

Q1

Sunday, March 13, 2022

3:29 PM

(a) 1 |  $P \wedge P$

2 | P       $\wedge \bar{E}$  1

(b) 1. |  $q \wedge p$

2. | q       $\wedge \bar{E}$  1

3. | P       $\wedge \bar{E}$  1

4. |  $p \wedge q$        $\wedge I$  2,3

(c) 1. |  $(p \wedge q) \wedge r$

2. | r       $\wedge \bar{E}$  1

3. |  $(p \wedge q)$        $\wedge E$  1

1 |  $\wedge$        $\wedge \Sigma$  2

4.	<u>q</u>	$\wedge E 3$
5.	p	$\wedge E 3$
6.	$(q \wedge r)$	$\wedge I 2, 4$
7.	$p \wedge (q \wedge r)$	$\wedge I 6, 5$

(d) 1.  $\frac{(p \wedge q) \rightarrow r}{}$

2.	<u>p</u>	
3.	<u>q</u>	
4.	$p \wedge q$	$\wedge I 2, 3$
5.	$(p \wedge q) \rightarrow r$	$R 1$
6.	r	$\rightarrow E 4, 5$

0.    | | |       $\rightarrow E \ 4, 5$
7.    |       $q \rightarrow r$        $\rightarrow I \ 3, 6$
8.    |       $p \rightarrow (q \rightarrow r)$        $\rightarrow I \ 2, 7$

- (e) 1.  $(p \rightarrow q) \wedge (p \rightarrow r)$
2.  $p \rightarrow q$        $\wedge E \ 1$
3.  $p \rightarrow r$        $\wedge E \ 1$
4.    |  $p$
5.    |  $q$        $\rightarrow E \ 2, 4$
6.    |  $r$        $\rightarrow E \ 3, 4$
7.    |  $(q \wedge r)$        $\wedge I \ 5, 1$

$$7. \quad | \quad | (q \wedge r) \quad \wedge I \quad 5, 6$$

$$8. \quad | \quad P \rightarrow (q \wedge r)$$

Assume that  $\exists$  a natural deduction proof w/ assumption  $\{\varphi_1, \varphi_2, \dots, \varphi_n\}$  and conclude  $\psi$

Then we have

1.	$\boxed{\{\varphi_1, \varphi_2, \dots, \varphi_n\}}$	
	$\vdots$	
3.	$\boxed{\psi}$	
4.	$(\varphi_1 \wedge \varphi_2 \wedge \dots \wedge \varphi_n) \rightarrow \psi$	$\rightarrow I \ 1, 3$

This rows 1~4 shows a proof w/ no assumptions and the desired result. One assumption is that line 2 (could be many lines) exist.

Now assume that  $\exists$  a natural deduction proof w/o assumption showing that  $(\varphi_1 \wedge \varphi_2 \wedge \dots \wedge \varphi_n) \rightarrow \psi$

Then we have

1.	$\dots$	
2.	$(\varphi_1 \wedge \varphi_2 \wedge \dots \wedge \varphi_n) \rightarrow \psi$	
3.	$\boxed{\{\varphi_1, \varphi_2, \dots, \varphi_n\}}$	
4.	$(\varphi_1 \wedge \varphi_2 \wedge \dots \wedge \varphi_n)$	$\wedge I \ 3$
5.	$\psi$	$\rightarrow E \ 2, 4$

This rows 3~5 is a proof w/ the desired assumption & result. One proving this time is that line 1 (could be many lines) exist.

Q3

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(a) 1.  $\boxed{P \leftrightarrow (q \leftrightarrow q)}$

2.  $\boxed{q}$

3.  $\boxed{q} \quad R \ 2$

4.  $\boxed{q}$

5.  $\boxed{q} \quad R \ 4$

6.  $\boxed{(q \leftrightarrow q)} \quad \leftrightarrow I \ 2, 3, 4, 5$

7.  $\boxed{P} \quad \leftrightarrow E \ 1, 6$

(b) 1.  $\boxed{P \leftrightarrow q}$

2.  $\boxed{P}$

3.  $\boxed{q} \quad \leftrightarrow E \ 1, 3$

4.  $\boxed{q}$

5.  $\boxed{P} \quad \leftrightarrow E \ 1, 4$

6.  $\boxed{q \leftrightarrow P} \quad \leftrightarrow I \ 2, 3, 4, 5$

$$6. \quad | \quad q \leftrightarrow p \leftrightarrow I \quad 2, 3, 4, 5$$

$$7. \quad | \quad \overbrace{q \leftrightarrow p}$$

$$8. \quad | \quad \overbrace{\quad}^P$$

$$9. \quad | \quad \overbrace{q}^P \leftrightarrow E \quad 7, 9$$

$$10. \quad | \quad \overbrace{q}^P$$

$$11. \quad | \quad \overbrace{P}^q \leftrightarrow E \quad 7, 11$$

$$12. \quad | \quad P \leftrightarrow q \leftrightarrow I \quad 8, 9, 10, 11$$

$$13. \quad | \quad (P \leftrightarrow q) \leftrightarrow (q \leftrightarrow p) \leftrightarrow I \quad 1, 6, 7, 12$$

1. Assume prime is finite ( $1 \dots n$  all all primes)
2.  $P = p_1 \times \dots \times p_n$
3.  $q = P + 1$  Disjunction is true
4.  $q$  is prime  $\downarrow$
5.  $q$  not in set of prime
6.  $q$  is in set of prime  
 $(\rightarrow E 1, 4)$
7. Prime is not finite (infinite)  
 $\neg I 1 \sim 6$
4.  $q$  is not prime
5. Thm 1 (R)
6. Lem 1 (R)
7.  $\exists p_i \mid q \quad E 4, 5$
8.  $P = p_1 \times \dots \quad R 2.$
9.  $p_i \mid P$
10.  $p_i \mid (q - P) \quad E 6, 8, 9$
11.  $p_i \mid 1$
12.  $p_i = 1$
13.  $p_i$  is prime  $R 1$
14.  $p_i \neq 1$
15.  $p_i = 1 \wedge p_i \neq 1$
16. Prime is not finite (infinite)  $\neg I 1 \sim 15$

- (a) 1.  $\neg\neg P, \neg\neg Q$
2.  $\neg P$
3.  $\neg\neg P$  R 1
4.  $\neg P \wedge \neg\neg P$   $\wedge I$  2, 3
5.  $P$  RAA 2~4
6.  $\neg Q$
7.  $\neg\neg Q$  R 1
8.  $\neg Q \wedge \neg\neg Q$   $\wedge I$  6, 7
9.  $Q$  RAA 6~8
10.  $P \wedge Q$   $\wedge I$  5, 10
11.  $\neg(P \wedge Q)$
12.  $(P \wedge Q) \wedge \neg(P \wedge Q)$  R 10
13.  $\neg\neg(P \wedge Q)$   $\rightarrow I$  11-12

(b).	1.	$\neg P \rightarrow \neg q$	
	2.	$\neg \neg q$	
	3.	$\neg q$	
	4.	$\neg \neg q$	$R \ 2$
	5.	$(\neg q) \wedge (\neg \neg q)$	$\wedge I \ 3, 4$
	6.	$q$	$RAA \ 2 \sim 5$
	7.	$\neg P$	
	8.	$\neg P \rightarrow \neg q$	$R \ 1$
	9.	$\neg q$	$R \ 6$
	10.	$q \wedge \neg q$	$\wedge I \ 7, 9$
	11.	$\neg \neg P$	$\neg I \ 7 \sim 10$
	12.	$\neg \neg q \rightarrow \neg \neg P$	$\rightarrow I \ 2 \sim 11$
	13.	$(\neg P \rightarrow \neg q) \rightarrow (\neg \neg q \rightarrow \neg \neg P)$	$\rightarrow I \ 1, 12$

- (c) 1. | P
2. |  $\neg P$
3. | P R 1
4. |  $P \wedge \neg P$   $\wedge I$  2, 3
5. |  $\neg \neg P$   $\neg I$  2, 4
6. |  $P \rightarrow (\neg \neg P)$

Q7

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1.	$\neg P$	
2.	$P$	
3.	$\neg P$	R I
4.	$P \wedge \neg P$	$\wedge I$ 2, 3
5.	$q$	$E F Q$ 4
6.	$P \rightarrow q$	$\rightarrow I$ 2~5
?	$\neg P \rightarrow (P \rightarrow q)$	$\rightarrow I$ 1~6

Q8

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1.

$\alpha \wedge \neg \alpha$

2.

$\beta$

$\bar{E}FQ$

1

1.

$\alpha \wedge \neg \alpha$

2.

$\neg \alpha$

$\wedge E$

1

3.

$\alpha \rightarrow \beta$

$\neg E$

2

4.

$\alpha$

$\wedge E$

2

5.

$\beta$

$\rightarrow E$

3

Diff process, same result

Q9

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1. |  $\neg\alpha$

2. |  $\alpha \rightarrow \beta \quad \neg E, I$

1. |  $\neg\alpha$

2. |  $\alpha$

3. |  $\neg\alpha \quad R \ I$

4. |  $\alpha \wedge \neg\alpha \quad \wedge I \ 2, 3$

5. |  $\beta \quad EFQ \ 4$

6. |  $\alpha \rightarrow \beta$

Diff process, same result