Philosophy 12, Problem Set 6

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1 Q1

1.1 a

Randomly pick items into the bag until we overshoot the maximumt weight W. Then remove the last item and check if what's in hte bag has at least V calories. If it does, we have shown that S exists. If not we take every item out and repeat the algorithm.
This is not a polynomial time algorithm. Since we are randomly selecting the items, it is possible to never find such a set S even if S exists.

1.2 b

Since each item can either be picked or not picked, there are 2^n ways to pick our items for the backpack. Now we can simply iterate through all 2^n options to see if any fits the requirement of having less weight than W and more calories than V.
This is also not a polynomial time algorithm since 2^n is exponential.

2 Q2

2.1 Each country has at least one color

Let i be an arbitrary country, then we have $c_i \vee m_i \vee y_i$. Therefore we can apply to all i:

$$\bigwedge_{i=1}^{n}(c_i \vee m_i \vee y_i)$$

2.2 Each country has at most one color

Let i be an arbitrary country, then we have $(c_i \wedge \neg m_i \wedge \neg y_i) \vee (m_i \wedge \neg y_i \wedge \neg c_i) \vee (y_i \wedge \neg c_i \wedge \neg m_i)$. Therefore we can apply to all i:

$$\bigwedge_{i=1}^{n} (c_i \wedge \neg m_i \wedge \neg y_i) \vee (m_i \wedge \neg y_i \wedge \neg c_i) \vee (y_i \wedge \neg c_i \wedge \neg m_i)$$

2.3 No adjacent countries have the same color

If two adjacent countries have the same color then $(c_i \wedge c_j) \vee (m_i \wedge m_j) \vee (y_i \wedge y_j)$ We can negate that for all such i, j

$$\neg \bigvee_{i,j \text{ adjacent}} (c_i \wedge c_j) \vee (m_i \wedge m_j) \vee (y_i \wedge y_j)$$

$$\equiv \bigwedge_{i,j \text{ adjacent}} \neg(c_i \wedge c_j) \wedge \neg(m_i \wedge m_j) \wedge \neg(y_i \wedge y_j)$$

Finally we can simply combine these, so our answer is $(2.1) \wedge (2.2) \wedge (2.3)$

3 Q5

The first statement is true because there are only two Republicans in the race: Reagan and Anderson. The second statement is where the problem begins. "A Republican" will win the election is true in the pollee's mind because Regan is a Republican. They have subsituted "a republican" with "Reagan" because he is decisively ahead of Carter. Therefore the final conclusion: "If it's not Regan who wins, it will be Anderson" doesn't make sense.
This shows that everyday English is not as strict as formal logic and contains a lot of implicit subsitutions of terms.

Q3

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1 Lemma 1

2 Lemma 2

3 Lemma 3

4 c is real

5 $c-1$ is real (E, 1, 4)

6 $0 \leq (c-1)^2$ (E, 2, 5)

7. $0 \leq c^2+1-2c$

8. $2c \leq c^2+1$

9. c is real $\to$ $2c \leq c^2+1$ (I, 4, 8)

Q4

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(a)

1. p
2. q
3. q (R 2)
4. $q \to q$ (I, 2, 3)
5. $p \to (q \to q)$ (I, 1, 4)

(b) $r \to (p \to (q \to p))$

1. r
2. p
3. q
4. p (R, 2)
5. $q \to p$ (I, 3, 4)
6. $p \to (q \to p)$ (I, 2, 5)
7. $r \to (p \to (q \to p))$ (I, 1, 6)

(c) $(p \to q) \to ((p \to (q \to r)) \to (p \to r))$

1. $p \to q$

1. $p \to q$
2. $p \to (q \to r)$
3. p
4. q (E, 1, 3)
5. $q \to r$ (E, 2, 3)
6. $p \to r$ (I, 3, 5)
7. $(p \to (q \to r)) \to (p \to r)$ (I, 2, 6)
8. $(p \to q) \to ((p \to (q \to r)) \to (p \to r))$ (I, 1, 7)

(d) $(p \to (q \to r)) \to (q \to (p \to r))$

1. $p \to (q \to r)$
2. q
3. p
4. $q \to r$ (E, 1, 3)
5. r (E, 2, 4)
6. $p \to r$ (I, 3, 5)
7. $q \to (p \to r)$ (I, 2, 6)
8. $(p \to (q \to r)) \to (q \to (p \to r))$ (I, 1, 7)

8. $(p \to (q \to r)) \to (q \to (p \to r))$ (I, 1, 7)

(e) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

1. $p \to (q \to r)$
2. $(p \to q)$
3. p
4. q (E, 2, 3)
5. $q \to r$ (E, 1, 3)
6. r (E, 4, 5)
7. $p \to r$ (I, 3, 6)
8. $(p \to q) \to (p \to r)$ (I, 2, 7)
9. $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$ (I, 1, 8)