Math 74, Week 6

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1 Mon Lec, 1a

Base case: $n = 1, n = 2, a_1 = 3, a_2 = 5$, base cases hold.
Assume that for some $n \in \mathbb{N}$, all $m \in \mathbb{N}1, 1 \leq m \leq n$ satisfies this identity, we have

$$a_{n+1} = 3a_n - 2a_{n-1} = 3(2^n + 1) - 2(2^{n-1} + 1) = 3 \times 2^n + 3 - 2 \times 2^{n-1} - 2$$

$$= 2^{n-1}(6 - 2) + 1 = 2^{n-1}(2^2) + 1 = 2^{n+1} + 1$$

Thus we have proven the inductive case. Q.E.D.

2 Mon Lec, 3c

Base case: $n = 0, m = 0, f_n = 1, f_m = 1, f_{n+m+1} = 2$, base case holds.
For any $n, m \in \mathbb{N}$, assume that the statement is true for all $n' \leq n, m' \leq m$.
Using our direct formula we have

$$f_{n+1} = \frac{1}{\sqrt{5}}(\phi^{n+1} - \bar{\phi}^{n+1})$$

$m + 1$ has a similar logic. Now consider $n + 1$ and f_{n+m+2}.
Using the direct formula, we know that $f_{n+m+2} = 1/\sqrt{5}(\phi^{n+m+2} - \bar{\phi}^{n+m+2})$

$$\frac{f_{n+m+2}}{f_{n+m+1}} = \frac{\phi^{n+m+2} - \bar{\phi}^{n+m+2}}{\phi^{n+m+1} - \bar{\phi}^{n+m+1}}$$

Now we take $(f_{n+1}f_m + f_{n+2}f_{m+1})/(f_nf_m + f_{n+1}f_{m+1})$

$$= \frac{(\phi^{n+1} - \bar{\phi}^{n+1})(\phi^m - \bar{\phi}^m) + (\phi^{n+1} - \bar{\phi}^{n+2})(\phi^{n+1} - \bar{\phi}^{m+1})}{(\phi^n - \bar{\phi}^n)(\phi^m - \bar{\phi}^m) + (\phi^{n+1} - \bar{\phi}^{n+1})(\phi^{m+1} - \bar{\phi}^{m+1})}$$

We can expand and cancel the terms through long division, finally leaving

$$\frac{\phi^{n+m+2} - \bar{\phi}^{n+m+2}}{\phi^{n+m+1} - \bar{\phi}^{n+m+1}}$$

, which is equal to the above fraction. Since $f_{n+m+1} = f_nf_m + f_{n+1}f_{m+1}$ by inductive hypothesis, we have now proven the inductive step. Q.E.D.

3 Mon Lec, 4a

4 Mon Dis, 6

First we claim that $n^2 - (n+1)^2 - (n+2)^2 + (n+3)^2 = 4$