Math 104, HW6

Tianshuang (Ethan) Qiu

October 10, 2021

1 Q1

Let $x_0 \in \mathbb{R}, \epsilon > 0$. We first define $\log_a a^k = k$. Choose $\delta = \log_a(\frac{\epsilon}{a^{x_0}} + 1)$
Let $|x - x_0| < \delta$, consider $|f(x) - f(x_0)| = |a^x - a^{x_0}| \leq |a^{|x|} - a^{|x_0|}|$. Since we know that $|x - x_0| < \delta$, we have $|x| < \delta + |x_0|$
Now we subsitute our inequality in:

$$|a^{|x|} - a^{|x_0|}| < |a^{\delta + |x_0|} - a^{|x_0|}| = |a^{x_0}(a^\delta - 1)| = |a^{x_0}(\frac{\epsilon}{a^{x_0}} + 1 - 1)| = |a^{x_0}\frac{\epsilon}{a^{x_0}}| = \epsilon$$

Therefore we have shown the epsilon delta property. ■