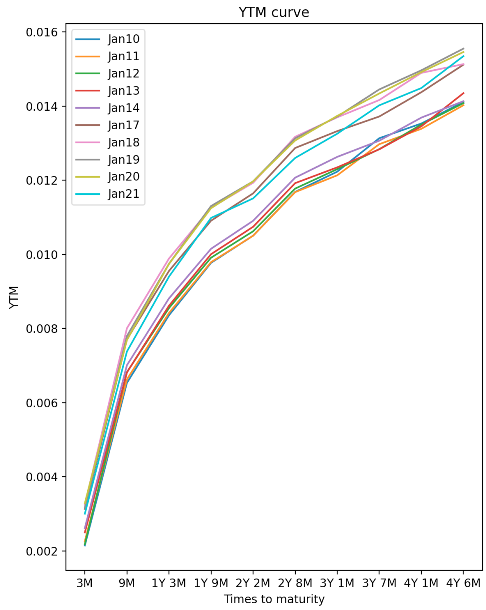
(a) For Yield to maturity, I basically used the Newton method to find the solution for the formula

$$DirtyPrice = \sum \frac{1}{(1+ytm)^i} * coupon + \frac{1}{(1+ytm)^n} * (coupon + facevalue)$$

or

$$dirtyprice = (coupon * \frac{1 - (\frac{1}{(1+ytm)^n})}{ytm}) + Facevalue * \frac{1}{(1+ytm)^n}$$

The way I deal with dirty price is using closed price plus the accrued interest. For the yields, I also multiply by two since the coupons are paid every half year.



(b) For spot rate, I used the bootstrapping method. For bond maturities less than 6 months, the formula is

$$r(T0) = \frac{-log(\frac{P}{N})}{T}$$

. For bond's maturities between six months and one year,

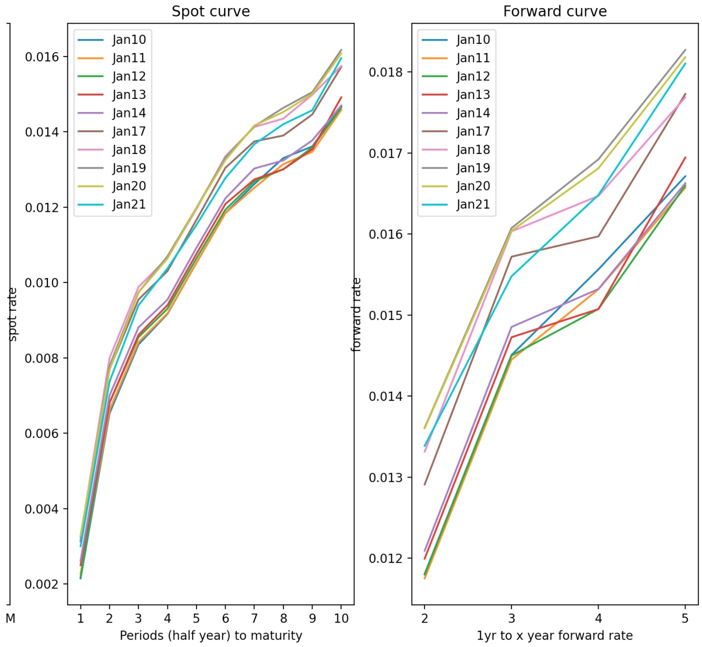
$$DirtyPrice = Coupon * e^{-r(t1)*t1} + (coupon + facevalue) * e^{-r(t2)*t2}$$

For maturities longer than 1 years ie. for n half years, the formula can be written as

$$DirtyPrice = \sum coupon * e^{-r(ti)*(ti)} + (coupon + facevalue) * e^{-r(tn)*tn}$$

for

$$0 < i < n$$



(c) For forward rate, we have the equation

$$r(t,T1,T2) = -\frac{logP(t,T2) - logP(t,T1)}{T2 - T1}$$

Also we know that

$$logP(t,T) = -r(t,T) * (T - t)$$

Then we can get

$$r(t,T1,T2) = \frac{r(t,T2) * (T2 - t) - r(t,T1) * (T1 - t)}{T2 - T1}$$

In this case, t is 0 and T1 is 1. So I used this formula to compute the forward rates.

5. For the daily log YTM, the covariance matrix is $\begin{bmatrix} 0.001655 & 0.001023 & 0.001023 & 0.000685 & 0.000569 \\ 0.001023 & 0.000715 & 0.000707 & 0.000553 & 0.000513 \\ 0.001023 & 0.000707 & 0.000723 & 0.000533 & 0.000492 \\ 0.000685 & 0.000553 & 0.000533 & 0.000558 & 0.000402 \\ 0.000569 & 0.000513 & 0.000492 & 0.000402 & 0.000653 \end{bmatrix}$

   For the daily log forward rate, the covariance matrix is $\begin{bmatrix} 0.000576 & 0.000551 & 0.000491 & 0.000504 \\ 0.000551 & 0.000595 & 0.000436 & 0.000432 \\ 0.000491 & 0.000436 & 0.000531 & 0.000360 \\ 0.000504 & 0.000432 & 0.000360 & 0.000671 \end{bmatrix}$

6. For YTM covariance, the eigenvectors are 1. $\begin{bmatrix} -0.6401534 & -0.43961343 & -0.4369919 & -0.3321034 & -0.3093364 \end{bmatrix}$ with eigenvalue 3.686186e-03. 2. $\begin{bmatrix} -0.553582147 & 0.050471692 & -0.002487172 & 0.271825430 & 0.785559778 \end{bmatrix}$ with eigenvalue 4.225323e-04. 3. $\begin{bmatrix} 0.26162295 & -0.06288551 & -0.02272312 & -0.83603629 & 0.47762519 \end{bmatrix}$ with eigenvalue 1.700624e-04. 4. $\begin{bmatrix} -0.36370956 & -0.05560517 & 0.88454573 & -0.23082964 & -0.17005867 \end{bmatrix}$ with eigenvalue 2.423589e-05. 5. $\begin{bmatrix} 0.2881305 & -0.8928304 & 0.1615375 & 0.2521492 & 0.1736695 \end{bmatrix}$ with

eigenvalue 9.837178e-07. The principle component is vector number 1. For daily log Forward rate covariance, the eigenvectors are 1. $\begin{bmatrix} -0.5350480 & -0.5089573 & -0.4558506 & -0.4968766 \end{bmatrix}$ with eigenvalue 1.986498e-03. 2. $\begin{bmatrix} -0.1023044 & -0.2637913 & -0.4864332 & 0.8266382 \end{bmatrix}$ with eigenvalue 2.589271e-04. 3. $\begin{bmatrix} -0.058949812 & -0.71064624 & 0.68098083 & 0.16664918 \end{bmatrix}$ with eigenvalue 1.216017e-04. And finally, the forth eigenvector is 4. $\begin{bmatrix} 0.8365300 & -0.4078707 & -0.3030645 & -0.2049659 \end{bmatrix}$ with eigenvalue 5.973533e-06. The principle component is vector number 1. It imply that the eigenvctors number one for both cases can explained the corresponding eigenvalues variation and they are the most important directions.

References and GitHub Link to Code

https://www.brookings.edu/research/fed-response-to-covid19/

https://thebusinessprofessor.com/en$_US/investments-trading-financial-markets/flat-yield-curve-definitionhttps://georgemdallas.wordpress.com/$2013/10/30$/principal-component-analysis-4-dummies-eigenvectors-eigenvalues-and-dimension-reduction/$

https://medium.com/@dareyadewumi650/understanding-the-role-of-eigenvectors-and-eigenvalues-in-pca-dimensionality-reduction-10186dad0c5c

Git HUb: https://github.com/Tianqiy3238/Apm466