Assignment 2 - Parallel LU Decomposition

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Project Files

```
root directory
        ├── utils.h .cpp         // APIs for log and L,12 norm validator
        ├── lu-omp.cpp           // Main function and LU
        ├── serial.sh            // compile and submit serial job on compute node
        ├── parallel.sh          // compile and submit parallel job on compute node
        ├── submit.sh            // compile and submit.batch on compute node
        ├── hpcgenerate.sh       // generate hpc dataset
        ├── hpcviewer.sh         // launch hpcviewer
        ├── plot.ipynb           // jupyter notebook to plot figures
        └── Makefile             // Makefile
```

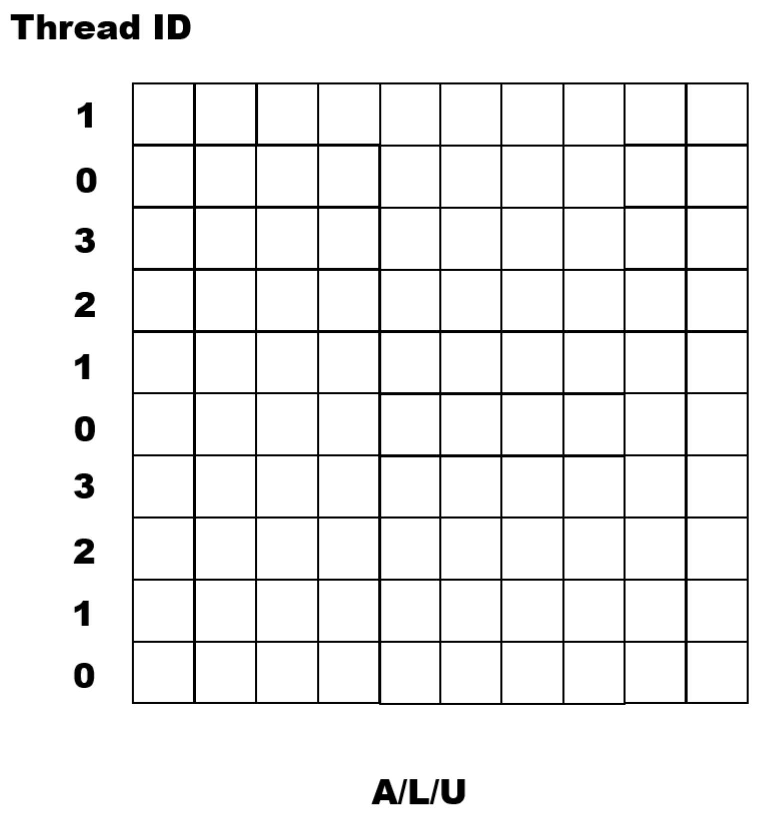
Algorithm Design & Implementation

Data Partition

In the program, there are three computing data, namely the matrix A, L, and U with the same size n * n. Use data type `double** A = new double*[n]` to present each row of a matrix. In the row data allocation and initialization phase, use `omp for` to iterate over rows. Set the schedule clause of `schedule static(1)` , and each thread allocates its row with `numa_alloc_local` , the code snip is like below:

```
# pragma omp parallel for schedule(static, 1)
for (int i = n - 1; i >= 0; --i) {
        A[i] = (double*)numa_alloc_local(sizeof(double) * n);
}
```

Under the `static(1)` clause, the program assigns each iteration from beginning to end in a round-robin way to each thread. If the iteration `i` decrease from `n - 1` to `0` , the row to the thread id assigned this row is like the graph below if 4 thread in total:



My purpose is to write a program with the best locality possible, namely, all threads access the data through its CPU socket instead of fetching from another node. Threads use `void *numa_alloc_local(size_t size)` to allocate the row, which will allocate in the memory that this thread belongs to.

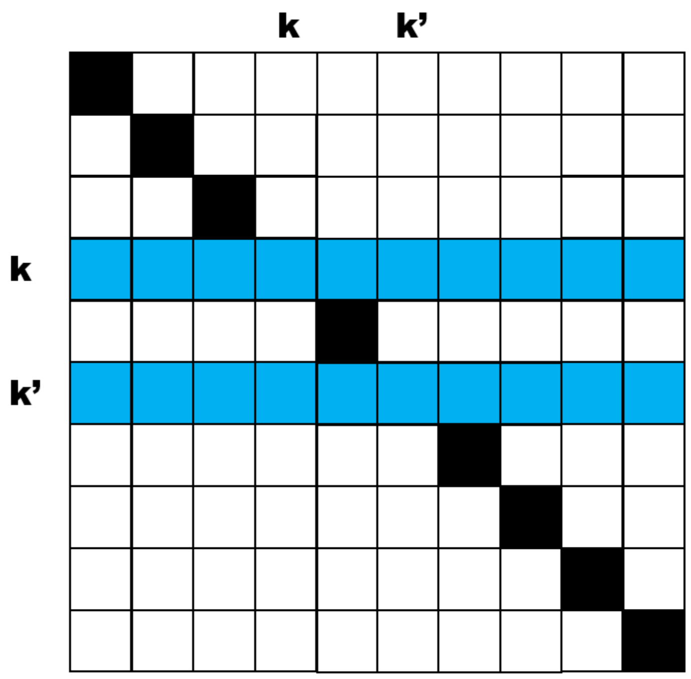
Parallelism Exploit & Synchronization

In the pseudo-code provided, the outer loop is impossible to parallel because of the swap operation over the row of `A`. The parallel can only be done in each iteration on the column of `A`, and synchronized at the end of each iteration.

In each iteration at column index `k`, there are four steps:

1 Find maximum column k' and swap(A[k], A[k'])

Find the maximum value of in the column `k` under row `k` as `k'`, then swap `A[k]` with `A[k']`. This step should be guaranteed done prior to other steps beginning. I tried to use a `omp reduction` to parallel the process of locating the maximum value column, but the difference is so trivial compared to the heavy compute of the other parts.

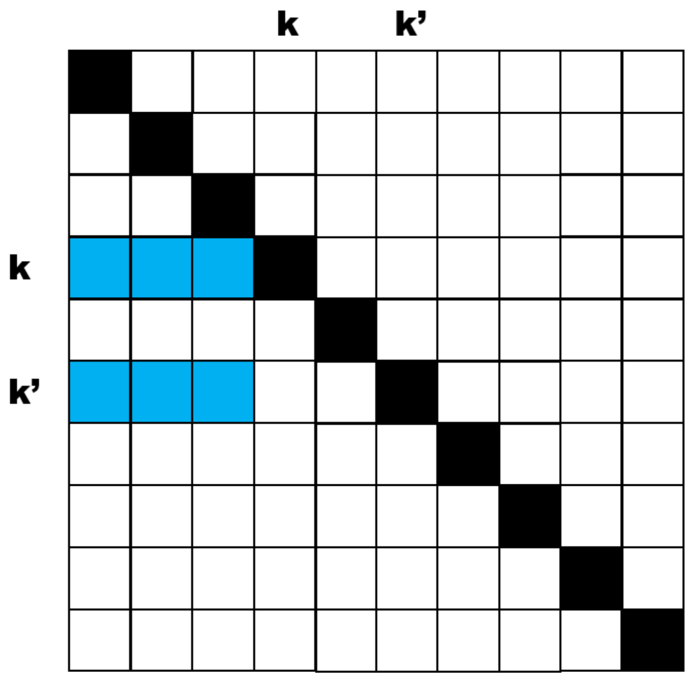


A

A omp parallel region can be created to cover the steps 2 ~ 4 below:

2 swap(L[k, :k], L[k', :k])

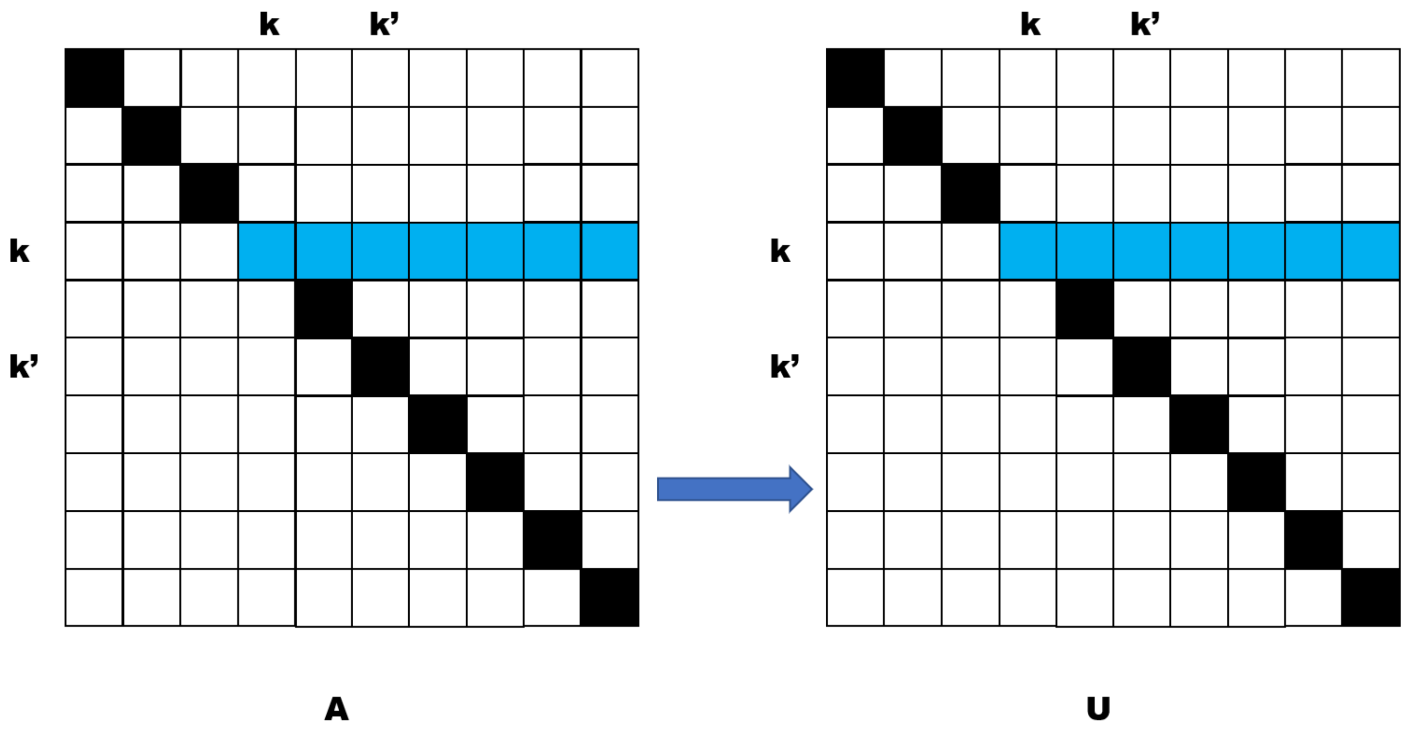
This step can be done in a `omp single` assigned to one thread.



L

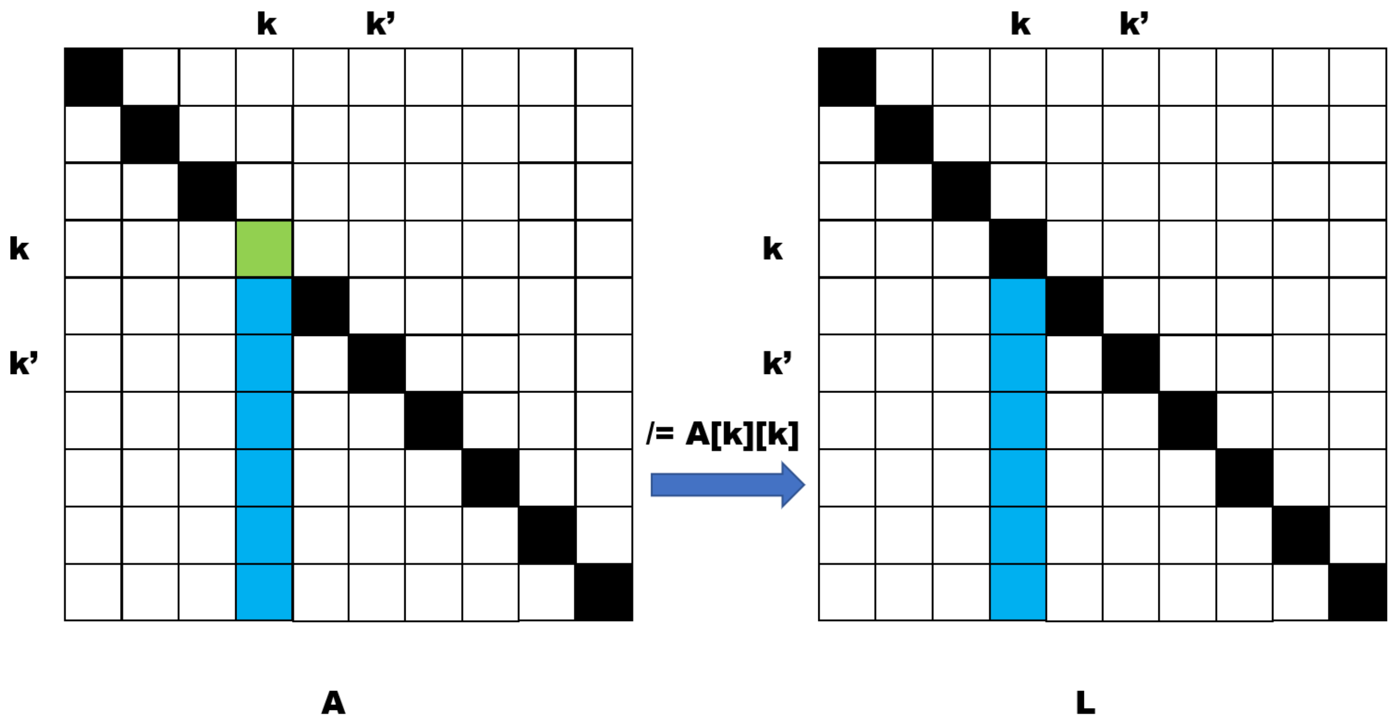
3 assign A[k, k:] to U[k, k:]

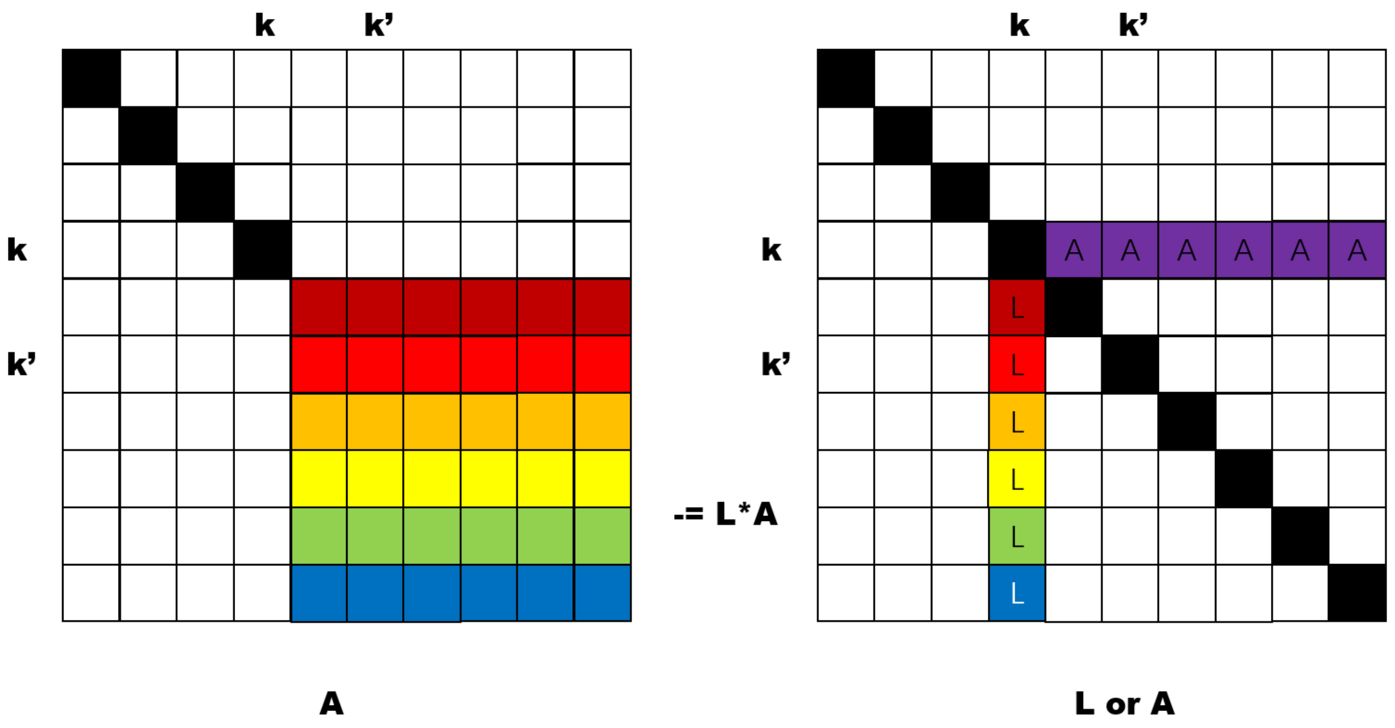
This step can be done in a `omp single` assigned to one thread.



4 `assign A[k:, k] / A[k][k] to L[k:, k]` then `A[i][j] -= L[i][k] * U[k][j]`

In this step, the colored rows of A from index `k + 1` to `n` are updated. For each row `i`, the pixel with the same row index and column index `k` at matrix `L`, namely `L[i][k]` paint as the same color with its corresponding row in `A`. Multiply `L[i][k]` with every element in the row segment `A[k][k+1:]` colored as purple in the graph, and subtract the product array to the A row by row.





This step is the most workload in one iteration. Use an `omp for` to iterate over rows. The code snip is like:

```
# pragma omp for nowait schedule(static, 1)
for (int i = n - 1; i > k; --i) {
    L[i][k] = (A[i][k] / A[k][k]);
    for (int j = k + 1; j < n; ++j) {
        A[i][j] -= L[i][k] * A[k][j];
    }
}
```

Notice that the loop from `n - 1` down to `k + 1`, which starts from `n - 1` just like the allocation phrase. The thread assignment in this loop will be the same as the allocation loop. The access of `L[i][k]` will be local and `A[i]` will be local access if not swapped from a row allocated on another node (in step 1).

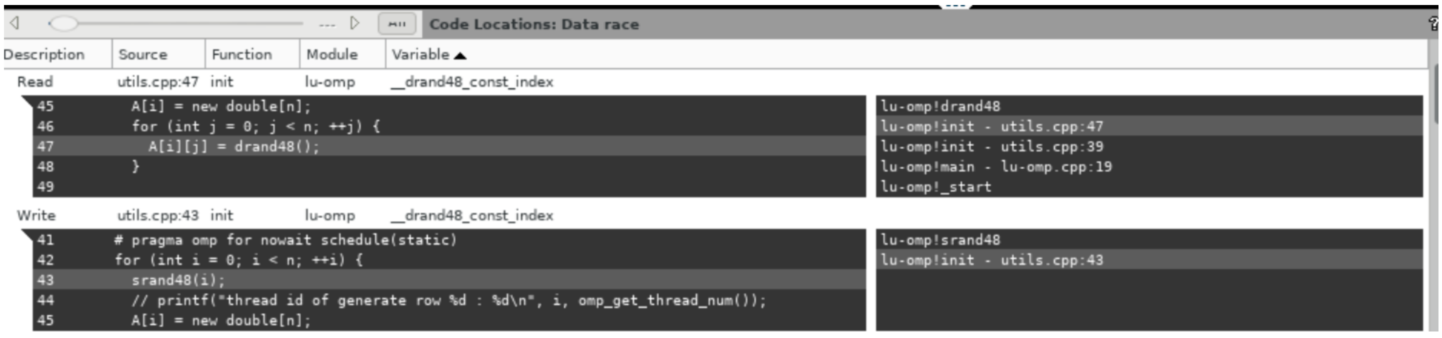
```
# pragma omp parallel
{
  // step 2 : swap(L[k, :k], L[k', :k])
  # pragma omp single nowait
    ...

  // step 3 : assign A[k, k:] to U[k, k:]
  # pragma omp single nowait
    ...

  // step 4 : assign A[k:, k] / A[k][k] to L[k:, k] then
  // A[i][j] -= L[i][k] * U[k][j], i and j in [k + 1, n]
  # pragma omp for nowait schedule(static, 1)
    ...
}
```

Experiment

Program Correctness & Data Races

Print out the `L2,1` norm of `PA - LU`, for a serial or parallel version of code, give arbitrary matrix size, the normalization is always 0 in the at least 6 digits after zero points.

Run `check.sh` to check the data races with **Intel tools**, a data races problem reported. Use `view.sh` to check with Intel's Inspector GUI.



Description	Source	Function	Module	Variable
Read	utils.cpp:47	init	lu-omp	__drand48_const_index
Write	utils.cpp:43	init	lu-omp	__drand48_const_index

```
45        A[i] = new double[n];
46        for (int j = 0; j < n; ++j) {
47          A[i][j] = drand48();
48        }
49
```

```
lu-omp!drand48
lu-omp!init - utils.cpp:47
lu-omp!init - utils.cpp:39
lu-omp!main - lu-omp.cpp:19
lu-omp!_start
```

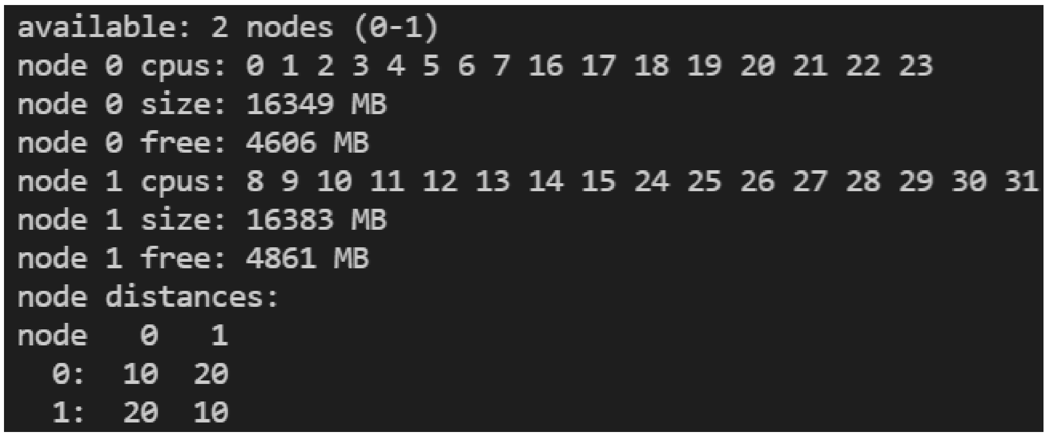
```
41      # pragma omp for nowait schedule(static)
42      for (int i = 0; i < n; ++i) {
43        srand48(i);
44        // printf("thread id of generate row %d : %d\n", i, omp_get_thread_num());
45        A[i] = new double[n];
```

```
lu-omp!srand48
lu-omp!init - utils.cpp:43
```

It seems a false alert, there will not be a data race condition, the correction check of computing normalization can confirm it.

Manage Threads and Data on NOTS

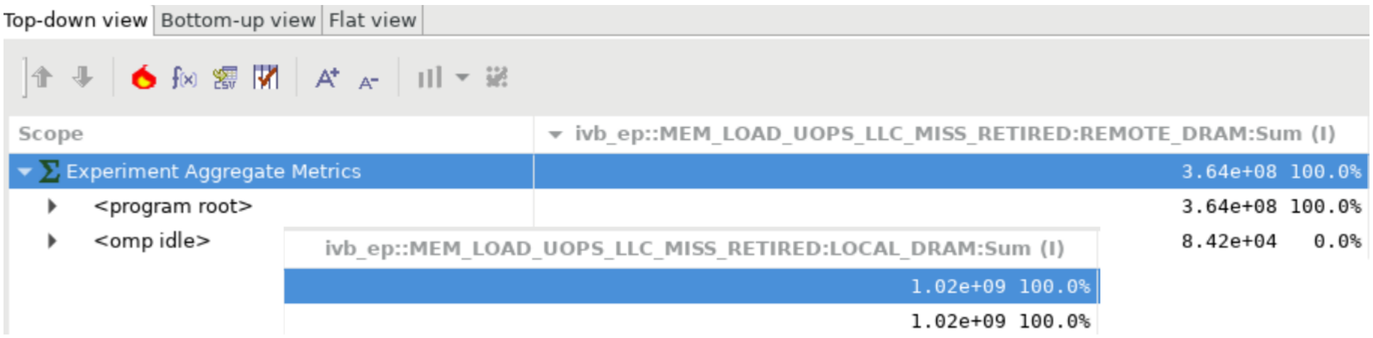
Run command `numactl --hardware` to see the NUMA information of a compute node.



```
available: 2 nodes (0-1)
node 0 cpus: 0 1 2 3 4 5 6 7 16 17 18 19 20 21 22 23
node 0 size: 16349 MB
node 0 free: 4606 MB
node 1 cpus: 8 9 10 11 12 13 14 15 24 25 26 27 28 29 30 31
node 1 size: 16383 MB
node 1 free: 4861 MB
node distances:
node   0   1
  0:  10  20
  1:  20  10
```

A compute node has 2 CPU sockets, a CPU node has 16 GB memory (32 GB in total). Node access to its memory distance is 10, across nodes is 20. In previous parts, I have demonstrated how to optimize in terms of NUMA. Use the helper function to print out the `omp_get_thread_num`, `sched_getcpu` and `numa_node_of_cpu` testified the runtime situation is as I expected.

HPCToolKit Detect for Remote DRAM



```
Scope                                ivb_ep::MEM_LOAD_UOPS_LLC_MISS_RETIRED:REMOTE_DRAM:Sum (I)
Experiment Aggregate Metrics                                             3.64e+08 100.0%
  <program root>                                                         3.64e+08 100.0%
  <omp idle>                                                             8.42e+04   0.0%

ivb_ep::MEM_LOAD_UOPS_LLC_MISS_RETIRED:LOCAL_DRAM:Sum (I)
                                                   1.02e+09 100.0%
                                                   1.02e+09 100.0%
```

Run `hpcgenerate.sh` to generate database, use two events of `ivb_ep::MEM_LOAD_UOPS_LLC_MISS_RETIRED:REMOTE_DRAM` and `ivb_ep::MEM_LOAD_UOPS_LLC_MISS_RETIRED:LOCAL_DRAM` to report the remote DRAM fetch. Calculate the ratio of remote fetch is:

$$\frac{remote}{remote + local} = \frac{3.64 * 10^8}{3.64 * 10^8 + 1.02 * 10^9} = 26.3\%$$

Parallel Efficiency

Use `parallel.sh` to submit parallel jobs on a compute node with parallel from 1 to 32, record time for different thread configuration.

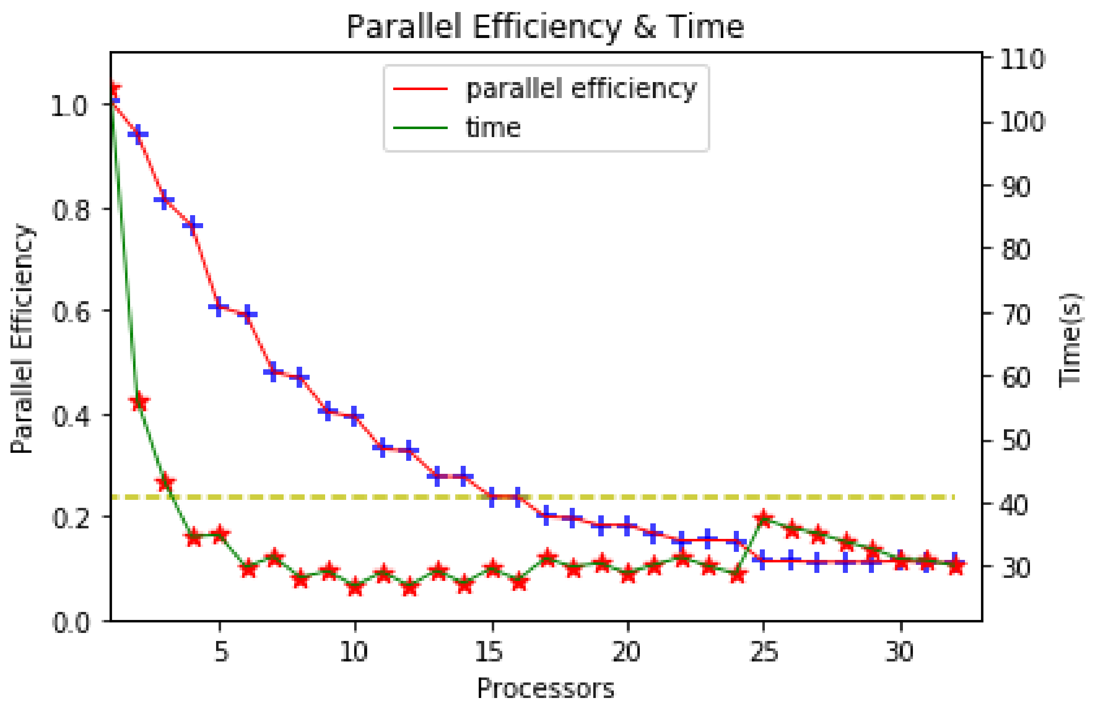
Use `serial.sh` to submit `lu-omp-serial` to a private node, the real time is `105.815s` . Parallel efficiency is computed as

$$\frac{S}{p * T(p)}$$

Where `S = 105.815s` , *T(p)* is the real time of the execution on *p* processors.

Processors	Time	Efficiency	Processors	Time	Efficiency
1	105.150	1.00	17	31.256	0.19
2	56.162	0.94	18	29.939	0.19
3	43.324	0.81	19	30.451	0.18
4	34.645	0.76	20	28.944	0.18
5	34.946	0.60	21	30.232	0.16
6	29.850	0.59	22	31.412	0.15
7	31.528	0.47	23	29.989	0.15
8	28.239	0.46	24	28.839	0.15
9	29.246	0.40	25	37.471	0.11

Processors	Time	Efficiency	Processors	Time	Efficiency
10	26.935	0.39	26	36.014	0.11
11	29.082	0.33	27	35.171	0.11
12	27.008	0.32	28	33.887	0.11
13	29.315	0.27	29	32.788	0.11
14	27.377	0.27	30	31.169	0.11
15	29.623	0.23	31	30.935	0.11
16	27.910	0.23	32	30.122	0.10



The parallel efficiency at `processors=16` is `23.7%` .