Math 74, Week 6

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1 Mon Lec, 1a

Base case: $n = 1, n = 2, a_1 = 3, a_2 = 5$, base cases hold.
Assume that for some $n \in \mathbb{N}$, all $m \in \mathbb{N}1, 1 \leq m \leq n$ satisfies this identity, we have

$$a_{n+1} = 3a_n - 2a_{n-1} = 3(2^n + 1) - 2(2^{n-1} + 1) = 3 \times 2^n + 3 - 2 \times 2^{n-1} - 2$$
$$= 2^{n-1}(6 - 2) + 1 = 2^{n-1}(2^2) + 1 = 2^{n+1} + 1$$

Thus we have proven the inductive case. Q.E.D.

2 Mon Lec, 3c

Base case: $n = 0, m = 0, f_n = 1, f_m = 1, f_{n+m+1} = 2$, base case holds.
For any $n, m \in \mathbb{N}$, assume that the statement is true for all $n' \leq n, m' \leq m$. Using our direct formula we have

$$f_{n+1} = \frac{1}{\sqrt{5}}(\phi^n - \bar{\phi}^n)$$

$m + 1$ has a similar logic. Now we can expand $f_n f_m + f_{n+1} f_{m+1}$ to

$$\frac{1}{5}(\phi^n - \bar{\phi}^n)(\phi^m - \bar{\phi}^m) + \frac{1}{5}(\phi^{n+1} - \bar{\phi}^{n+1})(\phi^{m+1} - \bar{\phi}^{m+1})$$

$$\frac{1}{5}(\phi^{m+n} - \phi^n\bar{\phi}^m - \bar{\phi}^n\phi^m + \bar{\phi}^{m+n}) + \frac{1}{5}(\phi^{m+n+2} - \phi^{n+1}\bar{\phi}^{m+1} - \bar{\phi}^{n+1}\phi^{m+1} + \bar{\phi}^{m+n+2})$$

3 Mon Dis, 6

First we claim that $n^2 - (n + 1)^2 - (n + 2)^2 + (n + 3)^2 = 4$