Math 74, Week 15

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1 Mon Lec, 2b

$g_2 = \sqrt{a_1a_2}$, so $(1+g_2)^2 = 1 + a_1a_2 + 2\sqrt{a_1a_2}$
Our left hand side should be $(1+a_1)(1+a_2) = 1+a_1a_2+a_1+a_2$. By Am-GM, $LHS \geq RHS$
Now we consider 3 elements. $g_3 = \sqrt[3]{a_1a_2a_3}$, and $(1+g_3)^3 = 1 + a_1a_2a_3 + 3\sqrt[3]{a_1a_2a_3} + 3(a_1a_2a_3)^{2/3}$.
Now LHS has $(1+a_1)(1+a_2)(1+a_3) = 1 + a_1 + a_2 + a_3 + a_1a_2 + a_1a_3 + a_2a_3 + a_1a_2a_3$ Here we can cancel the 1 on both sides, and by AM-GM we have $a_1 + a_2 + a_3 \geq 3\sqrt[3]{a_1a_2a_3}$. Now let the three terms be a_1a_2, a_1a_3, and a_2a_3. By AM-GM we have $a_1a_2 + a_1a_3 + a_2a_3 \geq 3\sqrt[3]{a_1^2a_2^2a_3^2}$.
Thus we have shown that $LHS \geq RHS$ term by term.

2 Mon Lec, 3c

Since our plane passes through the point $(5, 9, 12)$, we know that the equation of a plane can be given by $\frac{x}{r} + \frac{y}{s} + \frac{z}{t} = 1$. Furthermore we have $\frac{5}{r} + \frac{9}{s} + \frac{12}{t} = 1$. Now we apply the Hamonic Mean-GM inequality:

$$\frac{3}{\frac{5}{r} + \frac{9}{s} + \frac{12}{t}} \leq \sqrt[3]{\frac{rst}{420}}$$