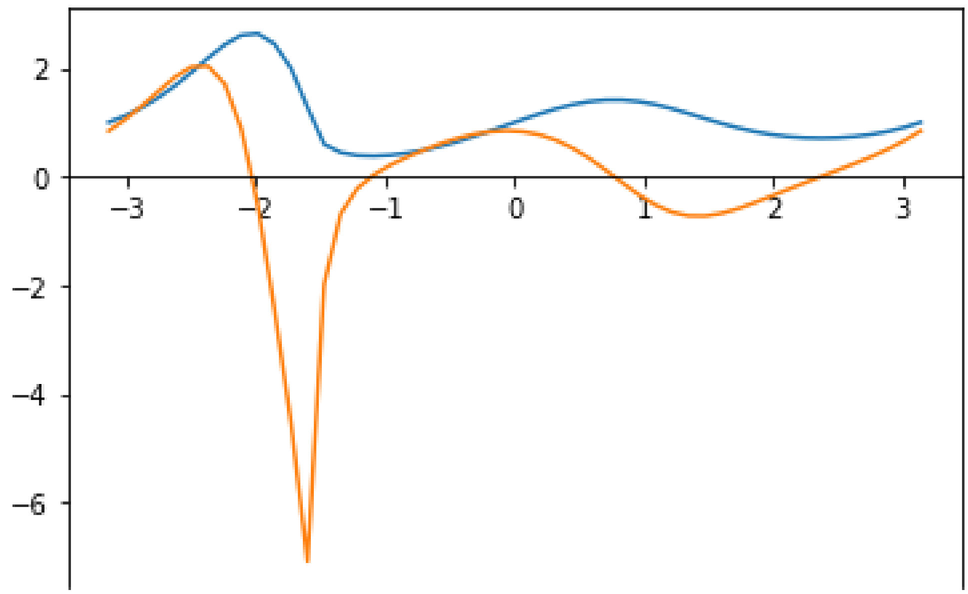
ps2_part1

January 21, 2019

Problem 1

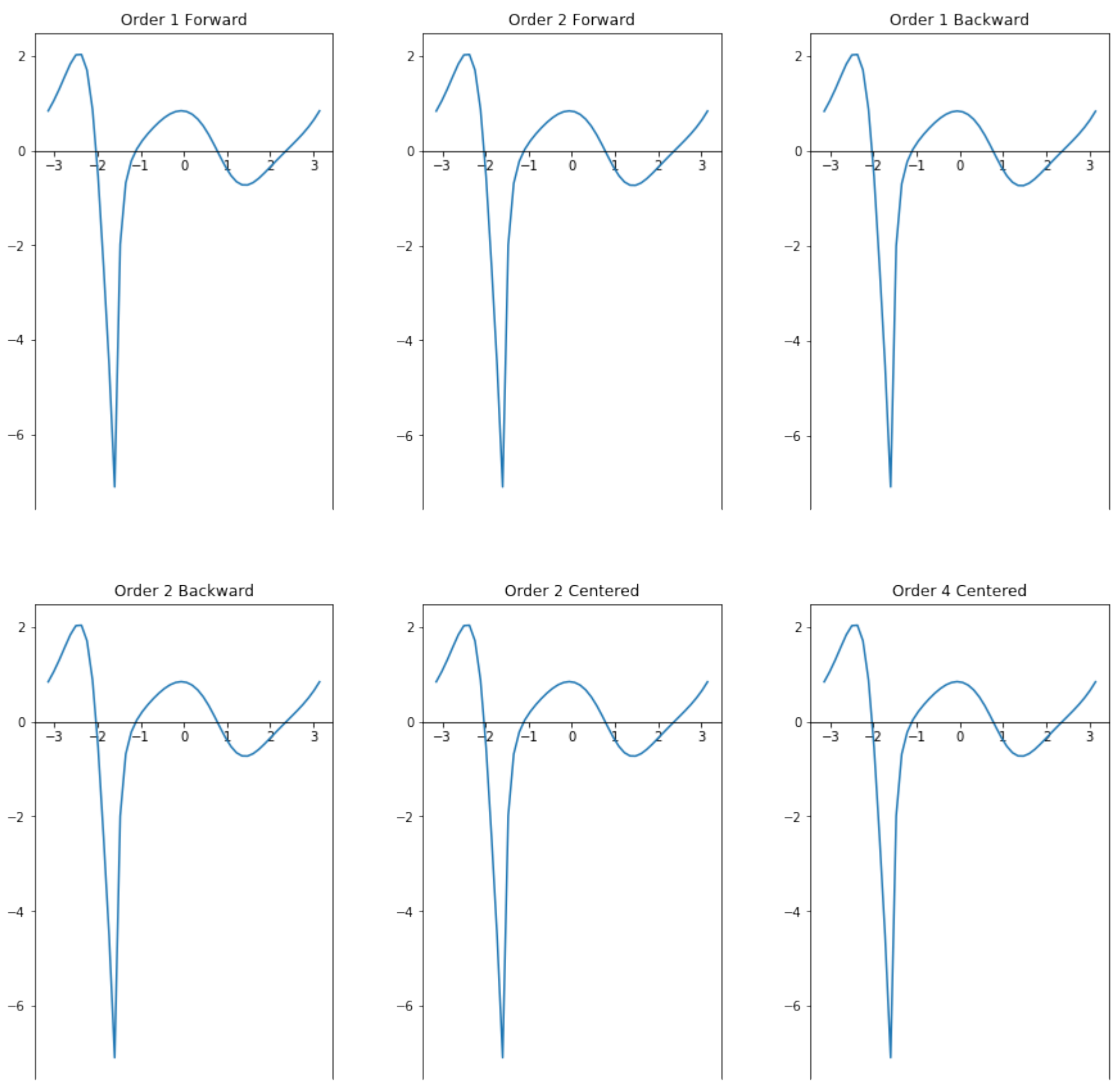
```
In [1]: import sympy as sy
        import math
        import numpy as np
        from matplotlib import pyplot as plt
        from pylab import *
        import warnings
        warnings.filterwarnings("ignore")
        x = sy.symbols('x')
        f = (sy.sin(x)+1)**(sy.sin(sy.cos(x)))
        g = sy.lambdify(x, f) #lambdify the original function

In [2]: f_p = sy.diff((sy.sin(x)+1)**(sy.sin(sy.cos(x))), x)
        g_p = sy.lambdify(x, f_p) #lambdify the derivative
        ax = plt.gca()
        ax.spines["bottom"].set_position("zero")
        xval = np.linspace(-math.pi, math.pi)
        yval1 = g(xval)
        yval2 = g_p(xval)
        ax.plot(xval, yval1)
        ax.plot(xval, yval2)

Out[2]: [<matplotlib.lines.Line2D at 0x11d6cb128>]
```



Problem 2

```
In [3]: def f1(f, x, h):
            return [(f(p + h) - f(p)) / h for p in x]

In [4]: def f2(f, x, h):
            return [(-3*f(p) + 4*f(p + h) - f(p + 2*h)) / (2*h) for p in x]

In [5]: def f3(f, x, h):
            return [(f(p) - f(p - h)) / h for p in x]

In [6]: def f4(f, x, h):
            return [(3*f(p) - 4*f(p - h) + f(p - 2*h)) / (2*h) for p in x]

In [7]: def f5(f, x, h):
            return [(f(p + h) - f(p - h)) / (2*h) for p in x]

In [8]: def f6(f, x, h):
            return [(f(p - 2*h) - 8*f(p - h) + 8*f(p + h) - f(p + 2*h)) / (12*h) for p in x]

In [9]: x_val = np.linspace(-math.pi, math.pi)

In [10]: y_val = [(math.sin(x) + 1) ** (math.sin(math.cos(x))) for x in x_val]

In [11]: def f(x):
             return (math.sin(x) + 1)**(math.sin(math.cos(x)))

         fig, ((ax1, ax2, ax3), (ax4, ax5, ax6)) = plt.subplots(2, 3, figsize=(15,15))
         for ax in [ax1, ax2, ax3,ax4, ax5, ax6]:
```

```
    ax.spines["bottom"].set_position("zero")
ax1.plot(x_val, f1(f, x_val, h=0.001))
ax2.plot(x_val, f2(f, x_val, h=0.001))
ax3.plot(x_val, f3(f, x_val, h=0.001))
ax4.plot(x_val, f4(f, x_val, h=0.001))
ax5.plot(x_val, f5(f, x_val, h=0.001))
ax6.plot(x_val, f6(f, x_val, h=0.001))

ax1.set_title("Order 1 Forward")
ax2.set_title("Order 2 Forward")
ax3.set_title("Order 1 Backward")
ax4.set_title("Order 2 Backward")
ax5.set_title("Order 2 Centered")
ax6.set_title("Order 4 Centered")
subplots_adjust(wspace = 0.3)
```



From the plot, we can see that the results of the six functions are almost the same as the result in problem 1.

Problem 3

I redifined the six functions in problem 2 to accept a single float point value x_0 as input.

```
In [12]: def f1(f, x_0, h):
             return (f(x_0 + h) - f(x_0))/ h
```

```
In [13]: def f2(f, x_0, h):
             return (-3*f(x_0) + 4*f(x_0 + h) - f(x_0 + 2*h)) / (2*h)
```

```
In [14]: def f3(f, x_0, h):
             return ((f(x_0) - f(x_0 - h)) / h)
```

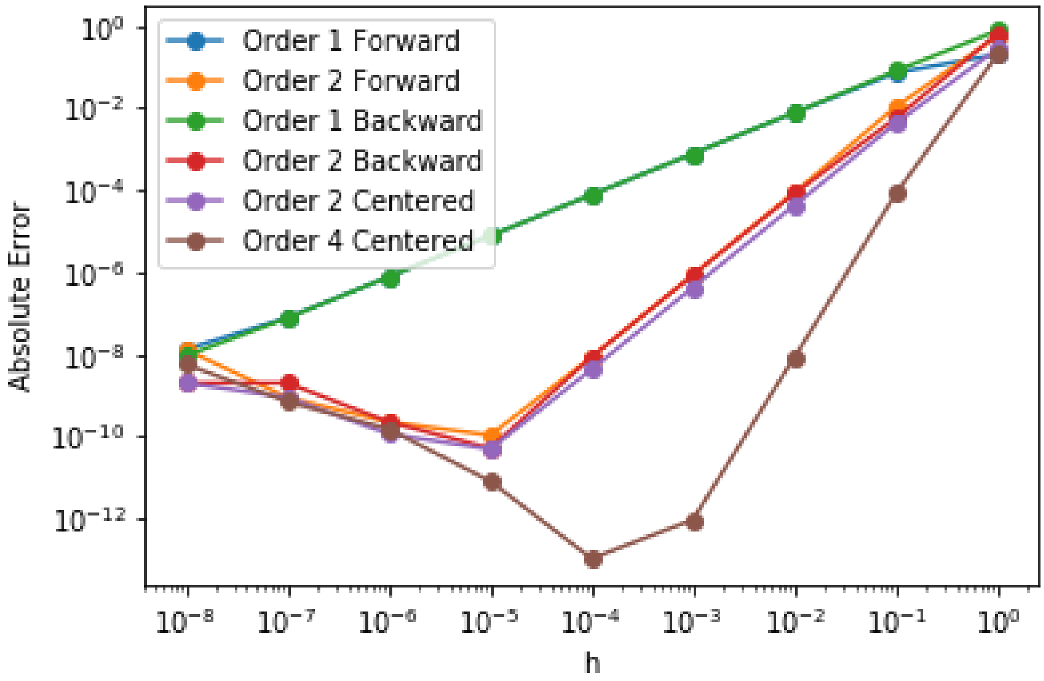
```
In [15]: def f4(f, x_0, h):
             return (3*f(x_0) - 4*f(x_0 - h) + f(x_0 - 2*h)) / (2*h)
```

```
In [16]: def f5(f, x_0, h):
             return (f(x_0 + h) - f(x_0 - h)) / (2*h)
```

```
In [17]: def f6(f, x_0, h):
             return (f(x_0 - 2*h) - 8*f(x_0 - h) + 8*f(x_0 + h) - f(x_0 + 2*h)) / (12*h)
```

```
In [18]: def ps3(x_0):
             h_range = np.logspace(-8,0,9)
             plt.plot(h_range, abs(g_p(x_0) - [f1(f, x_0, h) for h in h_range]), label="Order 
             plt.plot(h_range, abs(g_p(x_0) - [f2(f, x_0, h) for h in h_range]), label="Order 
             plt.plot(h_range, abs(g_p(x_0) - [f3(f, x_0, h) for h in h_range]), label="Order 
             plt.plot(h_range, abs(g_p(x_0) - [f4(f, x_0, h) for h in h_range]), label="Order 
             plt.plot(h_range, abs(g_p(x_0) - [f5(f, x_0, h) for h in h_range]), label="Order 
             plt.plot(h_range, abs(g_p(x_0) - [f6(f, x_0, h) for h in h_range]), label="Order 
             plt.legend(loc="upper left")
             plt.loglog()
             plt.xlabel("h")
             plt.ylabel("Absolute Error")
             plt.show()
```

```
In [19]: ps3(1)
```



Problem 4

```
In [20]: import pandas as pd
         data=np.load('plane.npy',encoding = "latin1")
         radar = pd.DataFrame(data, columns=['t','alpha','beta'])
         radar['alpha']

Out[20]: 0    56.25
         1    55.53
         2    54.80
         3    54.06
         4    53.34
         5    52.69
         6    51.94
         7    51.28
         Name: alpha, dtype: float64

In [21]: radar['alpha'] = np.deg2rad(radar['alpha'])
         radar['beta'] = np.deg2rad(radar['beta'])
         a = 500
         radar['alpha']

Out[21]: 0    0.981748
         1    0.969181
         2    0.956440
         3    0.943525
```

```
4    0.930959
5    0.919614
6    0.906524
7    0.895005
Name: alpha, dtype: float64
```

```
In [22]: radar['x(t)'] = (a * np.tan(radar['beta']) )/ \
                         (np.tan(radar['beta']) - np.tan(radar['alpha']))
```

```
In [23]: radar['y(t)'] = a * np.tan(radar['beta']) * np.tan(radar['alpha'])/ \
                         (np.tan(radar['beta']) - np.tan(radar['alpha']))
         radar['x_p(t)'] = None
         radar['y_p(t)'] = None
```

```
In [24]: radar['x_p(t)'][0] = (radar['x(t)'][1] - radar['x(t)'][0])/\
                              (radar['t'][1] - radar['t'][0])
         radar['y_p(t)'][0] = (radar['y(t)'][1] - radar['y(t)'][0])/\
                              (radar['t'][1] - radar['t'][0])
```

```
In [25]: radar['x_p(t)'][7] = (radar['x(t)'][7] - radar['x(t)'][6])/\
                              (radar['t'][7] - radar['t'][6])
         radar['y_p(t)'][7] = (radar['y(t)'][7] - radar['y(t)'][6])/\
                              (radar['t'][7] - radar['t'][6])
```

```
In [26]: for i in range(1, 7):
             radar['x_p(t)'][i] = (radar['x(t)'][i+1] - radar['x(t)'][i-1])/\
                                   (radar['t'][i+1] - radar['t'][i-1])
             radar['y_p(t)'][i] = (radar['y(t)'][i+1] - radar['y(t)'][i-1])/\
                              (radar['t'][i+1] - radar['t'][i-1])
```

```
In [27]: radar['speed'] = (radar['x_p(t)']**2 + radar['y_p(t)']**2)**(1/2)
```

```
In [28]: radar
```

Out[28]:

	t	alpha	beta	x(t)	y(t)	x_p(t)	y_p(t)	\
0	7.0	0.981748	1.178795	1311.271337	1962.456239	44.6651	12.6583	
1	8.0	0.969181	1.161866	1355.936476	1975.114505	45.3235	12.4449	
2	9.0	0.956440	1.144761	1401.918398	1987.346016	47.2803	12.8631	
3	10.0	0.943525	1.127308	1450.497006	2000.840713	48.361	13.0832	
4	11.0	0.930959	1.110378	1498.640350	2013.512411	46.651	12.4758	
5	12.0	0.919614	1.095020	1543.798955	2025.792234	49.7005	13.7391	
6	13.0	0.906524	1.077217	1598.041382	2040.990583	51.8986	14.6367	
7	14.0	0.895005	1.061509	1647.596093	2055.065571	49.5547	14.075	

	speed
0	46.4242
1	47.001
2	48.9988
3	50.0994

```
4   48.2904
5   51.5646
6    53.923
7   51.5148
```

```
In [29]: radar[['t','speed']]
```

```
Out[29]:       t    speed
         0   7.0  46.4242
         1   8.0   47.001
         2   9.0  48.9988
         3  10.0  50.0994
         4  11.0  48.2904
         5  12.0  51.5646
         6  13.0   53.923
         7  14.0  51.5148
```

Problem 5

```
In [43]: def Jac(func,pt,h):
             n=len(func)
             dim=len(pt)
             I=np.identity(dim)
             J = np.zeros((n,dim))
             for i,fu in enumerate(func):
                 for j,s in enumerate(pt):
                     f= sy.lambdify((x,y), fu, 'numpy')
                     right=pt+h*I[:,j]
                     left=pt-h*I[:,j]
                     J[i,j]=(f(right[0],right[1])-f(left[0],left[1]))/(2*h)
             return J
         x,y = sy.Symbol('x'), sy.Symbol('y')
         function = x**2, x**3-y
         X = [1,1]
         pt=[1,1]
         h=0.01
         Jac(function,pt,h)
```

```
Out[43]: array([[ 2.    ,  0.    ],
                [ 3.0001, -1.    ]])
```

Problem 7

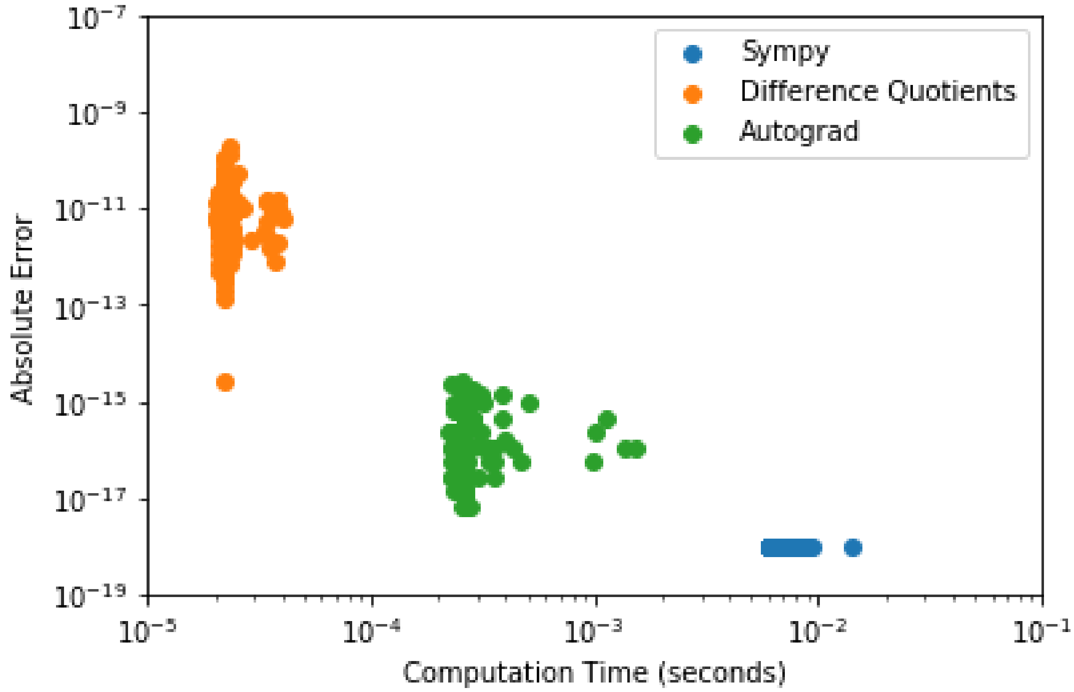
```
In [73]: from autograd import numpy as anp
         from autograd import grad
         import time
```

```
In [94]: def Time(N):
             t1 = np.zeros(N,dtype='float')
```

```python
        t2 = np.zeros(N,dtype='float')
        t3 = np.zeros(N,dtype='float')
        abs_e1 = np.array([1e-18] * N)
        abs_e2 = np.zeros(N,dtype='float')
        abs_e3 = np.zeros(N,dtype='float')

        y = lambda x: (anp.sin(x)+1)**(anp.sin(anp.cos(x)))
        auto_yprime = grad(y)
        for i in range(N):
            x = np.random.uniform(-math.pi, math.pi)
            time1 = time.clock()
            z =sy.symbols('z')
            yprime = sy.diff((sy.sin(z)+1)**sy.sin(sy.cos(z)), z)
            fprime = sy.lambdify(z, yprime, "numpy")
            prime = fprime(x)
            time2 = time.clock()
            t1[i] = time2 - time1
            time3 = time.clock()
            appr_prime = f6(g, x, h = 0.00001)
            time4 = time.clock()
            t2[i] = time4 - time3
            abs_e2[i] = abs(appr_prime - prime)
            time5 = time.clock()
            auto_appr_prime = auto_yprime(x)
            time6 = time.clock()
            t3[i] = time6 - time5
            abs_e3[i] = abs(auto_appr_prime- prime)
        return t1, t2, t3, abs_e1, abs_e2, abs_e3
```

In [95]:
```python
t1, t2, t3, abs_e1, abs_e2, abs_e3 = Time(200)

plt.scatter(t1, abs_e1, label='Sympy')
plt.scatter(t2 ,abs_e2, label='Difference Quotients')
plt.scatter(t3, abs_e3, label='Autograd')
plt.loglog()
plt.xlim(10**-5,10**-1)
plt.ylim(10**-19,10**-7)
plt.xlabel("Computation Time (seconds)")
plt.ylabel("Absolute Error")
plt.legend()
```

Out[95]: <matplotlib.legend.Legend at 0x123426cc0>



Sympy
Difference Quotients
Autograd
Absolute Error
Computation Time (seconds)
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-19}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}