CSC343 Assignment 3 Part 2

Malcolm MacDougall 1002440931, Tianxiang Chen 999473181

1.a) In relation V:

$LPR^+ = LPQRST$

$LR^+ = LRST$

$M^+ = LMO$

$MR^+ = LMNORST$

Each functional dependency in W violates BCNF since none are trivial in V and none form a superkey in V.

b) Starting with relationship V and $LPR \rightarrow Q$:

$LPR^+ = LPQRST$

We split V into V_1 with attributes LMNOPR and V_2 with attributes LPQRST.

$LR \rightarrow ST$ projected onto V_2 still violates BCNF since $LR^+ = LRST$,
so V_2 is replaced by V_3 and V_4 with attributes LPQR and LRST respectively.

Next, $M \rightarrow LMO$ violates BCNF with relation V_1, $M^+ = LMO$.
V_1 is replaced by V_5 and V_6 with attributes MNPR and LMO respectively.

$MR \rightarrow N$ violates BCNF in V_5 where $MR^+ = MNR$
Replace V_5 with V_7 and V_8 with attributes MPR and MNR respectively.

None of the functional dependencies result in a violation of BCNF when projected onto the remaining relations, so we are left with the following decomposition of V in BCNF:

- V_6 with attributes LMO and non-trivial functional dependencies $\{M \rightarrow LO\}$.
- V_3 with attributes LPQR and non-trivial functional dependencies $\{LPR \rightarrow Q\}$.
- V_4 with attributes LRST and non-trivial functional dependencies $\{LR \rightarrow ST\}$.
- V_8 with attributes MNR and non-trivial functional dependencies $\{MR \rightarrow N\}$.
- V_7 with attributes MPR and no non-trivial functional dependencies.

2.a) Begin by splitting up the right side of each functional dependency in T. This gives the set of functional dependencies {AB → C, AB → D, ACDE → B, ACDE → F, B → A, B → C, B → D, CD → A, CD → F, CDE → F, CDE → G, EB → D}.

Now determine which FDs are redundant and may be removed from the set:

AB → C: May be removed since B → C is in the set.

AB → D: May be removed since B → D is in the set.

ACDE → B: May not be removed since no other FD determines B.

ACDE → F: May be removed since CD → F is in the set.

B → A: May be removed since B → C, B → D and CD → A are in the set.

B → C: May not be removed, no other FD determines C (after AB → C is removed).

B → D: May not be removed since AB → D has been removed and EB → D is the only other FD which determines D but B does not determine E.

CD → A: May not be removed, B → A has been removed and no other FD determines A.

CD → F: May not be removed since CD does not determine E and CDE → F is the only other FD in the set which determines F.

CDE → F: May be removed since CD → F is in the set.

CDE → G: May not be removed since no other FD determines G.

EB → D: May be removed since B → D is in the set.

This leaves the functional dependencies {ACDE → B, B → C, B → D, CD → A, CD → F, CDE → G}. Now check the left side of each FD for redundant attributes. The only such attributes is A in ACDE → B, which may be removed since CD → A.

{CDE → B, B → C, B → D, CD → A, CD → F, CDE → G} is the resulting set of functional dependencies. It can be seen that the single attribute determined by each FD is unique for that FD, so this set has no redundant FDs. As well each FD has no redundant attributes on its left side, so this is a minimal basis for T. In alphabetical order:
{B → C, B → D, CD → A, CD → F, CDE → B, CDE → G}.

b) In the above minimal basis for T, attributes A, F and G only appear on the right side of FDs so these are not necessary for any key of P. Attribute E appears only on the left side of FDs and attribute H appears in no FD, so E and H are in every key of P. Attributes B, C and D may or may not be in a key of P since they appear both on the left and right of different FDs.

In P:
$EH^+ = EH$
$BEH^+ = ABCDEFGH$, BEH is a key for P. No superset containing BEH needs to be considered since it already determines every attribute.
$CEH^+ = CEH$
$DEH^+ = DEH$
$CDEH^+ = ABCDEFGH$, CDEH is also a key for P.

Thus BEH, CDEH and all supersets of these are the keys for P.

c) Begin by combining functional dependencies with the same left side in the computed minimal basis of T to get the set of functional dependencies
$\{B \rightarrow CD, CD \rightarrow AF, CDE \rightarrow BG\}$.

From these create three relations P_1, P_2 and P_3 with attributes ACDF, BCD and BCDEG respectively. P_2 is a subset of P_3 so it is removed.

Neither ACDF nor BCDEG form a superkey for P, so add another relation P_4 with attributes BEH to satisfy this requirement.

We are left with the following 3NF decomposition of P:
- P_1 with attributes ACDF and functional dependencies $\{CD \rightarrow AF\}$.
- P_3 with attributes BCDEG and functional dependencies $\{B \rightarrow CD, CDE \rightarrow BG\}$.
- P_4 with attributes BEH and no non-trivial functional dependencies.

d) This schema allows redundancy. B is not a key for P_3 so there may be many rows in P_3 with the same value for B which must necessarily also have the same value for CD.