

HW7

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8.16

Formulate the following as a CVX optimization problem:
Find the rectangle

$$R = \{ x \in \mathbf{R}^n \mid l \preceq x \preceq u \}$$

of maximum volume enclosed in the polyhedron

$$P = \{ x \mid Ax \preceq b \}$$

The volume can be expressed as:

Proposition 1.

$$v = \prod_{i=1}^n u_i - l_i \tag{1}$$

We want all the 2^n corners to be contained within the polyhedron. This every corner must meet the polyhedron constraint $Ac \preceq b$. Where c is the vector of corners. Each of these corners can be more succinctly represented as the vector based on the upper and lower values of each edge:

If we express x_i as $u_i - l_i$ then this system becomes

$$\sum_{i=1}^n a_{ij}(u_j - l_j) \leq b_i$$

The problem can be expressed as:

$$\begin{aligned} & \text{minimize} && \prod_{i=1}^n u_i - l_i \\ & \text{subject to} && \sum_{i=1}^n a_{ij}(u_j - l_j) \leq b_i \end{aligned}$$

The constraint is a posynomial as it is a summation of the monomial $a_{ij}(u_j - l_j)$.
To make the problem a non-linear geometric optimization problem, we take the log of the objective:

$$\begin{aligned} & \text{minimize} && \sum_{i=1}^n \log(u_i - l_i) \\ & \text{subject to} && \sum_{i=1}^n a_{ij}(u_j - l_j) \leq b_i \end{aligned}$$

8.24

We make use of the Cauchy-Schwarz inequality and substitute p knowing that $\|u\|_2 \leq p$:

Proposition 1.

$$u^T x_i \leq \|u\|_2 \|x_i\|_2 \quad (2)$$

$$\|u\|_2 \|x_i\|_2 \leq p \|x_i\|_2 \quad (3)$$

$$u^T y_j \leq \|u\|_2 \|y_j\|_2 \quad (4)$$

$$\|u\|_2 \|y_j\|_2 \leq p \|y_j\|_2 \quad (5)$$

$$-\|u\|_2 \|y_j\|_2 \geq -p \|y_j\|_2 \quad (6)$$

$$(7)$$

For x_i :

$$(a + u)^T x_i \geq b$$

$$a^T x_i + u^T x_i \geq b$$

$$a^T x_i + \|u\|_2 \|x_i\|_2 \geq b$$

$$a^T x_i + p \|x_i\|_2 \geq b$$

$$a^T x_i - b \geq -p \|x_i\|_2$$

For y_j :

$$(a + u)^T y_j \leq b$$

$$a^T y_j + u^T y_j \leq b$$

$$a^T y_j + \|u\|_2 \|y_j\|_2 \leq b$$

$$a^T y_j + p \|y_j\|_2 \leq b$$

$$a^T y_j - b \leq -p \|y_j\|_2$$

$$b - a^T y_j \geq p \|y_j\|_2$$

The optimization problem:

$$\begin{aligned} & \text{minimize} && p \\ & \text{subject to} && b - a^T y_j \geq p \|y_j\|_2 \\ & && a^T x_i - b \geq -p \|x_i\|_2 \\ & && \|a\|_2 \leq 1 \end{aligned}$$

Additional Exercises:

5.12

One heuristic is to treat P as noise.

5.18

We can reformulate the problem as the original object being less than or equal to some value z :

$$1 + \max_{k \neq y_i} f_k(x_i) - f_{y_i}(x_i) \leq z_i, z_i \geq 0$$

This can be represented by the following problem:

$$\begin{aligned} & \text{minimize} && \sum_i z_i + \mu \|A\|_F^2 \\ & \text{subject to} && 1 + \max_{k \neq y_i} f_k(x_i) - f_{y_i}(x_i) \leq z_i, \forall i \\ & && 1^T b = 0, z \geq 0 \end{aligned}$$

This can be reexpressed using the individual inequality constraints:

$$\begin{aligned} & \text{minimize} && \sum_i z_i + \mu \|A\|_F^2 \\ & \text{subject to} && 1^T b = 0, z \geq 0 \\ & && 1 + a_k^T x_i + b - y_i \leq z_i, k = 1, 2, \dots, y_i - 1, y_i + 1, \dots, K, i = 1, 2, \dots, m \end{aligned}$$