PS5_answer

February 12, 2019

```
In [1]: import numpy as np
        import pandas as pd
        import numpy.linalg as lin
        import scipy.stats as sts
        import scipy.integrate as intgr
        import scipy.optimize as opt
        import matplotlib
        import matplotlib.pyplot as plt
        import warnings
        warnings.filterwarnings("ignore")
```

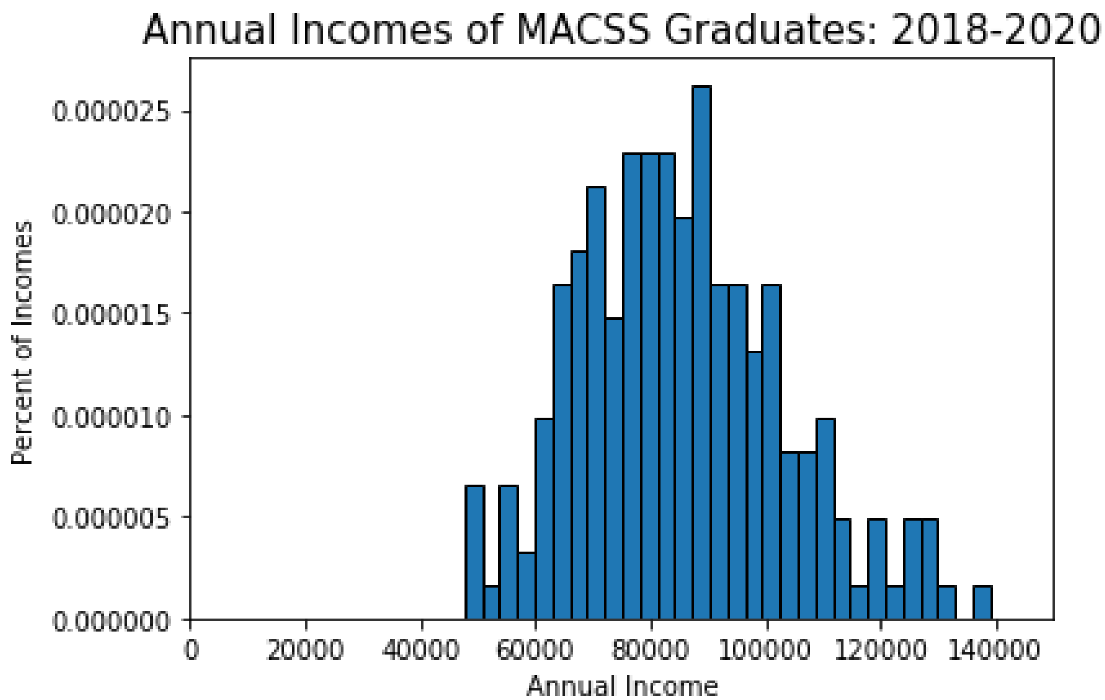
0.0.1 Problem 1

```
In [2]: df = pd.read_csv("data/incomes.txt",names=['Income'])
        df.head()
```

```
Out[2]:          Income
        0  53711.544399
        1  99731.233349
        2  84773.605417
        3  75184.025931
        4  73390.955933
```

a

```
In [3]: num_bins = 30
        plt.hist(df['Income'], num_bins, normed=True, edgecolor='k')
        plt.title('Annual Incomes of MACSS Graduates: 2018-2020', fontsize=15)
        plt.xlabel('Annual Income')
        plt.ylabel('Percent of Incomes')
        plt.xlim([1, 150000])
        plt.show()
```



b

```
In [4]: def trunc_lognorm_pdf(xvals, mu, sigma, cut_lb, cut_ub):

            if cut_ub == 'None' and cut_lb == 'None':
                prob_notcut = 1.0
            elif cut_ub == 'None' and cut_lb != 'None':
                prob_notcut = 1.0 - sts.lognorm.cdf(cut_lb, sigma, scale=np.exp(mu))
            elif cut_ub != 'None' and cut_lb == 'None':
                prob_notcut = sts.lognorm.cdf(cut_ub, sigma,scale=np.exp(mu))
            elif cut_ub != 'None' and cut_lb != 'None':
                prob_notcut = (sts.lognorm.cdf(cut_ub, sigma,scale=np.exp(mu)) -
                               sts.lognorm.cdf(cut_lb, sigma,scale=np.exp(mu)))

            pdf_vals    = ((1/(xvals*sigma * np.sqrt(2 * np.pi)) *
                            np.exp( - (np.log(xvals) - mu)**2 / (2 * sigma**2))) /
                            prob_notcut)

            return pdf_vals

In [5]: def data_moments(xvals):

            mean_data = xvals.mean()
            std_data = xvals.std()
```

```
        return mean_data, std_data
```

```
In [6]: def model_moments(mu, sigma, cut_lb, cut_ub):

            xfx = lambda x: x * trunc_lognorm_pdf(x, mu, sigma, cut_lb, cut_ub)
            (mean_model, m_m_err) = intgr.quad(xfx, cut_lb, cut_ub)
            x2fx = lambda x: ((x - mean_model) ** 2) * trunc_lognorm_pdf(x, mu, sigma, cut_lb,
            (var_model, v_m_err) = intgr.quad(x2fx, cut_lb, cut_ub)

            return mean_model, np.sqrt(var_model)
```

```
In [7]: def err_vec(xvals, mu, sigma, cut_lb, cut_ub, simple):

            mean_data, std_data = data_moments(xvals)
            moms_data = np.array([[mean_data], [std_data]])
            mean_model, std_model = model_moments(mu, sigma, cut_lb, cut_ub)
            moms_model = np.array([[mean_model], [std_model]])
            if simple:
                err_vec = moms_model - moms_data
            else:
                err_vec = (moms_model - moms_data) / moms_data

            return err_vec
```

```
In [8]: def criterion(params, *args):
            '''
            --------------------------------------------------------------------
            This function computes the GMM weighted sum of squared moment errors
            criterion function value given parameter values and an estimate of
            the weighting matrix.
            --------------------------------------------------------------------
            '''
            mu, sigma = params
            xvals, cut_lb, cut_ub, W = args
            err = err_vec(xvals, mu, sigma, cut_lb, cut_ub, simple=False)
            crit_val = err.T @ W @ err

            return crit_val
```

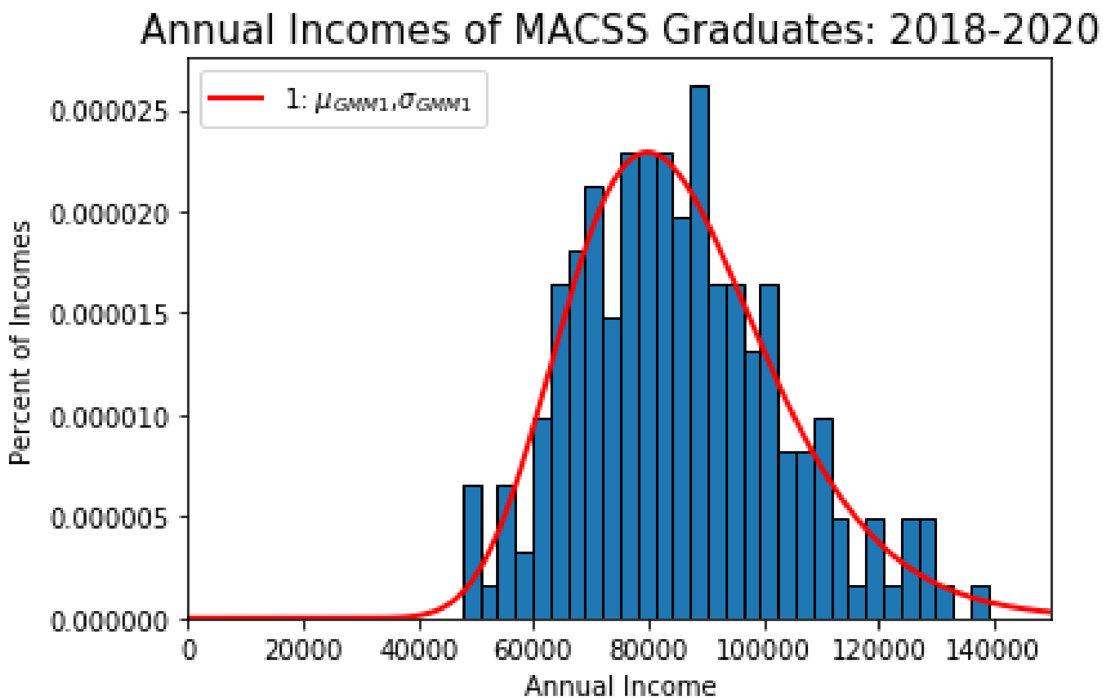
```
In [9]: mu_init = 11
        sig_init = 0.5
        params_init = np.array([mu_init, sig_init])
        W_hat = np.eye(2) #Identity matrix
        gmm_args = (df['Income'], 0.0, 150000.0, W_hat)
        results = opt.minimize(criterion, params_init, args=(gmm_args), tol=1e-14,
                               method='L-BFGS-B', bounds=((None, None), (1e-10, None)))
        mu_GMM1, sig_GMM1 = results.x
        print('mu_GMM1=', mu_GMM1, ' sig_GMM1=', sig_GMM1)
```

```
    mean_data, std_data = data_moments(df['Income'])
    mean_model, std_model = model_moments(mu_GMM1, sig_GMM1, 0.0, 150000.0)
    err1 = err_vec(df['Income'], mu_GMM1, sig_GMM1, 0.0, 150000.0, False).reshape(2,)
    print('Mean of points =', mean_data, ', Standard Deviation of points =', std_data)
    print('Mean of model =', mean_model, ', Standard Deviation =', std_model)
    print('Error vector=', err1)
    print('GMM criterion function:',results.fun)
```

```
mu_GMM1= 11.333533482547043  sig_GMM1= 0.21386191435240656
Mean of points = 85276.82360625808 , Standard Deviation of points = 18037.692869371564
Mean of model = 85276.82405894298 , Standard Deviation = 18037.692424757304
Error vector= [ 5.30841645e-09 -2.46491756e-08]
GMM criterion function: [[6.35761144e-16]]
```

```
In [10]: num_bins = 30
         plt.hist(df['Income'], num_bins, normed=True, edgecolor='k')
         plt.title('Annual Incomes of MACSS Graduates: 2018-2020', fontsize=15)
         plt.xlabel('Annual Income')
         plt.ylabel('Percent of Incomes')
         plt.xlim([1, 150000])


         dist_pts = np.linspace(0, 150000, 1000)
         plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_GMM1, sig_GMM1, 0.0, 150000),
                  linewidth=2, color='r', label='1: $\mu_{GMM1}$,$\sigma_{GMM1}$')
         plt.legend(loc='upper left')
         plt.show()
```



c

```
In [11]: def get_Err_mat2(pts, mu, sigma, cut_lb, cut_ub, simple=False):
             '''
             --------------------------------------------------------------------
             This function computes the R x N matrix of errors from each
             observation for each moment. In this function, we have hard coded R = 2.
             --------------------------------------------------------------------
             '''
             R = 2
             N = len(pts)
             Err_mat = np.zeros((R, N))
             mean_model, std_model = model_moments(mu, sigma, cut_lb, cut_ub)
             if simple:
                 Err_mat[0, :] = pts - mean_model
                 Err_mat[1, :] = ((mean_data - pts) ** 2) - std_model
             else:
                 Err_mat[0, :] = (pts - mean_model) / mean_model
                 Err_mat[1, :] = (((mean_data - pts) ** 2) - std_model) / std_model

             return Err_mat

In [12]: Err_mat =get_Err_mat2(df['Income'], mu_GMM1, sig_GMM1, 0.0, 150000.0, False)
         VCV2 = (1 / df['Income'].shape[0]) * (Err_mat @ Err_mat.T)
```

```
print("VCV2:","\n",VCV2)
W_hat2 = lin.inv(VCV2)
print("W_hat2:","\n",W_hat2)
```

```
VCV2:
 [[4.45167060e-02 1.67963797e+03]
 [1.67963797e+03 9.49413425e+08]]
W_hat2:
 [[ 2.40701669e+01 -4.25833101e-05]
 [-4.25833101e-05  1.12861743e-09]]
```

```
In [13]: gmm_args = (df['Income'], 0.0, 150000.0, W_hat2)
         params_init = np.array([mu_GMM1, sig_GMM1])
         results2 = opt.minimize(criterion, params_init, args=(gmm_args), tol=1e-14,
                                 method='L-BFGS-B', bounds=((None, None), (1e-10, None)))
         mu_GMM2, sig_GMM2 = results2.x
         print('mu_GMM2=', mu_GMM2, ' sig_GMM2=', sig_GMM2)

         mean_model2, std_model2 = model_moments(mu_GMM2, sig_GMM2, 0.0, 150000.0)
         err2 = err_vec(df['Income'], mu_GMM2, sig_GMM2, 0.0, 150000.0, False).reshape(2,)
         print('Mean of points =', mean_data, ', Standard Deviation of points =', std_data)
         print('Mean of model =', mean_model2, ', Standard Deviation of model =', std_model2)
         print('Error vector=', err2)
         print('GMM criterion function:',results2.fun)
```

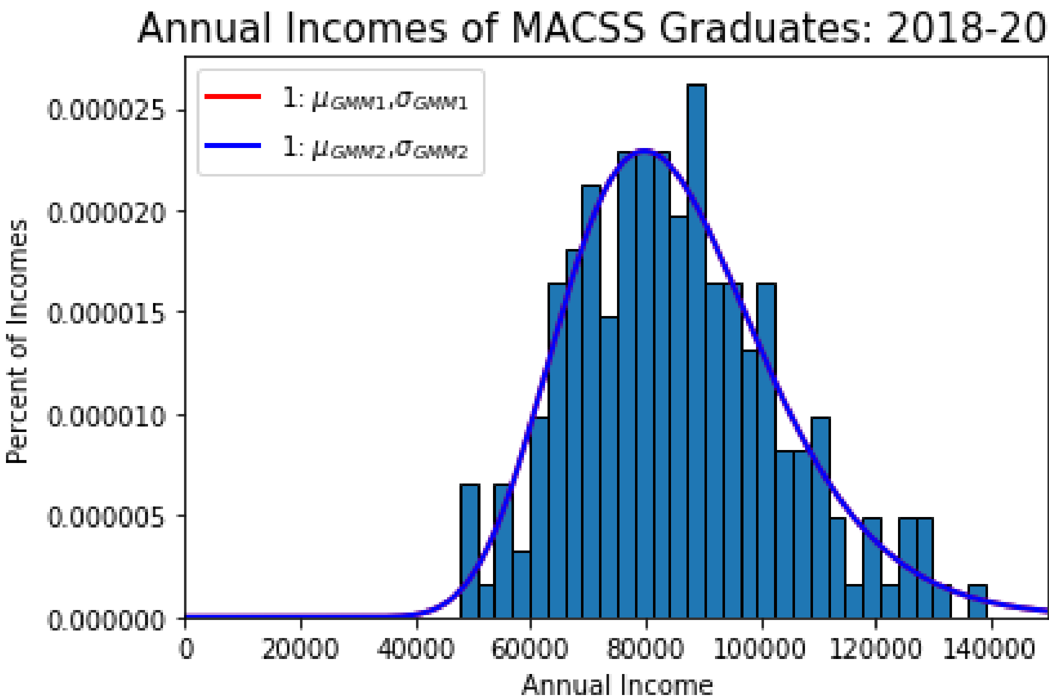
```
mu_GMM2= 11.333533472755567  sig_GMM2= 0.2138619137414848
Mean of points = 85276.82360625808 , Standard Deviation of points = 18037.692869371564
Mean of model = 85276.82324668276 , Standard Deviation of model = 18037.692248481748
Error vector= [-4.21656562e-09 -3.44217977e-08]
GMM criterion function: [[4.27941383e-16]]
```

```
In [14]: num_bins = 30
         plt.hist(df['Income'], num_bins, normed=True, edgecolor='k')
         plt.title('Annual Incomes of MACSS Graduates: 2018-2020', fontsize=15)
         plt.xlabel('Annual Income')
         plt.ylabel('Percent of Incomes')
         plt.xlim([1, 150000])


         plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_GMM1, sig_GMM1, 0.0, 150000),
                  linewidth=2, color='r', label='1: $\mu_{GMM1}$,$\sigma_{GMM1}$')
         plt.legend(loc='upper left')


         plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_GMM2, sig_GMM2, 0.0, 150000),
                  linewidth=2, color='b', label='1: $\mu_{GMM2}$,$\sigma_{GMM2}$')
```

```
plt.legend(loc='upper left')
plt.show()
```



d

```
In [15]: def data_moments3(xvals):
             '''
             --------------------------------------------------------------------
             This function computes the three data moments for GMM
             (binpct_1, binpct_2, binpct_3).
             --------------------------------------------------------------------
             '''
             bpct_1_dat = xvals[xvals <75000].shape[0] / xvals.shape[0]
             bpct_2_dat = (xvals[(xvals >=75000) & (xvals < 100000)].shape[0] /
                           xvals.shape[0])
             bpct_3_dat = xvals[xvals >= 100000].shape[0] / xvals.shape[0]

             return bpct_1_dat, bpct_2_dat, bpct_3_dat

In [16]: def model_moments3(mu, sigma, cut_lb, cut_ub):
             '''
             --------------------------------------------------------------------
             This function computes the three model moments for GMM
             (binpct_1, binpct_2, binpct_3, binpct_4).
```

```
    --------------------------------------------------------------------
    '''
    xfx = lambda x: trunc_lognorm_pdf(x, mu, sigma, cut_lb, cut_ub)
    (bpct_1_mod, bp_1_err) = intgr.quad(xfx, 0.0, 75000)
    (bpct_2_mod, bp_2_err) = intgr.quad(xfx, 75000, 100000)
    (bpct_3_mod, bp_3_err) = intgr.quad(xfx, 100000, 150000)

    return bpct_1_mod, bpct_2_mod, bpct_3_mod
```

In [17]:
```
def err_vec3(xvals, mu, sigma, cut_lb, cut_ub, simple):
    '''
    --------------------------------------------------------------------
    This function computes the vector of moment errors (in percent
    deviation from the data moment vector) for GMM.
    --------------------------------------------------------------------
    '''
    bpct_1_dat, bpct_2_dat, bpct_3_dat = data_moments3(xvals)
    moms_data = np.array([[bpct_1_dat], [bpct_2_dat], [bpct_3_dat]])
    bpct_1_mod, bpct_2_mod, bpct_3_mod = model_moments3(mu, sigma, cut_lb, cut_ub)
    moms_model = np.array([[bpct_1_mod], [bpct_2_mod], [bpct_3_mod]])
    if simple:
        err_vec = moms_model - moms_data
    else:
        err_vec = (moms_model - moms_data) / moms_data

    return err_vec
```

In [18]:
```
def criterion3(params, *args):
    '''
    --------------------------------------------------------------------
    This function computes the GMM weighted sum of squared moment errors
    criterion function value given parameter values and an estimate of
    the weighting matrix.
    --------------------------------------------------------------------
    '''
    mu, sigma = params
    xvals, cut_lb, cut_ub, W = args
    err = err_vec3(xvals, mu, sigma, cut_lb, cut_ub, simple=False)
    crit_val = err.T @ W @ err

    return crit_val
```

In [19]:
```
W_hat1_3 = np.eye(3)
gmm_args = (df['Income'], 0.0, 150000.0, W_hat1_3)

results_3 = opt.minimize(criterion3, params_init, args=(gmm_args),
                         method='L-BFGS-B', bounds=((1e-10, None), (1e-10, None)))
```

```
mu_GMM1_3, sig_GMM1_3 = results_3.x
print('mu_GMM1_3=', mu_GMM1_3, ' sig_GMM1_3=', sig_GMM1_3)
err3 = err_vec3(df['Income'], mu_GMM1_3, sig_GMM1_3, 0.0, 150000.0, False).reshape(3,

bpct_1_dat, bpct_2_dat, bpct_3_dat=data_moments3(df['Income'])
pert1, pert2, pert3 = model_moments3(mu_GMM1_3, sig_GMM1_3, 0.0, 150000.0)

print('Error vector=', err3)
print('GMM criterion function:',results_3.fun)
mm=pd.DataFrame({'Data Moments':[bpct_1_dat, bpct_2_dat, bpct_3_dat],
                'Model Momets':[pert1, pert2, pert3]},
                index=['Percent <75000','Percent 75000~100000','Percent >=100000'])
mm
```

```
mu_GMM1_3= 11.336705162764714  sig_GMM1_3= 0.21151353494384018
Error vector= [ 1.61522336e-07 -7.19526515e-08 -6.24018733e-08]
GMM criterion function: [[3.51606427e-14]]
```

Out[19]:

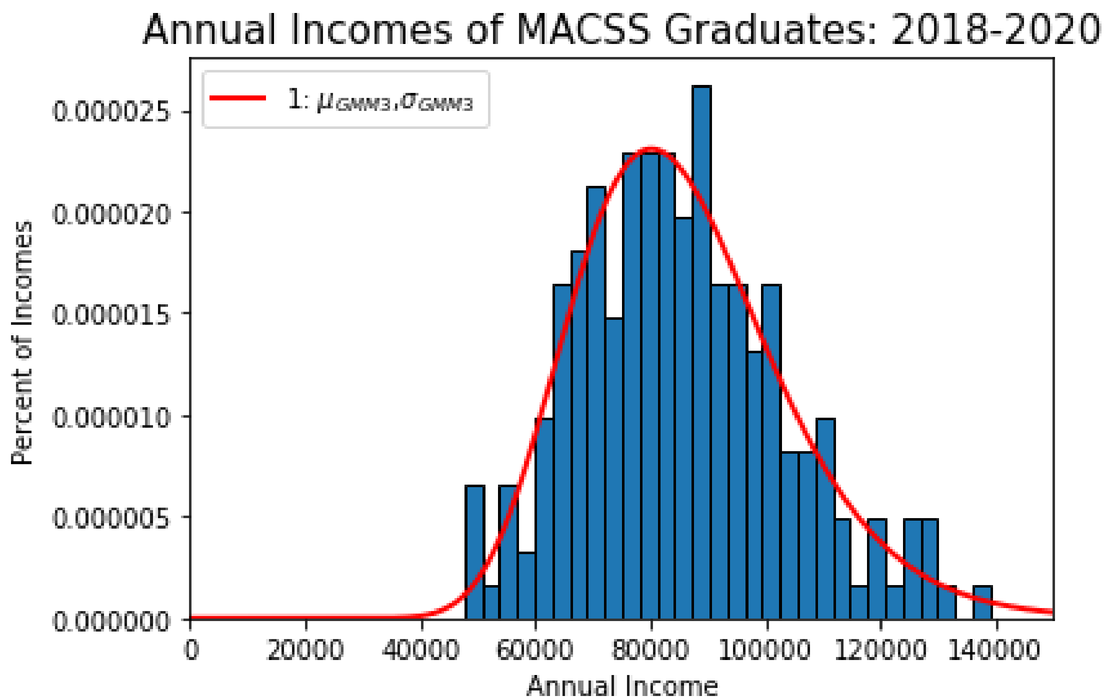
	Data Moments	Model Momets
Percent <75000	0.3	0.3
Percent 75000~100000	0.5	0.5
Percent >=100000	0.2	0.2

In [20]:

```
num_bins = 30
plt.hist(df['Income'], num_bins, normed=True, edgecolor='k')
plt.title('Annual Incomes of MACSS Graduates: 2018-2020', fontsize=15)
plt.xlabel('Annual Income')
plt.ylabel('Percent of Incomes')
plt.xlim([1, 150000])


plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_GMM1_3, sig_GMM1_3, 0.0, 150000),
         linewidth=2, color='r', label='1: $\mu_{GMM3}$,$\sigma_{GMM3}$')
plt.legend(loc='upper left')

plt.show()
```



e

```
In [21]: def get_Err_mat3(pts, mu, sigma, cut_lb, cut_ub, simple=False):
             '''
             --------------------------------------------------------------------
             This function computes the R x N matrix of errors from each
             observation for each moment. In this function, we have hard coded R = 3.
             --------------------------------------------------------------------
             '''
             R = 3
             N = len(pts)
             Err_mat = np.zeros((R, N))
             pct_1_mod, pct_2_mod, pct_3_mod = \
                 model_moments3(mu, sigma, cut_lb, cut_ub)
             if simple:
                 pts_in_grp1 = pts < 75000
                 Err_mat[0, :] = pts_in_grp1 - pct_1_mod
                 pts_in_grp2 = (pts >= 75000) & (pts < 100000)
                 Err_mat[1, :] = pts_in_grp2 - pct_2_mod
                 pts_in_grp3 = pts >= 100000
                 Err_mat[2, :] = pts_in_grp3 - pct_3_mod

             else:
                 pts_in_grp1 = pts < 75000
```

```
            Err_mat[0, :] = (pts_in_grp1 - pct_1_mod) / pct_1_mod
            pts_in_grp2 = (pts >= 75000) & (pts < 100000)
            Err_mat[1, :] = (pts_in_grp2 - pct_2_mod) / pct_2_mod
            pts_in_grp3 = pts >= 100000
            Err_mat[2, :] = (pts_in_grp3 - pct_3_mod) / pct_3_mod

        return Err_mat
```

```
In [22]: Err_mat3w = get_Err_mat3(df['Income'], mu_GMM1_3, sig_GMM1_3, 0.0, 150000.0, False)
         VCV2_3w = (1 / df['Income'].shape[0]) * (Err_mat3w @ Err_mat3w.T)
         print("VCV2_3:","\n",VCV2_3w)
         # We use the pseudo-inverse command here because the VCV matrix is poorly conditioned
         W_hat2_3 = lin.pinv(VCV2_3w)
         print("W_hat2_3","\n",W_hat2_3)
```

```
VCV2_3:
 [[ 2.33333258 -0.99999991 -0.9999999 ]
 [-0.99999991  1.00000014 -1.00000013]
 [-0.9999999  -1.00000013  4.0000005 ]]
W_hat2_3
 [[ 0.25761775 -0.14958453 -0.01246539]
 [-0.14958453  0.11911361 -0.07340718]
 [-0.01246539 -0.07340718  0.20221604]]
```

```
In [23]: gmm_args = (df['Income'], 0.0, 150000.0, W_hat2_3)
         results_4 = opt.minimize(criterion3, params_init, args=(gmm_args),method='L-BFGS-B')
         mu_GMM1_3w, sig_GMM1_3w = results_4.x
         print('mu_GMM1_3w=', mu_GMM1_3w, ' sig_GMM1_3w=', sig_GMM1_3w)
         err3w = err_vec3(df['Income'], mu_GMM1_3w, sig_GMM1_3w, 0.0, 150000.0, False).reshape

         pert1, pert2, pert3 = model_moments3(mu_GMM1_3w, sig_GMM1_3w, 0.0, 150000.0)

         print('Error vector=', err3w)
         print('GMM criterion function:',results_4.fun)
         mm=pd.DataFrame({'Data Moments':[bpct_1_dat, bpct_2_dat, bpct_3_dat],
                          'Model Momets':[pert1, pert2, pert3]},
                          index=['Percent <75000','Percent 75000~100000','Percent >=100000'])
         mm
```

```
mu_GMM1_3w= 11.333533482547043  sig_GMM1_3w= 0.21386191435240656
Error vector= [ 0.02420924 -0.01108994 -0.00858902]
GMM criterion function: [[0.00025207]]
```

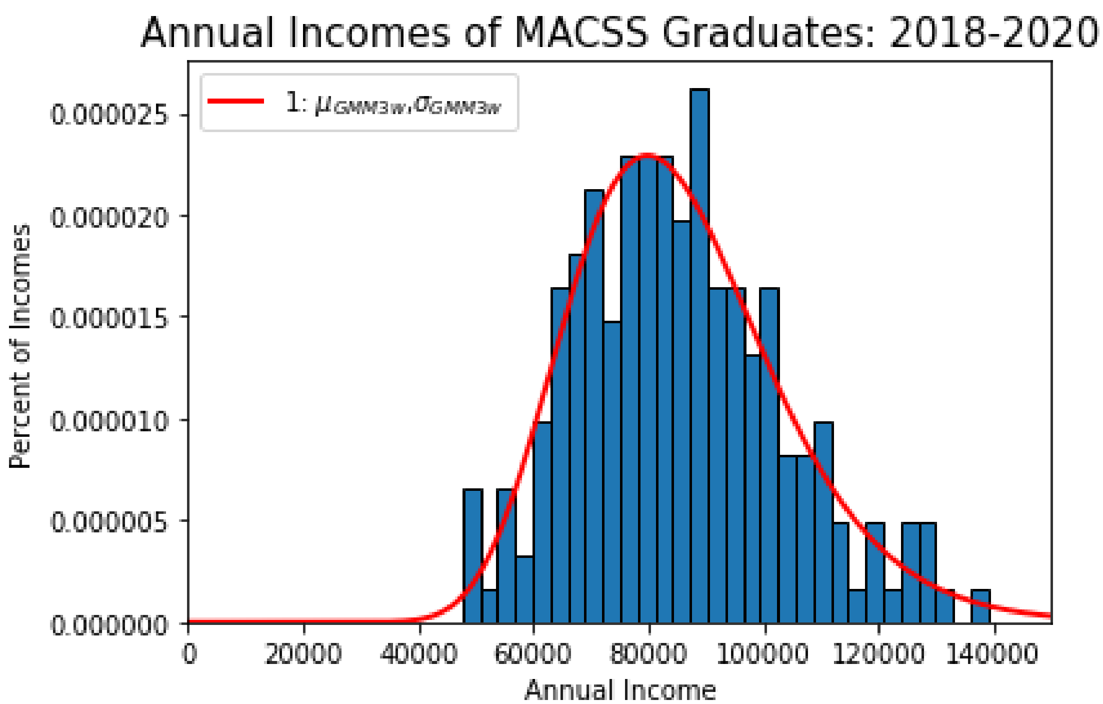
Out[23]:

	Data Moments	Model Momets
Percent <75000	0.3	0.307263
Percent 75000~100000	0.5	0.494455
Percent >=100000	0.2	0.198282

In [24]:
```
num_bins = 30
plt.hist(df['Income'], num_bins, normed=True, edgecolor='k')
plt.title('Annual Incomes of MACSS Graduates: 2018-2020', fontsize=15)
plt.xlabel('Annual Income')
plt.ylabel('Percent of Incomes')
plt.xlim([1, 150000])


plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_GMM1_3w, sig_GMM1_3w, 0.0, 150000),
         linewidth=2, color='r', label='1: $\mu_{GMM3w}$,$\sigma_{GMM3w}$')
plt.legend(loc='upper left')

plt.show()
```



f

In [25]:
```
mu=[mu_GMM1,mu_GMM2,mu_GMM1_3,mu_GMM1_3w]
sig=[sig_GMM1,sig_GMM2,sig_GMM1_3,sig_GMM1_3w]
GMMV=[results.fun,results2.fun,results_3.fun,results_4.fun]
```

In [26]:
```
pd.DataFrame({"mu":mu,"sigma":sig,"GMM Fn Value":GMMV}
             ,index=['2 moments+Identity Matrix','2 moments+Weighted Matrix',
                    '3 moments+Identity Matrix','3 moments+Weighted Matrix']).astype(f
```

Out[26]:
```
                                   mu     sigma  GMM Fn Value
2 moments+Identity Matrix   11.333533  0.213862  6.357611e-16
2 moments+Weighted Matrix   11.333533  0.213862  4.279414e-16
3 moments+Identity Matrix   11.336705  0.211514  3.516064e-14
3 moments+Weighted Matrix   11.333533  0.213862  2.520739e-04
```

The best fitting model is from (c) as it has the lowest minimized GMM function value.
This model uses the average and standard deviation of income as two moments with two-step optimal weighting matrix.

0.1 Problem 2

In [27]:
```python
df=pd.read_csv("data/sick.txt").astype('float64')
df.head()
```

Out[27]:

	sick	age	children	avgtemp_winter
0	1.67	57.47	3.04	54.10
1	0.71	26.77	1.20	36.54
2	1.39	41.85	2.31	32.38
3	1.37	51.27	2.46	52.94
4	1.45	44.22	2.72	45.90

In [28]:
```python
def err_vec(df,b0,b1,b2,b3):

    #Model Moments
    yhat=b0+b1*df['age']+b2*df['children']+b3*df['avgtemp_winter']

    #Data Moments
    y=df['sick']

    err_vec = yhat - y

    return err_vec
```

In [29]:
```python
def criterion(params, *args):
    '''
    --------------------------------------------------------------------
    This function computes the GMM sum of squared moment errors
    criterion function value given parameter values and a weighting matrix.
    --------------------------------------------------------------------
    '''

    b0,b1,b2,b3 = params
    df, W = args
    err = err_vec(df,b0,b1,b2,b3)
    crit_val = err.T @ W @ err

    return crit_val
```

```
In [30]: b0, b1, b2, b3 = 1, 0, 0, 0
         params_init = np.array([b0, b1, b2, b3])
         W_hat = np.eye(df.shape[0])
         gmm_args = (df, W_hat)
         results = opt.minimize(criterion, params_init, args=(gmm_args), tol=1e-14,method='L-BI
         b0, b1, b2, b3 = results.x
```

```
In [31]: print("Estimate of b0",b0)
         print("Estimate of b1",b1)
         print("Estimate of b2",b2)
         print("Estimate of b3",b3)
         print('GMM criterion function:',results.fun)
```

```
Estimate of b0 0.2516448636180223
Estimate of b1 0.012933469667256711
Estimate of b2 0.40050098511048643
Estimate of b3 -0.009991708483433188
GMM criterion function: 0.00182128980560192
```