Math 74, Week 10

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October 31, 2021

1 Mon Lec, 6c

$$z^{n-1} - 1 = \prod_{k=0}^{n-1}(z - \omega_k)$$

As shown in class, the complex roots are evenly spaced across the unit circle, $2\pi/n$ apart. So we have

$$\omega_k = e^{2\pi i \frac{k}{n}}$$

2 Mon Lec, 6f

$(z-1)(z^{n-1}+z^{n-2}+...+z^2+z+1) = (z^n+z^{n-1}+...+z^2+z)-(z^{n-1}+z^{n-2}+...+z+1) = z^n-1$

Therefore we can switch our statement to $\frac{z^n-1}{z-1}$
Now we subsitute our answer from the previous question in, and since $\omega_0 = 1$, it cancels out with the first term.
We can factor the expression into

$$\prod_{k=1}^{n-1}(z - \omega_k)$$

where

$$\omega_k = e^{2\pi i \frac{k}{n}}$$

Essentially the same as 6c but with $z = 1$ removed.

3 Mon Lec, 7b

3.1 6c

Sum: $-\frac{0}{1} = 0$
Product: $(-1)^n \frac{1}{1} = (-1)^n$

3.2 6f

Sum: $-\frac{1}{1} = -1$
Product: $(-1)^n \frac{0}{1} = 0$

4 Mon Dis, 1a

$$|A_0A1|...|A_0A_8| = \prod_{k=0}^{8} |1-\omega_k| = |\prod_{k=0}^{8}(1-\omega_k)|$$

The last equivalency is due the fact that multiplication of the modulus is equal to the modulus of the product.
We have proven above that $(z-1)(z^{n-1}+z^{n-2}+...+z^2+z+1) = z^n-1$, so consider

$$\frac{z^n-1}{z-1} = \frac{(z-1)(z^{n-1}+z^{n-2}+...+z^2+z+1)}{z-1} = z^{n-1}+z^{n-2}+...+z^2+z+1$$

Now using the roots of the polynomial we know that $z^9-1 = (z-1)(z-\omega)...(z-\omega^8)$, in this case we have divided out $z-1$, so we have

$$z^8+z^7+...+1 = (z-\omega)...(z-\omega^8)$$

Let $z=1$, and we have $9 = (z-\omega)...(z-\omega^8)$, since $|9| = 9$, we have shown that $|A_0A1|...|A_0A_8| = 9$

5 Mon Dis, 3f

Let $x = y-2$, so $x^3 = y^3-6y^2+12y-8$. Now we plug y back

$$y^3-6y^2+12y-8 = -6(y-2)^2-12y+24-6$$

$$y^3-6y^2+12y-8 = -6y^2+12y-6$$
$$y^3 = 2$$

Now since we know that $2^3 = 8$, we can directly solve: $y_1 = \sqrt[3]{2}, x_1 = \sqrt[3]{2}+2$
Then we factor $(y^3)/(y-\sqrt[3]{2}) = y^2+\sqrt[3]{2}y+\sqrt[3]{4}$ Now we apply the quadratic formula to get

$$y_2 = \frac{-1-\sqrt{3}i}{\sqrt[3]{2}}, y_3 = \frac{-1+\sqrt{3}i}{\sqrt[3]{2}}$$

So we have $x_2 = \frac{-1-\sqrt{3}i}{\sqrt[3]{2}} - 2, x_3 = \frac{-1+\sqrt{3}i}{\sqrt[3]{2}} - 2$