HW5

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CSCI 5254 - Convex Optimization

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6.1

Proposition 1.

$$log(x+1) \leq x \tag{1}$$
$$-log(x+1) \geq x \tag{2}$$
$$x > -1 \tag{3}$$

Proposition 2.

$$-log(1-x) \text{ is convex} \tag{4}$$

Proposition 3.

$$\phi(||u||_\infty) = -a^2 log(1 - \frac{||u||_\infty}{a^2}) \tag{5}$$

Left inequality working backwards:

$$||u||_2^2 \leq -a^2 \sum_{i=1}^{m} log(1 - \frac{u_i^2}{a^2})$$
$$\sum_{i=1}^{m} \frac{|u_i|^2}{a^2} \leq -\sum_{i=1}^{m} log(1 - \frac{u_i^2}{a^2})$$

Given proposition 1:

$$\sum_{i=1}^{m} \frac{|u_i|^2}{a^2} \leq -\sum_{i=1}^{m} log(1 - \frac{u_i^2}{a^2})$$

true when:

$$-\frac{u_i^2}{a^2} \geq -1$$

Right inequality:

$$\text{Given: } u_i^2 \leq ||u_i||_\infty^2$$

$$\sum_{i=1}^{m} -log(1-\frac{u_i^2}{a^2}) \leq \sum_{i=1}^{m} -log(1-\frac{||u_i||_\infty^2}{a^2}) \text{ given proposition 1}$$

$$\sum_{i=1}^{m} -log(1-\frac{u_i^2}{a^2}) \leq \frac{u_i^2}{||u||_2^\infty} \sum_{i=1}^{m} -log(1-\frac{||u_i||_\infty^2}{a^2}) \text{ given } \frac{u_i^2}{||u||_2^\infty} \geq 1$$

$$-a^2 \sum_{i=1}^{m} log(1-\frac{u_i^2}{a^2}) \leq -a^2 \frac{u_i^2}{||u||_2^\infty} \sum_{i=1}^{m} log(1-\frac{||u_i||_\infty^2}{a^2})$$

$$-a^2 \sum_{i=1}^{m} log(1-\frac{u_i^2}{a^2}) \leq \frac{u_i^2}{||u||_2^\infty} \phi(||u||_\infty)$$

6.9

To show convexity, the following level set must be convex:

$$S_\alpha = \{\, t_i \mid \max_{i=1,\ldots,k} |\frac{p(t_i)}{q(t_i)} - y_i| \leq \alpha \,\}$$