Philosophy 12, Problem Set 5

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1 Q1

The function is true if and only if there are between (inclusive) 2 and 3 true values in $x_1, x_2, ..., x_5$. Therefore we can simply consider the following cases: there are 2 trues or (exclusive) there are 3 trues.
For an algorithmic approach, we can simply negate the cases where they are negative, or more explicity, when they sum to 0, 1, 4 or 5.
$f(x_1, x_2, x_3, x_4, x_5) = 0$, then we have $\neg(x_1 \lor x_2 \lor x_3 \lor x_4 \lor x_5)$
$f(x_1, x_2, x_3, x_4, x_5) = 1$, then we have

- $\neg(x_2 \lor x_3 \lor x_4 \lor x_5) \land x_1$
- $\neg(x_1 \lor x_3 \lor x_4 \lor x_5) \land x_2$
- $\neg(x_1 \lor x_2 \lor x_4 \lor x_5) \land x_3$
- $\neg(x_1 \lor x_2 \lor x_3 \lor x_5) \land x_4$
- $\neg(x_1 \lor x_2 \lor x_3 \lor x_4) \land x_5$

$f(x_1, x_2, x_3, x_4, x_5) = 4$, then we have

- $x_2 \land x_3 \land x_4 \land x_5 \land x_1$
- $x_1 \land x_3 \land x_4 \land x_5 \land x_2$
- $x_1 \land x_2 \land x_4 \land x_5 \land x_3$
- $x_1 \land x_2 \land x_3 \land x_5 \land x_4$

- $x_1 \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5$

  $f(x_1, x_2, x_3, x_4, x_5) = 5$, then we have $x_1 \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5$

Finally we combine them and negate the whole thing:

$$\neg((\neg(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5)) \vee (\neg(x_2 \vee x_3 \vee x_4 \vee x_5) \wedge x_1) \vee (\neg(x_1 \vee x_3 \vee x_4 \vee x_5) \wedge x_2)$$

$$\vee (\neg(x_1 \vee x_2 \vee x_4 \vee x_5) \wedge x_3) \vee (\neg(x_1 \vee x_2 \vee x_3 \vee x_5) \wedge x_4) \vee (\neg(x_1 \vee x_2 \vee x_3 \vee x_4) \wedge x_5) \vee (x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_1)$$

$$\vee (\neg(x_1 \vee x_3 \vee x_4 \vee x_5) \wedge x_2) \vee (x_1 \wedge x_2 \wedge x_4 \wedge x_5 \wedge x_3) \vee (x_1 \wedge x_2 \wedge x_3 \wedge x_5 \wedge x_4) \vee (x_1 \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5)$$

$$\vee (x_1 \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5))$$

2 Q5

2.1 a

$\neg((p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg q \wedge \neg r))$
$\equiv \neg(p \wedge q \wedge r) \wedge \neg(\neg p \wedge q \wedge r) \wedge \neg(\neg q \wedge \neg r)$
$\equiv (\neg p \vee q \vee r) \wedge (p \vee q \vee r) \wedge (q \vee r)$

2.2 b

$(p \rightarrow (q \wedge r)) \wedge \neg(q \leftrightarrow r) \wedge ((q \vee r) \rightarrow p)$
$\equiv (\neg p \vee (q \wedge r)) \wedge \neg((\neg q \vee r) \wedge (\neg r \vee q)) \wedge (\neg(q \vee r) \vee p)$
$\equiv (\neg p \vee q) \wedge (\neg p \vee r) \wedge (\neg(\neg q \vee r) \vee \neg(\neg r \vee q)) \wedge (p \vee (\neg q \wedge \neg r))$
$\equiv (\neg p \vee q) \wedge (\neg p \vee r) \wedge ((\neg r \wedge q) \vee (r \wedge \neg q)) \wedge (p \vee \neg q) \wedge (p \vee \neg r)$
$((\neg r \wedge q) \vee (r \wedge \neg q)) \equiv ((\neg r \wedge q) \vee r) \wedge ((\neg r \wedge q) \vee \neg q)$
$\equiv (\neg r \vee r) \wedge (r \vee q) \wedge (\neg q \vee \neg r) \wedge (\neg q \vee q)$
Therefore the final expression is

$$(\neg p \vee q) \wedge (\neg p \vee r) \wedge (r \vee q) \wedge (\neg q \vee \neg r) \wedge (p \vee \neg q) \wedge (p \vee \neg r)$$

2.3 c

$(p \implies (q \implies r)) \implies ((p \implies q) \implies (p \implies r))$
$\equiv \neg(\neg p \vee (\neg q \vee r)) \vee (\neg(\neg p \vee q) \vee (\neg p \vee r))$
$\equiv (p \wedge \neg(\neg q \vee r)) \vee (p \wedge \neg q) \vee (\neg p \vee r)$
$\equiv (p \wedge q \wedge \neg r) \vee (p \wedge \neg q) \vee \neg p \vee r$
$\equiv ((p) \wedge (p \vee \neg q) \wedge (q \vee p) \wedge (\neg r \vee p) \wedge (\neg r \vee \neg q)) \vee \neg p \vee r$

$\equiv ((p) \wedge (p \vee \neg q) \wedge (\neg r \vee \neg q \vee \neg p)) \vee r$
$\equiv (p \vee r) \wedge (p \vee \neg q \vee r)$

A lot of terms are removed since $1 \wedge x \equiv x$, and $p \vee \neg p \equiv 1$.

3 Q6

3.1 a

Write the complete truth table of the formula, then for each row that the formula is true, write clause that is the conjunction of all the truth values of the variables. Finally combine each row clause with a disjunction. Thus we have a DNF that describes the formula and is equivalent to it.

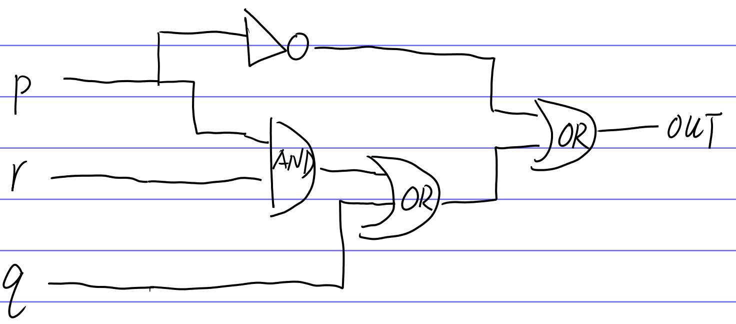
3.2 b

$(p \iff q) \iff r$
$\equiv \neg(\neg p \vee q) \vee r$
$\equiv (p \wedge \neg q) \vee r$
$(p \vee q) \wedge (p \vee r) \wedge (p \vee s)$
$\equiv (p \vee (q \wedge r)) \wedge (p \vee s)$
$\equiv p \vee (q \wedge r \wedge s)$

3

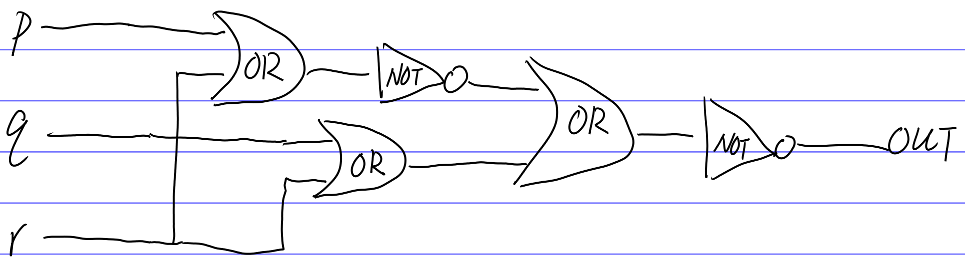
(a) $p \to (\neg q \to (p \wedge r))$

$\equiv \neg p \vee (q \vee (p \wedge r))$

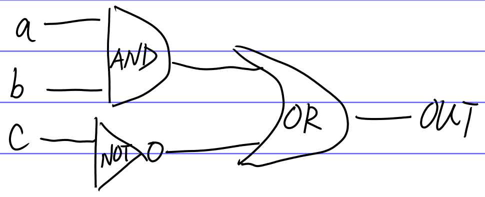


(b) $\neg((p \vee r) \to (q \vee r))$

$\equiv \neg(\neg(p \vee r) \vee (q \vee r))$



4.



$x_1 \wedge x_2$

$\equiv \neg(\neg x_1 \vee \neg x_2)$

$\equiv (\neg x_1) \downarrow (\neg x_2)$

$\equiv (x_1 \downarrow x_1) \downarrow (x_2 \downarrow x_2)$

$(a \wedge b) \vee \neg c$

$\equiv ((a \downarrow a) \downarrow (b \downarrow b)) \vee (c \downarrow c)$

$\equiv \neg(((a \downarrow a) \downarrow (b \downarrow b)) \downarrow (c \downarrow c))$

$\equiv (((a \downarrow a) \downarrow (b \downarrow b)) \downarrow (c \downarrow c))$

$\downarrow (((a \downarrow a) \downarrow (b \downarrow b)) \downarrow (c \downarrow c))$