FUSION OF SPATIO-TEMPORAL AND MULTI-SCALE FREQUENCY FEATURES FOR DRY ELECTRODES MI-EEG DECODING

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ABSTRACT

Dry-electrode Motor Imagery (MI) Electroencephalography (EEG) enables fast, comfortable, real-world Brain Computer Interface by eliminating gels and shortening setup for at-home and wearable use. However, dry recordings pose three main issues: lower Signal-to-Noise Ratio with more baseline drift and sudden transients; weaker and noisier data with poor phase alignment across trials; and bigger variances between sessions. These drawbacks lead to larger data distribution shift, making features less stable for MI-EEG tasks. To address these problems, we introduce STGMFM, a tri-branch framework tailored for dryelectrode MI-EEG, which models complementary spatiotemporal dependencies via dual graph orders, and captures robust envelope dynamics with a multi-scale frequency mixing branch, motivated by the observation that amplitude envelopes are less sensitive to contact variability than instantaneous waveforms. Physiologically meaningful connectivity priors guide learning, and decision-level fusion consolidates a noise-tolerant consensus. On our collected dryelectrode MI-EEG, STGMFM consistently surpasses competitive CNN/Transformer/graph baselines. Codes are available at https://github.com/Tianyi-325/STGMFM

Index Terms— EEG, motor imagery, dry electrodes, graph neural network, frequency mixing

1. INTRODUCTION

Non-invasive Electroencephalography (EEG) is the most accessible neural sensing modality for daily-life Brain Computer Interface (BCI) applications. Among non-invasive options, dry-electrode systems avoid conductive gel, shorten setup/cleanup, and better fit out-of-lab use [1,2]. This practicality directly makes it suitable for rehabilitation at home, longitudinal self-training and wearable assistive interaction, where user comfort, compliance, and scalability are critical. However, dry recordings differ systematically from wet

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data: the unstable skin-electrode coupling often elevates and fluctuates impedance, lowering Signal-to-Noise (SNR) and amplifying motion/contact artifacts [1, 2]. Besides, repositioning across days leads to cross-session drift. What is more, inter-subject variability is exacerbated due to hair and pressure differences. In practice, dry-cap protocols also limit the number and duration of trials to reduce discomfort and time burden, which increases the premium on data-efficient and noise-robust decoding and makes frequent recalibration impractical. Currently, most existing Motor Imagery (MI) decoders were designed/validated under wet-electrode assumptions, and most of them transfer poorly to dry conditions due to the noise characteristics, distribution shift, and trial scarcity [3]. As a result, explicit algorithmic tailoring for dry-electrode MI remains under-explored.

Non-invasive BCIs with dry electrodes ease setup, but they typically suffer lower SNR and contact variability, which depresses MI accuracy and calls for noise-robust, data-efficient models [1, 2, 4]. Classical MI pipelines relied on Common Spatial Pattern/bandpower and linear classifiers or Riemannian geometry on covariances [3, 5–7], while deep CNNs improved end-to-end decoding yet still assume relatively clean, well-calibrated signals [8-13]. Graph formulations encode inter-channel relations and have shown benefits for spatio-temporal fusion and cross-session robustness [14]—most notably ST-GF by Wang et al. [15]; multi-branch/domain-generalization schemes further diversify representations via decision fusion [16]. In time-series modeling, recent "mixers" learn multi-period structure with compact, multi-resolution tokenization, including TimeMixer and its successor TimeMixer++ by Wang et al. [17, 18], as well as related sequence learners [19, 20]. Connectivity priors such as PLV provide phase-synchrony graphs that are amplitude-invariant and physiologically meaningful [21, 22].

Although these advances are compelling, most were devised and tuned with wet-electrode data rather than dry EEG. Therefore, they fail to remediate the low-SNR, contact-motion-artifact instability. They also emphasize localized patterns and lack multi-view learning, limiting the learning

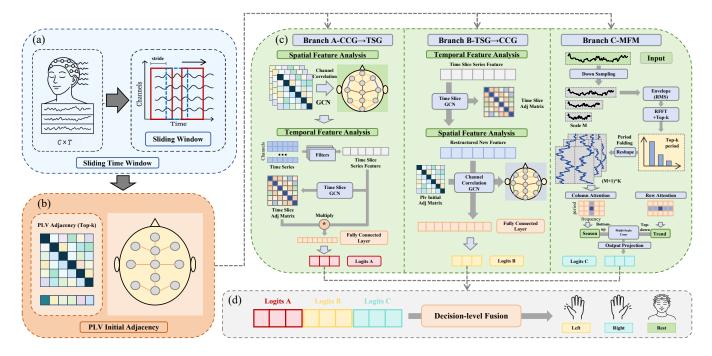


Fig. 1. Pipeline of the proposed model. Raw EEG is windowed into overlapping segments and processed by three branches. **A** (CCG→TSG) aligns functional connectivity via a PLV-initialized channel graph before aggregating on a time-slice graph. **B** (TSG→CCG) first reconstructs intra-trial continuity and then fuses channels on the CCG. **C** (MFM) operates on the amplitude envelope (RMS) with multi-resolution imaging and cross-scale mixing. Linear head fuses branch logits at the decision level.

of deep, invariant features to bridge cross-session/subject shifts. In addition, multi-scale spatio and temporal structure is under-explored, leaving deep cross-scale cues untapped. Motivated by these gaps, we propose STGMFM, which unifies dual ordered spatio-temporal graph branches (CCG-TSG and TSG—CCG) so that spatial-first and temporal-first propagation can hedge against different noise pathways. And we augments them with a multi-cale frequency mixer that learns multi-resolution envelope dynamics, aligned with ERD/ERS to offer a robust temporal cue when instantaneous waveform detail is unreliable. Physiological connectivity priors (PLV) provide a relatively detailed and precise starting point that is then adapted end-to-end to accommodate subject/session variability, while decision-level fusion forms a simple consensus that resists over-reliance on any single branch. In experiments on a dry-electrode MI-EEG dataset, STGMFM outperforms CNN/transformer/graph baselines, proving it to be a practical path toward dry-electrode BCIs.

Main contributions: (1) We introduce a PLV-initialized EEG graph that creates a more detailed and precise prior adjacency, suitable for low-SNR dry recordings. (2) We design a dual-order spatio-temporal graph to yield complementary evidence. (3) We introduce a lightweight Multi-Scale Frequency Mixer that extracts phase-invariant, multi-scale temporal cues aligned with ERD/ERS. (4) We employ a stable training recipe (decision-level fusion, L1/L2 regularization, cosine annealing) that reduces overfitting and over-reliance.

2. METHODS

2.1. Overview

We present a triple-branch framework for cross-subject MI EEG decoding. Raw trials are segmented with overlapping windows. Branches A/B pair a channel-correlation graph (CCG) and a time-slice graph (TSG) in opposite orders (CCG \rightarrow TSG vs. TSG \rightarrow CCG), yielding complementary biases—spatial alignment before temporal aggregation vs. temporal stabilization before spatial fusion. Branch C consumes amplitude envelopes to build a lightweight multi-resolution representation of rhythmic/trend dynamics. Each branch outputs logits, and a shallow head fuses them at the decision level, delivering robust generalization without fragile feature-level alignment. Figure 1 illustrates the overall pipeline.

2.2. Notation and Sliding Windowing

Let a trial be $\mathbf{X} \in \mathbf{R}^{C \times T}$ with C channels and T time steps. Using window length W_n and stride Str, the number of windows is

$$W = \left\lfloor \frac{T - W_n}{\text{Str}} \right\rfloor + 1. \tag{1}$$

After segmentation, we obtain a 4-D tensor $\mathbf{X} \in \mathbf{R}^{N \times W \times C \times T_w}$ for batch size N and per-window length $T_w = W_n$. Win-

dowing provides natural nodes for the TSG and a consistent statistical unit for the graph modules and the envelope branch, avoiding drift on long unsegmented sequences.

2.3. PLV-driven Initial Channel Graph

To encode functional connectivity as a prior, we build an initial adjacency from the phase-locking value (PLV). For channels i and j with analytic phases $\phi_i(t)$ and $\phi_j(t)$,

$$PLV(i,j) = \left| \frac{1}{T} \sum_{t=1}^{T} e^{i(\phi_i(t) - \phi_j(t))} \right|.$$
 (2)

We remove self-loops, sparsify by per-row Top-k (or a threshold), symmetrize, and degree-normalize to obtain $\tilde{\mathbf{A}} = \mathbf{D}^{-1/2}\mathbf{A}\mathbf{D}^{-1/2}$. The same $\tilde{\mathbf{A}}$ initializes the CCG in both branches. During training, a small learnable increment adapts $\tilde{\mathbf{A}}$ to subject-specific variability while retaining interpretability. At the same time, it also provides more detailed and precise adjacency. Fig. 2 shows the before/after learning comparison of adjacency under PLV-initial and the basic spatial-neighborhood prior.

2.4. Branch A: CCG→TSG

Branch A first performs graph propagation on the channel graph to align truly cooperating electrodes and suppress mismatched ones. With node features $\mathbf{H}^{(l)} \in \mathbf{R}^{C \times d}$, one CCG layer reads

$$\mathbf{H}^{(l+1)} = \sigma\left(\tilde{\mathbf{A}} \mathbf{H}^{(l)} \mathbf{W}_s^{(l)}\right), \quad l = 0, \dots, K_s - 1. \quad (3)$$

We visualized the changes in spatial connectivity before and after learning, see Fig. 2. The resulting spatially denoised representation then passes through a shared temporal convolutional block (depthwise–pointwise 1D convolution with GELU/normalization) and is aggregated over the *time-slice graph*. Let $\mathbf{U}^{(l)} \in \mathbf{R}^{W \times d}$ be slice-level features and $\tilde{\mathbf{S}} \in \mathbf{R}^{W \times W}$ the learnable slice adjacency; one TSG layer is

$$\mathbf{U}^{(l+1)} = \sigma(\tilde{\mathbf{S}} \ \mathbf{U}^{(l)} \ \mathbf{W}_t^{(l)}), \quad l = 0, \dots, K_t - 1.$$
 (4)

Placing temporal aggregation on top of a connectivity-aligned representation improves the SNR of time reconstruction by reducing sensitivity to phase misalignment across channels. A global average pooling (GAP) and a linear head yield \mathbf{z}_A . We insert a lightweight shared temporal block between CCG and TSG to align feature spaces and add local temporal expressiveness without disturbing the graph inductive bias.

2.5. Branch B: TSG→CCG

Branch B starts by reconstructing intra-trial continuity on the slice graph. Continuous MI segments become tightly clustered while non-task fragments are attenuated. The graph-filtered features are then rearranged to the channel dimension

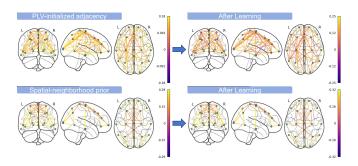


Fig. 2. Before/after training comparison of adjacency under two priors: PLV initialization and the spatial-neighborhood prior

and passed through the CCG propagation with $\hat{\mathbf{A}}$. By stabilizing time first, B prevents short-term temporal noise from being prematurely diffused spatially. A shared temporal block and classifier head produce \mathbf{z}_B . Shared temporal block is also placed between TSG and CCG.

2.6. Branch C: Multi-Scale Frequency Mixer

The third branch operates on the amplitude envelope (RMS) (e.g., μ/β band), which makes ERD/ERS modulation explicit and phase-invariant. We first form multi-resolution "time images" by selecting dominant periods/frequencies (rfft) and mapping the 1D signal to a compact time–frequency lattice. We then decouple periodic (seasonal) patterns from slower trend evolution using dual-axis processing, and finally carry out multi-scale and multi-resolution mixing to aggregate evidence. This imaging—decoupling—mixing pipeline turns long dependencies into local ones on the image plane and yields a compact cross-scale representation. A lightweight configuration (small channel width and a single block, with one down-sampling and Top-1 dominant period) suits the low-sample EEG regime. The branch outputs \mathbf{z}_C .

2.7. Decision-level Fusion

Let $[\cdot;\cdot]$ denote concatenation. We fuse logits with a shallow linear head,

$$\mathbf{z} = \mathbf{W} \left[\mathbf{z}_A; \mathbf{z}_B; \mathbf{z}_C \right] + \mathbf{b}, \qquad \hat{y} = \arg \max_k \mathbf{z}_A.$$
 (5)

We avoid gating/temperature mechanisms: in small EEG datasets they introduce extra degrees of freedom that are hard to calibrate across subjects and can overfit. A fixed linear combiner preserves class-wise alignment ability while remaining stable.

2.8. Objective and Optimization

We train with cross-entropy and combine ℓ_1 sparsity on graph-increment parameters with ℓ_2 weight decay (AdamW).

Table 1. Avg±Std results on dry-EEG under three evaluation protocols. We report Accuracy (ACC), Cohen's kappa, and F1.

Method	Cross Session			Cross Subject			Cross-Subject + Single-Session Fine-tuning		
	ACC(%)	kappa	F1(%)	ACC(%)	kappa	F1(%)	ACC(%)	kappa	F1(%)
ShallowNet [9]	47.30 ± 7.68	0.2095 ± 0.1154	45.39 ± 7.85	50.38 ± 10.92	0.2621 ± 0.1643	48.55 ± 12.13	52.06 ± 11.30	0.2809 ± 0.1695	50.18 ± 12.03
EEGNet [8]	47.67 ± 11.78	0.2152 ± 0.1767	45.96 ± 12.41	52.21 ± 10.81	0.2936 ± 0.1625	50.17 ± 11.85	54.94 ± 10.23	0.3241 ± 0.1535	52.71 ± 11.62
EEG-TCNet [11]	44.52 ± 11.86	0.1678 ± 0.1779	40.53 ± 13.40	46.25 ± 10.71	0.1936 ± 0.1415	41.65 ± 11.26	52.66 ± 9.12	0.2899 ± 0.1368	48.15 ± 11.36
EEGConformer [12]	48.55 ± 8.62	0.2282 ± 0.1295	46.07 ± 9.62	50.23 ± 9.61	0.2534 ± 0.1441	48.01 ± 10.81	55.83 ± 11.64	0.3374 ± 0.1746	53.82 ± 12.81
BaseNet [23]	46.07 ± 11.16	0.1911 ± 0.1674	41.80 ± 13.52	52.05 ± 10.21	0.2960 ± 0.1535	48.59 ± 12.55	53.97 ± 10.22	0.3095 ± 0.1534	50.14 ± 12.05
LMDANet [13]	46.13 ± 10.70	0.1920 ± 0.1604	43.58 ± 11.58	47.89 ± 8.87	0.2184 ± 0.1333	45.78 ± 10.46	51.33 ± 11.26	0.2701 ± 0.1689	49.03 ± 12.17
STGENet [15]	47.08 ± 10.58	0.2034 ± 0.1602	43.01 ± 12.60	47.66 ± 8.70	0.2155 ± 0.1253	44.96 ± 9.63	50.83 ± 10.28	0.2604 ± 0.1552	48.75 ± 11.21
STGMFM	$\textbf{49.25} \pm \textbf{4.16}$	0.2368 ± 0.0643	$\textbf{47.50} \pm \textbf{5.16}$	$\textbf{57.26} \pm \textbf{8.34}$	0.3592 ± 0.1247	$\textbf{56.52} \pm \textbf{8.08}$	$\textbf{59.81} \pm \textbf{6.71}$	$\textbf{0.3972} \pm \textbf{0.1006}$	$\textbf{59.22} \pm \textbf{6.72}$

The total loss is

$$\mathcal{L} = \mathcal{L}_{CE} + \lambda_s \|\Delta \tilde{\mathbf{A}}\|_1 + \lambda_t \|\Delta \tilde{\mathbf{S}}\|_1 + \beta \sum_{\theta} \|\theta\|_2^2.$$
 (6)

Cosine-annealed learning rates promote smooth convergence and better generalization.

3. EXPERIMENTS AND RESULTS

3.1. Dataset and Protocols

We collected a dry-electrode MI-EEG dataset with a 23channel cap at 250 Hz. 19 subjects each completed two sessions on different days; within each session, three repeated runs were recorded for a three-class MI task. We evaluate three realistic protocols: (i) Cross-Session: train on one session and test on the other session from the same subject; (ii) Cross-Subject: leave one subject out for testing, train on the remaining subjects; (iii) Cross-Subject + Single-Session Fine-tuning: pre-train on other subjects and adapt using one session of the target subject. Trials are segmented using windows of 125 samples (0.5 s) with a stride of 125, yielding nine time slices per trial. We train 1000 epochs with AdamW (initial learning rate 2×10^{-3}), batch size 32, dropout 0.2, cosine-annealing schedule, and L1/L2 regularization on graph weights and classifier. All experiments run on a NVIDIA RTX 4090.

3.2. Baselines and Overall Performance

We compare against some representative models (ShallowNet [9], EEGNet [8], EEG-TCNet [11], EEGConformer [12], BaseNet [23], LMDANet [13], STGENet [15].) We report 3 averaged metrics over 19 subjects: Accuracy, Cohen's kappa and F1. Table 1 summarize the results. STGMFM attains the best accuracy across all three protocols.

3.3. Ablation and Component Analysis (on Cross-Subject)

We quantify the contribution of each component using the Cross-Subject protocol. Table 2 reports the results.

Modules Analysis: (i) Dual graph orders (A: CCG→ TSG; B: TSG→CCG): The two orders inject complementary inductive biases that hedge against distinct error pathways

Table 2. Ablation study on dry-EEG under Cross Subject. We report Avg±Std results for Accuracy (ACC), Cohen's kappa, and F1. Best results are **bold**.

Variant	ACC(%)	kappa	F1(%)
Only A&B	54.19 ± 8.87	0.3132 ± 0.1327	53.53 ± 8.78
Only C	51.14 ± 9.16	0.2669 ± 0.1376	50.11 ± 9.45
No PLV initialization	56.76 ± 8.71	0.3517 ± 0.1278	56.11 ± 9.03
Spatial adjacency fixed	52.20 ± 9.48	0.2834 ± 0.1420	51.64 ± 9.50
No L1/L2 regularization	54.32 ± 9.46	0.3153 ± 0.1416	53.56 ± 9.38
Gated fusion	52.89 ± 9.43	0.2936 ± 0.1412	52.22 ± 9.35
STGMFM (full)	$\textbf{57.26} \pm \textbf{8.34}$	$\textbf{0.3592} \pm \textbf{0.1247}$	$\textbf{56.52} \pm \textbf{8.08}$

in dry-EEG: A first stabilizes spatial connectivity then consolidates temporal relations while B emphasizes slice-wise temporal consistency. (ii) Multi-Scale Frequency-Mixer (C): MFM focuses on multi-scale envelopes and long-range modulations with reduced phase sensitivity, recovering ERD/ERS-like patterns that are blurred by dry-electrode noise. (iii) PLV-initialized, learnable graphs: PLV provides physiologically plausible priors that narrow the search space and improve early-epoch stability, yet learnability preserves subject/session adaptability. At the same time, it also provides more detailed and precise adjacency. (iv) Regularization and simple fusion: L1 prevents spurious edges/feature dominance, while L2 and cosine annealing smooth optimization.

4. CONCLUSION

We introduced STGMFM, a tri-branch fusion network for dry-electrode MI-EEG. Two complementary graph orders (CCG→TSG and TSG→CCG) and a Multi-Scale Frequency-Mixer jointly model spatio-temporal dependencies and envelope regularities. With PLV-initialized learnable graphs, cosine-annealed training, and L1/L2 regularization, STGMFM surpasses strong CNN/Transformer/GCN baselines across cross-session, cross-subject, and pre-train & adapt protocols on a 23-channel dry-EEG dataset. Ablations confirm the role of each module under low-SNR, contact variability: dual graph orders curb noise propagation, MFM captures robust ERD/ERS-like envelopes, and simple decision-level fusion generalizes better than gated schemes. Future work will explore adaptive PLV estimation, subject-aware fusion, and channel pruning for on-device inference.

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