Math 74, Week 4

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1 Lec Mon, 1c

1.1 a

Since each term is the product of x^a, y^b, z^c, and $a + b + c = 2020$, we can simplify this problem into dogs and biscuits. $\binom{2020+3-1}{3-1} = \binom{2022}{2}$

1.2 b

Before combining, we expand each term by picking one variable from each of the 2020 $(x + y + z)$ multiplied together. So we have 3^2020.

1.3 c

We can reach the same result by subtracting the amount where there is only x, or only y, or only z, or xy, xz, yz.
For the first three, there is only 1 way for that to happen since that variable has to be raised to 2020. For xz, we have $a + b = 2020$, feeding 2018 biscuits to 2 dogs. We need to subtract 2 since we have already counted having only one term. Therefore $\binom{2018+2-1}{2-1} = 2019$.
Adding them together we have $1 \times 3 + 2019 \times 3 = 6060$

2 Dis Mon, 1a

LHS is the amount of ways to choose a team with k people and a captain from a group with n people. It chooses the team first: $\binom{n}{k}$. Then from that team we choose a captain with k ways to do it.

RHS calculates the amount of ways to choose a captain first: n, then the rest of the team: $\binom{n-1}{k-1}$. Both sides calculate the same thing. Therefore LHS $=$ RHS.

Q.E.D.

3 Dis Mon, 4

$$x + \frac{1}{x} = 7$$

$$(x + \frac{1}{x})^2 = 49$$

$$x^2 + \frac{1}{x^2} + 2 = 49$$

$$x^2 + \frac{1}{x^2} = 47$$