HW7

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8.16

Formulate the following asa CVX optimization problem:
Find the rectangle

$$R = \{\, x \in \mathbf{R}^n \mid l \preceq x \preceq u \,\}$$

of maximum volume enclosed in the polyhedron

$$P = \{\, x \mid Ax \preceq b \,\}$$

The volume can be expressed as:

Proposition 1.

$$v = \prod_{i=1}^{n} u_i - l_i \tag{1}$$

We want all the 2^n corners do be contained within the polyhedron. This every corner must meet the polyhedron constraint $Ac \preceq b$. Where c is the vector of corners. Each of these corners can be be more succinctly represented as the vector based on the upper and lower values of each edge:
If we express x_i as $u_i - l_i$ then this system becomes

$$\sum_{i=1}^{n} a_{ij}(u_j - l_j) \leq b_i$$

The problem can be expressed as:

$$\begin{array}{ll} \text{minimize} & \displaystyle\prod_{i=1}^{n} u_i - l_i \\ \text{subject to} & \displaystyle\sum_{i=1}^{n} a_{ij}(u_j - l_j) \leq b_i \end{array}$$

The constraint is a posynomial as it is a summation of the monomial $a_{ij}(u_j - l_j)$.
To make the problem a non-linear geometric optimization problem, we take the log of the objective:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} log(u_i - l_i) \\ \text{subject to} \quad & \sum_{i=1}^{n} a_{ij}(u_j - l_j) \leq b_i \end{aligned}$$