Q1

Saturday, March 19, 2022 4:14 PM

¬(Some digit occurs inf many times)

All occurs finite times

Decimal expansion has finite length

Decimal form of $\pi$ is rational

$\pi$ is rational & irrational

$(\pi \in \mathbb{Q}) \wedge \neg(\pi \in \mathbb{Q})$

Some digit occurs inf. many times

Q2

Saturday, March 19, 2022 4:18 PM

(a)

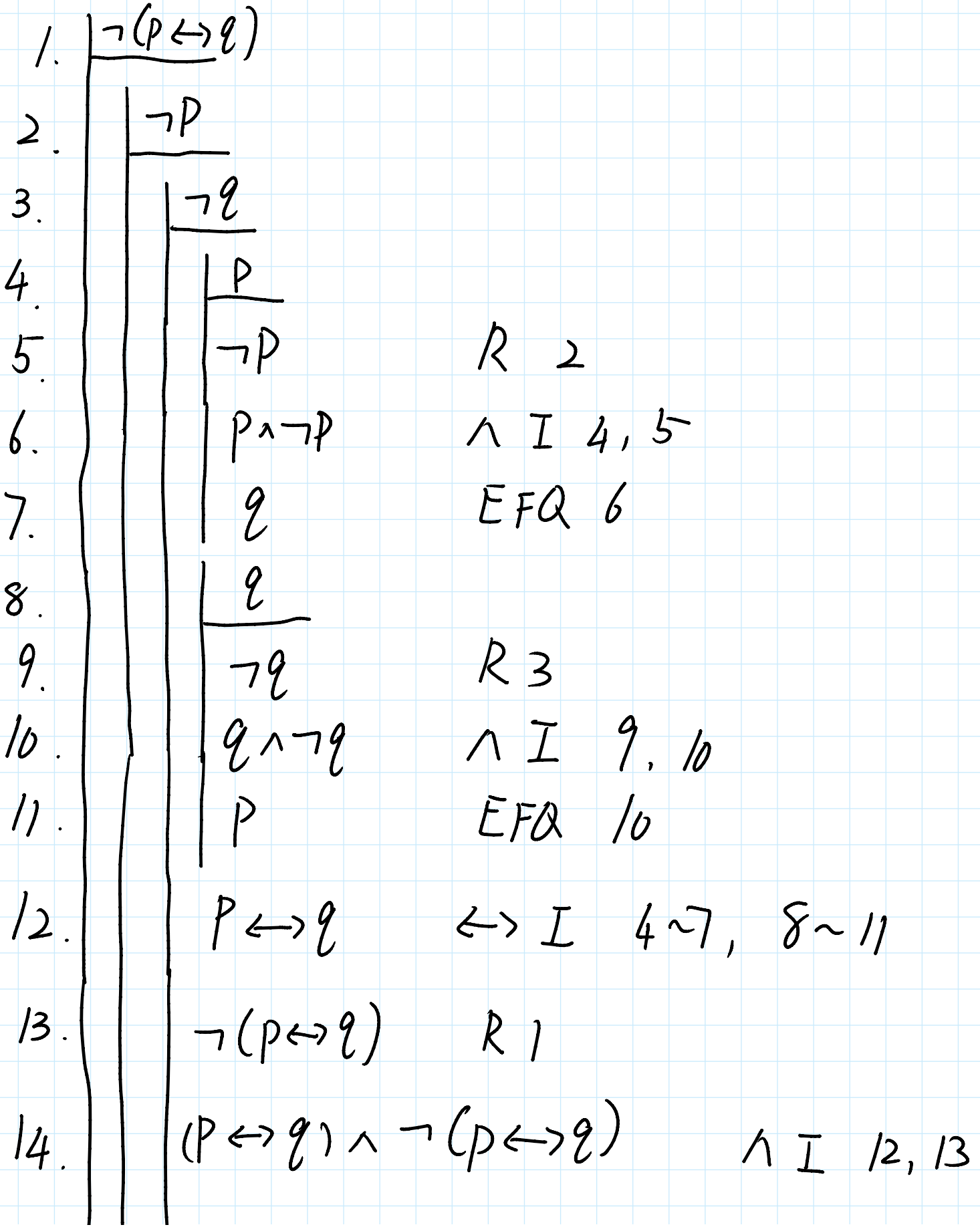
| | | |
|---|---|---|
| 1. | $\neg p$ | |
| 2. | $\quad p$ | |
| 3. | $\quad \neg p$ | R 1 |
| 4. | $\quad p \wedge \neg p$ | $\wedge$ I 2, 3 |
| 5. | $\quad q$ | RAA 4 |
| 6. | $p \rightarrow q$ | $\rightarrow$ I 2 - 5 |

(b)

| | | | |
|---|---|---|---|
| 1. | $\neg\neg P$ | | |
| 2. | $\quad\neg P$ | | |
| 3. | $\quad\neg\neg P$ | R | 1 |
| 4. | $\quad(\neg P) \wedge (\neg\neg P)$ | $\wedge$I | 2, 3 |
| 5. | $\quad P$ | RAA | 2~4 |
| 6. | $\neg\neg P \rightarrow P$ | $\rightarrow$I | 1~5 |

Q3

Saturday, March 19, 2022 4:22 PM



15. $q$ RAA 3~14

16. $q$

17. $P$

18. $P$

19. $q$ R 16

20. $q$

21. $P$ R 17

22. $P \leftrightarrow q$ $\leftrightarrow$ I 18~19, 20~21

23. $\neg(P \leftrightarrow q)$ R 1

24. $(P \leftrightarrow q) \wedge \neg(P \leftrightarrow q)$ $\wedge$ I 22, 23

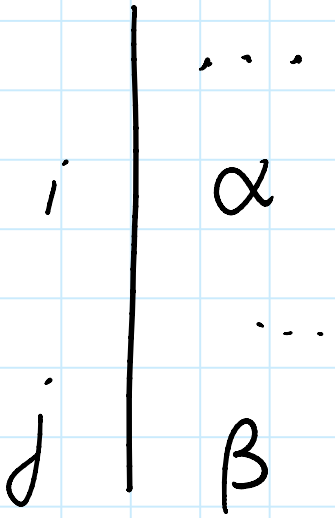
25. $\neg P$ $\neg$ I 17~24

26. $q \leftrightarrow \neg P$ $\leftrightarrow$ I, 2~15, 16~25

Q4

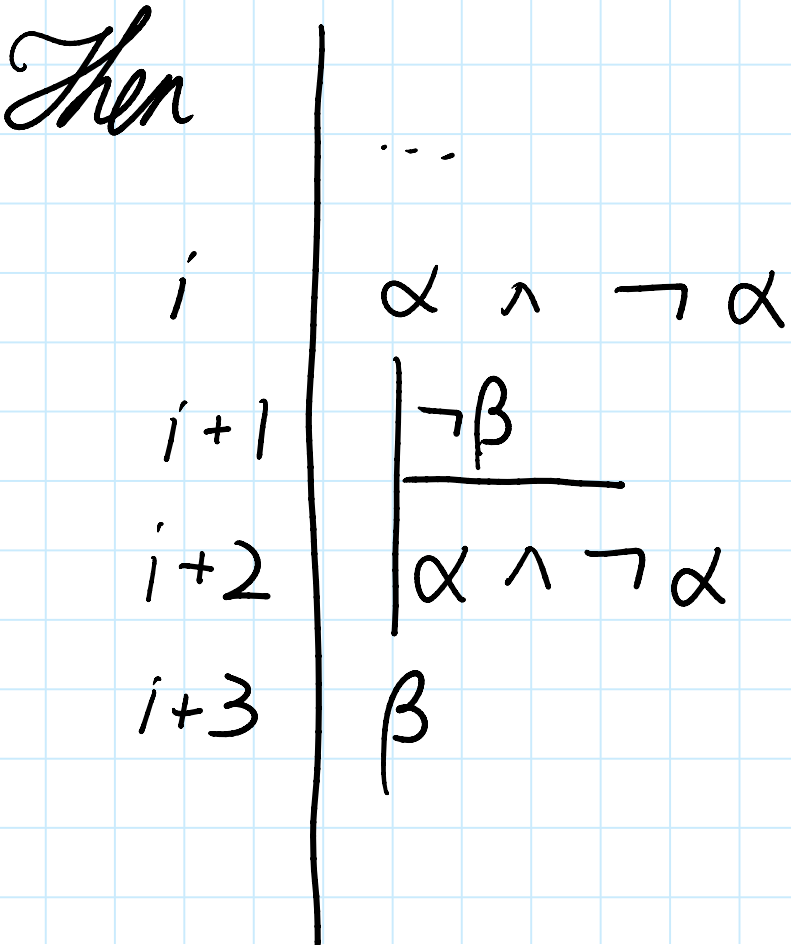
Saturday, March 19, 2022 4:29 PM

Consider




$E \vdash Q \quad i$

Then




$R \quad i$

$RAA \quad i+1 \sim i+2$

Q5

Sunday, March 20, 2022 3:35 PM

(a)

| | | |
|---|---|---|
| 1. | $\neg P \wedge \neg Q$ | |
| 2. | $\neg P$ | $\wedge E$ 1. |
| 3. | $\neg Q$ | $\wedge E$ 1. |
| 4. | $P \vee Q$ | |
| 5. | $P$ | |
| 6. | $\neg P$ | R 2 |
| 7. | $(P \wedge \neg P)$ | $\wedge I$ 5, 6 |
| 8. | $Q$ | |
| 9. | $\neg Q$ | R 3 |
| 10. | $Q \wedge \neg Q$ | $\wedge I$ 8, 9 |
| 11. | $P$ | EFQ 10 |
| 12. | $\neg P$ | R 2 |
| 13. | $P \wedge \neg P$ | $\wedge I$ 11, 12 |
| 14 | $(P \wedge \neg P)$ | $\vee E$ 4~7, 8~13 |
| 15. | $\neg (P \vee Q)$ | $\neg I$ 4~14 |
| 16. | $\neg (P \vee Q)$ | |

16. | | $\neg(p \vee q)$

17. | | | $p$

18. | | | $p \vee q$ $\vee$ I 17

19. | | | $\neg(p \vee q)$ R 16

20. | | | $(p \vee q) \wedge \neg(p \vee q)$ $\wedge$ I 18, 19

21. | | $\neg p$

22. | | | $q$

23. | | | $p \vee q$ $\vee$ I 22

24. | | | $\neg(p \vee q)$ R 16

25. | | | $(p \vee q) \wedge \neg(p \vee q)$ $\wedge$ I 24, 25

26. | | $\neg q$

27 | | $\neg p \wedge \neg q$ $\wedge$ I 21, 26

28. | $(\neg p \wedge \neg q) \leftrightarrow \neg(p \vee q)$ $\leftrightarrow$ I 1~28

(b)

| | | |
|---|---|---|
| 1. | $\neg(P \wedge Q)$ | |
| 2. | $\neg(\neg P \vee \neg Q)$ | |
| 3. | $\neg P$ | |
| 4. | $\neg P \vee \neg Q$ | $\vee$ I 3 |
| 5. | $\neg(\neg P \vee \neg Q)$ | R 2 |
| 6. | $(\neg P \vee \neg Q) \wedge \neg(\neg P \vee \neg Q)$ | $\wedge$ I 4, 5 |
| 7. | $P$ | RAA 3~6 |
| 8. | $\neg Q$ | |
| 9. | $\neg P \vee \neg Q$ | $\vee$ I 8 |
| 10. | $\neg(\neg P \vee \neg Q)$ | R 2 |
| 11. | $(\neg P \vee \neg Q) \wedge \neg(\neg P \vee \neg Q)$ | $\wedge$ I 9, 10 |
| 12. | $Q$ | RAA 8~11 |
| 13. | $P \wedge Q$ | $\wedge$ I 7, 12 |
| 14. | $\neg(P \wedge Q)$ | R 1 |
| 15. | $(P \wedge Q) \wedge \neg(P \wedge Q)$ | $\wedge$ I 13, 14 |

15. | | $(p \wedge q) \wedge \neg(p \wedge q)$ $\wedge$ I 13, 14

16. | $\neg p \vee \neg q$

Q6

Sunday, March 20, 2022 5:12 PM

1. A has 3 friends or 3 enemies
2. | 3 Friends (B, C, D)
3. | (B, C Friends ∨ B, D Friends ∨ C, D Friends) ∨ BCD Enemies
4. | | Former
5. | | A and the two form 3 mutual friends
6. | | ∃ 3 mutual friends or 3 mutual enemies ∨I 5
7. | | Latter
8. | | B, C, D form 3 mutual enemy
9. | | ∃ 3 mutual friends or 3 mutual enemies ∨I 8
10. | | ∃ 3 mutual friends or 3 mutual enemies ∨E 3, 4~6, 7~9
11. | 3 Enemies (B, C, D)
12. | (B, C Enemies ∨ B, D Enemies ∨ C, D Enemies) ∨ BCD Friends
13. | | Former
14. | | A and the two form 3 mutual enemies
15. | | ∃ 3 mutual friends or 3 mutual enemies ∨I 14
16. | | Latter
17. | | B, C, D form 3 mutual frieds
18. | | ∃ 3 mutual friends or 3 mutual enemies ∨I 17
19. | | ∃ 3 mutual friends or 3 mutual enemies ∨E, 11, 13~15, 16~18
20. | ∃ 3 mutual friends or 3 mutual enemies ∨E, 1, 2~10, 11~19

Q7

Sunday, March 20, 2022 5:34 PM

(a)

| | | |
|---|---|---|
| 1. | $(q \vee p)$ | |
| 2. | $\quad q$ | |
| 3. | $\quad p \vee q$ | $\vee$ I 2 |
| 4. | $\quad p$ | |
| 5. | $\quad p \vee q$ | $\vee$ I 4 |
| 6. | $p \vee q$ | $\vee$ E 1, 2~3, 4~5 |

(b)

| | | |
|---|---|---|
| 1. | $(p \to r) \wedge (q \to r)$ | |
| 2. | $p \to r$ | $\wedge$ E 1 |
| 3. | $q \to r$ | $\wedge$ E 1 |
| 4. | $(p \vee q)$ | |
| 5. | $\quad p$ | |
| 6. | $\quad p \to r$ | R 2 |
| 7. | $\quad r$ | $\to$ E 5, 6 |
| 8. | $\quad q$ | |
| 9. | $\quad q \to r$ | R3 |
| 10. | $\quad r$ | $\to$ I 8, 9 |

| | | | |
|---|---|---|---|
| 10. | r | →E | 8, 9 |
| 11. | r | ∨E | 4, 5~7, 8~10 |
| 12. | (p∨q)→r | →I | 4~11 |

(c)

| | | | |
|---|---|---|---|
| 1. | p∨¬p | | |
| 2. | ¬p | | |
| 3. | p→q | ¬E | 2 |
| 4. | (p→q)∨(q→p) | ∨I | 3 |
| 5. | p | | |
| 6. | q | | |
| 7. | p | R | 5 |
| 8. | q→p | →I | 6~7 |
| 9. | (p→q)∨(q→p) | ∨I | 8 |
| 10. | (p→q)∨(q→p) | ∨E | 1, 2~4, 5~9 |