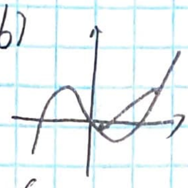
5.

(b) 

Convex

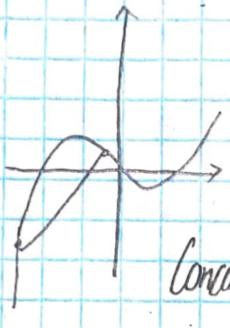
Graph is below the line when $x_1, x_2 > 0$

$$f\left(\tfrac{1}{2}x_1 + \tfrac{1}{2}x_2\right) \overset{?}{\leq} \tfrac{1}{2}f(x_1) + \tfrac{1}{2}f(x_2)$$

$$\left(\frac{x_1+x_2}{2}\right)^3 \overset{?}{\leq} \frac{x_1^3+x_2^3}{2}$$

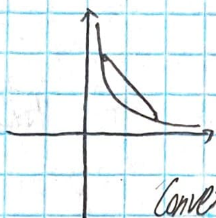
$$\left(\frac{\sqrt[3]{x_1^3}+\sqrt[3]{x_2^3}}{2}\right)^3 \leq \frac{x_1^3+x_2^3}{2}$$

PM (Power $\frac{1}{3}$) AM $\therefore$ It is true



Concave

Graph is above line when $x_1, x_2 < 0$, the same derivation as above but the $-$ sign flips the inequality

(c) 

Convex

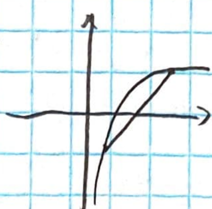
Graph below the line

$$f\left(\tfrac{1}{2}x_1 + \tfrac{1}{2}x_2\right) \overset{?}{\leq} \tfrac{1}{2}f(x_1) + \tfrac{1}{2}f(x_2)$$

$$\frac{2}{x_1+x_2} \overset{?}{\leq} \frac{1}{2}\left(\frac{1}{x_1}+\frac{1}{x_2}\right)$$

HM AM

$\Rightarrow$ Therefore it is true

(f) 

Graph is above the line

$$f\left(\tfrac{1}{2}x_1 + \tfrac{1}{2}x_2\right) \overset{?}{\geq} \tfrac{1}{2}f(x_1) + \tfrac{1}{2}f(x_2)$$

$$\log_3\left(\tfrac{1}{2}(x_1+x_2)\right) \overset{?}{\geq} \tfrac{1}{2}\log_3(x_1) + \tfrac{1}{2}\log_3(x_2)$$

$$\log_3\left(\frac{x_1+x_2}{2}\right) \overset{?}{\geq} \log_3(x_1x_2)^{\frac{1}{2}}$$

This is true due to AM-GM w/ x_1, x_2

Mon DIS

3(c)

$$P_r = \left(\frac{60^r + 75^r + 90^r}{3}\right)^{1/r}$$

Since $r < 0$,

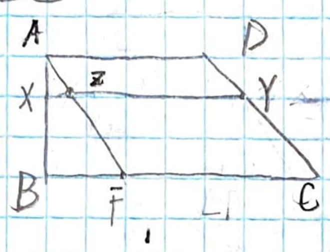
$$60^r \le 60^r + 75^r + 90^r \le 3(60^r)$$

$$60 \le P_r \le 60 \quad \text{since} \quad \lim_{r \to -\infty} \left(\frac{60^r}{3}\right)^{1/r} = 60$$

$\therefore$ P_r approaches 60 as $r \to -\infty$

Wed DIS

2b



Let $\overline{AX} = 3l$, $\overline{XB} = 7l$, $AF \parallel DC$, F on BC, Z is AF intersect XY

$\because$ AFCD is a parallelogram,

$\overline{CD} = \overline{AF}$, $AD = FC$

$\therefore BF = BC - AD$

$\because XZ \parallel BF$

$\therefore XZ : BF = AX : XB = 3 : 7$

$\therefore XZ = \frac{3}{7} BF = \frac{3}{7}(BC - AD)$

$\therefore XY = AD + \frac{3}{7}(BC - AD)$