# Math 104, HW12

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# 1 Q1

## 1.1 a

First we know that $\sqrt{x^2} = |x|$. Since we know that $\frac{1}{n} \to 0$ and $\frac{1}{n^2} \to 0$, we can show uniform convergence by the following.
Let $\epsilon > 0$, pick $N$ such that for all $n > N$, $|\frac{1}{n^2}| < \epsilon$. Now since $|x| < 1$ and the domain of the square root being positive,

$$f_n(x) = \sqrt{x^2 + \frac{1}{n}} \leq \sqrt{x^2 + \frac{2}{\sqrt{n}}|x| + \frac{1}{n}} \leq \sqrt{(x + \frac{1}{\sqrt{n}})^2}$$

By our first statement the above expression is equal to $|(x + \frac{1}{\sqrt{n}})|$. By our definition of $N$,

$$||(x + \frac{1}{\sqrt{n}})| - |x|| \leq |x + \frac{1}{\sqrt{n}} - x| = |\frac{1}{\sqrt{n}}| < \frac{1}{N} < \epsilon$$

Thus we have $f_n \to |x|$ uniformly.

## 1.2 b

$f_n(x) = \sqrt{x^2 + \frac{1}{n}}$, and by the power rule we know that

$$f_n'(x) = \frac{2x}{\sqrt{x^2 + \frac{1}{n}}}$$