Math 104, HW8

Tianshuang (Ethan) Qiu

October 29, 2021

1 Q1

For $x \neq 1$, $\frac{1}{x}$ is differentiable, and according to the inverse theorem, it is equal to $-\frac{1}{x^2}$
Since $\sin'(x) = \cos(x)$, and sin is well defined on all of $\mathbb{R}$ (the codomain of $\frac{1}{x} : \mathbb{R} \setminus 0 \to \mathbb{R}$), we can apply the chain rule to the second factor: $(\sin(\frac{1}{x}))' = \cos(\frac{1}{x})(-\frac{1}{x^2})$
Now since $x^2 : \mathbb{R} \to \mathbb{R}$ is continuous in all its domain, we attempt to differentiate at an arbitrary point x_0:

$$\lim_{x \to x_0} \frac{(x^2) - (x_0^2)}{x - x_0} = \lim_{x \to x_0} \frac{(x + x_0)(x - x_0)}{x - x_0} = \lim_{x \to x_0} (x + x_0) = 2x_0$$

Therefore $(x^2)' = 2x$
Finally we use the product rule and the derivative is

$$2x \sin(\frac{1}{x}) + x^2 \cos(\frac{1}{x})(-\frac{1}{x^2}) = 2x \sin(\frac{1}{x}) - \cos(\frac{1}{x})$$

2 Q2

We claim that $f'(0)$ exists and is equal to 0.
Consider the definition of the derivative:

$$\lim_{x \to 0} \frac{x^2 \sin(\frac{1}{x}) - 0}{x - 0}$$

Since this funciton is defined on every point but 0, we have $f' : \mathbb{R} \setminus 0 \to \mathbb{R}$
Now we can simplify to:

$$\lim_{x \to 0} x \sin(\frac{1}{x})$$

Now we apply the squeeze theorem with $-1 \leq \sin(s) \leq 1$, and

$$-x \leq x \sin(\frac{1}{x}) \leq x$$

Since $-x, x$ both converge to 0, our derivative also converges to 0.

3 Q3

We will use the fact that the