PHIL 12A – Spring 2022
Problem Set 4

Due February 20, 2022

75 points.

2 Basic Theory of Propositional Logic

2.1 Truth Functions

2.1.1 Truth Functions I

1. (15 points) Consider the truth function $\mathrm{full} : 2^5 \to 2$ (short for 'full house'):

$$\mathrm{full}(x_1, x_2, x_3, x_4, x_5) = \begin{cases} 1 & \text{if } 2 \leq x_1 + x_2 + x_3 + x_4 + x_5 \leq 3 \\ 0 & \text{otherwise} \end{cases}.$$

   Find a formula that defines full.

2.1.2 Truth Functions II

2. *Extra credit.* (5 points) Let $\mathcal{L}_{\to}(\{p, q\})$ be the set of formulas of $\mathcal{L}(\{p, q\})$ in which the only connective is $\to$. How many *equivalence classes* of formulas of $\mathcal{L}_{\to}(\{p, q\})$ are there? Justify your answer.

2.1.3 Digital Circuits

3. (10 points) Draw circuits corresponding to the following formulas that use only NOT, AND, and OR gates:

   (a) $p \to (\neg q \to (p \wedge r))$;

   (b) $\neg((p \vee r) \to (q \vee r))$

4. (10 points) Given the circuit in Figure 1, draw an equivalent circuit using only NOR gates.

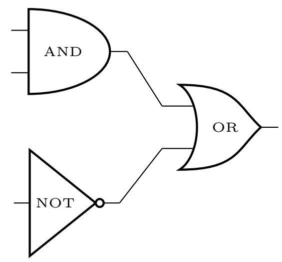

Figure 1: circuit for problem 4.

2.2 Algorithms and Combinatorial Problems

2.2.1 Algorithms I

5. (15 points) Convert each of the following formulas into an equivalent formula in CNF, showing each step of the CNF algorithm:

   (a) $\neg((p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg q \wedge \neg r))$;

   (b) $(p \to (q \wedge r)) \wedge \neg(q \leftrightarrow r) \wedge ((q \vee r) \to p)$;

   (c) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$.

6. (15 points)

   (a) Describe an algorithm for converting any given formula into an equivalent formula in DNF.

   (b) Run your algorithm on the following formulas:

      i. $(p \leftrightarrow q) \leftrightarrow r$;

      ii. $(p \vee q) \wedge (p \vee r) \wedge (p \vee s)$.

2.2.2 Algorithms II

7. (10 points) On the resolution algorithm. For each of the CNF formulas produced in problem 5, use resolution to test whether the formulas is satisfiable, showing each step of the resolution algorithm.