PHIL 12A – Spring 2022
Problem Set 4

Due February 20, 2022

65 points.

2 Basic Theory of Propositional Logic

2.1 Truth Functions

2.1.1 Truth Functions I

1. (15 points) Consider the truth function full : $2^5 \to 2$ (short for 'full house'):

$$\text{full}(x_1, x_2, x_3, x_4, x_5) = \begin{cases} 1 & \text{if } 2 \leq x_1 + x_2 + x_3 + x_4 + x_5 \leq 3 \\ 0 & \text{otherwise} \end{cases}.$$

   Find a formula that defines full.

2.1.2 Truth Functions II

2. *Extra credit.* (5 points) Let $\mathcal{L}_{\to}(\{p, q\})$ be the set of formulas of $\mathcal{L}(\{p, q\})$ in which the only connective is $\to$. How many *equivalence classes* of formulas of $\mathcal{L}_{\to}(\{p, q\})$ are there? Justify your answer.

2.1.3 Digital Circuits

3. (10 points) Draw circuits corresponding to the following formulas that use only NOT, AND, and OR gates:

   (a) $p \to (\neg q \to (p \wedge r))$;

   (b) $\neg((p \vee r) \to (q \vee r))$

4. (10 points) Given the circuit in Figure 1, draw an equivalent circuit using only NOR gates.

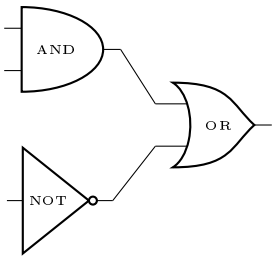

Figure 1: circuit for problem 4.

2.2 Normal Forms

2.2.1 Conjunctive Normal Form

5. (15 points) For each of the following formulas, find an equivalent formula in CNF. (NB: You may use the CNF algorithm if you would like, but there might be more efficient approaches.)

   (a) $\neg((p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg q \wedge \neg r))$;

   (b) $(p \to (q \wedge r)) \wedge \neg(q \leftrightarrow r) \wedge ((q \vee r) \to p)$;

   (c) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$.

2.2.2 Disjunctive Normal Form

6. (15 points)

   (a) Describe an algorithm for converting any given formula into an equivalent formula in DNF.

   (b) Using your algorithm or otherwise, for each of the following formulas, find an equivalent formula in DNF.

      i. $(p \leftrightarrow q) \leftrightarrow r$;

      ii. $(p \vee q) \wedge (p \vee r) \wedge (p \vee s)$.