Math 74, Week 15

Tianshuang (Ethan) Qiu

December 1, 2021

1 Mon Lec, 2b

$g_2 = \sqrt{a_1 a_2}$, so $(1+g_2)^2 = 1 + a_1 a_2 + 2\sqrt{a_1 a_2}$
Our left hand side should be $(1+a_1)(1+a_2) = 1 + a_1 a_2 + a_1 + a_2$. By Am-GM, $LHS \geq RHS$
Now we consider 3 elements. $g_3 = \sqrt[3]{a_1 a_2 a_3}$, and $(1+g_3)^3 = 1 + a_1 a_2 a_3 + 3\sqrt[3]{a_1 a_2 a_3} + 3(a_1 a_2 a_3)^{2/3}$.
Now LHS has $(1+a_1)(1+a_2)(1+a_3) = 1 + a_1 + a_2 + a_3 + a_1 a_2 + a_1 a_3 + a_2 a_3 + a_1 a_2 a_3$ Here we can cancel the 1 on both sides, and by AM-GM we have $a_1 + a_2 + a_3 \geq 3\sqrt[3]{a_1 a_2 a_3}$. Now let the three terms be $a_1 a_2$, $a_1 a_3$, and $a_2 a_3$. By AM-GM we have $a_1 a_2 + a_1 a_3 + a_2 a_3 \geq 3\sqrt[3]{a_1^2 a_2^2 a_3^2}$.
Thus we have shown that $LHS \geq RHS$ term by term.

2 Mon Lec, 3c

Since our plane passes through the point $(5, 9, 12)$, we know that the equation of a plane can be given by $\frac{x}{r} + \frac{y}{s} + \frac{z}{t} = 1$. Furthermore we have $\frac{5}{r} + \frac{9}{s} + \frac{12}{t} = 1$. Now we apply the Hamonic Mean-GM inequality:

$$\frac{3}{\frac{5}{r} + \frac{9}{s} + \frac{12}{t}} \leq \sqrt[3]{\frac{rst}{540}}$$

Now from the equation of the plane we know that $LHS = 3$, so now $\sqrt[3]{\frac{rst}{540}} \geq 3$, $\frac{rst}{540} \geq 27$ Finally, since the volume of this terahedron is equal to $\frac{1}{2}rst$, we know that $V \geq 7290$.
When the terms $\frac{5}{r}, \frac{9}{s}, \frac{12}{t}$ are equal, we have $V = 7290$. Furthermore their sum is equal to 1. Therefore they are eaech a third. $r = 15, s = 27, t = 36$.

3 Wed Dis, 3a

We know that both sides of the equation is positive, so the inequality is equivalent to us taking the natural log of both sides

$$\ln x^x \geq \ln \left(\frac{x+1}{2}\right)^{x+1}$$

$$x \ln x \geq (x+1) \ln \frac{x+1}{2}$$

Now consider the function $y \ln(y)$ and two values $1, x$. By Jensen's inequality we have $(\ln(1) + x\ln(x))/2 \geq \frac{x+1}{2} \ln \frac{x+1}{2}$, which is identical to our initial statement when we multiply both sides by 2.