PS7_ANSWER

February 27, 2019

In [1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
#plt.style.use('seaborn')
import statsmodels.api as sm
from sklearn.model_selection import train_test_split
from sklearn import preprocessing
from sklearn.linear_model import LogisticRegression
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split, LeaveOneOut, KFold, cross_val_sco
from sklearn import metrics
from sklearn.metrics import classification_report, mean_squared_error
from scipy.interpolate import LSQUnivariateSpline
```

In [2]:
```
df = pd.read_csv("data/strongdrink.txt")
df.head()
```

Out[2]:
```
   cultivar   alco  malic   ash   alk  magn  tot_phen  flav  nonfl_phen  \
0         1  14.23   1.71  2.43  15.6   127      2.80  3.06        0.28
1         1  13.20   1.78  2.14  11.2   100      2.65  2.76        0.26
2         1  13.16   2.36  2.67  18.6   101      2.80  3.24        0.30
3         1  14.37   1.95  2.50  16.8   113      3.85  3.49        0.24
4         1  13.24   2.59  2.87  21.0   118      2.80  2.69        0.39

   proanth  color_int   hue  OD280rat  proline
0     2.29       5.64  1.04      3.92     1065
1     1.28       4.38  1.05      3.40     1050
2     2.81       5.68  1.03      3.17     1185
3     2.18       7.80  0.86      3.45     1480
4     1.82       4.32  1.04      2.93      735
```

In [3]:
```
df["cultivar"].value_counts()
```

Out[3]:
```
2    71
1    59
3    46
Name: cultivar, dtype: int64
```

a. Multinomial logistic regression

```
In [4]: y = df["cultivar"]
        X = df[["alco","malic","tot_phen","color_int"]]
        X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.25,random_state
```

```
In [5]: clf = LogisticRegression(solver='newton-cg',multi_class='multinomial').fit(X_train, y_
```

```
In [6]: pd.DataFrame({"j=1":np.append(clf.intercept_[0],clf.coef_[0]),
                      "j=2":np.append(clf.intercept_[1],clf.coef_[1])},
                       index=["beta0","beta1",'beta2','beta3','beta4'])
```

```
Out[6]:              j=1        j=2
        beta0 -24.011332  22.801680
        beta1   1.700433  -1.467985
        beta2  -0.265610  -0.333051
        beta3   1.223894   0.664006
        beta4   0.022748  -0.922709
```

```
In [7]: y_pred = clf.predict(X_test)
        print(classification_report(y_test, y_pred))
```

```
             precision    recall  f1-score   support

          1       0.87      1.00      0.93        13
          2       1.00      0.90      0.95        21
          3       1.00      1.00      1.00        10

avg / total       0.96      0.95      0.96        44
```

The error rates are 13%, 0%, and 0% for group 1,2, and 3, respectively.
The model is best at predicting the third group (highest f1-score).
The one with the most observations is the second group. So the most accurately predicted category is not the one with the most observations.

b. LOOCV

```
In [8]: Xvars = df[['alco', 'malic', 'tot_phen', 'color_int']].values
        yvars = df['cultivar'].values
        N_loo = Xvars.shape[0]
        loo = LeaveOneOut()
        loo.get_n_splits(Xvars)
        MSE_vec = np.zeros(N_loo)
        y_test_lst = np.zeros(N_loo)
        y_pred_lst = np.zeros(N_loo)
```

```
In [9]: for train_index, test_index in loo.split(Xvars):
            X_train, X_test = Xvars[train_index], Xvars[test_index]
            y_train, y_test = yvars[train_index], yvars[test_index]
            LogReg = LogisticRegression()
            LogReg.fit(X_train, y_train)
            y_pred = LogReg.predict(X_test)
            y_pred_lst[test_index] = y_pred
            y_test_lst[test_index] = y_test
            MSE_vec[test_index] = (y_test != y_pred)
            print('MSE for test set', test_index, ' is', MSE_vec[test_index])
```

```
MSE for test set [0]  is [0.]
MSE for test set [1]  is [0.]
MSE for test set [2]  is [0.]
MSE for test set [3]  is [0.]
MSE for test set [4]  is [0.]
MSE for test set [5]  is [0.]
MSE for test set [6]  is [0.]
MSE for test set [7]  is [0.]
MSE for test set [8]  is [0.]
MSE for test set [9]  is [0.]
MSE for test set [10]  is [0.]
MSE for test set [11]  is [1.]
MSE for test set [12]  is [0.]
MSE for test set [13]  is [0.]
MSE for test set [14]  is [0.]
MSE for test set [15]  is [0.]
MSE for test set [16]  is [0.]
MSE for test set [17]  is [0.]
MSE for test set [18]  is [0.]
MSE for test set [19]  is [0.]
MSE for test set [20]  is [0.]
MSE for test set [21]  is [1.]
MSE for test set [22]  is [1.]
MSE for test set [23]  is [1.]
MSE for test set [24]  is [1.]
MSE for test set [25]  is [1.]
MSE for test set [26]  is [0.]
MSE for test set [27]  is [1.]
MSE for test set [28]  is [0.]
MSE for test set [29]  is [0.]
MSE for test set [30]  is [0.]
MSE for test set [31]  is [0.]
MSE for test set [32]  is [1.]
MSE for test set [33]  is [0.]
MSE for test set [34]  is [1.]
MSE for test set [35]  is [0.]
MSE for test set [36]  is [0.]
```

```
MSE for test set [37]  is [1.]
MSE for test set [38]  is [1.]
MSE for test set [39]  is [0.]
MSE for test set [40]  is [0.]
MSE for test set [41]  is [1.]
MSE for test set [42]  is [0.]
MSE for test set [43]  is [1.]
MSE for test set [44]  is [0.]
MSE for test set [45]  is [0.]
MSE for test set [46]  is [0.]
MSE for test set [47]  is [0.]
MSE for test set [48]  is [0.]
MSE for test set [49]  is [0.]
MSE for test set [50]  is [0.]
MSE for test set [51]  is [0.]
MSE for test set [52]  is [0.]
MSE for test set [53]  is [0.]
MSE for test set [54]  is [0.]
MSE for test set [55]  is [0.]
MSE for test set [56]  is [0.]
MSE for test set [57]  is [0.]
MSE for test set [58]  is [0.]
MSE for test set [59]  is [0.]
MSE for test set [60]  is [0.]
MSE for test set [61]  is [1.]
MSE for test set [62]  is [0.]
MSE for test set [63]  is [1.]
MSE for test set [64]  is [0.]
MSE for test set [65]  is [1.]
MSE for test set [66]  is [1.]
MSE for test set [67]  is [1.]
MSE for test set [68]  is [0.]
MSE for test set [69]  is [0.]
MSE for test set [70]  is [0.]
MSE for test set [71]  is [0.]
MSE for test set [72]  is [0.]
MSE for test set [73]  is [0.]
MSE for test set [74]  is [0.]
MSE for test set [75]  is [0.]
MSE for test set [76]  is [0.]
MSE for test set [77]  is [0.]
MSE for test set [78]  is [0.]
MSE for test set [79]  is [0.]
MSE for test set [80]  is [0.]
MSE for test set [81]  is [0.]
MSE for test set [82]  is [0.]
MSE for test set [83]  is [1.]
MSE for test set [84]  is [0.]
```

```
MSE for test set [85]  is [0.]
MSE for test set [86]  is [0.]
MSE for test set [87]  is [0.]
MSE for test set [88]  is [0.]
MSE for test set [89]  is [0.]
MSE for test set [90]  is [0.]
MSE for test set [91]  is [0.]
MSE for test set [92]  is [0.]
MSE for test set [93]  is [0.]
MSE for test set [94]  is [0.]
MSE for test set [95]  is [0.]
MSE for test set [96]  is [0.]
MSE for test set [97]  is [0.]
MSE for test set [98]  is [1.]
MSE for test set [99]  is [0.]
MSE for test set [100]  is [0.]
MSE for test set [101]  is [0.]
MSE for test set [102]  is [0.]
MSE for test set [103]  is [0.]
MSE for test set [104]  is [0.]
MSE for test set [105]  is [0.]
MSE for test set [106]  is [0.]
MSE for test set [107]  is [0.]
MSE for test set [108]  is [0.]
MSE for test set [109]  is [0.]
MSE for test set [110]  is [0.]
MSE for test set [111]  is [0.]
MSE for test set [112]  is [0.]
MSE for test set [113]  is [0.]
MSE for test set [114]  is [0.]
MSE for test set [115]  is [0.]
MSE for test set [116]  is [0.]
MSE for test set [117]  is [0.]
MSE for test set [118]  is [0.]
MSE for test set [119]  is [0.]
MSE for test set [120]  is [0.]
MSE for test set [121]  is [1.]
MSE for test set [122]  is [0.]
MSE for test set [123]  is [0.]
MSE for test set [124]  is [0.]
MSE for test set [125]  is [0.]
MSE for test set [126]  is [0.]
MSE for test set [127]  is [0.]
MSE for test set [128]  is [0.]
MSE for test set [129]  is [0.]
MSE for test set [130]  is [1.]
MSE for test set [131]  is [0.]
MSE for test set [132]  is [0.]
```

```
MSE for test set [133]  is [0.]
MSE for test set [134]  is [1.]
MSE for test set [135]  is [0.]
MSE for test set [136]  is [0.]
MSE for test set [137]  is [0.]
MSE for test set [138]  is [1.]
MSE for test set [139]  is [0.]
MSE for test set [140]  is [0.]
MSE for test set [141]  is [0.]
MSE for test set [142]  is [0.]
MSE for test set [143]  is [0.]
MSE for test set [144]  is [0.]
MSE for test set [145]  is [0.]
MSE for test set [146]  is [0.]
MSE for test set [147]  is [0.]
MSE for test set [148]  is [0.]
MSE for test set [149]  is [0.]
MSE for test set [150]  is [0.]
MSE for test set [151]  is [0.]
MSE for test set [152]  is [0.]
MSE for test set [153]  is [0.]
MSE for test set [154]  is [0.]
MSE for test set [155]  is [0.]
MSE for test set [156]  is [0.]
MSE for test set [157]  is [0.]
MSE for test set [158]  is [0.]
MSE for test set [159]  is [0.]
MSE for test set [160]  is [0.]
MSE for test set [161]  is [0.]
MSE for test set [162]  is [0.]
MSE for test set [163]  is [0.]
MSE for test set [164]  is [0.]
MSE for test set [165]  is [0.]
MSE for test set [166]  is [0.]
MSE for test set [167]  is [0.]
MSE for test set [168]  is [0.]
MSE for test set [169]  is [0.]
MSE for test set [170]  is [0.]
MSE for test set [171]  is [0.]
MSE for test set [172]  is [0.]
MSE for test set [173]  is [0.]
MSE for test set [174]  is [0.]
MSE for test set [175]  is [0.]
```

```
In [10]: print(classification_report(y_test_lst, y_pred_lst))
```

```
             precision    recall  f1-score   support
```

```
        1.0       0.84      0.78      0.81        59
        2.0       0.83      0.89      0.86        71
        3.0       0.96      0.93      0.95        46

avg / total       0.86      0.86      0.86       176
```

Error rate for each type is 0.16, 0.17, 0.04 respectively. The most precise estimation is j=3.

```
In [11]: MSE_loo = MSE_vec.mean()
         MSE_loo_std = MSE_vec.std()
         print('Test estimate MSE loocv = {}.'.format(MSE_loo))
```

```
Test estimate MSE loocv = 0.13636363636363635.
```

(c)

```
In [12]: X=df[["alco", "malic", "tot_phen", "color_int"]].values
         y=df["cultivar"].values
         k = 4
         kf = KFold(n_splits=k, random_state=10, shuffle=True)
         kf.get_n_splits(X)

         MSE_vec_kf = np.zeros(k)

         k_ind = int(0)
         for train_index, test_index in kf.split(X):
             print('k index=', k_ind)
             X_train, X_test = X[train_index], X[test_index]
             y_train, y_test = y[train_index], y[test_index]
             LogReg = LogisticRegression(multi_class='multinomial', solver='newton-cg')
             LogReg.fit(X_train, y_train)
             y_pred = LogReg.predict(X_test)
             MSE_vec_kf[k_ind] = (y_test != y_pred).mean()
             print('MSE for test set', k_ind, ' is', MSE_vec_kf[k_ind])
             print(classification_report(y_test, y_pred))
             k_ind += 1
```

```
k index= 0
MSE for test set 0  is 0.1590909090909091
             precision    recall  f1-score   support

          1       0.71      1.00      0.83        12
          2       1.00      0.75      0.86        24
          3       0.78      0.88      0.82         8

avg / total       0.88      0.84      0.84        44
```

```
k index= 1
MSE for test set 1  is 0.11363636363636363
             precision    recall  f1-score   support

          1       1.00      0.76      0.87        17
          2       0.69      1.00      0.81        11
          3       1.00      0.94      0.97        16

avg / total       0.92      0.89      0.89        44

k index= 2
MSE for test set 2  is 0.045454545454545456
             precision    recall  f1-score   support

          1       0.94      1.00      0.97        15
          2       0.94      0.94      0.94        16
          3       1.00      0.92      0.96        13

avg / total       0.96      0.95      0.95        44

k index= 3
MSE for test set 3  is 0.045454545454545456
             precision    recall  f1-score   support

          1       0.88      1.00      0.94        15
          2       1.00      0.90      0.95        20
          3       1.00      1.00      1.00         9

avg / total       0.96      0.95      0.95        44
```

```
In [13]: print('the average error rate for category 1 is {:2f}%'.format((1- (0.71 + 1.00 + 0.94
         print('the average error rate for category 2 is {:2f}%'.format((1- (1.00 + 0.69 + 0.94
         print('the average error rate for category 3 is {:2f}%'.format((1- (0.78 + 1.00 + 1.00
```

```
the average error rate for category 1 is 11.750000%
the average error rate for category 2 is 9.250000%
the average error rate for category 3 is 5.500000%
```

Compared to those from part (b), the error rates become slightly larger.

Compared to those from part (a), the error rate for category 1 become slightly smaller, but those for category 2 and category 3 become much larger.

```
In [14]: MSE_kf = MSE_vec_kf.mean()
         MSE_kf_std = MSE_vec_kf.std()
```

```
print('test estimate MSE k-fold=', MSE_kf)
print('test estimate MSE standard err=', MSE_kf_std)
```

```
test estimate MSE k-fold= 0.09090909090909091
test estimate MSE standard err= 0.04821182598999188
```

2. Splines and interpolation

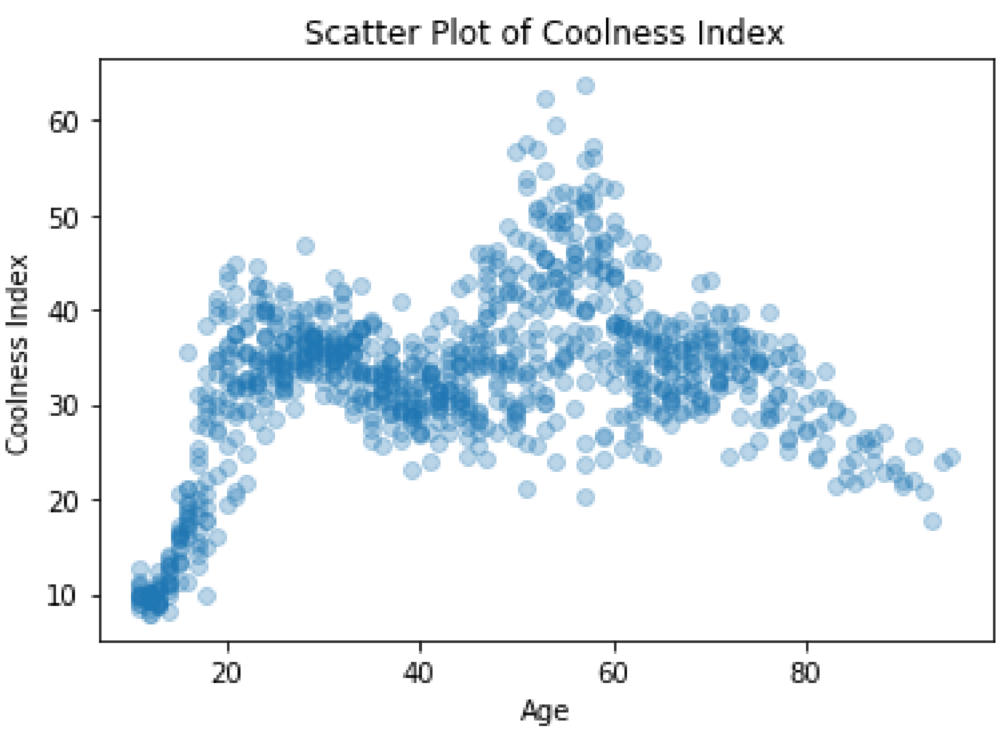
(a)

```
In [15]: df1 = pd.read_csv("data/CoolIndex.txt", names=["Age", "Cool"])
         df1.head()
```

Out[15]:

	Age	Cool
0	11.0	10.981602
1	11.0	11.364925
2	11.0	10.190227
3	11.0	9.903725
4	11.0	8.997918

```
In [16]: plt.scatter(x =df1['Age'], y =df1['Cool'], alpha=0.3, label="Points")
         plt.title('Scatter Plot of Coolness Index')
         plt.xlabel('Age')
         plt.ylabel('Coolness Index')
         plt.show()
```



(b)

```
In [17]: df1["G1"] = np.where((df1['Age'] >= 11) & (df1['Age'] < 22),1,0)
         df1["G2"] = np.where((df1['Age'] >= 22) & (df1['Age'] < 40),1,0)
         df1["G3"] = np.where((df1['Age'] >= 40) & (df1['Age'] < 59),1,0)
         df1["G4"] = np.where((df1['Age'] >= 59) & (df1['Age'] < 77),1,0)
         df1["G5"] = np.where((df1['Age'] >= 77) & (df1['Age'] <= 95),1,0)
```

```
In [18]: X=df1[["G1","G2","G3","G4","G5"]]
         reg=sm.OLS(df1['Cool'], X, missing='drop').fit()
         print(reg.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   Cool   R-squared:                       0.429
Model:                            OLS   Adj. R-squared:                  0.427
Method:                 Least Squares   F-statistic:                     178.7
Date:                Wed, 27 Feb 2019   Prob (F-statistic):          3.73e-114
Time:                        10:50:59   Log-Likelihood:                -3214.5
No. Observations:                 956   AIC:                             6439.
Df Residuals:                     951   BIC:                             6463.
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
G1            20.1025      0.562     35.746      0.000      18.999      21.206
G2            34.4758      0.431     80.006      0.000      33.630      35.321
G3            37.6351      0.424     88.814      0.000      36.804      38.467
G4            35.2254      0.485     72.560      0.000      34.273      36.178
G5            27.2964      0.936     29.175      0.000      25.460      29.132
==============================================================================
Omnibus:                       80.102   Durbin-Watson:                   1.236
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              101.718
Skew:                           0.714   Prob(JB):                     8.17e-23
Kurtosis:                       3.719   Cond. No.                         2.21
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
In [19]: for i in range(5):
             print("Beta_{} = {}".format(i+1, reg.params[i]))
```

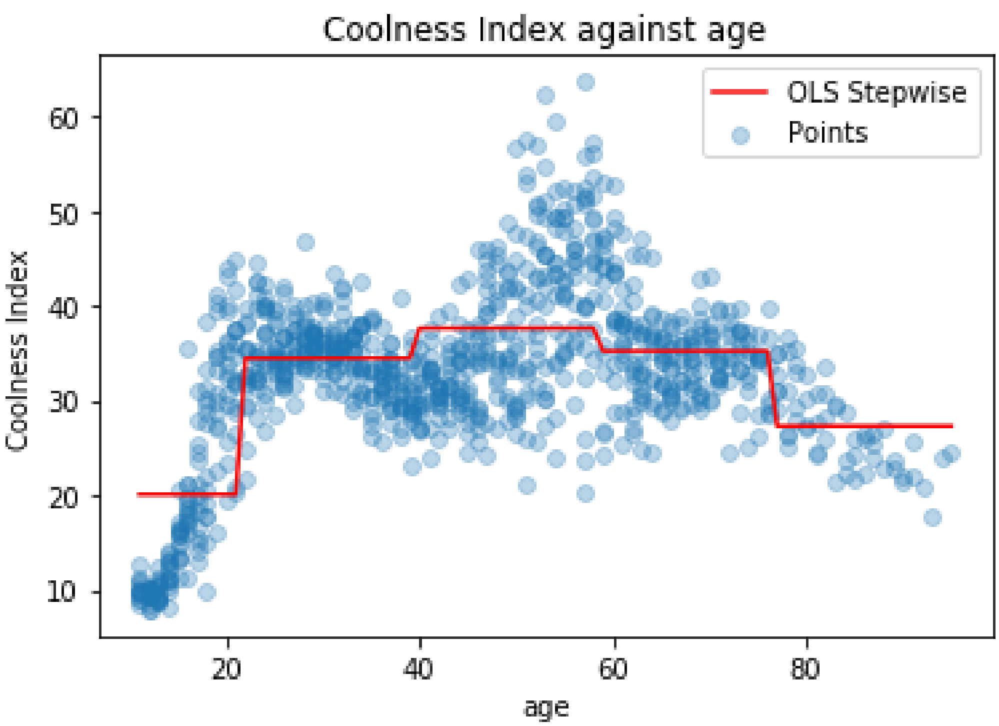
```
Beta_1 = 20.102457252090748
Beta_2 = 34.47578807755938
```

```
Beta_3 = 37.63510549244961
Beta_4 = 35.22540004024275
Beta_5 = 27.296378244321282
```

```
In [20]: print('The predicted coolness of a 73-year old from the stepwise function is', reg.pa
```

```
The predicted coolness of a 73-year old from the stepwise function is 35.22540004024275
```

```
In [21]: plt.scatter(df1['Age'], df1['Cool'], alpha=0.3, label="Points")
         plt.plot(df1['Age'], reg.predict(), 'r', label = "OLS Stepwise")
         plt.legend()
         plt.xlabel("age")
         plt.ylabel("Coolness Index")
         plt.title("Coolness Index against age")
         plt.show()
```



(c)

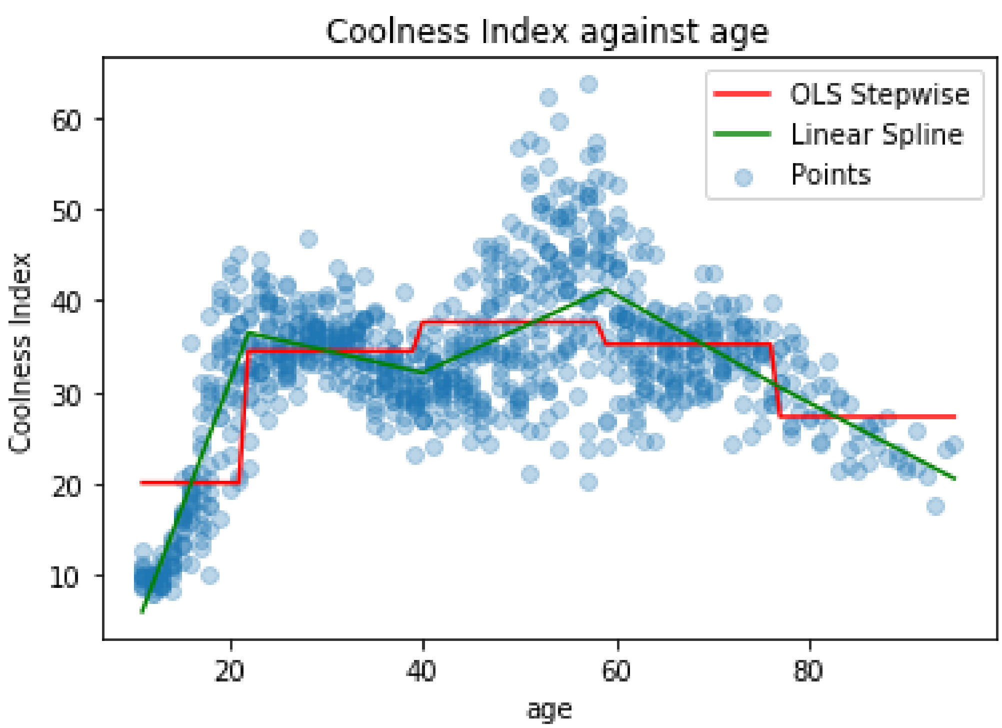
```
In [22]: df2=df1.groupby('Age').mean()
         df2['Age']=df2.index
         df2.head()
```

Out[22]:

	Cool	G1	G2	G3	G4	G5	Age
Age							
11.0	10.110237	1	0	0	0	0	11.0
12.0	9.365623	1	0	0	0	0	12.0
13.0	10.015882	1	0	0	0	0	13.0
14.0	11.747109	1	0	0	0	0	14.0
15.0	15.434739	1	0	0	0	0	15.0

In [23]:
```
knots=[22, 40, 59, 77]
linear_spline=LSQUnivariateSpline(df2.Age.values, df2.Cool.values, t=knots, k=1)
```

In [24]:
```
plt.scatter(df1.Age, df1.Cool, alpha=0.3, label="Points")
plt.plot(df1.Age, reg.predict(), "r", label = "OLS Stepwise")
plt.plot(df2.Age, linear_spline(df2.Age), "g", label = "Linear Spline")
plt.legend()
plt.xlabel("age")
plt.ylabel("Coolness Index")
plt.title("Coolness Index against age")
plt.show()
```



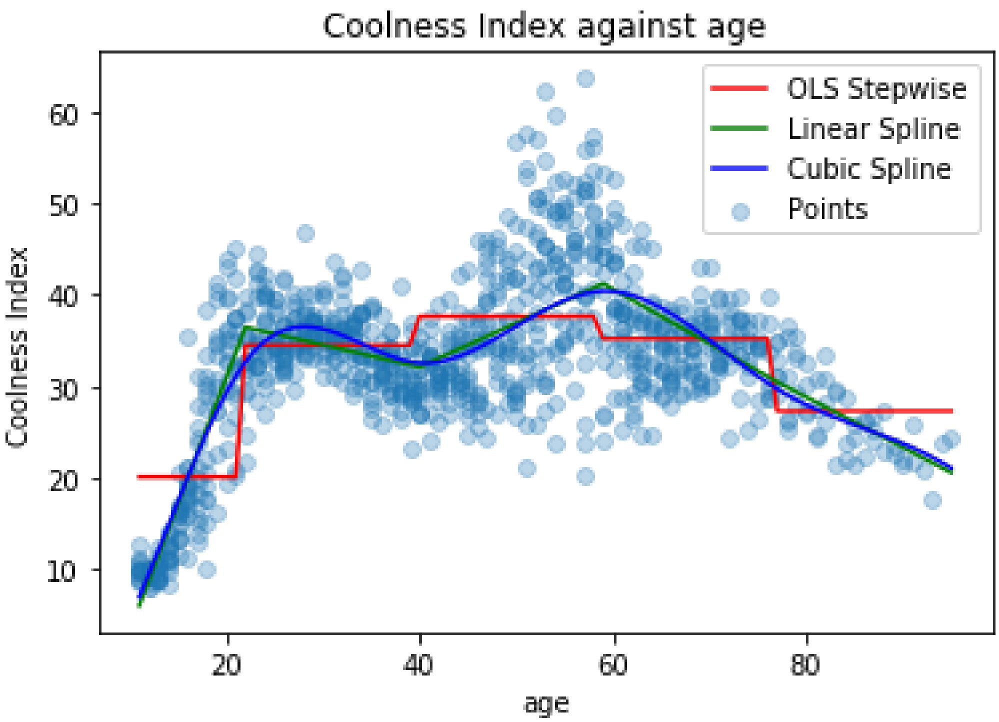
In [25]:
```
print('The predicted coolness of a 73-year old from the linear spline is', linear_spl
```

```
The predicted coolness of a 73-year old from the linear spline is 32.86784862349653
```

(d)

```
In [26]: cubic_spline = LSQUnivariateSpline(df2.Age.values, df2.Cool.values, knots, k=3)
```

```
In [27]: plt.scatter(df1.Age, df1.Cool, alpha=0.3, label="Points")
         plt.plot(df1.Age, reg.predict(), "r", label = "OLS Stepwise")
         plt.plot(df2.Age, linear_spline(df2.Age), "g", label = "Linear Spline")
         plt.plot(df2.Age, cubic_spline(df2.Age), "b", label = "Cubic Spline")
         plt.legend()
         plt.xlabel("age")
         plt.ylabel("Coolness Index")
         plt.title("Coolness Index against age")
         plt.show()
```




```
In [28]: print('The predicted coolness of a 73-year old from the cubic spline is', cubic_spline
```

```
The predicted coolness of a 73-year old from the cubic spline is 32.642301066279764
```