Abstract

Plasma physics involves complex phenomena characterized by multiple scales, for example, the propagation of electromagnetic fields, the interaction between charged particles and electromagnetic fields, and the drift of fluids. To numerically simulate plasmas, discretization schemes that can automatically adapt to the scale of interest and remain stable regardless of the system scales are desirable. In this thesis, we discuss asymptotic-preserving (AP) schemes for the Euler-Maxwell system parametrized by the dimensionless Debye length λ in a 3D setting for multi-species plasma modeling. The key capability of the AP scheme is the uniform performance, in the sense of stability and convergence, for a series of problems that are parametrized by arbitrary λ and constitute a singular perturbation as $\lambda \to 0$. As the first part of the thesis, the AP property, still being a vague concept throughout literature, is clarified by two aspects: asymptotic-stability and asymptotic-consistency, which respectively refer to the unconditional stability on the asymptotic parameter and the convergence of the limit scheme to the limit model. Two propositions discussing some sufficient conditions for general AP schemes are presented, hopefully providing deeper insights into the AP property and some guidance for devising AP schemes. In the sequel, we reproduce the results in Degond et al. (2012), based on which we extend the 1D two-fluid discrete Euler-Maxwell model to a more general model in a 3D setting that involves multiple species, friction terms, and possibly other source terms. The 3D discrete model for the Euler-Maxwell system is implemented based on the frameworks of Finite Volume Method (FVM) and Finite Integration Technique (FIT) for the Euler system and the Maxwell system respectively. Details about the procedure of time-stepping and the assembly of the linear system due to implicitness are provided. An essential aspect lies in the choice of right boundary conditions that are compatible with both non-limit and limit models. The discrete model is tested on a case that is adapted from Fuchs (2021) due to the difficulty arising from the insulating solid domain. As verification, the scheme is checked on the subsystems and is compared with the corresponding 1D discrete model. The numerical results on the fully-coupled system with λ ranging in $[0, 1]$ manifest a continuous transition from the non-neutral model ($\lambda = \mathcal{O}(1)$) to the quasi-neutral model ($\lambda = 0$), which, as well as the convergence of the solutions over time, implies that the scheme is AP.

Paper plan:

1. Three-species model in a special situation [electrons, neutral, ions]
[Neglect generation & recombination]

Friction terms: (3.25a) (3.27a) → Eq. (4.5) —
Full plasma model

$$\begin{tikzcd} {} \arrow[r, "\lambda \to 0"] \arrow[d, "h"] & {} \arrow[d, "h"] \\ {} \arrow[r, "\lambda \to 0"] & {} \end{tikzcd}$$

- AP: general concept (short)

[Where to discuss boundary conditions]

2. Scaling [based on Degond's work]
→ Section 4.1. [justify scaling of friction terms "$\lambda \sim g$"]
Issue: Scaling of friction term ?
Focus on $\lambda \to 0$: quasi-neutral limit

For the sake of simplicity we confine ourselves to two species (e^-, ion$^+$). Three-species model → Roman's thesis

3. Previous work: AP discretization in 1D

3.1. 1D plasma model

3.2. Staggered grid finite differences

3.3. Semi-implicit AP timestepping

4. Discretization in 3D

4.1. Primal-dual staggered meshes

4.2. Discrete Maxwell's equations: DEC

4.3. Finite-volume Discretization of Euler Equations

4.4. Discrete boundary conditions

4.5. Stabilization outside plasma domain

4.6. Semi-implicit AP timestepping

5. Numerical experiments

[LaTeX style: thesis T.-W. Yu]