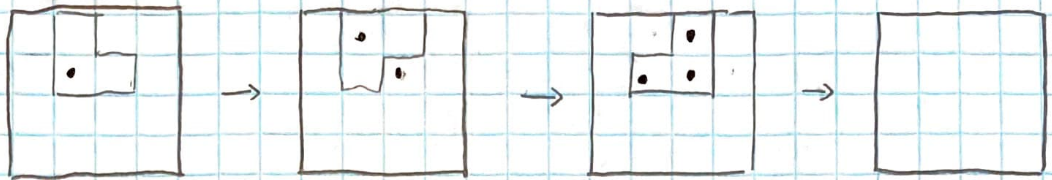
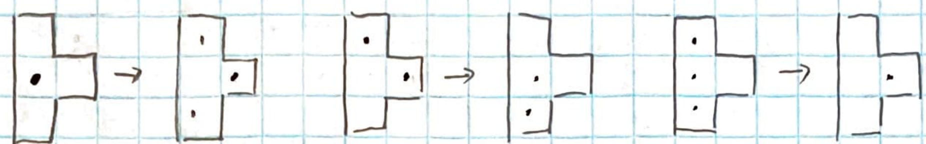
1.

(a)

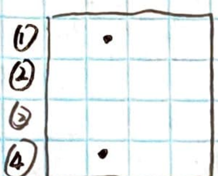


(b)



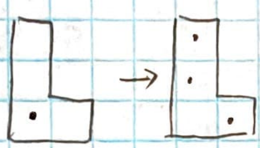
1-0 → 2-1   1-1 → 2-0   3-0 → 0-1

This preserves the parity of the difference between adjacent rows / columns



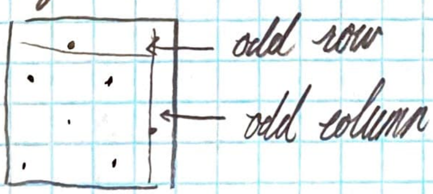
①-② is odd, so is ③-④, but in order to solve the board, they need to be 0 (even). ∴ it is impossible

(c)



1-0 → 2-1

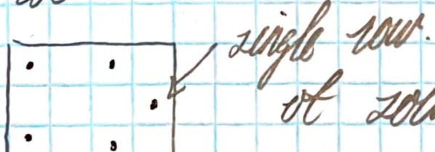
Similar to (b), this preserves the parity of the diff between adjacent rows / columns.



Once again we have an odd row & an odd column. ∴ it is impossible

(d)

This pattern preserves the parity of the points in any row/col.

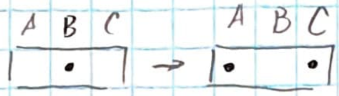


We have a single row and therefore cannot solve it.

(e) A

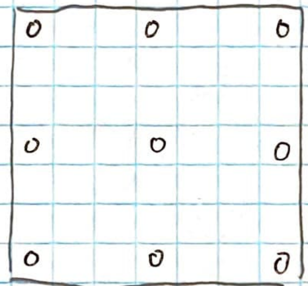
•	B	C	A	B	C	A	B	C	A
B	C	A	B	C	A	B	C	A	B
C	A	B	C	A	B	C	A	B	•

C



In any operation, the parity of A-B, B-C, A-C is preserved. Since we have 1A, 1C, A-B is odd, and thus it is not solvable.

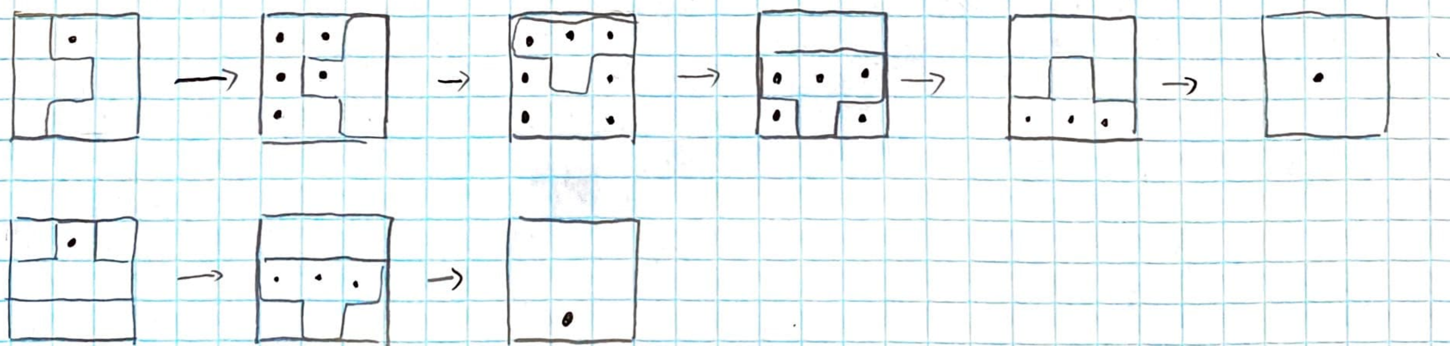
2.



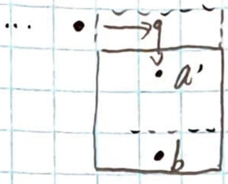
We can remove the marked squares. After removal we just tile the row horizontally and everything else vertically.

Any other removal cannot be tiled because when we color ~~it~~ it in A, B, C colors, we need to have the board w/ equal numbers of each color. However, any other tile removed would lead to an uneven number when the board is flipped.

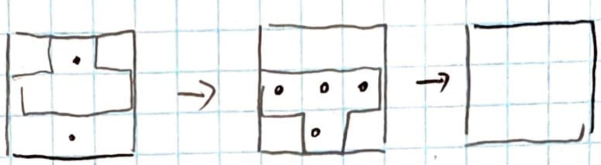
Ex 12



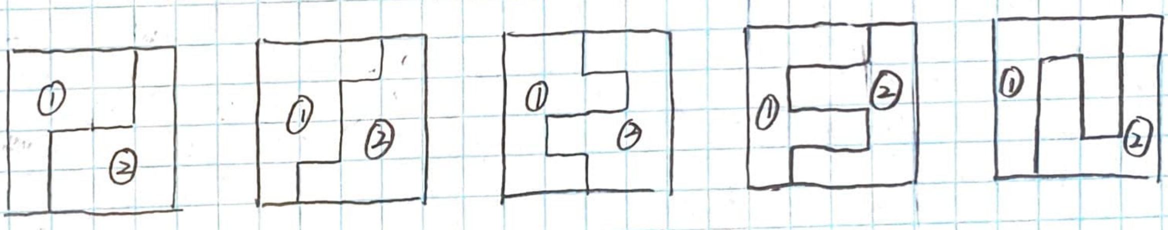
In the previous problem we have an algorithm for moving a dot 1 square down. We can rotate it to make it go in any direction. Let the two points be a, b.



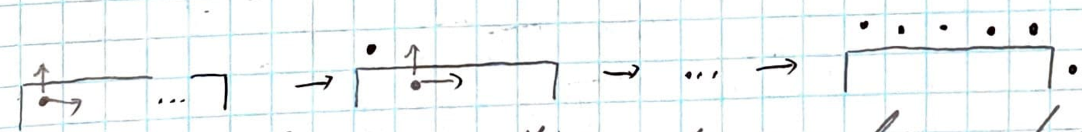
We draw a 3 by 3 grid around b like the one on the left, move a to a'. This is possible because none of the 3×3 boxes around any of the intermediate a's contain b.

Now do 

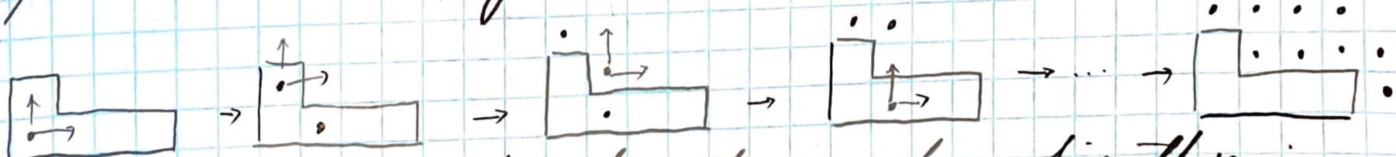
Thus it is solved.

4a. 

Fri lec

a. 

Clone the first man, this creates one above and one to the side. Keep cloning anyone still in the prison. After n moves the prison is now empty.

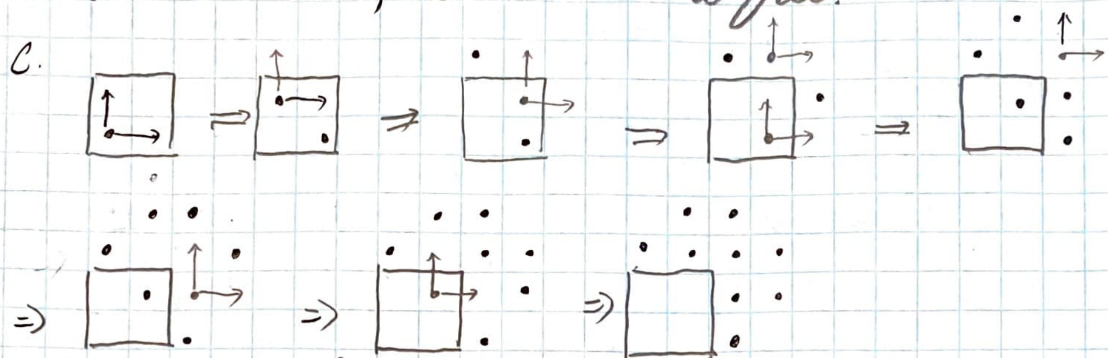
b. 

Clone the first man, then clone the one above. Since there is a clone blocking the right clone, we clone the blocker.

Each time the blocker is preventing escape, clone him. After

$n+(n-1)=2n-1$ moves, the clones are free.

c.



Thus all clones have escaped.

d. Using the weight method we ~~assig~~ assign

$$\begin{array}{llll} 1/4 & \cdots & & \\ 1/2 & 1/4 & \cdots & \\ 1 & 1/2 & 1/4 & \cdots \end{array}$$

to the squares.

Total weight $= 2+1+\frac{1}{2}+\cdots = 4$

Weight inside box $= 1+\frac{1}{2}\times 2+\frac{1}{4}\times 3+\frac{1}{8}\times 2+\frac{1}{16} = 3\frac{1}{16}$

$\therefore$ Weight outside $= 4-3\frac{1}{16} = \frac{15}{16} < 1$

If it is possible than the square weights of the clones at the end must add up to 1. If we have clones in all squares, we would still be 1/16 short.

$\therefore$ This problem is not solvable.

1 Mon Dis, 2c

Let $a \leq b \leq c \leq d$, we can do this since $\mathbb{N}$ is ordered. We rearrange $a+b+c+d = abcd$ to $a+b+c = d(abc-1)$
Since we have assumed an order, we have $a+b+c \leq 3d$, so $abc - 1 \leq 3$, $abc \leq 4$
Now in this case abc can only be $1, 2, 3, 4$. We consider each of these cases:
$abc = 1 \implies a = 1, b = 1, c = 1$, solving for our original equation gives $3 = 0d$, which does not have a solution.
$abc = 2$, by our assumed order we have $a = 1, b = 1, c = 2$, and we solve $2d = 4 + d$, $d = 4$, this gives a solution.
$abc = 3$, by our assumed order we have $a = 1, b = 1, c = 3$, and we solve $3d = 5 + d$, which does not have an integer solution.
$abc = 4$, by our assumed order we have $a_0 = 1, b_0 = 1, c_0 = 4$, or $a_1 = 1, b_1 = 2, b_2 = 2$. Solving the first equation gives $d = 2$, which contradicts our assumption of $c \leq d$, so this is not a solution. Solving the latter gives $4d = 5 + d$, which does not have an integer solution.
So finally we have our solution: rearrangements of $1, 1, 2, 4$ is the only natural solution to this system.

2 Mon Dis, 4g

$$(7, 29) \to (7+1, 29+1) = (8, 30) \to (4, 15)$$
$$(4, 15) \to (4+11, 15+11) = (15, 26)$$
$$(7, 29) \to (7+19, 29+19) = (26, 48)$$
$$((4, 15), (15, 26)) \to (4, 26)$$
$$((4, 26), (26, 48)) \to (4, 48)$$
$$(4, 48) \to (2, 24) \to (1, 12)$$