PHIL 12A - Spring 2022
Problem Set 1

100 points.

1 Syntax and Semantics of Propositional Logic

1.1 Semi-Formal Introduction to Propositional Logic

1.1.1 What is Propositional Logic?

1. **[8 POINTS]** For each of the following forms of argument, give an example of a specific argument of that form, by plugging in propositions for p and q. Then explain whether or not you think the form of argument in question is a good form of argument.

   (1) If p, then q.

   (2) It is not the case that p.

   (3) Therefore, it is not the case that q.

   (1) It is more likely that p than it is that q.

   (2) It is at least as likely that q as it is that r.

   (3) Therefore, it is more likely that p than it is that r.

   [PRACTICE]

   (1) It is not the case that both p and q.

   (2) p.

   (3) Therefore, it is not the case that q.

1.1.2 Truth-Functional Connectives

2. **[8 POINTS]** Is the unary propositional connective 'Pete heard that ... truth functional? Justify your answer by providing a truth table or explaining why there is no truth table for this connective.

3. **[8 POINTS]** Is the binary propositional connective 'neither ... nor ..., as in 'Kate is neither in the library nor Kate is in the gym', truth functional? Justify your answer by providing a truth table or explaining why there is no truth table for this connective.

1.1.3 The Truth-Functional Conditional

4. **[8 POINTS]** On the truth-functional conditional, from Herbert Enderton's *A Mathematical Introduction to Logic* (p. 29): "An advertisement for a tennis magazine states, "If I'm not playing tennis, I'm watching tennis. And if I'm not watching tennis, I'm reading about tennis." We can assume that the speaker cannot do more than one of these activities at a time. What is the speaker doing? (Translate the given sentences into our formal language; consider the possible truth assignments.)"

5. **[8 POINTS]** On the truth-functional conditional: This exercise is due to Raymond Smullyan, a logician famous for devising logic puzzles. On the Island of Knights and Knaves, each person is either a knight or a knave and not both. The knights always tell the truth, while the knaves always lie. You arrive at the island in search of gold. When you ask a person on the island if the island has any gold, the person replies, "If I am a knight, then there is gold on the island." From this, can you determine if there is gold on the island, and whether the person is a knight or a knave? Justify your answer.

1.1.4 Valid Forms of Argument I

6. **[8 POINTS]** For each of the following arguments, indicate whether it is an instance of a valid form of argument or an instance of an invalid form of argument. Display the form of argument using letters in place of propositions. If the argument is an instance of a valid form of argument, indicate whether or not the argument is sound.

   (1) If rabbits are fish, then mice are fish.

   (2) If mice are fish, then unicorns exist.

   (3) Therefore, if rabbits are fish, then unicorns exist.

   (1) Rabbits are fish or mice are fish.

   (2) Rabbits are fish or unicorns exist.

   (3) Therefore, Rabbits are fish, and unicorns exist or mice are fish.

   [PRACTICE]

   (1) Sacramento is the capital of Florida or Carson City is the capital of Nevada.

   (2) Sacramento is the capital of Florida or Salem is the capital of Oregon.

   (3) Therefore, Sacramento is the capital of Florida or both Carson City is the capital of Nevada and Salem is the capital of Oregon.

1.1.5 Valid Forms of Argument II

7. **[8 POINTS]** For each of the following forms of argument, decide whether it is valid or invalid. If it is valid, present a full truth table showing that in every row in which the premises are true, the conclusion is also true. If it is invalid, present one row of a truth table in which the premises are true, while the conclusion is false.

(1) q or it is not the case that p.

(2) If it is not the case that r, then it is not the case that p.

(3) Therefore, q.

(1) p and it is not the case that p.

(2) Therefore, q.

[PRACTICE]

(1) If r, then p.

(2) If it is not the case that r, then q

(3) Therefore, p or q.

1.1.6 Validity and Soundness

8. **[12 POINTS]** For each of the following questions, either provide an example argument or explain why such an argument cannot be given.

   (i) Provide an instance of a valid form of argument with (at least) one true premise and a false conclusion.

   (ii) Provide an instance of a valid form of argument that has a true conclusion and that is not sound.

   (iii) Provide a sound argument with a false conclusion.

1.2 Syntax of the Propositional Language

1.2.1 Syntax of the Propositional Language I

9. **[8 POINTS]** On translating from English: Exercise 2.6 of Logic in Action.

10. **[8 POINTS]** On translating from English: This exercise is drawn from L.T.F. Gamut's Logic, Language, and Meaning, Vol. 1 (p. 40-41). Translate each of the following English sentence into the language of propositional logic, and provide a key of what your propositions p, q, r, s, p_1, p_2, etc. stand for:

    (a) **[PRACTICE]** It is not the case that Tiana comes if Allie or Dominique comes.

    (b) **[PRACTICE]** It is not the case that Cain is guilty and Abel is not.

    (c) **[PRACTICE]** This has not been written with a pen or a pencil.

    (d) **[PRACTICE]** I am going to the beach or the movies on foot or by bike.

    (e) **[PRACTICE]** Charles and Elsa are brother and sister or nephew and niece.

    (f) Shamik goes to work by car, or by bike and train.

    (g) The party is not on, if the weather is bad and too many are sick, .

    (h) Jasmine is going to school, and if it is raining so is Seth.

(i) If it isn't summer, then it is damp or cold, if it is evening or night.

(j) If you do not help me if I need you, I will not help you if you need me.

(k) If you stay with me if I won't drink any more, then I will not drink any more.

(l) We are going, unless it is raining.

(m) You don't mean it, and if you do, I don't believe you.

Example problem. Consider the English sentence 'If I have lost if I cannot make a move, then I have lost'. We can translate this into $(\neg p \to q) \to q$ where p stands for 'I can make a move' and q stands for 'I have lost'.

1.2.2 Syntax of the Propositional Language II

11. **[8 POINTS]** Give construction sequences for the formulas $((p \to q) \vee (q \to p))$ and $\neg\neg(p \vee \neg p)$.

12. **[8 POINTS]** On construction trees: Exercises 2.8 and 2.9 of Logic in Action.