Math 74, Week 8

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1 Mon Dis, 2c

Let $a \leq b \leq c \leq d$, we can do this since $\mathbb{N}$ is ordered. We rearrange $a + b + c + d = abcd$ to $a + b + c = d(abc - 1)$
Since we have assumed an order, we have $a + b + c \leq 3d$, so $abc - 1 \leq 3$, $abc \leq 4$
Now in this case abc can only be $1, 2, 3, 4$. We consider each of these cases:
$abc = 1 \implies a = 1, b = 1, c = 1$, solving for our original equation gives $3 = 0d$, which does not have a solution.
$abc = 2$, by our assumed order we have $a = 1, b = 1, c = 2$, and we solve $2d = 4 + d$, $d = 4$, this gives a solution.
$abc = 3$, by our assumed order we have $a = 1, b = 1, c = 3$, and we solve $3d = 5 + d$, which does not have an integer solution.
$abc = 4$, by our assumed order we have $a_0 = 1, b_0 = 1, c_0 = 4$, or $a_1 = 1, b_1 = 2, b_2 = 2$. Solving the first equation gives $d = 2$, which contradicts our assumption of $c \leq d$, so this is not a solution. Solving the latter gives $4d = 5 + d$, which does not have an integer solution.
So finally we have our solution: rearrangements of $1, 1, 2, 4$ is the only natural solution to this system.

2 Mon Dis, 4g

We first treat each letter as a unique character. Since the word has 11 characters we have 11! ways of rearranging it.
Next we observe that "S" is repeated 4 times, "I" is repeated 4 times, and

"P" twice, so we divide them, leaving us with:

$$\frac{11!}{4! \times 4! \times 2!}$$