Math 74, Week 12

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1 Mon Lec, 5

1.1 a

$|K| = 6$, let e be the identity transformation and r be a rotation of $\frac{360}{6} = 60°$. The other elements are: r^2, r^3, r^4, r^5

1.2 b

Since D_6 is the group of all symmetries, K is only the rotational ones, and D_6 also contains reflective symmetries. Therefore K is a subgroup of D_6

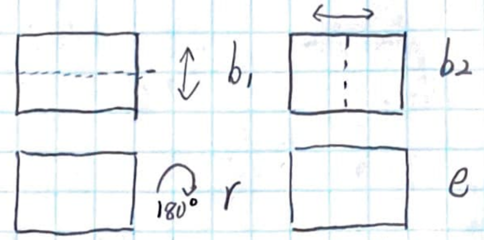
1.3 c

Let our generator $\omega = r$, $\omega^0 = e, \omega^1 = r, \omega^2 = r^2$, and so on. It is a generator because its powers can generate all the elements of our gorup.

1.4 d

Let our generator $\omega = r^5$, $\omega^0 = e, \omega^1 = r^5, \omega^2 = r^4, \omega^3 = r^3$ etc.

1a.



	e	b_1	b_2	r
e	e	b_1	b_2	r
b_1	b_1	e	r	b_2
b_2	b_2	r	e	b_1
r	r	b_2	b_1	e

1d.

e

	e
e	e

no rotational or reflective symmetry

5.

Take an equilateral triangle and add "legs" to it. This way it is no longer reflectively ~~rotate~~ symmetric.

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(b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 4 & 1 & 3 & 5 & 2 \end{pmatrix}$

$1 \to 7 \to 7$

$2 \to 6 \to 6$ ...

(c) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 2 & 3 & 6 & 4 \end{pmatrix}$

$5 \to 4$ $6 \to 4$

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 7 & 8 & 2 & 4 & 6 & 5 & 3 \end{pmatrix}$

$8 \to 3 \to 8 \to 3$

$7 \to 2 \to 4 \to 5$

$6 \to 6$ ...

$5 \to 7 \to 2 \to 4$

4. $(1342)(123) = (1\ 4\ 3\ 2)$

$(1534269)^{-1} = (9\ 6\ 2\ 4\ 3\ 5\ 1)$

5a. The order of a permutation P is the smallest integer s such that $P^s = I$ (Identity).

Let $P = P_1 P_2 \dots P_n$, $x_1 \in P_1 \cup P_2 \dots \cup P_n$. If $x_1 \in P_1$, s must be a multiple of $|P_1|$, the same is true for all $P_2, P_3 \dots P_n$. S must be a multiple of $|P_2| \dots |P_n|$ to ensure that P^s maps x to x.

Therefore $|P| = \mathrm{lcm}(P_1 \dots P_n)$

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6b. $(1\ 3\ 6\ 7\ 4\ 8\ 10) \Rightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 2 & 6 & 8 & 5 & 7 & 4 & 10 & 9 & 1 \end{pmatrix}$

$\Rightarrow (1\ 3)(1\ 6)(1\ 7)(1\ 4)(1\ 8)(1\ 10)$ even

9e. $(25)^{-1} = (5\ 2)$

$[(25)(63)]^{-1} = (36)(5\ 2)$

$[(25)(256)]^{-1} = (6\ 5\ 2)(5\ 2)$

$(153429)^{-1} = (9\ 2\ 4\ 3\ 5\ 1)$

$[(1342)(123)]^{-1} = (3\ 2\ 1)(2\ 4\ 3\ 1)$

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 2 & 5 & 6 & 7 & 8 & 1 & 4 \end{pmatrix}^{-1} = [(3\ 5\ 7\ 1)(4\ 6\ 8)]^{-1} = (8\ 6\ 4)(1\ 7\ 5\ 3)$

6. Since block 16 (the empty block) is at the same spot, and shifts can be written as a product of transpositions, any shift up must have a shift down. the same for left and right. Thus the total oddity must be

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

is the identity, reachable

4	3	2	1
5	6	7	8
12	11	10	9
13	14	15	

$$\left.\begin{array}{l} = (1\ 4)(2\ 3) \\ \\ = (9\ 12)(10\ 11) \end{array}\right\}$$ even permutations, reachable

10	9	8	7
11	2	1	6
12	3	4	5
13	14	15	

$= (1\ 10\ 3\ 8\ 6\ 2\ 9\ 12\ 5\ 11\ 4\ 7)$

size 12 cycle $\Rightarrow$ odd # of transpositions, not reachable

8	14	11	3
12	2	15	9
6	4	13	1
7	10	5	

$= (1\ 8\ 9\ 6\ 2\ 14\ 10\ 4\ 3\ 11\ 13\ 7\ 15\ 5\ 12)$

size 15 cycle $\Rightarrow$ even #, solvable

7b. Assuming that the rest of the puzzle is complete:

1	2	3	4
5	6	7	8
9	10	11	12
?	?	?	

We know that the only even permutation are 13 14 15, 14 15 13, 15 13 14. One is already the identity, for the rest we do the following

1	2	3	4
5	6	7	8
9	10	11	12
14	15	13	

$\Rightarrow$

1	2	3	4
5	6	7	8
14	9	10	11
15	13		12

$\Rightarrow$

1	2	3	4
5	6	7	8
14	13	9	11
15		10	12

$\Rightarrow$

1	2	3	4
5	6	7	8
13		9	11
14	15	10	12

$\Rightarrow$

1	2	3	4
5	6	7	8
13	9	10	11
14	15		12

$\Downarrow$

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

$\Rightarrow$

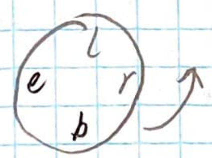
Since $(15\ 13\ 14) = (14, 15, 13)^2$, applying this algorithm twice yields the desired result.

Mon Lec

Consider a group of size 4, since it is a group, it is closed and has an identity

$\times$	e	l	r	b
e	e	l	r	b
l	l			
r	r			
b	b			

Each element has to appear in a row. If $l \cdot l \neq e$, then lb or lr must be e. In this case it is the rotation group since it is isomorphic to $n \bmod 4$, the numbers rotate like on a wheel.



If $l \cdot l = e$, then it is the rock group each element times itself is the identity and $lr = b$ since neither is the identity and e has already appeared. Thus it must be isomorphic to the rock group