PHIL 12A – Spring 2022
Problem Set 2

Due February 6, 2022

60 points.

1 Syntax and Semantics of Propositional Logic

1.1 Semantics of the Propositional Language

1.1.1 Semantics of the Propositional Language I

1. **[14 POINTS]** On truth table semantics: Exercise 2.22 of Logic in Action. For each part, if the formulas are logically equivalent, a full truth table should be provided. If they are not, a single row demonstrating their non-equivalence will suffice.

1.1.2 Semantics of the Propositional Language II

2. **[10 POINTS]** On valid argument forms: Exercise 2.20 of Logic in Action. If you determine that the inference is valid, a full truth table should be provided. If the inference is invalid, a carefully chosen single row will suffice.

3. **[12 POINTS]** On valid argument forms. For each of the following English arguments, represent the form of the argument using a set of formulas for the premises and a single formula as the conclusion. Then indicate whether the form of argument is valid. This problem is adapted from an exercise in Elliot Mendelson's *Introduction to Mathematical Logic*, 5th Edition (p. 17):

   (a) "If vinegar and baking soda were included in the batter, the red velvet cake would rise. The red velvet cake did not rise. Therefore, if the batter contained vinegar, then it did not contain baking soda."

   (b) "If Kovak wins the election, then taxes will increase so long as her party maintains control of the legislature. Therefore, if taxes do not increase, Kovak lost the election and her party lost control of the legislature."

   (c) "If $a = 0$ and $b = 0$, then $a + b = 0$. But $a + b \neq 0$. Hence, $a \neq 0$ or $b \neq 0$."

   (d) "If Jones drove the car, Smith is innocent. If Brown did not fire the gun, then Smith is not innocent. Hence, if Brown fired the gun, then Jones did not drive the car."

4. **[12 POINTS]** Use truth tables to justify your answers.

   (a) Determine whether each formula is logically implied by the following collection of formulas:

   $$\neg p, \quad q \to p, \quad p \to r$$

   i. $q \to r$
   ii. $\neg q$
   iii. $\neg r$

   (b) Determine whether each formula is logically implied by the following collection of formulas:

   $$p \vee r, \quad q \to \neg r, \quad q \vee \neg r$$

   i. $p \to q$
   ii. $p \to (r \wedge \neg q)$
   iii. $(\neg p \vee r) \to q$

5. **[12 POINTS]** On consequence: This problem is adapted from an exercise in Elliot Mendelson's *Introduction to Mathematical Logic*, 5th Edition (p. 8). Which of the following formulas are logical consequences of $p \leftrightarrow q$? (No justification is required. Just write *yes* or *no* for each part.)

   (a) p
   (b) q
   (c) $p \to q$
   (d) $\neg p \to \neg q$
   (e) $p \vee q$
   (f) $p \vee \neg q$
   (g) $\neg p \vee \neg q$
   (h) $p \wedge q$
   (i) $(p \vee q) \to (p \wedge q)$
   (j) $p \to (\neg q \to \neg p)$
   (k) $\neg q \to (p \to (p \vee q))$
   (l) $\neg q \to (\neg p \to (p \vee q))$

   [PRACTICE] Which of the above formulas are logical consequences of $p \wedge q$? Of $p \vee q$?

1.2 Syntax of the Propositional Language

6. **[EXTRA CREDIT]** Let $\mathcal{L}$ be the language of propositional logic. For $\phi \in \mathcal{L}$, let p_ϕ be the number of parentheses (counting both left and right parentheses) occurring in ϕ, and let l_ϕ be the number of characters appearing in ϕ. For example, if $\phi = \neg(p \wedge q)$, then $p_\phi = 2$ and $l_\phi = 6$. Prove by structural induction that $2 \cdot p_\phi < l_\phi$ for any $\phi \in \mathcal{L}$.