Math 74, Week 6

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1 Mon Lec, 4c

Non of the number repeating means that it is a rearrangement of the set of remainders $\{1, 2, ...6\}$. We first show that if $a \not\equiv b \mod 7, \text{then} 4a \not\equiv 4b \mod 7$
Proof: Suppose for contradiction that $4a \equiv 4b \mod 7$, then we have $7 \mid 4(a-b)$. Since $\gcd(4, 7) = 1$, $7 \mid a - b$, $a \equiv b \mod 7$, ↯. We have a contradiction, therefore $4a \not\equiv 4b \mod 7$
Let $a = 1, b = 2, 3, 4, 5, 6$. We can see that $(4 \times 1)...(4 \times 6) \equiv 6! \mod 7$. Since $6!$ is coprime with 7, we can divide it, leaving us with

$$4^6 \equiv 1 \mod 7$$

2 Mon Lec, 5a

Using the formula

$$\phi(n) = n \prod_{p|n} (1 - \frac{1}{p})$$

$$\phi(10) = 10(1 - \frac{1}{2})(1 - \frac{1}{5}) = 4$$

3 Mon Lec, 6

We simplify

$$\frac{x + 2k}{3} \leq x + 1$$

$$x + 2k \leq 3x + 3$$

$$2x \geq 2k - 3$$
$$x \geq \frac{2k-3}{2}$$

So $(2k-3)/2 = 3, 2k = 9, k = 4.5$

4 Mon Dis, 2b

$$17^{1707} \bmod 11 \equiv 6^{1707} \bmod 11$$

By fermats little theorem $6^10 \equiv 1 \bmod 11$

$$6^1707 \equiv (6^10)^170 \times 6^7 \bmod 11 \equiv 6^7 \bmod 11$$

$$6^2 \equiv 3 \bmod 11$$

$$6^4 \equiv 9 \bmod 11$$

$$6^7 \equiv 3 \times 9 \times 6 \bmod 11 \equiv 8 \bmod 11$$

5 Mon Dis, 4c