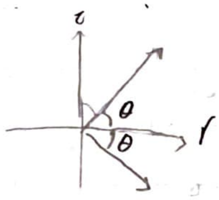
4.e



Let our complex number be represented as (r, θ). $z = r\cos\theta + r\sin\theta\, i$

$\bar{z} = r\cos\theta - r\sin\theta\, i$

$z \cdot \bar{z}$ takes the sum of these angles: $\theta - \theta = 0$, which is the positive real direction. The magnitude is the product of the two: $|z|^2$ because $|z| = |\bar{z}|$.

ρ^2	ρ	$\cdot 1$	ρ	ρ^2
ρ^3	ρ^2	ρ	ρ^2	ρ^3
ρ^4	ρ^3	ρ^2	ρ^3	ρ^4
ρ^5	ρ^4	ρ^3	ρ^4	ρ^5 ...

4.

(a) We assign values to the board, 1 is the ~~gol~~ goal, and ~~φ~~ $\rho = \frac{1}{\varphi}$

Getting closer: $\rho^{n+1} + \rho^n \stackrel{?}{=} \rho^{n-1} \Rightarrow \frac{1}{\phi^{n-1}} \stackrel{?}{=} \frac{1}{\phi^n} + \frac{1}{\phi^{n+1}}$

It is equal $\phi^2 = \phi + 1$

Further away: $\rho^{n-1} + \rho^n > \rho^{n+1}$ since $\rho < 1$

$\therefore$ when jumping away the sum of the value is a monovariant (can only decrease), when jumping towards the goal its an invariant

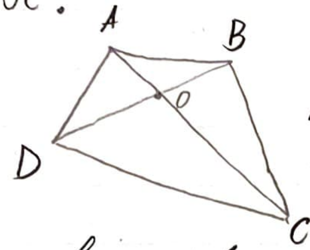
(b) If every row is occupied:

1st: $(\rho^5 + \rho^6 + \dots) + (\rho^6 + \rho^7 + \rho^8 + \dots) = \frac{\rho^5 + \rho^6}{1-\rho} = \varphi^2\left(\frac{1}{\varphi^5} + \frac{1}{\varphi^6}\right)$
$= \frac{\varphi + 1}{\varphi^4} = \frac{1}{\varphi^2} = \rho^2$

2nd: ρ^3, 3rd ρ^4 ...

All: $\frac{\rho^2}{1-\rho} = \frac{1}{\varphi^2} \cdot \varphi^2 = 1$

We have infinitely many spots occupied and we need all of them so we need infinitely many moves. But the game is finite, it is not possible.

5c. 

Let ABCD be an arbitrary quadrilateral, AC, BD intersect at O.

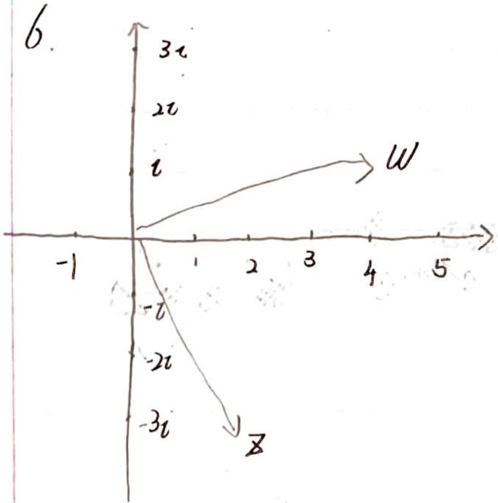
Consider $\triangle AOD$, by the $\triangle$ inequality we know that $AO+OD > AD$, Similarly, in $\triangle BOC$ we have $BO+OC > BC$

Now ~~we~~ we add to get $AO+OD+BO+OC > AD+BC$

$\because AC = AO+OC$, $BD = BO+OD$

$\therefore AC+BD > AD+BC$

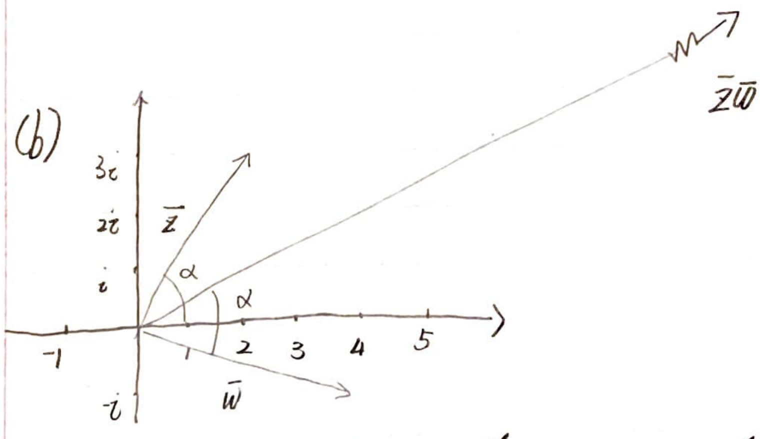
6.



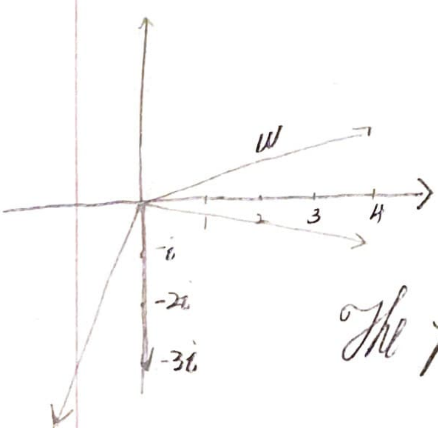
(a) $\bar{z} \cdot \bar{w} = (2+3i)(4-i) = 8+3-2i+12i = 11+10i$

$|-3iw| = |(-3i)(4-i)| = |-12i-3| = \sqrt{9+144} = \sqrt{153} = 3\sqrt{17}$

(b)



$\bar{z}\bar{w}$'s ~~ist~~ angle is the same as the sum of the two angles ($\bar{z}$, $\bar{w}$). So in the graph, the two α s are the same. The length is the product of the two lengths.



$|w| = \sqrt{17}$

$|3i| = 3$

$|-3iw| = 3\sqrt{17}$

The product ~~is~~ of the lengths = the solution.