Philosophy 12, Problem Set 5

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1 Q1

1.1 a

Resolvent of $c1, c2$, we have $V_1 = q \vee s$
Resolvent of $c1, c3$, we have $V_2 = p \vee \neg s$
Resolvent of $c2, c3$, we have $V_3 = \neg p \vee \neg q$
We have our new formula: $(p \vee q) \wedge (\neg p \vee s) \wedge (\neg q \vee \neg s) \wedge (q \vee s) \wedge (p \vee \neg s) \wedge (\neg p \vee \neg q)$
We continue the algorithm: the resolvent of $c1, c4$ is $(p \vee q \vee s)$, $c1, c5$ yields $(p \vee q \vee \neg s)$, $c1, c6$ yields $(q \vee \neg q) = 1$, and will be excluded.
$c2, c4$ yields $(\neg p \vee q \vee s)$, $c2, c5$ yields 1, $c2, c6$ yields $(s \vee \neg p \vee \neg q)$
$c3, c4$ yields 1, $c3, c5$ yields $(\neg s \vee p \vee \neg q)$, $c3, c6$ yields $(\neg s \vee \neg q \vee \neg p)$
Our new formula becomes $(p \vee q) \wedge (\neg p \vee s) \wedge (\neg q \vee \neg s) \wedge (q \vee s) \wedge (p \vee \neg s) \wedge (\neg p \vee \neg q) \wedge (p \vee q \vee s) \wedge (p \vee q \vee \neg s) \wedge (\neg p \vee q \vee s) \wedge (s \vee \neg p \vee \neg q) \wedge (\neg p \vee q \vee s) \wedge (s \vee \neg p \vee \neg q) \wedge (\neg s \vee p \vee \neg q) \wedge (\neg s \vee \neg q \vee \neg p)$
At this stage, any new resolution of any two sub-formulas result in an expression already in our formula. Thus our algorithm terminates, and the formula is satisfiable.

1.2 b

Resolvent of $c1, c2$: (q)
Resolvent of $c1, c3$: $(\neg r)$
Resolvent of $c1, c4$: $(r \vee \neg q)$
Consider our formula: $p \wedge (\neg p \vee q) \wedge (\neg p \vee \neg r) \wedge (r \vee \neg p \vee \neg q) \wedge (q) \wedge (\neg r) \wedge (r \vee \neg q)$

Resolvent of $q \wedge (r \vee \neg q)$: r. We have both $\neg r$ and r in our CNF, thus it is not satisfiable.

2 Q2

2.1 a

$$(p \vee \neg s) \wedge (\neg p \vee q \vee s) \wedge \neg s \wedge (s \vee \neg q)$$

Resolvent of $c1, c3$: $\neg s$ ($c1$ is subsumed)
Resolvent of $c2, c4$: $(\neg p \vee s)$ ($c2$ is subsumed)
Our formula becomes: $\neg s \wedge (s \vee \neg q) \wedge (\neg p \vee s)$
Resolvent of $c1, c2$: $\neg q$ ($c2$ is subsumed)
Resolvent of $c1, c3$: $\neg p$
Thus our formula is $\neg s \wedge \neg q \wedge \neg p$. There can no longer be more resolvents. Our algorithm terminates and thus the expression is satisfiable.

2.2 b

$s = 0, q = 0, p = 0$

3 Q3

Assume that the algorithm fails and that the result after subsumption yields that the expression is satisfiable but in reality it is not.
Then there must exist subformulas C, C' such that $C \leq C'$. Our algorithm removes C' when C is in the formula. Our assumption implies that at least one such step fails.
Consider an arbitrary valuation of C such that $\hat{V}(C) = 1$. Since C' is subsumed by C, it is a disjunction of several other atomic expressiosn. Thus we have $C' = C \vee ....$ Now we can subsitute the valuation of C that gives $\hat{V}(C) = 1$. From our formula above, we have $C' = 1 \vee ... = 1$.
Thus it is impossible for any subsuming formula to be satisfiable and the original to be not satisfiable. Our assumption is in correct and the algorithm works.