PHIL 12A – Spring 2022
Problem Set 6

50 points.

2 Basic Theory of Propositional Logic

2.3 Algorithms and Combinatorial Problems

2.3.3 Algorithms III

1. (10 points) Suppose you are packing for a backpacking trip and trying to decide which snacks to bring. Your home pantry contains m snack items, each of which has a certain weight w_i and a calorie value v_i. Your backpack can only hold a maximum weight of W, and for your journey you need a minimum of V calories. Therefore, you need to answer the question: is there is some set S of items from your pantry such that the sum of the weights of the items in S is less than or equal to W, while the sum of the calorie values of the items in S is greater than or equal to V?

   (a) Describe a non-deterministic algorithm for deciding the question. Is it a nondeterministic polynomial-time algorithm? Explain your answer.

   (b) Describe a deterministic algorithm for answering the question. Is it a polynomial-time algorithm? Explain your answer.

2.3.4 Combinatorial Problems

2. (10 points) Suppose you are given a (finite) map and three colored pencils—say cyan, magenta, and yellow. You are asked if there is a way to color the map so that (i) each country is colored with *exactly one* color, and (ii) no two adjacent countries (countries whose borders touch) are colored with the same color.

   Explain how you can encode this problem with a formula of propositional logic that is satisfiable if and only if there is such a coloring of the map.

   Suggestion: number the countries from $1, \ldots, n$, and for each i in $\{1, \ldots, n\}$, let c_i mean that country i is colored cyan, m_i mean that country i is colored magenta, and y_i mean that country i is colored yellow.

3 Natural Deduction

3.1.1–3.1.4 Conditional

3. (10 points) Outline a natural deduction proof that formalizes the conditional introduction and conditional elimination steps of the proof of Lemma 3 below.

   Lemma 1. If a is a real number, then $a - 1$ is a real number.

   Lemma 2. If b is a real number, then $0 \leq b^2$.

   Lemma 3. If c is a real number, then $2c \leq c^2 + 1$.

   Proof. Suppose c is a real number. Then by Lemma 1, $c - 1$ is a real number, so by Lemma 2, we have $0 \leq (c-1)^2$. Then since $(c-1)^2 = c^2 - 2c + 1 = (c^2+1) - 2c$, we have $0 \leq (c^2+1) - 2c$ and hence $2c \leq c^2 + 1$. This completes the proof. □

4. (10 points) Give natural deduction proofs of the following formulas (from no assumptions):

   (a) $p \to (q \to q)$;

   (b) $r \to (p \to (q \to p))$;

   (c) $(p \to q) \to ((p \to (q \to r)) \to (p \to r))$;

   (d) $(p \to (q \to r)) \to (q \to (p \to r))$;

   (e) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$.

5. (10 points) Consider the following passage from Van McGee's paper "A Counterexample to Modus Ponens" (*Journal of Philosophy*, Vol. 82, No. 9, 1985, p. 462):

   > Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:
   >
   > > If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.
   > >
   > > A Republican will win the election.
   >
   > Yet they did not have reason to believe
   >
   > > If it's not Reagan who wins, it will be Anderson.

   Question: Does McGee's example show that for the 'if...then' of ordinary English, it is possible for the premises of a modus ponens argument to be true and yet the conclusion false? Or does it show something else? Explain your answer.