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Programming Assignment 1

1) Brutal force algorithm analysis:

Using brutal force to find the closet pair of points is straightforward but time-inefficient. Assuming we have N points in total, the brutal force algorithm basically employs two nested loops parsing through each point and calculate the distance between this point and all other points:

```
For i=1 to N  do
        For j=i to N do
                Comparing distance between point[i] and point[j] with current
                smallest distance .
                If smaller, make it the new smallest distance.
```

Obviously, such reckless algorithm makes a great number of comparisons=N-1+N-2+N-3…..+1=O(n^2)

2) Divide and conquer algorithm analysis:

To divide a big problem into two smaller ones makes it easier to be solved. Based on such idea comes the divide and conquer algorithm. First, we mergesort all points in terms of their x-coordinates; and then we divide these points into multiple sub-sets in the same way we do in mergesort. As a result, we acquire many sub-sets containing two points, and then we continue merging these sub-sets together while record the closest pair of points we find. Finally we get back to the set of points we start with and during such process the closest pair of points is found. To be more specific, the process of merging two sets and finding closest pair of points in them is as following:

d1=the distance of two closest points in set1;

d2=the distance of two closest points in set2;

m=median x-coordinate of these points;

d=min(d1,d2);

merge all points in terms of their y-coordinates;

for i=start to end of set;

if abs(point[i].x-m)<=d then

for j=point[i -6] to point[i+6] do

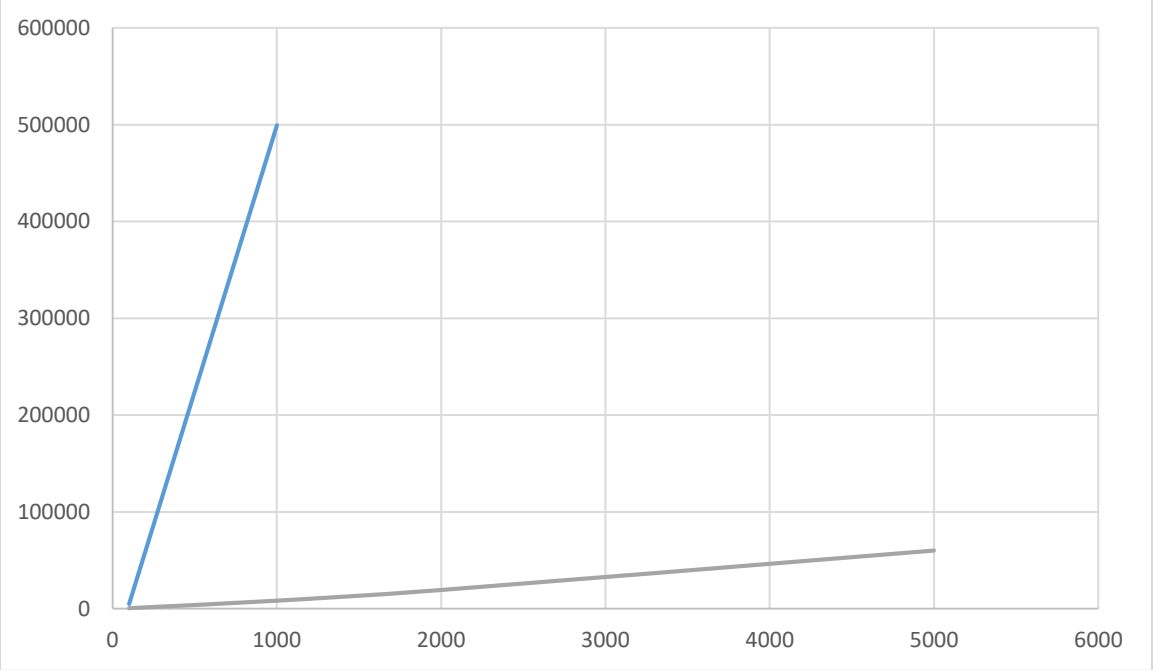
d3=distance between two points (excluding itself);

d=min(d,d3);

return d; (coordinates are recorded as well by other means but it is critical to this algorithm)

Since there are at most 6 other points that can generate distance smaller than d with point[i] but we do not know which side, left of right to the median, is point[i] at, we check in total 12 points to make sure we do not miss any candidate points.

As we know, merging operation is O(n) and comparison is approximately 12n; Therefore, we can describe the entire algorithm with a relation function T(N)=T(N/2)+cn, where c is a real number. According to Master Theorem, overall complexity is O(nlogn).



In the plot above, blue line represents the number of comparison conducted by brutal force and gray line represents the number of comparison conducted by divide and conquer algorithm. It is clear that as the number of points increases, the number of comparison increases dramatically by brutal force whereas slowly by divide and conquer.