Philosophy 12, Problem Set 1

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1 Q1

1.1 1

No it is not, when $\phi = 0, \psi = 1$, $\phi \implies \psi$ is ture, but $\psi \implies \phi$ is false. Therefore they are not equivalent.

1.2 2

ϕ	ψ	$\phi \implies \psi$	$\neg\psi \implies \neg\phi$
1	1	1	1
1	0	0	0
0	0	1	1
0	1	1	1

Since every line is the same, they are equivalent.

1.3 3

Let $\phi = 0, \psi = 1$, $(\phi \implies \psi) = 0$ so $\neg(\phi \implies \psi) = 1$. However $\phi \vee \neg\psi = 0$. They are not equivalent.

1.4 4

ϕ	ψ	$\neg(\phi \implies \psi)$	$\phi \wedge \neg\psi$
1	1	0	0
1	0	1	1
0	0	0	0
0	1	0	0

Since every line is the same, they are equivalent.

1.5 5

Let $\phi = 0, \psi = 1$, $(\phi \iff \psi) = 0$ so $\neg(\phi \iff \psi) = 1$. However $\neg\phi \iff \neg\psi = 0$. They are not equivalent.

1.6 6

ϕ	ψ	$\neg(\phi \iff \psi)$	$\neg\phi \iff \psi$
1	1	0	0
1	0	1	1
0	0	0	0
0	1	1	1

Since every line is the same, they are equivalent.

1.7 7

ϕ	ψ	$(\phi \wedge \psi) \iff (\phi \vee \psi)$	$\phi \iff \psi$
1	1	0	0
1	0	0	0
0	0	1	1
0	1	0	0

Since every line is the same, they are equivalent.

2 Q2

2.1 1

q	r	$\neg(q \wedge r)$	$\neg r$
1	1	0	0
1	0	1	1
0	0	1	1
0	1	1	0

In all possible rows, there is no such row where both premises are true and the conclusion false, therefore this is a valid consequence.

2.2 2

p	q	r	$\neg p \vee \neg q \vee \neg r$	$q \vee r$
1	1	1	1	1
1	1	0	0	1
1	0	1	1	1
1	0	0	1	0
0	1	1	1	1
0	1	0	1	1
0	0	1	1	1
0	0	0	1	0

In all possible rows, there is no such row where all premises are true and the conclusion false, therefore this is a valid consequence.

3 Q3

3.1 a

Let "vinegar is included in the batter" be p, "baking soda is included in the batter" be q, "the velvet cake rises" be r.
Then we have $(p \wedge q) \implies r$. Our conclusion is that $\neg r \implies (\neg p \implies q)$. This statement is valid because either the vinegar or the baking soda must be absent. If the batter contained vinegar, then it must not have baking soda.

3.2 b

Let "Kovak wins the election" be p, "the taxes increase" be q, "her party maintains control of the legislature" be r.
Then we have $p \implies (r \implies q)$. Our conclusion is that $\neg q \implies (\neg p \wedge \neg r)$.
This statement is not valid because Kovak could have won the election but her party did not maintain control. This satisfies the premise but contradicts the conclusion.

3.3 c

Let $a = 0$ be p, $b = 0$ be q, $a + b = 0$ be r.
Then we have $(p \wedge q) \implies r$. Our conclusion is $\neg r \implies (\neg q \vee \neg p)$.
This statement is valid because it is the contrapositive of the original.

3.4 d

Let "Jones drove the car" be p, "Smith is innocent" be q, "Brown fired the gun" be r.
Then we have $(p \implies q) \wedge (\neg r \implies \neg q)$. Our conclusion is $r \implies \neg q$.
This statement is invalid because in the case that Brown fired the gun, Smith is innocent, and Jones drove the car, all premises are met. However the conclusion is not: Jones did drive the car. Therefore the conclusion is invalid.

4 Q4

4.1 a

p	q	r	$\neg p$	$q \implies p$	$p \implies r$	$q \implies r$	$\neg q$	$\neg r$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	0
0	1	0	0	0	1	1	0	1
0	1	1	1	0	1	1	0	0
1	0	0	0	1	0	1	1	0
1	0	1	0	1	1	1	1	1
1	1	0	0	1	0	0	0	1
1	1	1	0	1	1	1	0	0

We can see that the first two lines are the only cases where all 3 premises are true. In these cases both (i) and (ii) are true but (iii) is false on line 2. Therefore (i) and (ii) are logically implied.

4.2 b

p	q	r	$p \vee r$	$q \implies \neg r$	$q \vee \neg r$	$p \implies q$	$p \implies (r \vee \neg q)$	$(\neg p \vee r) \implies q$
0	0	0	0	1	1	1	1	0
0	0	1	1	1	0	1	1	0
0	1	0	0	1	0	1	1	1
0	1	1	1	0	1	1	1	1
1	0	0	1	1	1	0	1	1
1	0	1	1	1	0	0	1	0
1	1	0	1	1	1	1	0	1
1	1	1	1	0	1	1	1	1

We can see that there are only 2 rows in which all 3 prermises are satisfied. In these rows we observe that only (iii) is always true. Therefore only (iii) is logically implied.

5 Q5

5.1 a

No

5.2 b

No

5.3 c

Yes

5.4 d

Yes

5.5 e

No

5.6 f

Yes

5.7 g

No

5.8 h

No

5.9 i

Yes

5.10 j

Yes

5.11 k

Yes

5.12 l

No

6 Q6

6.1 Base case

Consider the proposition ϕ. It has length 1 and no brackets, therefore we know that $p_\phi = 0, l_\phi = 1$. The base case holds.

6.2 Inductive case

Assume that $2p_\psi < l_\psi$ holds for all $\psi \in L$ such that $l_\psi \leq n$ for some natural n.
Let A, B be arbitrary formulas of length n. By our hypothesis we know that they satisfy $2p_A < l_A$ (same for B). Furthermore, we know that formulas can be formed by joining two sub-formulas with $(A \wedge B)$, $(A \vee B)$, $(A \implies B)$ or $\neg A$. In the first three cases, our total amount of brackets is $p_A + p_B + 2$ and our length is $l_A + l_B + 3$. Now we can multiply the first statement by 2: $2p_A + 2p_B + 4$. Now we can observe the second statement. By our inductive hypothesis we know that $l_A + l_B + 3 > 2p_A + 2p_B + 3$. However, since there can only be an integer number of characters, we have $l_A \geq 2p_A + 1$. The same is true for l_B. Therefore we have $l_A + l_B + 3 \geq 2p_A + 2p_B + 3 + 2 > 2p_A + 2p_B + 3$
In the last case, we have a total of p_A brackets and a total lengtht of $l_A + 1$. It is obviously true due to our inductive hypothesis.
Thus we have proven the base case and the inductive case.
Q.E.D.