HW7

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CSCI 5254 - Convex Optimization

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8.16

Formulate the following asa CVX optimization problem:
Find the rectangle

$$R = \{ x \in \mathbf{R}^n \mid l \preceq x \preceq u \}$$

of maximum volume enclosed in the polyhedron

$$P = \{ x \mid Ax \preceq b \}$$

The volume can be expressed as:

Proposition 1.

$$v = \prod_{i=1}^{n} u_i - l_i \tag{1}$$

We want all the 2^n corners do be contained within the polyhedron. This every corner must meet the polyhedron constraint $Ac \preceq b$. Where c is the vector of corners. Each of these corners can be be more succinctly represented as the vector based on the upper and lower values of each edge:
If we express x_i as $u_i - l_i$ then this system becomes

$$\sum_{i=1}^{n} a_{ij}(u_j - l_j) \leq b_i$$

The problem can be expressed as:

$$\begin{aligned} \text{minimize} \quad & \prod_{i=1}^{n} u_i - l_i \\ \text{subject to} \quad & \sum_{i=1}^{n} a_{ij}(u_j - l_j) \leq b_i \end{aligned}$$

The constraint is a posynomial as it is a summation of the monomial $a_{ij}(u_j - l_j)$.
To make the problem a non-linear geometric optimization problem, we take the log of the objective:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} log(u_i - l_i) \\ \text{subject to} \quad & \sum_{i=1}^{n} a_{ij}(u_j - l_j) \leq b_i \end{aligned}$$

8.24

We make use of the Cauchy-Schwarz inequality and sustite p knowing that $||u||_2 \leq p$:

Proposition 1.

$$u^t x_i \leq ||u||_2 |x_i||_2 \tag{2}$$
$$||u||_2 ||x_i||_2 \leq p||x_i||_2 \tag{3}$$
$$u^t y_j \leq ||u||_2 |y_j||_2 \tag{4}$$
$$||u||_2 |y_j||_2 \leq p||y_j||_2 \tag{5}$$
$$-||u||_2 ||y_j||_2 \geq -p||y_j||_2 \tag{6}$$
$$\tag{7}$$

For x_i:

$$\begin{aligned} (a+u)^T x_i &\geq b \\ a^T x_i + u^T x_i &\geq b \\ a^T x_i + ||u||_2 |x_i||_2 &\geq b \\ a^T x_i + p|x_i||_2 &\geq b \\ a^T x_i - b &\geq -p|x_i||_2 \end{aligned}$$

For x_i:

$$\begin{aligned} (a+u)^T y_j &\leq b \\ a^T y_j + u^T y_j &\leq b \\ a^T y_j + ||u||_2 ||y_j||_2 &\leq b \\ a^T y_j + p||x_i||_2 &\leq b \\ a^T y_j - b &\leq -p||y_j||_2 \\ b - a^T y_j &\geq p||y_j||_2 \end{aligned}$$

The optimization problem:

$$\begin{aligned} \text{minimize} \quad & p \\ \text{subject to} \quad & b - a^T y_j \geq p||y_j||_2 \\ & a^T x_i - b \geq -p|x_i||_2 \\ & ||a||_2 \leq 1 \end{aligned}$$

Additional Exercises:

5.12

One heurisitc estiamte an initial $\hat{x}$ using the huber penalty function. We then use that $\hat{x}$ to estimate a $\hat{P}$ by aligning the indices of Ax and y to find a permutation matrix. Then using that same permutation we reoptimized for $\hat{x}$. We repeat this algorithm until the euclidean norm of the distance between the $\hat{x}_\tau$ and $\hat{x}_{\tau-1}$ is below some tolerance, τ being the current iteration step.

TODO CODE CODE AND EVALUATION

5.18

We can reformulate the problem as the original object being less than or equal to some value z:

$$1 + max_{k \neq y_i} f_k(x_i) - f_{y_i}(x_i) \leq z_i, z_i \geq 0$$

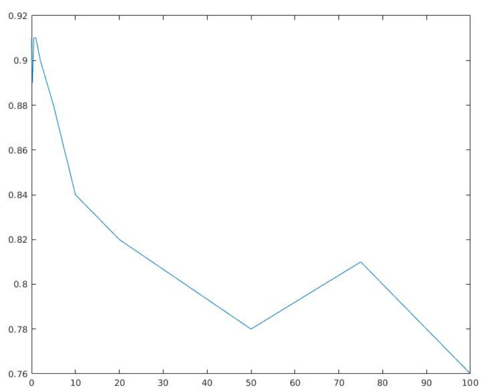
This can be represented by the following problem:

$$\begin{array}{ll} \text{minimize} & \sum_i z_i + \mu ||A||_F^2 \\ \text{subject to} & 1 + max_{k \neq y_i} f_k(x_i) - f_{y_i}(x_i) \leq z_i, \forall i \\ & 1^T b = 0, z \geq 0 \end{array}$$

This can be reexpressed using the individual inequality constraints:

$$\begin{array}{ll} \text{minimize} & \sum_i z_i + \mu ||A||_F^2 \\ \text{subject to} & 1^T b = 0, z \geq 0 \\ & 1 + a_k^T x_i + b - y_i \leq z_i, k = 1, 2, , y_i - 1, y_i + 1, , K, i = 1, 2, , m \end{array}$$

TODO CODE CODE AND EVALUATION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