

1 | Lemma 1

2 | Lemma 2

3 | Lemma 3

4 | C is real

5 | $C-1$ is real $(E, 1, 4)$

6 | $0 \leq (C-1)^2$ $(E, 2, 5)$

7. | $0 \leq C^2 + 1 - 2C$

8. | $2C \leq C^2 + 1$

9. | C is real $\rightarrow 2C \leq C^2 + 1$ $(I, 4, 8)$

(a)

1		<u>P</u>	
2		<u>q</u>	
3		q	(R 2)
4		q → q	(I, 2, 3)
5		p → (q → q)	(I, 1, 4)

(b) $r \rightarrow (p \rightarrow (q \rightarrow p))$

1.		<u>r</u>	
2.		<u>p</u>	
3.		<u>q</u>	
4.		p	(R, 2)
5.		q → p	(I, 3, 4)
6.		p → (q → p)	(I, 2, 5)
7.		r → (p → (q → p))	(I, 1, 6)

(c) $(p \rightarrow q) \rightarrow ((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow r))$

1. | | p → q

1.	$P \rightarrow q$	
2.	$P \rightarrow (q \rightarrow r)$	
3.	P	
4.	q	(E, 1, 3)
5.	$q \rightarrow r$	(E, 2, 3)
6.	$P \rightarrow r$	(I, 3, 5)
7.	$(P \rightarrow (q \rightarrow r)) \rightarrow (P \rightarrow r)$	(I, 2, 6)
8.	$(p \rightarrow q) \rightarrow ((P \rightarrow (q \rightarrow r)) \rightarrow (P \rightarrow r))$	(I, 1, 7)

(d) $(p \rightarrow (q \rightarrow r)) \rightarrow (q \rightarrow (p \rightarrow r))$

1.	$P \rightarrow (q \rightarrow r)$	
2.	q	
3.	P	
4.	$q \rightarrow r$	(E, 1, 3)
5.	r	(E, 2, 4)
6.	$P \rightarrow r$	(I, 3, 5)
7.	$q \rightarrow (P \rightarrow r)$	(I, 2, 6)
8.	$(P \rightarrow (q \rightarrow r)) \rightarrow (q \rightarrow (P \rightarrow r))$	(I, 1, 7)

$$8. \mid (p \rightarrow (q \rightarrow r)) \rightarrow (q \rightarrow (p \rightarrow r)) \quad (I, 1, 7)$$

$$(e) \quad (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$$

1.	$p \rightarrow (q \rightarrow r)$	
2.	$(p \rightarrow q)$	
3.	p	
4.	q	$(E, 2, 3)$
5.	$q \rightarrow r$	$(E, 1, 3)$
6.	r	$(E, 4, 5)$
7.	$p \rightarrow r$	$(I, 3, 6)$

$$8. \mid (p \rightarrow q) \rightarrow (p \rightarrow r) \quad (I, 2, 7)$$

$$9. \mid (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)) \quad (I, 1, 8)$$

Philosophy 12, Problem Set 6

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1 Q1

1.1 a

Randomly pick items into the bag until we overshoot the maximum weight W . Then remove the last item and check if what's in the bag has at least V calories. If it does, we have shown that S exists. If not we take every item out and repeat the algorithm.

This is not a polynomial time algorithm. Since we are randomly selecting the items, it is possible to never find such a set S even if S exists.

1.2 b

Since each item can either be picked or not picked, there are 2^n ways to pick our items for the backpack. Now we can simply iterate through all 2^n options to see if any fits the requirement of having less weight than W and more calories than V .

This is also not a polynomial time algorithm since 2^n is exponential.

2 Q2

2.1 Each country has at least one color

Let i be an arbitrary country, then we have $c_i \vee m_i \vee y_i$. Therefore we can apply to all i :

$$\bigvee_{i=1}^n (c_i \vee m_i \vee y_i)$$

2.2 Each country has at most one color

Let i be an arbitrary country, then we have $(c_i \wedge \neg m_i \wedge \neg y_i) \vee (m_i \wedge \neg y_i \wedge \neg c_i) \vee (y_i \wedge \neg c_i \wedge \neg m_i)$. Therefore we can apply to all i :

$$\bigwedge_{i=1}^n (c_i \wedge \neg m_i \wedge \neg y_i) \vee (m_i \wedge \neg y_i \wedge \neg c_i) \vee (y_i \wedge \neg c_i \wedge \neg m_i)$$

2.3 No adjacent countries have the same color

If two adjacent countries have the same color then $(c_i \wedge c_j) \vee (m_i \wedge m_j) \vee (y_i \wedge y_j)$
We can negate that for all such i, j

$$\begin{aligned} & \neg \bigvee_{i,j \text{ adjacent}} (c_i \wedge c_j) \vee (m_i \wedge m_j) \vee (y_i \wedge y_j) \\ & \equiv \bigwedge_{i,j \text{ adjacent}} \neg(c_i \wedge c_j) \wedge \neg(m_i \wedge m_j) \wedge \neg(y_i \wedge y_j) \end{aligned}$$

Finally we can simply combine these, so our answer is $(2.1) \wedge (2.2) \wedge (2.3)$

3 Q5

The first statement is true because there are only two Republicans in the race: Reagan and Anderson. The second statement is where the problem begins. “A Republican” will win the election is true in the pollee’s mind because Regan is a Republican. They have subsituted “a republican” with “Reagan” because he is decisively ahead of Carter. Therefore the final conclusion: “If it’s not Regan who wins, it will be Anderson” doesn’t make sense.

This shows that everyday English is not as strict as formal logic and contains a lot of implicit substitutions of terms.