Homework 3

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1 Ross 8.2

1.1 a

Since $n^2 + 1 > n \forall n \in \mathbb{R}$, the fraction $\frac{n}{n^2+1} < 1$. Furthermore since $n \geq 0$ and $n^2 > 0$ the fraction is positive. So we have $< \frac{n}{n^2+1} < 1$. It is bounded
Now we need to show that it is monotonically decreasing, and we will do this via induction.
Base case: $a_1 = \frac{1}{2}, a_2 = \frac{2}{5}, a_2 < a_1$
Inductive case: assume that $a_n < a_{n-1}$, consider

$$a_{n+1} = \frac{n+1}{(n+1)^2 + 1}$$

We take $a_{n+1} - a_n$

$$= \frac{n+1}{(n+1)^2+1} - \frac{n}{n^2+1}$$

$$= \frac{(n+1)(n^2+1) - n((n+1)^2+1)}{(n^2+1)((n+1)^2+1)}$$

$$= \frac{n^3 + n + n^2 + n - n(n+1)^2 - n}{(n^2+1)((n+1)^2+1)}$$

$$= \frac{n^3 + n + n^2 + n - n^3 - n - 2n^2 - n}{(n^2+1)((n+1)^2+1)}$$