Math 104

Tianshuang (Ethan) Qiu

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1 Wed Lec, 1a

Prove that $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$

1.1 algebraic

We first expand the expression:

$$LHS = \frac{n!}{(n-k)!k!} + \frac{n!}{(n-k-1)!(k+1)!}$$

Then we simplify:

$$LHS = \frac{n!}{(n-k)(n-k-1)!k!} + \frac{n!}{(k+1)(n-k-1)!k!}$$

$$LHS = \frac{(n!(k+1)) + (n!(n-k))}{(n-k)(k+1)(n-k-1)!k!}$$

$$LHS = \frac{n!(n+1)}{(k+1)!(n-k)!}$$

$$LHS = \frac{(n+1)!}{(k+1)!(n-k)!}$$

Now we expand the right side:

$$RHS = \frac{(n+1)!}{(k+1)!(n-k)!}$$

We see thatt RHS = LHS.
Q.E.D.

1.2 Combinatorial

The right hand side calculates the number of bitstrings of length $n+1$ with $k+1$ 0's. Since each bit can either end in 0 or 1, we can split them into different cases.
If the the last is 0, there are k 0's left in the substring of length n. So to calculate this, we use $\binom{n}{k}$.
If the last bit is 1, there are still $k+1$ 0's left in the substring before it. Using the formula, we get $\binom{n}{k+1}$
Adding them together equals the right hand side. They are evaluating the same thing.
Q.E.D.

2 Wed Lec, 4b

Let people be the pigeons and each letter is a different hole.
Number of pigeons = number of names = 33
Number of holes = number of letters - 30
Since the amount of pigeons is greater than the amount of holes, some hole must have at least 2 pigeons by PHP. Therefore some ending letter must be shared by at least 2 names.

3 Wed Lec, 5

3.1 a

We can form a segment from any two points. So the total amount of segments is $\binom{50}{2} = \frac{50!}{2!48!} = 1225$

3.2 b

Since no three points are colinear, we can choose any 3 points to form a triangle. So the total amount is $\binom{50}{3} = \frac{50!}{3!47!} = 19600$

3.3 c

Similar to part (b), we choose any 4 points to form a quadrilateral. The total amount is $\binom{50}{4} = \frac{50!}{46!4!} = 230300$.