AMATH 482: HOME WORK 1

TIANRUI ZHOU

AMath Department, University of Washington, Seattle, WA
tianrz5@uw.edu

Abstract. This report is about solving a Finding Submarine problem. The content will be divided into five sections: Introduction and Overview, Theoretical Background, Algorithm Implementation and Development, Computational Results, and Summary and Conclusions. They will elaborate on how I used Fourier Transform and Filters to process the raw noisy data, thereby locating and analyzing path of the submarine. I used Python code as my data processing tool and generated visual results with it. I will provide detailed explanations of my understanding of the mathematical foundations of the methods and algorithms. I will present my computational results and my conclusion for this project.

1. Introduction and Overview

The goal is to locate a moving submarine, which is a new technology emitting unknown acoustic frequency, in the Puget Sound using noisy acoustic data. The data given is broad spectrum recording of acoustics data, in a subdata.npy file, obtained over 24 hours in half-hour increments.[2]

There are a few tasks I need to solve for this project. First, I have to determine the frequency signature, which is the center frequency, generated by the submarine, by calculating the average of the Fourier transform. Then, use the result of the frequency signature to design and implement a Filter to denoise the data given, so that a more robust path of the submarine can be created.[2]

To enable future tracking of a submarine by an aircraft, I have to plot the x, y coordinates visualization of the submarine during the 24 hour period.[2]

2. Theoretical Background

The primary objective of my program is to analyze the submarine acoustic pressure signal over time, find the frequency signature first, and then apply it to Filter and extract a more robust path of the submarine using Filter.

Each timestamp within the dataset corresponds to a 3D measurement of acoustic pressure with the size $64 \times 64 \times 64$. The raw data files contain a matrix with 49 columns of data, which stand for 49 timestamps.

At each timestamp, the one-dimensional array of acoustic pressure measurements is reshaped into a three-dimensional grid. Then, the Fourier transform (`np.fft.fftn`) is applied to the reshaped data and transfers it to the frequency domain. To determine the frequency signature, it is done

Date: January 26, 2024.

by averaging the Fourier transform for every timestamps. This averaged Fourier transform provides information for dominant frequencies generated by the submarine over the observation period.

$$F(u, v, w) = \iiint_{-\infty}^{\infty} f(x, y, z) \cdot e^{-2\pi i(ux+vy+wz)} \, dx \, dy \, dz$$

[1] The above is the formula for $3D$ Fourier Transform.

In order to find a more robust path for the submarine, I decided to reduce noise at each timestamp using a 3D Gaussian filter. This filter is centered at frequency signature that was already calculated, and operates by selectively keeping the lower frequency signals around the center, which corresponds to overall motion patterns; At the same time, it suppresses the noisy (higher frequency) signals. The application of this filter is then followed by the inverse Fourier transform (`np.fft.ifftn`) to shift the filtered signal back to the spatial domain.

To extract the current location of the submarine, the highest frequency in the frequency domain serves as a symbol for the information of current location. Through this idea, a more robust and denoised trajectory can be determined.

3. Algorithm Implementation and Development

In the python code, I used some software packages to help me do the calculation and analysis.

The `NumPy`[3] library is used for numerical operations and array manipulations. For example, the `np.zeros` function is creating a 3D array filled with zeros as its entries; `np.reshape` reshapes the dimension of the array, and `np.fft.fftn` computes the N-dimensional Fourier transform of the signal; `np.fft.fftshift` function shifts the zero-frequency components to the center of the spectrum, and `np.abs` computes the absolute values element-wise.

The `Plotly`[5] library is employed for creating interactive, publication-quality 3D plots. The `go.Scatter3d` function is specifically used to construct a 3D scatter plot for my data provided.

`Matplotlib`[4] is applied for creating 2D plots.

The use of these software packages allows me to design the analysis of the data emitted by the submarine in a efficient way, particularly when it comes to the application of Fourier Transform and Filter techniques.

4. Computational Results

My computational result includes the frequency signature, a $3D$ plot to show a more robust path of the submarine through filtering, and visualizing $2D$ plot for the x, y coordinates of the submarine during the 24 hour period.

4.1. **Frequency Signature.** The frequency signature, representing the highest amplitude point in the frequency domain, was determined through the averaging of the Fourier transform over the 24-hour period. Through my calculation in my code, the result of the frequency signature is $(39, 49, 10)$, which is a collection of index in frequency domain. The information of the index helps me construct and design the $3D$ gaussian filter.

4.2. $2D$ **and** $3D$ **path Over Time.** In order to have a visualization of the filtered path of the submarine's movement over the 24-hour period, I plot the coordinates (x, y) in spacial domain $[-10, 10]$ at each timestamp. And I also plot the the coordinates (x, y) for the unfiltered data, in order to do some comparison.

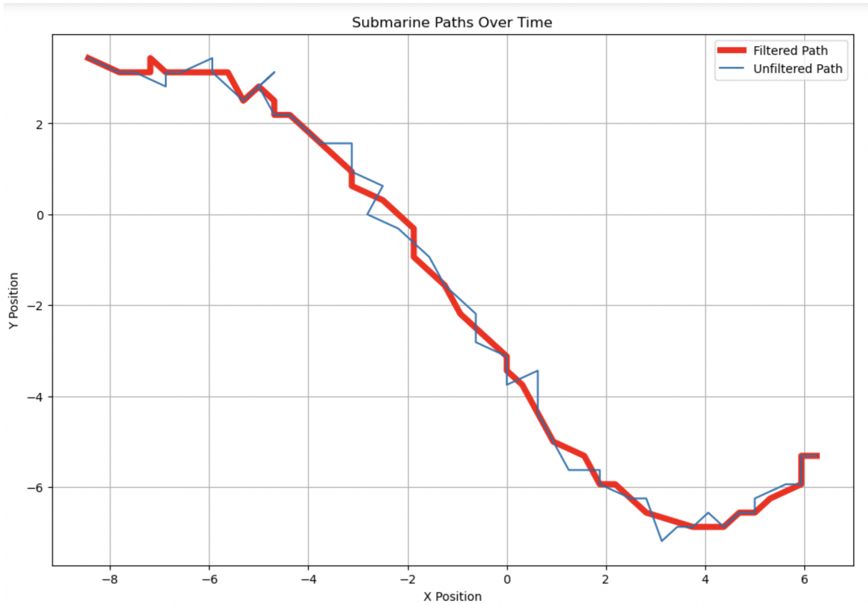

FIGURE 1. Filtered and unfiltered path of the submarine in the xy plane over time.

In Figure 1, it is clear to see that the red curve, which is the filtered path, is smoother than the blue cure, which is the unfiltered path. This is a good sign that my gaussian filter successfully filters out some noise, and so a more robust path is generated.

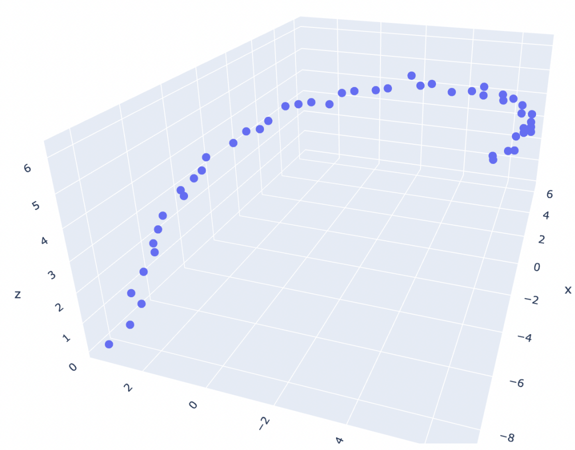

FIGURE 2. 3D plot of the filtered path of the submarine.

In Figure 2, it plots the filtered path of the submarine in a $3D$ visualization. Through employing the 3D Gaussian filter centered at the frequency signature enhances the robustness of the path of the submarine under water.

5. Summary and Conclusions

In conclusion, this project aimed to analyze the acoustic pressure measurements from a submarine, in order to create a more robust path for it. By using mathematical theory of Fourier transform and Filter on the acoustic data over a 24-hour period, I successfully extracted the frequency signature of the signal. This frequency signature plays an important role in my design of the 3D Gaussian filtering process, which efficiently denoise the noisy part of the original path of the submarine. [2]

In my Figure 1 and Figure 2, the visualization in the spatial domain, it is clear to find that the gaussian filter really make a big difference to the original path, and we can tell the filtered curve is smoother than the unfiltered path on the 2D plot.

This project provides design techniques for trajectory tracking and optimization, which may be useful for submarines or similar emerging technologies. Additionally, it can transform raw numerical data into visual representations, such as 3D models or 2D plots.

Acknowledgements

The author is thankful to Prof. Eli Shlizerman and TA Saba Heravi for useful discussions about the general theory part for this project. We are also thankful to peer Ziyi Zhao for meaningful discussion of logic of the coding part, and helping me debug my code successfully.

References

[1] 2d and 3d fourier transforms - yale university, 2022. PDF document.
[2] Directions, reminders and policies. PDF, 2024. File: AMATH482582$_{H}omework1.pdf$.
[3] C. Harris, K. Millman, S. van der Walt, et al. Array programming with numpy. *Nature*, 585:357–362, 2020.
[4] J. D. Hunter. Matplotlib: A 2d graphics environment. *Computing in Science & Engineering*, 9:90–95, 2007.
[5] P. T. Inc. Plotly: Interactive, browser-based plotting for python, 2022.