Homework 2

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1 Ross 4.15

For this we first prove that $\frac{1}{n} > 0 \forall n \in \mathbb{N}$. First we multiply both sides by n, since $n \in \mathbb{N}, n > 0$, the sign does not change.
We get $LHS = 1$ $RHS = 0$, since $1 > 0$ is an axiom, therefore we know that $1/n > 0$.
Now since we have proven $\frac{1}{n} \geq 0 \forall n \in \mathbb{N}$, by the ordered field axioms $a \leq b$.
Q.E.D.

2 Ross 4.16

Let the set in question be denoted by S. We will prove the claim via contradiction. We break its negative down into two cases: $\sup S < a$ or $\sup S > a$. For the former, let $x = \sup S | x \in \mathbb{R}$. By the denseness of rationals we see that there exists $q \in \mathbb{Q} s.t. x \leq q \leq a$. By the definition of this set we have $q \in S$. Therefore we have just found a member of this set that is greater than the supremum. This is a contradiciton so x cannot be less than a.
For the latter, we once again let $x = \sup S | x \in \mathbb{R}$. Consider a. $a < x$ and by definition of S, $\forall s \in S, s < a$. We have found an upperbound that is less than our supremum. That is a contradiction so x cannot be greater than a.
$\sup S = a$ Q.E.D.

3 Ross 8.2

3.1 a

Claim: $a_n \to 0$
Proof: $a_n = \frac{n}{n^2+1} \leq \frac{n+1/n}{n^2+1} = \frac{1}{n}$ $(n \neq 0)$
Let $\epsilon > 0$, we select our $N = \frac{1}{\epsilon}$. $\forall n > N, |a_n - 0| < \frac{1}{n} < \epsilon$, thus the sequence converges.

3.2 c

Claim: $c_n \to \frac{4}{7}$
Proof: $|c_n - \frac{4}{7}| = |\frac{28n+21}{49n-35} - \frac{4(7n-5)}{49n-35}|$

$$= \frac{28n + 21 - 28n + 20}{49n - 35}$$

$$= \frac{41}{49n - 35} \leq \frac{41}{49n}$$

Let $\epsilon > 0$, we select our $N = \frac{41}{49\epsilon}$. For all $n > N, |c_n - \frac{4}{7}| \leq \frac{41}{49n} < \epsilon$

3.3 e

Claim: $s_n \to 0$
Proof: $|s_n - 0| = |\frac{1}{n} sinn| \leq \frac{1}{n}$.
Let $\epsilon > 0$, we select our $N = \frac{1}{epsilon}$. For all $n > N, |s_n - 0| \leq \frac{1}{n} < \epsilon$

4 Ross 8.5

4.1 a