Comparison between Different Parameters Identification Criteria using the Bayesian Lasso

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Overview

Bayesian Lasso

Regularization

Models with small sample sizes and large number of variables often lead to low model generalizability.

Regularization has the potential to create a better balance between model simplicity and model fit in such conditions.

The lasso (least absolute shrinkage and selection operator; Tibshirani, 1996) method has been used in many fields and increasingly in social sciences (Lindstrøm & Dahl, 2020).

Consider a regression model with J predictors $\mathbf{X}$ to outcome data $\mathbf{Y}$ by minimizing the estimation function

$$L^{lasso}(\beta) = (|\mathbf{Y} - \mathbf{X}\beta|)^{\mathbf{2}} + \lambda \sum_{\mathbf{j=1}}^{\mathbf{J}} |\beta_{\mathbf{j}}| \tag{1}$$

- β : $\mathbf{J} \times \mathbf{1}$ vector of regression coefficients for the J predictors
- $L^{lasso}(\beta)$ and $(|\mathbf{Y} - \mathbf{X}\beta|)^{\mathbf{2}}$: loss function of lasso regression and least square difference, respectively.
- $\lambda \sum_{j=1}^{J} |\beta_j|$: lasso penalty function with the tuning parameter $\lambda \geq 0$.

Bayesian Lasso

The Bayesian lasso has been increasingly used in the social sciences

- lasso can be readily applied in Bayesian analyses by using the double exponential priors (Park & Casella, 2008)
- the Bayesian lasso can provide estimates of standard errors and intervals that are difficult to obtain under a frequentist framework
- the tuning parameters can be more conveniently estimated with other coefficients simultaneously under the Bayesian lasso paradigm (Hans, 2009; Park & Casella, 2008)

Applications of Lasso and Bayesian Lasso

Network Analysis and Structural Equation Modeling

- graphical lasso network models (Costantini et al., 2019)
- network analysis with an adaptive lasso method (Marcus, Preszler, & Zeigler-Hill, 2017)
- Bayesian lasso confirmatory factor analysis (CFA; Chen et al., 2020; Pan, Ip, & Dubé, 2017
- exploratory mediation analysis (Serang et al., 2017)
- Bayesian adaptive lasso for ordinal regression with latent variables (Feng, Wu, & Song, 2017)
- regularized multiple-indicators and multiple-caused (MIMIC) models (Jacobucci, Brandmaier, & Kievit, 2019)

Dilemma in Practice

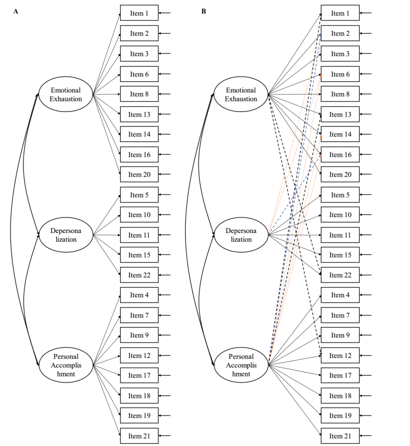
Decision rules for parameter identification (i.e., for determining whether a coefficient is non-zero and included in the model) tend to vary greatly across studies.

At least three criteria for parameter identification—the thresholding rule, the *p*-value rule, and the credible interval rule—have been used in the literature.

In this paper, we compare the three criteria for identifying parameter for inclusion into a CFA model using the Bayesian lasso.

Dilemma in Practice

To illustrate, we use a data set regarding burnout in elementary school male teachers ($N = 372$) (Byrne, 1994, 2012). Participants were asked to respond to the 22-item Maslach Burnout Inventory (MBI, 7-point Likert scale ; Maslach & Jackson, 1981).



Research Goals

Apply the Bayesian lasso to regularize both cross-loadings or residual correlations (both within- and across-factor), to evaluate the performance of the three criteria under different conditions.

Provide further recommendations on the decision rules for using the Bayesian lasso CFA.

Suppose $y_1, y_2, ..., y_n$ are independent random observations, and each $y_i = (y_{i1}, y_{i2}, ..., y_{ip})^T$ satisfies the following factor analysis model:

$$y_i = \mu + \Lambda\omega_i + \epsilon_i, i = 1, 2, ..., n, \tag{2}$$

- $\mu : p \times 1$ vector of intercepts.
- $\Lambda : p \times q$ factor loading matrix, reflects the relation of observed variables in y_i with the $q \times 1$ latent factors in ω_i.
- $\omega_i \sim N[0, \Phi]$.
- $\epsilon_i : p \times 1$ random vector of measurement errors, $\sim N[0, \Psi]$, independent of ω_i.

Bayesian Lasso CFA

The theory being tested simply does not fit the data well due to the assumptions:

- no cross-loading
- zero residual covariances

Under the Bayesian framework, the strict assumptions are relaxed through the assignment of priors to the corresponding parameters.

Bayesian lasso:

- double exponential priors for cross-loadings and residual covariances matrix.
- simultaneously identifie non-negligible cross-loadings and residual covariances in a joint estimation procedure.

Bayesian Lasso CFA

Two-steps methods: reanalyze the data with the identified freed (non-zero) parameters without regularization (Muthén & Asparouhov, 2013; Serang et al., 2017; Zhang et al., 2021).

Note that such methods make use of the benefits of regularization in variable selection instead of model estimation.

In this way, the identification of "significantly large" non-zero entries require careful operationalization.

Different Ways of Parameter Identification

The Thresholding Rule

Frequentist Lasso:

- 0, 0.001 (Liang & Jacobucci, 2020; Serang et al., 2017; Serang & Jacobucci, 2021; Yuan & Liu, 2020).

Bayesian Lasso:

- 0.1 ($|\beta| \geq 0.1$, Guo et al., 2012; Hoti & Sillanpää, 2006; Feng, Wu, & Song, 2017).

The cutoff value can also be justified from a substantive standpoint.

- cross-loading of less than 0.1 can be considered to have little practical importance (Muthén & Asparouhov, 2012).
- correlation coefficient as 0.1 is a typical value of low-effect size (Cohen, 1988).

The p-value and Interval Rule

Frequentist Lasso:

- In regression models, R-package covTest (Lockhart et al., 2014) can be used to obtain p-values.
- In network analysis, Epskamp et al. (2018) demonstrated how frequentist-lasso regularization can provide confidence intervals using a bootstrap method.

Bayesian Lasso:

- it is relatively straightforward to obtain p-values.
- credible intervals such as the HPD interval can also be calculated using Markov Chain Monte Carlo (MCMC; Gilks, Richardson, & Spiegelhalter, 1996) algorithms

Multiple Adjustment

Epskamp et al. (2018) pointed out the α level would be corrected to 0.000003 even with a small 20-node network if the Bonferroni correction is used.

Pan et al. (2017) adopted the nominal α level at 0.05 when calculating the HPD intervals of residual covariances and demonstrated that a 95% HPD interval could maintain low Type I error rates.

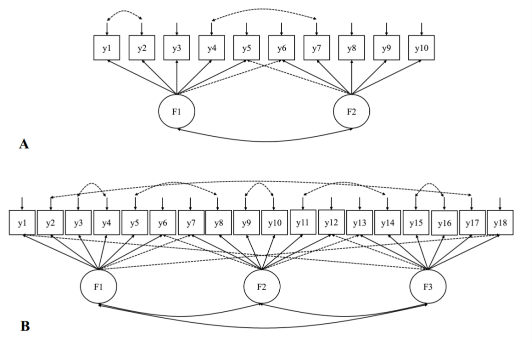
In this paper, the nominal α level at 0.05 is used throughout.

Simulation Study

Simulation Study

To investigate the performance of three criteria, we manipulated the following factors and generated 100 datasets per condition.

- Sample size: 200, 500, 1000
- Model Size: 2 factors and 10 items, 3 factors and 18 items
- Effect Sizs: 0, 0.1, 0.2, 0.3 for cross-loadings; 0, 0.1, 0.3, 0.7 for residual correlations



Model Estimation

To avoid the possible confounding effect, the conditions of non-zero cross-loadings and non-zero residual correlations were separately generated and analyzed.

- M_1: model with some non-zero cross-loadings (Fig. 2) and diagonal residual covariance matrix
- M_2: model with some non-zero, off-diagonal residual covariance entries but no cross-loading.

Model Estimation

Priors for the intercepts and factor covariance matrix:

$$\boldsymbol{\mu} \sim N(\boldsymbol{\mu}_0, \mathbf{H}_{\boldsymbol{\mu}0}), \boldsymbol{\Phi}^{-1} \sim Wishart(\mathbf{R}_0, \rho_0) \tag{3}$$

For M_1, the loadings for the j-th item $\Lambda_j = \binom{\Lambda_j^m}{\Lambda_j^c}$ where Λ_j^m and Λ_j^c respectively represent main loadings and cross-loadings

$$\Lambda_j^m \sim N(\Lambda_{0j}, \mathbf{H}_{0j}), \Lambda_j^c | \psi_{jj} \sim N(0, \psi_{jj}\mathbf{D}_{\tau_j}) \tag{4}$$

$$\psi_{jj}^{-1} \sim Gamma(a_{0j}, b_{0j}), \mathbf{D}_{\tau_j} = diag(\tau_{j1}^2, ..., \tau_{jK}^2) \tag{5}$$

$$\tau_{jk}^2 \sim Gamma(1, \frac{\gamma^2}{2}), \gamma^2 \sim Gamma(a_{lj}, b_{lj}) \tag{6}$$

For M_2, $\Lambda_j \sim N(\Lambda_{0j}, \mathbf{H}_{0j})$. Following Khondker et al. (2013) and Wang (2012), graphical lasso priors are specified for the inverse of the residual variance-covariance matrix.

Sensitivity Analysis

We adopted three sets of values for hyperparameters, results were similar.

Set	$\boldsymbol{\mu}_0$	$\mathbf{H}_{\mu 0}$	$\mathbf{R}_0$	ρ_0	$\boldsymbol{\Lambda}_{0j}$	$\mathbf{H}_{0j}$	a_{0j}	b_{0j}	a_{lj}	b_{lj}	$a_{\lambda 0}$	$b_{\lambda 0}$
1	0	$4\mathbf{I}$	$\mathbf{I} + 0.1$	K+2	0	$4\mathbf{I}$	1	0.01	1	0.01	1	0.01
2	0	$4\mathbf{I}$	$\mathbf{I} + 0.5$	K+8	0	$\mathbf{I}$	1	0.1	1	0.1	1	0.1
3	0	$100\mathbf{I}$	$\mathbf{I}$	K+2	0	$100\mathbf{I}$	1	0.01	1	0.01	1	0.01

Parameter Identification

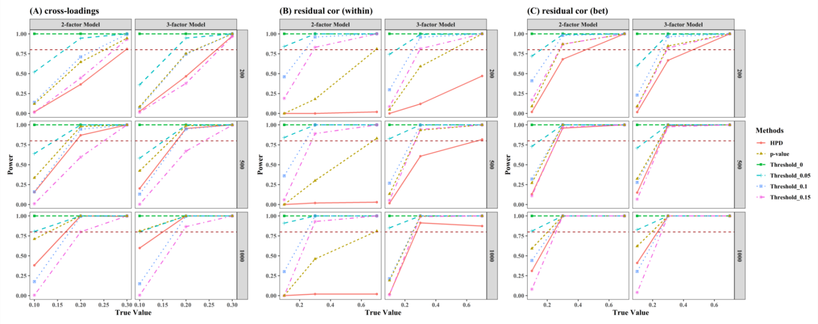
- Thresholds of magnitude 0, 0.05, 0.1, and 0.15 with the decision rule to include if the absolute value of the standard estimate is larger than the cutoff.
- A *p*-value with α=0.05, with the decision rule to include if *p* ¡ 0.05. The *p*-value can be different from the frequentist p-value, it is one-tailed and is based on MCMC samples rather than the z-test.
- A 95% HPD interval, with the decision rule to include if the point 0.0 is outside the 95% HPD interval.

Evaluation Criteria

- Power: the probability of correctly identifying the cross-loadings/residual correlations when the parameters are non-zero (Muthén & Asparouhov, 2012).
- Type-I error rates: the probability of erroneously identifying the cross-loadings/residual correlations when the parameters are zero.
- The ratio of correct identification to the total number of identified parameters (Yuan & Liu, 2020).

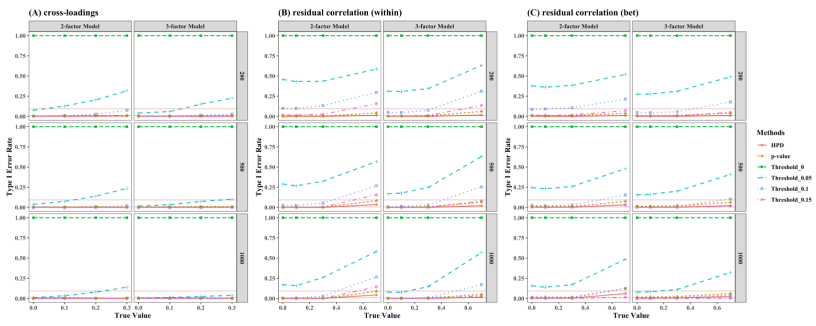
Results: Power

- The HPD interval and *p*-value rule have similar power problems in detecting within-factor residual correlations.
- The thresholding rule is more robust to sample sizes.



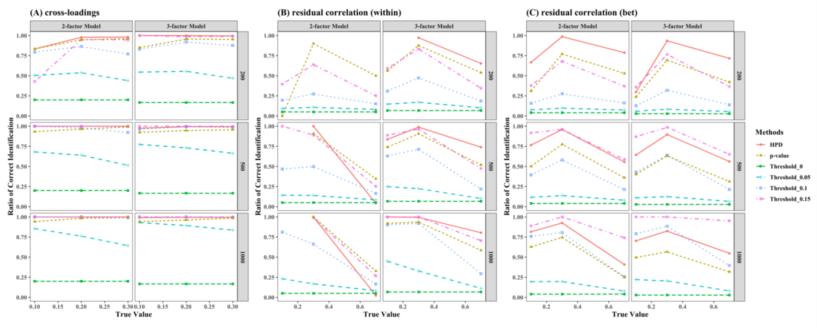
Results: Type I Error Rate

The general pattern was almost a mirror image of that of power.



Results: The Ratio of Correct Identification

- The more conservative the rule, the higher the ratio of correct identification.
- The metric was higher for cross-loadings compared to residual correlations, which was partly caused by the relatively large proportion of zero residual correlations in the generative models.



Detect Cross-loadings & Residual Covariances Simultaneously

- The co-existence of cross-loadings and residual correlations may be expected in practice.
- The phenomenon of low power of p-value and HPD interval methods was still present for within-factor residual correlations.
- Compared to performance when only one kind of parameter was present, we found the HPD interval and p-value methods were more sensitive to model size and provided lower power.

Discussion

Recommendation

Sample Size	Parameters	Threshold 0.1		Threshold 0.15		HPD Interval		*p*-value	
		Power	Type I	Power	Type I	Power	Type I	Power	Type I
200	Cross-loadings[1]	×	√	×	√	×	√	×	√
	Residual correlations	√	×	√	√	×	√	×	√
500	Cross-loadings	√	√	×	√	√	√	√	√
	Residual correlations	√	√	√	√	×	√	×	√
1000	Cross-loadings	√	√	√	√	√	√	√	√
	Residual correlations	√	√	√	√	×	√	×	√

Note: Type I: Type I Error Rate; √: acceptable in most conditions, ×: unacceptable in many conditions, shaded: the best criterion in the corresponding condition.

[1]None of the criteria can provide sufficient power under this condition.

Usage of The Thresholding Rule

- The 0.1 cutoff value, which we recommend for detecting cross-loading, can be a candidate for application to parameter identification for path coefficients in SEM.
- We conjecture that using a cutoff of 0.15 for other correlation parameters would be appropriate if the Bayesian lasso model is adopted.
- For exploratory analysis in SEM in which the purpose is to extract as many potentially important relationships as possible, the cutoff value can be lowered.

Limitations and Future Research

- Limited model sizes
- Extension of the lasso method (e.g., adaptive lasso).
- Comparison with one-step approach such as the spike-and-slab method.

Thanks for listening!