Q1

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(a)

1	$p \wedge p$	
2	p	$\wedge E$ 1

(b)

1.	$q \wedge p$	
2.	q	$\wedge E$ 1
3.	p	$\wedge E$ 1
4.	$p \wedge q$	$\wedge I$ 2, 3

(c)

1.	$(p \wedge q) \wedge r$	
2.	r	$\wedge E$ 1
3.	$(p \wedge q)$	$\wedge E$ 1
4.	[illegible]	$\wedge E$ [illegible]

4. q $\wedge E\ 3$

5. p $\wedge E\ 3$

6. $(q \wedge r)$ $\wedge I\ 2, 4$

7. $p \wedge (q \wedge r)$ $\wedge I\ 6, 5$

(d) 1. $(p \wedge q) \rightarrow r$

2. p

3. q

4. $p \wedge q$ $\wedge I\ 2, 3$

5. $(p \wedge q) \rightarrow r$ $R\ 1$

6. r $\rightarrow E\ 4, 5$

6.	r	$\to E$ 4,5
7.	$q \to r$	$\to I$ 3,6
8.	$p \to (q \to r)$	$\to I$ 2,7

(e)

1.	$(p \to q) \wedge (p \to r)$	
2.	$p \to q$	$\wedge E$ 1
3.	$p \to r$	$\wedge E$ 1
4.	p	
5.	q	$\to E$ 2,4
6.	r	$\to E$ 3,4
7	$(q \wedge r)$	$\wedge I$ 5,6

7. $(q \wedge r)$ $\wedge I$ 5,6

8. $p \to (q \wedge r)$

Q2

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Assume that $\exists$ a natural deduction proof w/ assumption $\{\varphi_1, \varphi_2, \ldots, \varphi_n\}$ and conclude ψ

Then we have

1. $\{\varphi_1, \varphi_2, \ldots, \varphi_n\}$

$\ldots$

3. ψ

4. $(\varphi_1 \wedge \varphi_2 \wedge \cdots \varphi_n) \rightarrow \psi$ $\quad \rightarrow I$ 1, 3

Thus rows 1~4 shows a proof w/ no assumptions and the desired result. One assumption is that line 2 (could be many lines) exist.

Now assume that $\exists$ a natural deduction proof w/o assumption showing that $(\varphi_1 \wedge \varphi_2 \wedge \cdots \varphi_n) \rightarrow \psi$

Then we have

1. $\ldots$

2. $(\varphi_1 \wedge \varphi_2 \wedge \cdots \varphi_n) \rightarrow \psi$

3. $\{\varphi_1, \varphi_2, \ldots \varphi_n\}$

4. $(\varphi_1 \wedge \varphi_2 \wedge \cdots \varphi_n)$ $\quad \wedge I$ 3

5. ψ $\quad \rightarrow E$ 2, 4

Thus rows 3~5 is a proof w/ the desired assumption & result. One premise this time is that line 1 (could be many lines) exist).

Q3

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(a)

1.	$P \leftrightarrow (q \leftrightarrow q)$	
2.	q	
3.	q	R 2
4.	q	
5.	q	R 4
6.	$(q \leftrightarrow q)$	$\leftrightarrow$I 2, 3, 4, 5
7.	P	$\leftrightarrow$E 1. 6

(b)

1.	$P \leftrightarrow q$	
2.	P	
3.	q	$\leftrightarrow$E 1, 3
4.	q	
5.	P	$\leftrightarrow$E 1, 4
6.	$q \leftrightarrow P$	$\leftrightarrow$I 2, 3, 4, 5

6.	$q \leftrightarrow p$	$\leftrightarrow I$ 2, 3, 4, 5
7.	$q \leftrightarrow p$	
8.	p	
9.	q	$\leftrightarrow E$ 7, 9
10.	q	
11.	p	$\leftrightarrow E$ 7, 11
12.	$p \leftrightarrow q$	$\leftrightarrow I$ 8, 9, 10, 11
13.	$(p \leftrightarrow q) \leftrightarrow (q \leftrightarrow p)$	$\leftrightarrow I$ 1, 6, 7, 12

Q5

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1. Assume prime is finite (1 ... n are all primes)
2. $p = p_1 \times \dots p_n$
3. $q = p + 1$

Disjunction is true

4. q is prime
5. q not in set of prime
6. q is in set of prime ($\neg E$ 1, 4)
7. Prime is not finite (infinite) $\neg I$ 1~6

4. q is not prime
5. Thm 1 (R)
6. Lem 1 (R)
7. $\exists\, p_i \mid q$ E 4,5
8. $p = p_i \times \cdots$ R 2.
9. $p_i \mid p$
10. $p_i \mid (q - p)$ E 6, 8, 9
11. $p_i \mid 1$
12. $p_i = 1$
13. p_i is prime R 1
14. $p_i \neq 1$
15. $p_i = 1 \wedge p_i \neq 1$
16. Prime is not finite (infinite) $\neg I$ 1~15

Q6

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(a)

1.	$\neg\neg p, \neg\neg q$	
2.	$\neg p$	
3.	$\neg\neg p$	R 1
4.	$\neg p \wedge \neg\neg p$	$\wedge$I 2, 3
5.	p	RAA 2~4
6.	$\neg q$	
7.	$\neg\neg q$	R1
8.	$\neg q \wedge \neg\neg q$	$\wedge$I 6, 7
9.	q	RAA 6~8
10.	$p \wedge q$	$\wedge$I 5, 10
11.	$\neg(p \wedge q)$	
12	$(p\wedge q) \wedge \neg(p \wedge q)$	R 10
13.	$\neg\neg(p \wedge q)$	$\neg$I 11-12

(b).

1.	$\neg p \to \neg q$	
2.	$\neg\neg q$	
3.	$\neg q$	
4.	$\neg\neg q$	R 2
5.	$(\neg q) \wedge (\neg\neg q)$	$\wedge$I 3, 4
6.	q	RAA 2~5
7.	$\neg p$	
8.	$\neg p \to \neg q$	R 1
9.	$\neg q$	R 6
10.	$q \wedge \neg q$	$\wedge$I 7, 9
11.	$\neg\neg p$	$\neg$I 7~10
12.	$\neg\neg q \to \neg\neg p$	$\to$I 2~11
13.	$(\neg p \to \neg q) \to (\neg\neg q \to \neg\neg p)$	$\to$I 1, 12

(c)

1.	P	
2.	$\neg P$	
3.	P	R 1
4.	$P \wedge \neg P$	$\wedge$I 2, 3
5.	$\neg\neg P$	$\neg$I 2, 4
6.	$P \to (\neg\neg P)$	

Q7

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1.	$\neg p$	
2.	p	
3.	$\neg p$	R 1
4.	$p \wedge \neg p$	$\wedge$I 2, 3
5.	q	EFQ 4
6.	$p \to q$	$\to$I 2~5
7.	$\neg p \to (p \to q)$	$\to$I 1~6

Q8

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1.	$\alpha \wedge \neg\alpha$	
2.	β	EFQ 1

1.	$\alpha \wedge \neg\alpha$	
2.	$\neg\alpha$	$\wedge$E 1
3.	$\alpha \to \beta$	$\neg$E 2
4.	α	$\wedge$E 2
5.	β	$\to$E 3

Diff process, same result

Q9

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1.	$\neg \alpha$	
2.	$\alpha \to \beta$	$\neg E$, 1

1.	$\neg \alpha$	
2.	α	
3.	$\neg \alpha$	R 1
4.	$\alpha \wedge \neg \alpha$	$\wedge I$ 2, 3
5.	β	EFQ 4
6.	$\alpha \to \beta$	

Diff process, same result

4.

(a)

1.	φ	
2.	φ	R 1
3.	ψ	
4.	φ	R 3
5.	$\varphi \leftrightarrow \varphi$	$\leftrightarrow$I 1-2, 3-4

(b) $\varphi \sim \psi \equiv \varphi \leftrightarrow \psi$

1.	$\varphi \leftrightarrow \psi$	
2.	φ	
3.	$\varphi \leftrightarrow \psi$	R 1
4.	ψ	$\leftrightarrow$E 2, 3
5.	ψ	
6.	$\varphi \leftrightarrow \psi$	R 1
7.	φ	$\leftrightarrow$E 5, 6
8.	$\psi \leftrightarrow \varphi$	$\leftrightarrow$I 2-4, 5, 7

$\therefore \psi \sim \varphi$

(c) $\varphi \sim \psi \equiv \varphi \leftrightarrow \psi$ $\quad \psi \sim \chi \equiv \psi \leftrightarrow \chi$

1.	$\varphi \leftrightarrow \psi$	
2.	$\psi \leftrightarrow \chi$	
3.	φ	
4.	$\varphi \leftrightarrow \psi$	R 1
5.	$\psi \leftrightarrow \chi$	R 2
6.	ψ	$\leftrightarrow$E 3, 4
7.	χ	$\leftrightarrow$E 5, 6
8.	$\varphi \rightarrow \chi$	$\rightarrow$I 3, 7